

Simulation Results of Channel Capacity using Parity Bits for Error Correction

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1 Introduction

Claude Shannon was a major figure in creating the mathematical discipline of information theory. This paper analyzes how emperical results line up with the statements made by this mathematical model of information. More formally, we will be analyzing the effect of using parity bits for error correction and how well they are at approaching the formal limit on the conveyance of data.

2 Mathematical Description

The official description of conveyable information is the mutual information between to system of events. Let A be the source alphabet. Let B be the reception alphabet. Let Q be the transmission matrix (IE the probability of receiving $b_j \in B$ given a transmission of $a_i \in A$. Let these entries be represented as $q_{i,j}$ respectively. For our example we will only consider the simplier channel known as the binary symettric channel. Thus

$$Q = \begin{pmatrix} q_{0,0} & q_{0,1} \\ q_{0,1} & q_{1,1} \end{pmatrix} = \begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix}$$

Given such a channel, the probability of bit errors can be represented by a Bernoulli Trial.

The mutual information between two system of events (definition given elsewhere) is defined as following:

$$\begin{aligned} I(A, B) &= \sum_{i=1}^m \sum_{j=1}^n Pr(A_i \cup B_j) \log \left(\frac{Pr(A_i \cup B_j)}{Pr(A_i)Pr(B_j)} \right) \\ &\dots = \sum_{i=1}^m Pr(A_i) \sum_{j=1}^n q_{i,j} \log \left(\frac{q_{i,j}}{\sum_{t=1}^m Pr(A_t)q_{t,j}} \right) \end{aligned}$$

It has been proved elsewhere that the maximal channel capacity (maximum $I(A,B)$ based on $Pr(A_i)$ – probability that a source letter is transmitted) for a BSC is when $Pr(A_1 = 0) = Pr(A_2 = 1) = \frac{1}{2}$. Furthermore for these values,

$$I(A, B) = p * \log(2 * p) + (1 - p) * \log(2 * (1 - p))$$

For example, if $p = .99$ then $I(A, B) = .919$.

3 Problem Statement

To test this model of channel capacity and mutual information, a Matlab program was written that would satisfy the important requirements listed above ($Pr(A_i) = \frac{1}{2}$, Q as described above). The necessary error correction was implemented using parity bits. The Matlab code is attached below.

```

1  PRECISION = 100;
2  RETRY_COUNT = 10000;
3
4  %first avg transmission
5  %second error rate of transmission
6  %third avg channel capacity
7  A = zeros(3, 100);
8  A = zeros(2, PRECISION);
9
10 temp = 0;
11 for i = 1:PRECISION
12     for j = 1:RETRY_COUNT
13         temp = transmit(generate_random_msg(10,4), i/PRECISION, 20);
14         if temp == -1
15             A(2,i) = A(2,i) + 1;
16         else
17             A(1,i) = A(1,i) + temp;
18         end
19     end
20     %compute avg ignoring failed transmissions
21     temp = RETRY_COUNT - A(2,i);
22     if temp == 0
23         %do nothing because A(1,i) will be zero also
24     else
25         A(1,i) = A(1,i)/(RETRY_COUNT-A(2,i));
26     end
27 end
28
29 %compute channel cap graphics_toolkit
30 %min bytes computed by testing for zero error
31 min_bytes = transmit(generate_random_msg(10,4), 0, 20);
32 %remember to subtract parity bytes because not in original msg
33 A(3,:) = (ones(1,PRECISION)*(min_bytes-10))./A(1,:);
34
35 figure
36 %bits of info needed for transmission
37 a = subplot(3,1,1);
38 plot(A(1,:))
39 ylabel(a,'Number of Bits Needed for Transmission')
40 xlabel(a,'Error Chance (1-Success Percentage)')
41 %number of errors in transmission
42 b = subplot(3,1,2);
43 plot(A(2,:))
44 ylabel(b,'Number of Errant Messages')
45 xlabel(b,'Error Chance (1-Success Percentage)')
46 %channel capacity
47 c = subplot(3,1,3);
48 plot(A(3,:))
49 ylabel(c,'Channel Capacity')
50 xlabel(c,'Error Chance (1-Success Percentage)')

```

Listing 1: Main Simulator

```

1  function bit_count = transmit(msg, err_prob, retry_lim)
2      msg = insert_parity_bit(msg);
3
4      num_transmissions = 0;
5      transmitted_msg = 0;
6      for i = 1:size(msg)(1)
7          transmitted_msg = transmit_msg(msg(i,:),err_prob);
8          %check rough equality with parity bit
9          %guarantees proper transmission for one or less errors
10         temp_c = 0;
11         while transmitted_msg(1) != parity(transmitted_msg(2:end)) && temp_c < retry_lim
12             num_transmissions = num_transmissions + 1;
13             temp_c = temp_c + 1;
14             transmitted_msg = transmit_msg(msg(i,:),err_prob);
15         end
16         %msg would fail using given parity scheme and error probability
17         if isequal(transmitted_msg, msg(i,:)) == false
18             bit_count = -1;
19             return
20         end
21         %always one transmission
22         num_transmissions = num_transmissions + 1;
23     end
24
25     %num trans multiplied by the size of packet transmitted
26     bit_count = num_transmissions * size(msg)(2);
27 end

