Cyclicality of Investment Volatility: Implications of Specification Choice

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June 1, 2023

Abstract

I estimate the conditional volatility of aggregate investment and its components as an autoregressive-moving-average process (ARMA) using standard generalized autoregressive conditional heteroskedasticity (GARCH) estimators. I show that the aggregate volatility of total and equipment investment are acyclical and exhibit low persistence, while the volatility of structure investment is countercyclical and exhibits high persistence. I benchmark my results to prior estimates of the cyclicality of the volatility of aggregate investment in US data and simulated environments. This comparative analysis shows that other specifications in the literature are sensitive to sample selection and outliers.

1 Introduction

Firms face nonlinear frictions (e.g. fixed costs, adverse selection) when they invest, which prevent them from continuously adjusting their capital stock (Caballero *et al.* (1995), Baley and Blanco (2021)). When the firms cannot adjust their capital stocks frictionlessly, their investment patterns exhibit periods of inaction followed by moments of activity. In structural investment models, firms' implied propensity to invest explains this observed behavior. When firms' propensity to invest varies over time, the aggregate investment rate exhibits conditional heteroskedasticity. Specifically, aggregate shocks and past investment decisions determine the conditional volatility of the aggregate investment rate. Bachmann *et al.* (2013) (BCE, hereafter) document that periods of heightened aggregate investment volatility follow protracted periods of high aggregate investment. While their specifications measure the state-dependence of the volatility of aggregate investment, their specifications only indirectly measure the cyclicality and persistence of the volatility of aggregate investment.

In this paper, I estimate the conditional volatility of aggregate investment and its components as an autoregressive-moving-average process (ARMA) using standard generalized autoregressive conditional heteroskedasticity (GARCH) estimators. Unlike the specifications in BCE, these alternative specifications do not measure the volatility of aggregate investment as a function of the lagged average of aggregate investment. As such, they offer a more direct estimate of the persistence of the conditional volatility of aggregate investment. Using the implied volatility of aggregate investment recovered from these alternative specifications, I test whether volatility systematically varies across business cycles. I show that the aggregate volatility of total and equipment investment are acyclical and exhibit low persistence, while the volatility of structure investment is countercyclical and exhibits high persistence.

First, I motivate the choice to estimate the conditional volatility of aggregate investment as an ARMA process. To do so, I examine the squared residuals of a univariate autoregression on the aggregate investment rate. I present the auto-correlation functions of these squared residuals, alongside the auto-correlation functions of aggregate investment. The squared residuals of aggregate investment exhibit low persistence. I then estimate the correlation between these squared residuals and output across different horizons. The squared residuals of total and equipment investment are acyclical, while the squared residuals of structure investment are countercylical. These results do not motivate the choice of BCE to use the lagged average of aggregate investment, a highly persistent and cyclical variable, to measure the persistence and cyclicality of the conditional heteroskedasticity of aggregate investment. Instead, these results motivate the use of alternative specifications which estimate aggregate investment's conditional volatility's autoregressive and moving-average components directly.

Second, I estimate the implied aggregate investment volatility using alternative GARCH estimators. These specifications, which assume that aggregate investment volatility follows an ARMA process, estimate the auto-regressive component and moving average component of the volatility of aggregate investment. I find that this autoregressive component is small and statistically insignificant for total and equipment investment, and large and statistically significant for structure investment. That said, the moving average component is positive and significant for all components and total investment. This suggests that past innovations are informative of the future investment volatility, but that the conditional volatility of aggregate investment exhibits low persistence.

Third, I benchmark the performance of these alternative GARCH estimators against the BCE specifications in US data and simulated environments. First, I show that the BCE specification is isomorphic to an asymmetric GARCH(∞) estimator, which estimates the relationship between the conditional volatility of aggregate investment and previous investment residuals. This implicit weighting scheme explains why BCE specifications are sensitive to sample selection and outliers. Second, I compare the performance of the two specifications in environments simulated using the canonical heterogenous firm models of Khan and Thomas (2008) and Winberry (2021). These comparisons clarify the use of the BCE specification. The BCE specifications test whether the volatility of aggregate investment depends on past investment behavior, but does not measure the cyclicality or persistence of the volatility of aggregate investment.

This project informs previous empirical and quantitative work linking aggregate dynamics to microeconomic firm investment dynamics. The former motivated the latter. Specifically, Caballero *et al.* (1995) document that higher-order moments of US aggregate investment rate are non-gaussian. In simulated environments, non-gaussian higher-order moments can result from firms responding to gaussian shocks in the face of non-convex investment costs. BCE show that aggregate investment is more sensitive following periods of increased investment. Under certain structural assumptions, these papers present direct evidence of the relevance of lumpy investment dynamics for analyzing higher-order moments of country-level investment data.

This project also contributes to the literature that studies firms' investment models with non-convex adjustment costs. Researchers evaluate the performance of these models indirectly, through simulated method of moments, matching the time-series dynamics of the interest-rate demand elasticity of investment across firms. Following the insights of Winberry (2021) and Baley and Blanco

(2021), researchers have been able to match the underlying distributional dynamics of the microeconomic data, but have not evaluated whether these models can match the cyclicality and persistence of the conditional volatility of the aggregate investment rate.

This project also contributes to the heterogenous agent literature more broadly. The use of conditional heteroskedasticity as an empirical target for structural models is not limited to investment models. Berger and Vavra (2015) also observe that aggregate durable expenditures exhibit conditional heteroskedasticity. As such, this work offers guidance on how to best estimate and benchmark future models, wherein state dependence may produce aggregate conditional heteroskedasticity.

