

Geometric phases and polarization patterns in multiple light scattering

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Multiple light scattering is widely used to characterize dense colloidal systems as well as in deep tissue imaging; experiments are often interpreted via a theory of diffusion of the light intensity within a sample, neglecting the vector nature of the electromagnetic wave. Recent experiments on diffuse backscattering with linearly polarized light from colloidal suspensions of micron size particles were found to display strong intensity variations with fourfold rotational symmetry when observed through an arbitrarily oriented linear analyzer. We show that these polarization patterns are manifestations of a Berry phase of the multiple scattered beam.

The quality of imaging in strongly scattering media such as biological tissue is enhanced if polarization discrimination is used to filter the light. Rather surprisingly, it is found that circularly and linearly polarized light do not display the same quality and resolution in imaging [1]. These puzzling observations have motivated a detailed characterization of the backscattering characteristics of strongly scattering media. Systematic experiments characterizing of the scattering properties of dilute colloids of latex beads in solution [2] as a model of tissue scattering found surprisingly rich results. In the experiments a beaker of a strongly scattering suspension is illuminated by a polarized light source focused to a small point. The surface of the beaker is then imaged with various analyzers. With a linear analyzer and a linearly polarized beam the experiments on $2\mu\text{m}$ diameter latex beads showed strong variations in intensity exhibiting a fourfold symmetry about the incident spot, somewhat like the petals of a daisy, Fig. (1). An additional striking result is that when the analyzer is rotated an angle $\pi/4$ the pattern of intensity rotates just one half this angle, $\pi/8$, without changing shape. While analytic approaches [3] have been successful in treating problems in the propagation of the intensity of multiple scattered light, many treatments of the evolution of the polarization state have been purely numerical, [4–6] using Monte-Carlo techniques to trace light through a multiply scattering medium. Analytical work has been based on the idea that polarization states should be rapidly randomized and that polarization dependent effects should be weak and transient [7]. We show in this report that simple geometric considerations allow one to gain a qualitative understanding of the observed polarization patterns. The patterns are due to Berry phases in the multiply scattered beam.

The evolution of the plane of polarization of light propagating in a *smoothly* disordered medium was first treated by Rytov [8] in the eikonal approximation. The geometry of the propagation was then rediscovered by Berry [9] and applied to many other wave phenomena, including quantum mechanics. The results of these studies are best illustrated by experiments in which light propagates along a tortuous fiber optic [10, 11]. While propagating along a uniform fiber the polarization state of the light evolves in such a way to minimize twisting [12] of

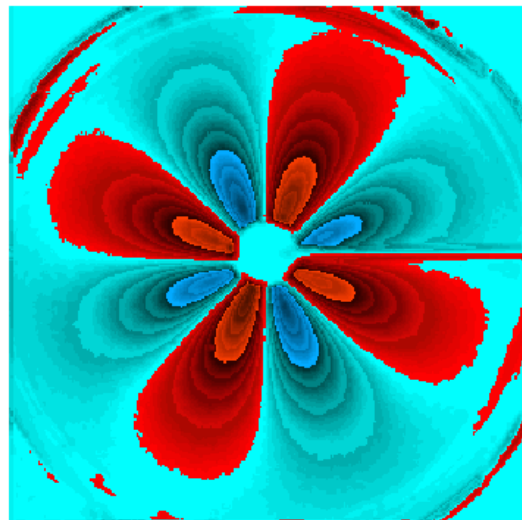


FIG. 1: False color image of polarization patterns [18] on the surface of a beaker of a colloidal solution [16]. Linearly polarized light is incident at a spot at the center of the image. The surface of the beaker is imaged with a camera through a linear analyzer. The experimentalists have plotted an element of the Mueller matrix characterizing the transfer function between the incident and backscattered light as a function of position on the surface. For $2\mu\text{m}$ diameter beads the other elements of this matrix implied in the scattering of linearly polarized light are circularly symmetric; the image thus reflects the variation of brightness of a backscattered linearly polarized beam observed with a linear polarizer. Red corresponds to brighter than average regions, blue to darker areas.

the polarization vector \mathbf{E} , in fact one speaks of the evolution of \mathbf{E} by *parallel transport*. This evolution law is locally trivial, however it leads to a global rotation of the polarization state. This effect of coupling of polarization to the propagation path is now generally considered as a simple example of a *geometric phase*. The main conclusion of the present letter is that identical phenomena are to expected even in the absence of the guiding fiber, as for instance is the case for light multiply scattered in a colloidal suspension; In this report we shall firstly justify the use of the Rytov-Berry result, valid for the evolution of the polarization in a continuous medium, in the case of multiple scattering by distinct particles. We then show that a geometric phase naturally leads to fourfold

symmetric polarization patterns in retro-diffusion.