```

Listing 2: Transmission Simulator

```

1 function msg = transmit_msg(msg_in, err_prob)
2     msg = zeros(size(msg_in));
3     rnd_num = 0;
4     for i = 1:size(msg_in)(2)
5         rnd_num = rand();
6
7         if rnd_num < err_prob
8             if msg_in(i) == 0
9                 msg(i) = 1;
10            else
11                msg(i) = 0;
12            end
13        else
14            msg(i) = msg_in(i);
15        end
16    end
17 end

```

Listing 3: Per-packet Transmission Simulator

```

1 function msg = insert_parity_bit(msg_in)
2     c = size(msg_in)(1);
3     msg = zeros(size(msg_in)(1),size(msg_in)(2) + 1);
4     for i = 1:c
5         %https://www.mathworks.com/matlabcentral/newsreader/view\_thread/331396
6         msg(i,:) = [parity(msg_in(i,:)) msg_in(i,:)];
7     end
8 end

```

Listing 4: Per-packet Parity Bit Inserter

```

1  function par = parity(packet)
2      num_ones = 0;
3      for i = 1:size(packet)(2)
4          if packet(i) == 1
5              num_ones = num_ones + 1;
6          end
7      end
8      if (mod(num_ones,2) == 0)
9          par = 0;
10     else
11         par = 1;
12     end
13 end

```

Listing 5: Parity Bit Message Inserter

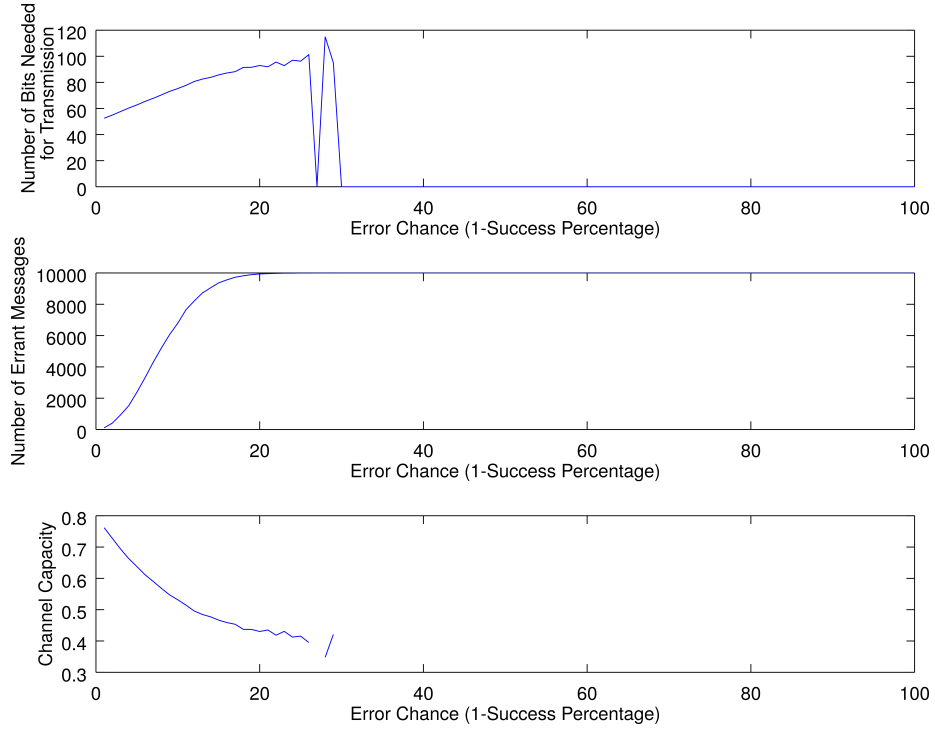
```

1  function msg = generate_random_msg(num_packets, packet_len)
2      msg = zeros(num_packets,packet_len);
3      for i = 1:num_packets
4          for j = 1:packet_len
5              msg(i,j) = floor(2*rand());
6          end
7      end
8  end

```

Listing 6: Generator of Random Messages with Packets

4 Results



Because the error correction was implemented using parity bits, the maximum channel capacity possible is $\frac{4}{5}$. This is since one new bit is added to each four bit transmission package thus lengthening the transmitted message at a $\frac{4}{5}$ rate.

5 Analysis