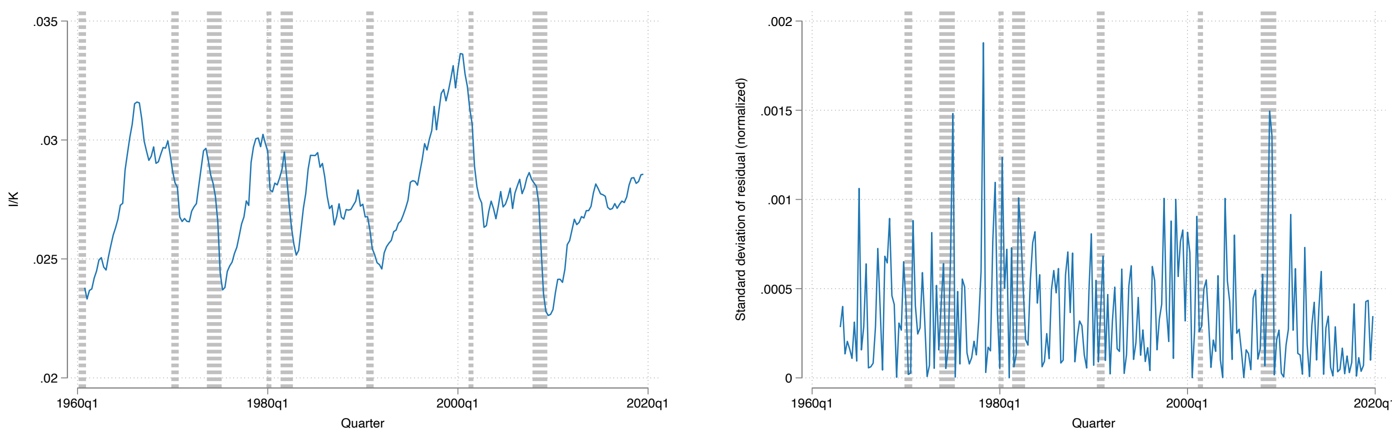
2 Data and Framework

This section summarizes the data and empirical specifications used in the subsequent analysis. I construct quarterly aggregate investment rates for total, equipment, and structure investment using data from the Bureau of Economic Analysis (BEA), using the process employed in BCE. The analysis sample spans 59 years from 1960 through the end of 2019. Appendix 8 discusses the data construction and validation process in detail.

Figure I – US Non-Residential Private Fixed Investment

(A) Total Investment

(B) Standard Deviation of Residual



Notes: Panel A reports the U.S. aggregate investment rate calculated following the procedure in BCE. Panel B reports the standard deviation of the residual, which is calculated by taking the square root of the squared residuals of a univariate autoregression of the aggregate investment rate with a lag order of 6. Appendix 8 discusses the data construction process in detail.
Sources: BEA and author calculations.

Figure I plots the U.S. aggregate investment rate. The aggregate investment rate exhibits observable volatility clustering, suggesting that the underlying data generating process is heteroskedastic. This clustering is most apparent when one views the standard deviation of a univariate autoregression of the aggregate investment rate. Panel B reports the standard deviation of the residual, which is calculated by taking the square root of the squared residuals of a univariate autoregression of the aggregate investment rate with a lag order of 6. The standard deviation exhibits limited, short-lived volatility clusters. This observable feature of the data motivates the use of GARCH models to explain variation in the time series.

This paper features two families of GARCH estimators. The BCE estimator is a GARCH-X estimator, which includes explanatory variables outside of the autoregressive and moving average terms. The alternative estimators presented here are standard GARCH estimators, which include a combination of autoregressive and moving average terms. These specifications can be described by the following system of equations:

$$x_t = \sum_{j=1}^{p} \phi_j x_{t-j} + \varepsilon_t, \tag{1}$$

$$\varepsilon_t = \sigma_t e_t, \tag{2}$$

$$\sigma_t^2 = f_v(\sigma_{t-1}^2, \varepsilon_{t-1}^2, \mathbf{m}_t). \tag{3}$$

with a conditional mean function of $f_m(.)$, a conditional variance function of $f_v(.)$, conditional variance covariates $\mathbf{m}_t$, conditional mean lag order p, and i.i.d. structural innovations $e_t \sim N(0,1)$. The estimation procedure proceeds in two steps. First, I estimate the autoregression of the aggregate investment rate with the same lag order as BCE. Second, I estimate the conditional variance function $\hat{\sigma}_t$ using the squared residuals ε_t^2 recovered from the first stage. The conditional variance function can be estimated using a least-squared or maximum likelihood estimator. This paper presents the following two alternative GARCH estimators:

$$\sigma_t^2 = \alpha + \beta_a \varepsilon_{t-1}^2, \tag{4}$$

$$\sigma_t^2 = \alpha + \beta_g \sigma_{t-1}^2 + \beta_a \varepsilon_{t-1}^2. \tag{5}$$

The first estimator is a GARCH(0,1) with a single moving average term, and the second estimator is a GARCH(1,1) with a moving average and autoregressive term. GARCH models with an autoregressive coefficient cannot be estimated using OLS, because lagged volatility σ_{t-1} is a latent variable. As these estimators include latent variables, I estimate them using MLE. The original BCE specification estimates $\hat{\sigma}_t$ as a function of the lagged average of aggregate investment using OLS. BCE estimates $\hat{\sigma}_t$ using the following specification:

$$\left(\sigma_t^{BCE}\right)^2 = \alpha + \eta \bar{x}_{t-1}^k. \tag{6}$$

$$\hat{\eta} = \frac{\mathbb{C}ov\left(\hat{\varepsilon_t}^2, x_{t-1}^k\right)}{\mathbb{V}\left(x_{t-1}^k\right)} \tag{7}$$

where the lag order k is chosen to maximize the AIC for their initial sample (1960q1-2005q4). The parameter η measures the relationship between the conditional volatility and the lagged average of the aggregate investment rate. They estimate $\hat{\eta}$ in two steps. In the first step, they estimate equation 1 using OLS and recover the residuals ε_t. In the second step, they square the residuals and regress them onto an intercept and the lagged average of aggregate investment. As structural innovations have a zero mean, this second step estimates equation 6, where the coefficient of interest represents the linear relationship between the volatility of the residual and the lagged average of aggregate investment.