If the polarization state of a light beam is to evolve by parallel transport we require that helicity flipping events are rare; such events are for instance generated by large angle deflections such as reflections at an interface. Specular reflection at a surface preserves the linear polarization state of an incident beam and can not lead to fourfold symmetric patterns in the back scattered beam. It has been shown both numerically [5, 6] and analytically [13] that helicity flipping events occur on a characteristic length scale which is somewhat larger than the length over which the beam is deviated in the case of strongly forward scattering. From now on we neglect these events. Furthermore using the Born approximation it has been shown [13, 14] that under multiple scattering conditions the polarization vector of the forward scattered beam evolves so that $\mathbf{E}_j \sim \mathbf{E}_{j-1} - (\mathbf{E}_{j-1} \cdot \mathbf{t}_j) \mathbf{t}_j$, where \mathbf{E}_j is the polarization vector after the j 'th collision and \mathbf{t}_j is the direction of propagation of the light. In the limit of many small angle scattering events this evolution law is equivalent to parallel transport of the polarization vector. This result is valid for scattering from micron sized latex particles, used in the experiments.

The angle of rotation of a polarization vector due to a geometric phase is calculated from the propagation direction $\mathbf{t}(s)$ expressed as a function of the path length s . It is identical to the solid angle enclosed by the path $\mathbf{t}(s)$ on a sphere [9, 11]. We now proceed by translating the backscattering geometry into an ensemble of paths on the unit sphere, $\{\mathbf{t}\}$ in order to apply this result. As shown in Fig. (2) backscattered light corresponds to a path from the south to north poles of the sphere, $\mathbf{t}(s)$, describing the direction of propagation. We take as a reference state light scattered to the left, polarized in the plane of the page. We see that an original polarization vector \mathbf{E}_A is parallel transported around the sphere, Fig. (2, bottom) so that the initial and final vectors are antiparallel. The indicated path Fig (2, top) is scattered preferentially to the left; since the real space scattering of the beam is linked to the tangent curve by the equation $\mathbf{r}(s) = \int_0^s \mathbf{t}(s') ds'$, the path $\mathbf{t}(s)$ must remain largely on the western latitude as shown in the Fig (2, bottom). We now change the point of observation on the surface of the sample, generating a second trajectory $\mathbf{t}'(s)$. This new trajectory together with the reference path form a closed loop on the sphere which allow us to apply the Berry result. As we change the point of observation on the sample, B, and wind an angle of 2π about the incident beam, A, the new path $\mathbf{t}'(s)$ on the sphere sweeps out a solid angle of 4π . We thus deduce that the polarization at the surface of the sample rotates *two full turns* as we move just once about the incident beam. Since a linear analyzer is sensitive to the angle of rotation modulo π we understand that there are four radial directions in which an analyzer detects a maximum in the intensity. We also understand that if the analyzer rotates an angle θ then the intensity pattern rotates just $\theta/2$.