Following BCE, I assume that the underlying process is stationary, because the investment rate is a finite ratio by definition. If the underlying process is nonstationary, estimates of conditional heteroskedasticity (sensitivity, elsewhere) will be biased upwards (see Lamoureux and Lastrapes (1990)).

As such, estimates on HP-filtered data are also included to evaluate the importance of stochastic trends in this setting. The estimates of the BCE specification are not sensitive to the choice of estimator. Appendix 8 presents a replication of the BCE empirical analysis using MLE, as a robustness exercise. For clarity, I use OLS to estimate the BCE specification and MLE to estimate the alternative GARCH estimators in the benchmark analysis.

3 Empirical Motivation

To motivate my analysis, I examine the squared residuals of a univariate autoregression on the aggregate investment rate in two steps. First, I estimate the autocorrelations of ε_t^2 and x_t. If these autocorrelation functions exhibit significant differences in persistence, then the use of alternative GARCH specifications will provide more direct estimates of the persistence of the conditional variance of aggregate investment, relative to the BCE specification. Second, I estimate the correlation between ε_t^2 and log gross domestic product y. If the squared residuals are cyclical, then σ_t^2 is likely cyclical. If the squared residuals are acyclical or countercyclical, it is unlikely that σ_t^2 exhibits the same level of cyclicality as the aggregate investment rate.

Figure II plots the autocorrelation function for the aggregate investment series and the squared residuals. While the aggregate investment rate exhibits high autocorrelation across many lags, the squared residuals of the aggregate investment rate series are not persistent. The difference in the persistence of investment and the squared residuals suggests that alternative specifications may provide improved estimates of the persistence of the conditional volatility of aggregate investment

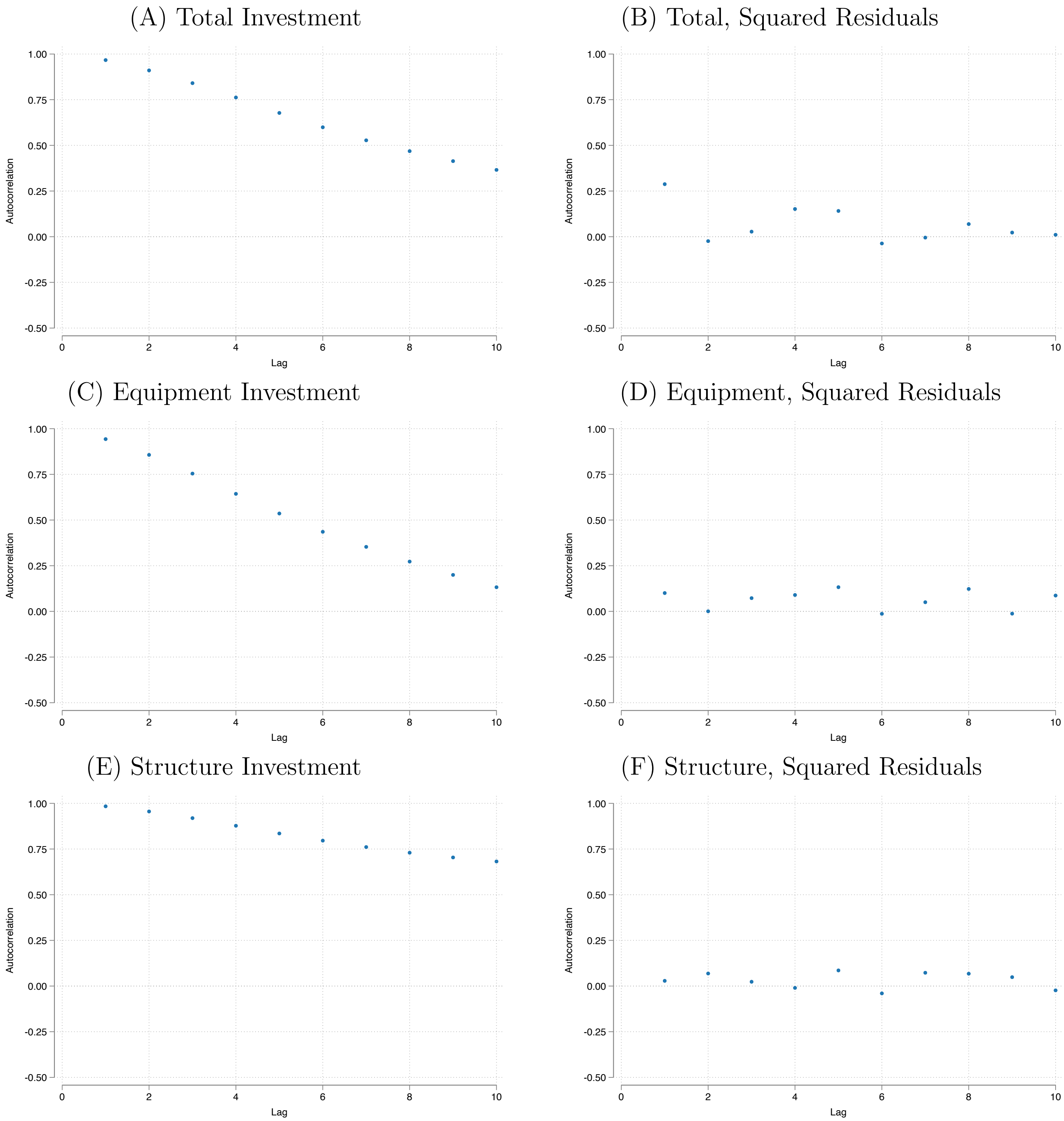
Figure III plots the correlation between a set of output lags and the squared residuals of equipment, structure, and total investment. The squared residuals of the aggregate investment rate series are not procyclical. Equipment and total investment are not cyclical regardless of the filter choice. Unfiltered structure investment is counter-cyclical. When structure investment is hp-filtered, it is not cyclical. For all series, the cyclicality of lagged investment does not coincide with the cyclicality of the squared residuals. These results motivate the use of alternative GARCH specifications to measure the persistence and cyclicality of aggregate investment rate series.

4 Conditional Volatility of Aggregate Investment

In this exercise, I estimate the cyclicality of the conditional volatility of the aggregate US investment rate. Table XVI reports the estimation results for the GARCH and BCE specifications on unfiltered data. Table XVII reports the estimation results for the GARCH and BCE specifications on HP-filtered data. Figure IV plots the cyclicality of the alternative GARCH specifications against the BCE specification.

The estimates in Table XVI and able XVII illicit three key insights. First, the autoregressive coefficient β_g is insignificant for equipment and total investment. The coefficient for structure investment is positive and significant, suggesting meaningful persistence of the conditional variance. Second, the moving average coefficient is significant for total, structure, and equipment investment. Third, the conditional volatility exhibits different business cycle properties, depending on whether lagged aggregate investment is included as an explanatory variable.

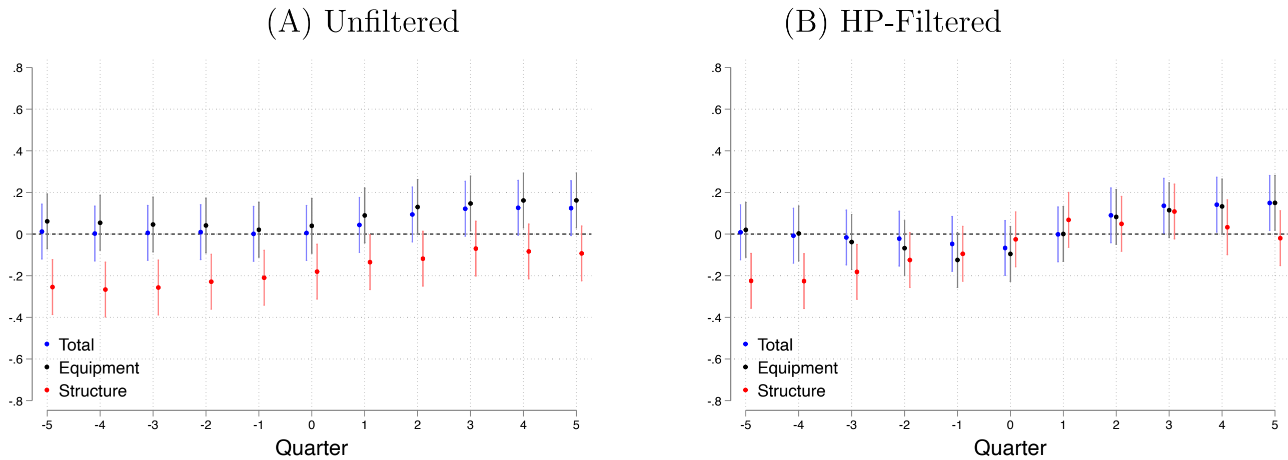
Figure II – Autocorrelation of Aggregate Investment and Squared Residuals



Notes: This figure features six autocorrelation functions. Panels A, C, E plot the autocorrelation function for U.S. aggregate total investment, U.S. aggregate equipment investment, and U.S. structure investment, respectively. Panels B, D, F plot the autocorrelation function for the squared residuals of univariate autoregressions that maximize the AIC. The autoregression of total and structure investment rate residuals has a lag order of 6. The autoregression of the equipment investment rate residuals has a lag order of 7.
Sources: BEA and author calculations.

When estimated using the BCE specification, the conditional volatility exhibits different business cycle properties. In the case of equipment and total investment, the behavior of σ_t^{BCE} is more cyclical and less persistent than the σ_t recovered from the alternative GARCH specifications. In the case of structure investment, the behavior of σ_t^{BCE} is either more or less countercyclical, depending

Figure III – Cyclicality of Squared Residuals



Notes: This figure features two subplots. Panel A plots correlograms of different output lags and the squared residuals of unfiltered U.S. aggregate total, equipment, and structure investment. Panel B plots correlograms of different output lags and the squared residuals of HP-filtered U.S. aggregate total, equipment, and structure investment. Total and structure investment rate residuals are recovered from an autoregression with 6 lags. Equipment investment rate residuals are recovered from an autoregression with 7 lags. The unfiltered correlation is estimated on output detrended using a second order deterministic filter. The hp-filtered correlation is estimated on output detrended using an HP-filter with a smoothing parameter of 1600. *Sources:* BEA and author calculations.