The geometry of this result is strongly reminiscent of

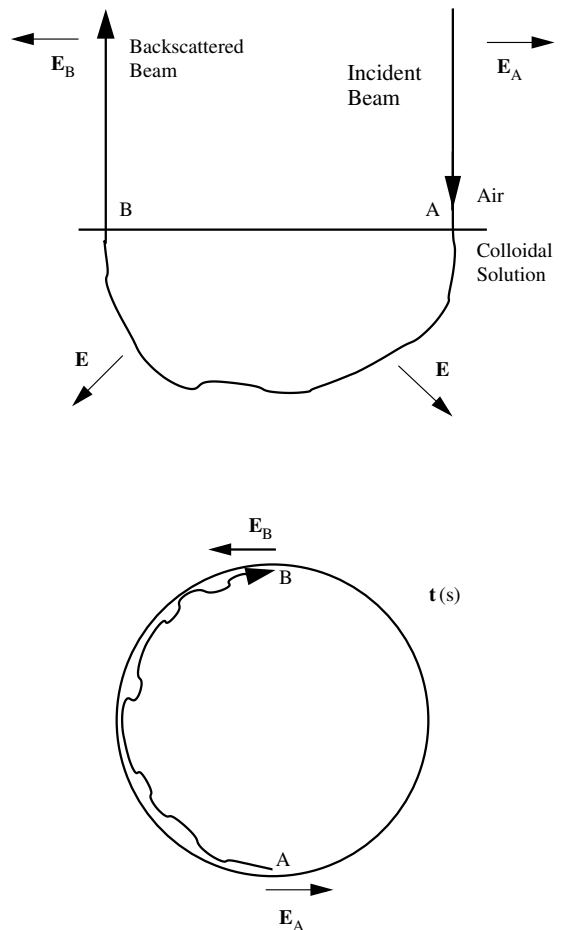


FIG. 2: *top*: Light is incident on a multiple scattering medium. The direction of propagation is randomly bent back and the light escapes from the surface. *bottom* The direction of propagation of the scattered light is plotted on a sphere. Incident light corresponds to the south pole, A. The escaping light corresponds to the north pole, B. The indicated path is scattered principally to the left so that on the sphere the path remains, on average on the western area of the sphere. As the point of observation goes around the incident beam, an angle of 2π , the path between the poles sweeps out an area on the sphere of 4π . Polarized light with polarization state \mathbf{E}_A is parallel transported to the state \mathbf{E}_B .

the *plate trick* demonstration for spinor rotations [15]. If one holds a plate horizontally in the palm of one's hand one can spin it about a vertical axis by performing a suitable contortion of the arm. Against all intuition the plate can be turned an arbitrarily large angle; for each single cycle of the arm the plate spins twice. Parameterizing the shape of the arm by its direction $\mathbf{t}(s)$ and plotting this on the sphere one sees that this trick can also be understood by the fact that during a cycle $\mathbf{t}(s)$ sweeps out a solid angle of 4π on a sphere translating giving two full rotations in a plane for each cycle.

Clearly the geometry of Fig. (2) is simplified; we have neglected fluctuations in the light paths. In order to calculate the full illumination state of the light at B one

should sum over all scattering paths from A to B; the path sketched in Fig. (2) is just one contribution to this sum. However, paths that exit at the same point of the surface are correlated so that the Berry phases corresponding to each path are also correlated and do not average to zero. The superposition leads to modifications in the geometry when imaging at small distances from the incident beam. For light which returns directly on axis, all paths around the sphere are equally likely and the polarization state is undetermined [14]. Away from the central spot the sum is dominated by the most direct paths, such as the path shown in the figure. We note that with a coherent light source the speckle structure of the radiation field can never be neglected. There are naturally strong fluctuations in the polarization state due to the existence of zeros in the instantaneous speckle pattern for each component of the polarization vector. The experimental image, Fig. (1), is in fact a time average so no speckles can be seen.

In this report we have considered the problem of scattering from particles in the Mie and Rayleigh-Gans scattering regimes in the limit of strong forward scattering where individual scattering events show weak polarization dependence. Small particles have very different scattering properties and show strong polarization dependence in the scattering. In this limit [4] the fourfold

symmetric pattern due to parallel transport is not seen. Our results can also be used to understand the scattering of circularly polarized light. In this case the Berry phase is just a simple phase shift of the backscattered beam, rather than a rotation in polarization plane. No polarization patterns are to be expected on the surface of a uniform colloid. We understand that the difference in the coupling of the Berry phase to linearly and circularly polarized light is partially responsible for the different imaging qualities of circularly polarized light in colloidal suspensions and tissue fantoms [17]. An object hidden deep under the surface of the beaker in Fig. (1) can only weakly modify the intensity at the surface. This weak modification is easily hidden by the strong variation due to the Berry phase.

Finally, a number of interesting variations can be played on the geometry of the figures observed in diffuse backscattering. Addition of an strongly optically active molecule in the solution should lead to systematic bending of the daisy giving rise to helical patterns in the polarization state on the surface of the colloidal solution. This would be of interest as a method of measuring path lengths in the solution as a function of the distance between the incident beam and the detection point.

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