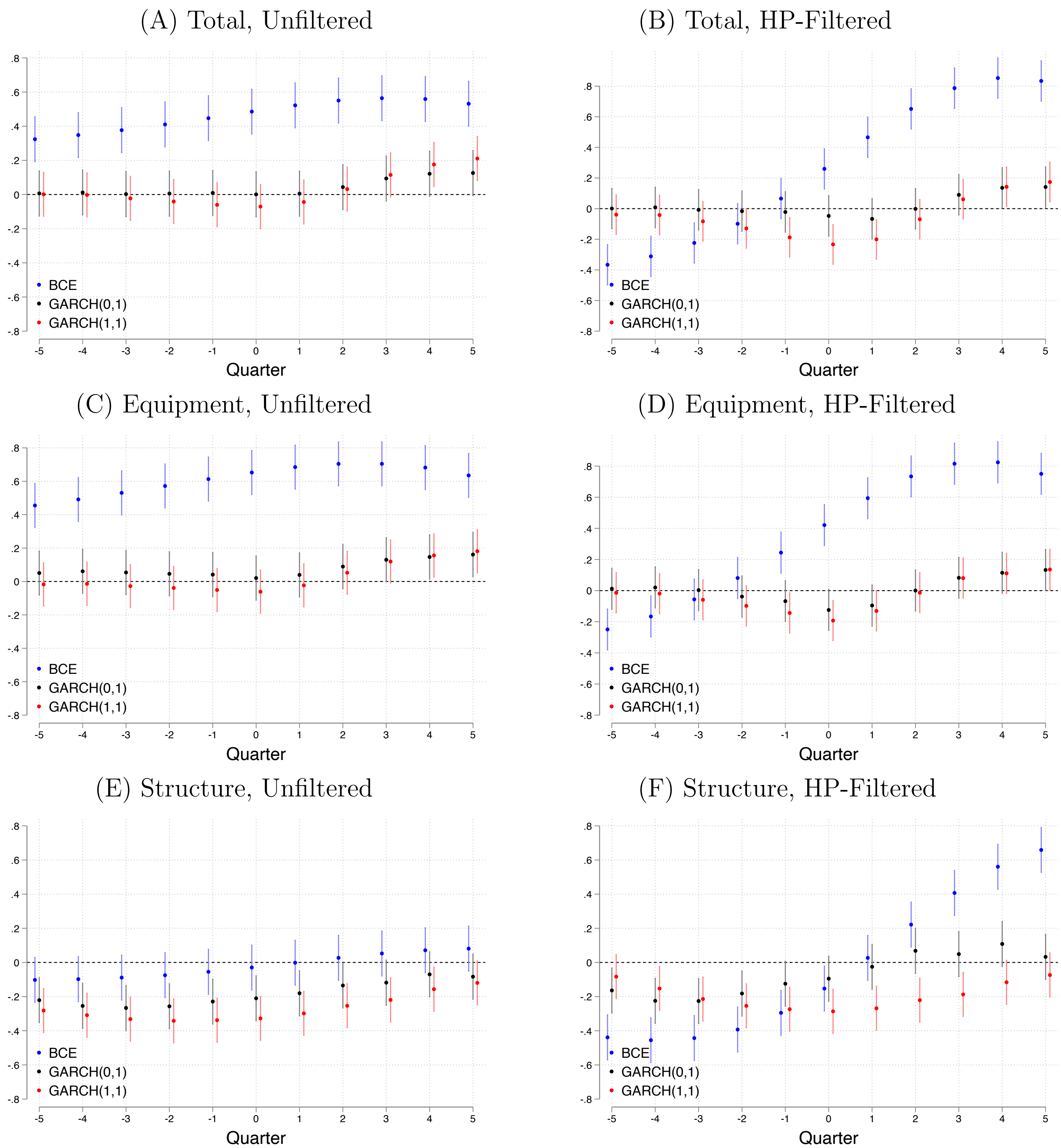
on whether the data is filtered beforehand, respectively. Moreover, σ_t^{BCE} is more persistent than estimates recovered from the alternative specifications for all series, regardless of filter choice.

This exercise also clarifies the inference associated with the BCE specification. The parameter η tests whether the conditional volatility of aggregate investment depends on the lagged average of aggregate investment. That said, the size of the η is not correlated with the cyclicality of the conditional volatility, contrary to the original interpretation of the specification. For example, η^{eq} is larger than η^{st}, but the volatility of equipment investment is less cyclical than the volatility of structure investment.

Figure V compares the implied conditional volatility of aggregate investment implied by the BCE specification against that of the GARCH(1,1) specification. The condition volatility implied by the BCE specification is strongly persistent and fails to capture the spikes in the standard deviation of residuals. The GARCH(1,1) specification captures the spikiness and limited persistence of the standard deviation of residuals. The BCE specification also implies that the conditional volatility of structure investment is lower in the latter portion of the sample, a feature that is not apparent in the standard deviation of residuals.

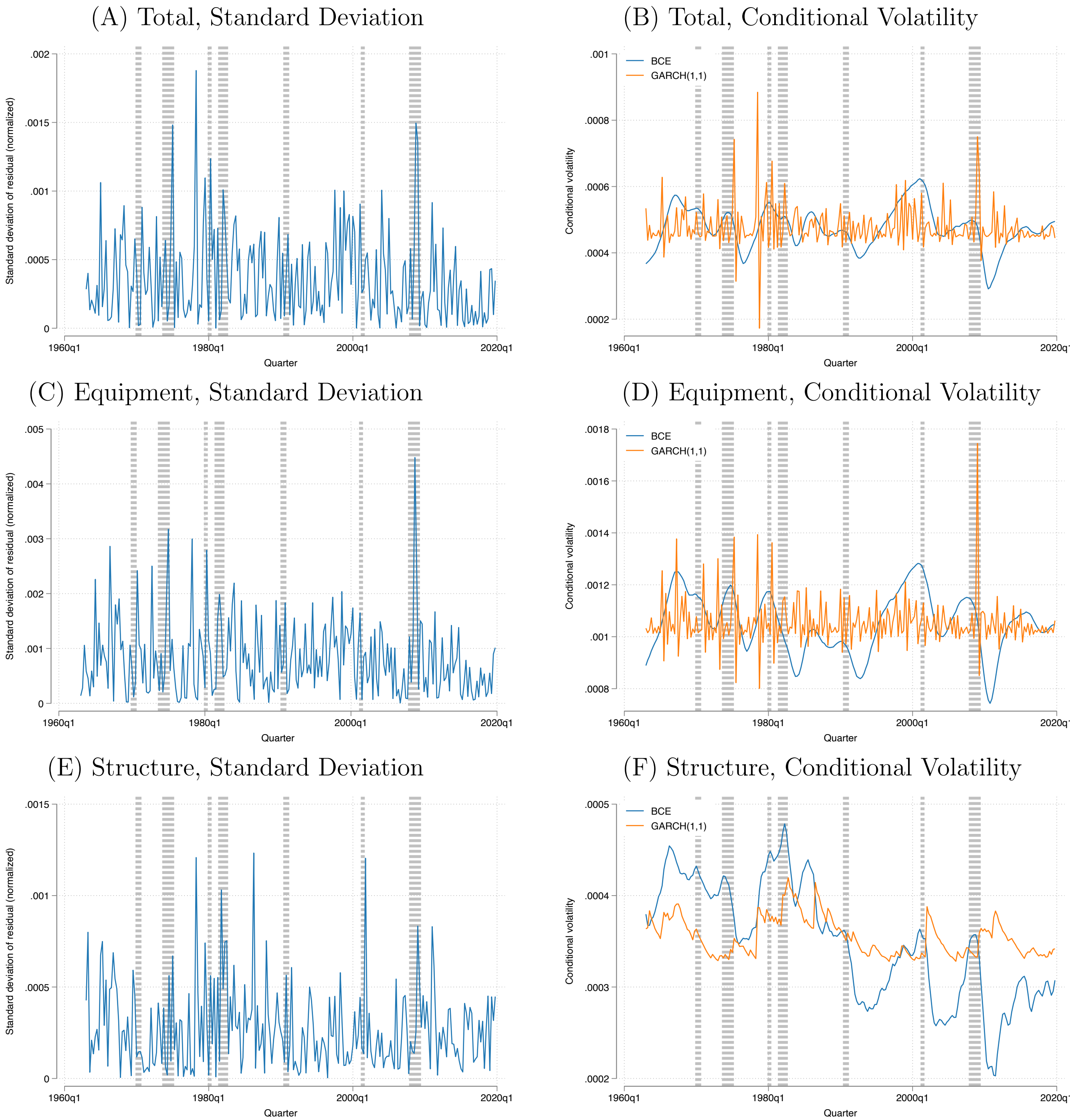
These results, taken together with the results of the previous subsection, suggest that the lagged average of aggregate investment does not measure the persistence and cyclicality of the conditional volatility of aggregate investment. If it were, then one should expect that the time series behavior of σ_t^{BCE} to track the dynamics depicted in Figure III.

Figure IV – Cyclicality of Conditional Volatility



Notes: This plot contains six subplots. Panels A and B plot correlograms of different output lags and the conditional volatility of U.S. aggregate total investment with different specification and filtering choices. Panesl C and D plot correlograms of different output lags and the conditional volatility of U.S. aggregate equipment investment with different specification and filtering choices. Panels E and F plot correlograms of different output lags and the conditional volatility of U.S. aggregate structure investment with different specification and filtering choices. The unfiltered correlation is estimated on output detrended using a second order deterministic filter. The hp-filtered correlation is estimated on output detrended using an HP-filter with a smoothing parameter of 1600.
Sources: BEA and author's calculations.

Figure V – Comparison of Conditional Variance



Notes: This plot contains six subplots. Panel A plots the normalized residuals, otherwise referred to as the standard deviation of residuals, from a univariate autoregression of lag order 6 against the lagged average of investment for the U.S. aggregate total investment rate series. Panel C plots the normalized residuals, otherwise referred to as the standard deviation of residuals, from a univariate autoregression of lag order 7 against the lagged average of investment for the U.S. aggregate equipment investment rate series. Panel E plots the normalized residuals, otherwise referred to as the standard deviation of residuals, from a univariate autoregression of lag order 6 against the lagged average of investment for the U.S. Subplots B, D, and F plot the conditional volatility recovered from BCE and GARCH(1,1) specfications for U.S. aggregate total investment rate series, U.S. aggregate equipment investment rate series, U.S. aggregate structure investment rate series, respectively.aggregate structure investment rate series.
Sources: BEA and author calculations.

5 Discussion

In this section, I discuss why the two families of estimators produce different estimates of the conditional volatility. First, I show that the BCE estimator is isomorphic to an asymmetric GARCH(∞) estimator. Using this insight, I then show that BCE's measure of conditional volatility's state dependence η is not a direct measure of cyclicality and is sensitive to outliers. As a result, BCE's findings rely on the investment volatility spikes in the data that their model fails to predict.

The relationship between these two families of GARCH estimators is not immediately apparent. To provide intuition for my analysis, I use Wold's theory to show that the BCE specification is actually a type of asymmetric GARCH(∞) estimator:

$$\left(\sigma_t^{BCE}\right)^2 = \alpha + \eta \bar{x}_{t-1}^k,$$

$$\left(\sigma_t^{BCE}\right)^2 = \alpha + \eta \left(\sum_{j=1}^{k-1} \left(\frac{\sum_{i=1}^{j} b_{j-i}}{k} \right) \varepsilon_{t-j} + \sum_{j=k}^{\infty} \left(\frac{\sum_{i=1}^{k} b_{j-i}}{k} \right) \varepsilon_{t-j} \right),$$

$$\left(\sigma_t^{BCE}\right)^2 = \alpha + \eta \sum_{j=1}^{\infty} \left(\frac{d_j}{k} \right) \varepsilon_{t-j}, \; d_j = \sum_{i=1}^{\min\{j,k\}} b_{j-i}, \tag{8}$$

where b_j is the moving average weight of a residual at lag i in equation 1 (see Engle (1990) for initial discussion of asymmetric GARCH). Posing the BCE specification in this form yields two insights. First, the weight of each residual does not decrease monotonically, if the lag order of the moving average (k) is greater than 1. This is the case for all specifications included in BCE's empirical analysis. Rather, the weight of a residual increases between the first lag and the k^{th} lag before decreasing. The BCE specification exhibits higher persistence by construction, relative to the alternative specifications. Moreover, the BCE specifications that maximize the AIC place equal weight on residuals in the recent and distant past. To demonstrate this feature of the specification, I use an AR(1) to estimate the residuals of aggregate investment. This specification provides an intuitive closed-form representation of the BCE specification.

$$x_t = \rho x_{t-j} + \varepsilon_t, \tag{9}$$

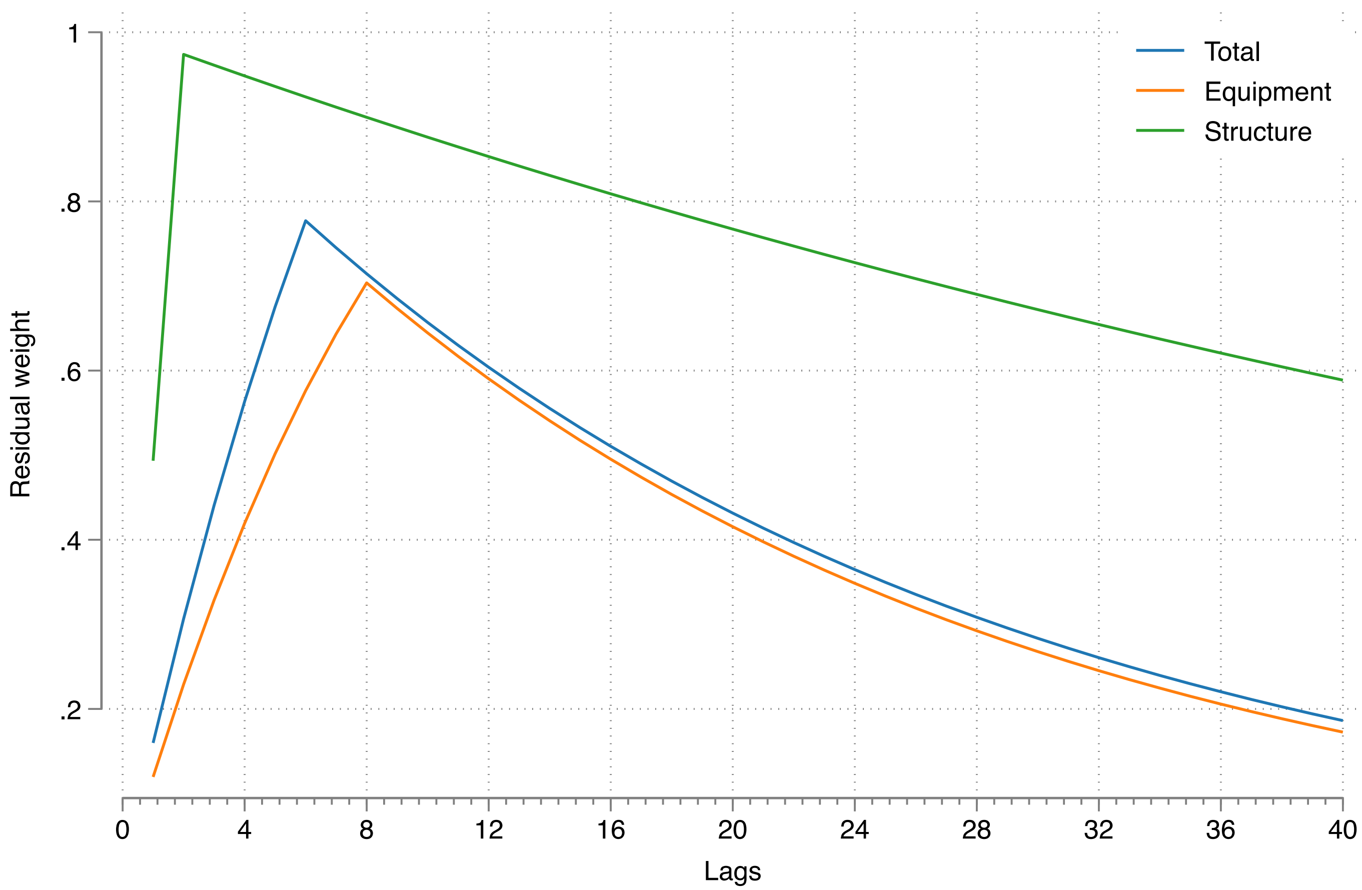
$$\varepsilon_t = \sigma_t e_t, \tag{10}$$

$$\left(\sigma_t^{BCE}\right)^2 = \alpha + \eta \sum_{j=1}^{\infty} \left(\frac{d_j}{k} \right) \varepsilon_{t-j}, \; d_j = \sum_{i=1}^{\min\{j,k\}} \rho^{j-i}, \tag{11}$$

Figure VI plots the residual weights of the following specifications. Outliers may lead to spurious asymmetries even in large samples in asymmetric GARCH models Carnero and Prez (2021), while outliers may lead to downward bias in the coefficients in symmetric GARCH models Kim and Meddahi (2020). Due to the size of their filter, the BCE specification places more weight on residuals from ten years ago than the residual from last quarter.

This features clarifies the economic intuition and statistical inference of the BCE specification. While the BCE specification tests for whether periods of heightened investment predict periods of

Figure VI – Weight of Residuals in BCE Specifications



Notes: This figure plots the implicit residual weights for the asymmetric GARCH(∞) representation of the BCE specifications. The weights correspond to a univariate autoregression with lag order of 1 for U.S. aggregate total investment, U.S. aggregate structure investment, and U.S. equipment investment. The lag order of lagged aggregate investment for the BCE specifications is 6 for total and equipment investment and 2 for structure investment.
Sources: BEA and author calculations.

increased volatility of investment, these periods do not coincide with the business cycle. The economic expansions of the 1990s and 2010s are the longest in the sample with durations of approximately 10 years. As a result, the BCE specification often places significant weight on residuals from previous expansions to predict future volatility. The implicit residual weighting scheme of BCE specifications also assumes that the effect of a single residual on the explanatory variable does not dissipate for multiple years. Figure VI can also be interpreted as a rescaled impulse response of conditional volatility to an exogenous shock to aggregate investment.

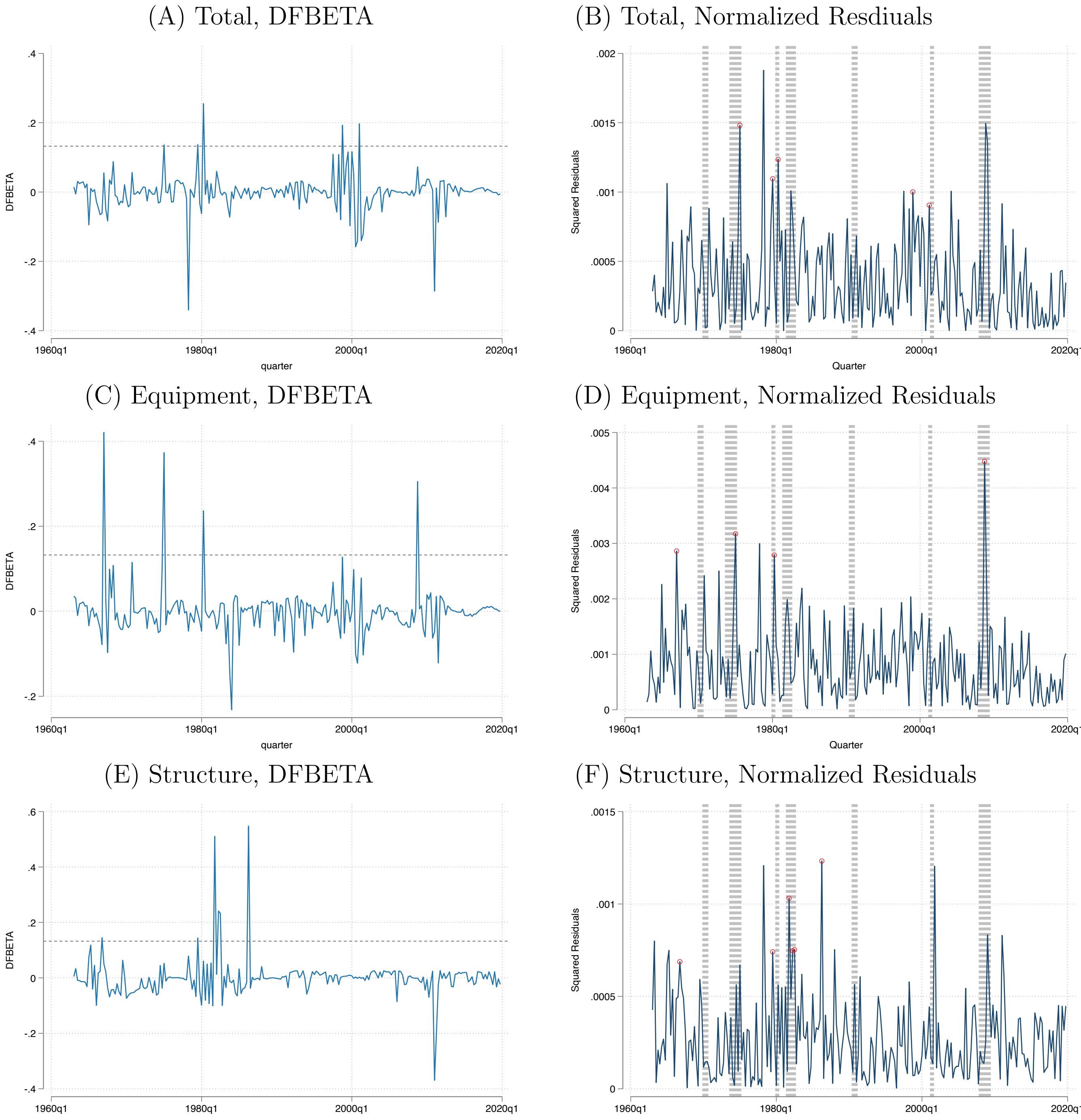
This implicit weighting scheme of the BCE specification also explains why η is not proportional to the cyclicality of the conditional volatility measured by the BCE specification. Different values of η do not change the persistence or shape of the impulse responses for an arbitrary shock to an economy at baseline. These features are determined by the selection of the lag order of the lagged average and the serial correlation of aggregate investment.

The functional form of the BCE estimator also increases the weight of large investment residuals on the conditional volatility of aggregate investment. It follows that their result may depend on a small number of influential observations. To measure the influence of each observation, I estimate the DFTBETA for each observation in the time series. The DFBETA measures the change in the coefficient caused by removing a certain observation, scaled by the standard deviation of the point estimate in the regression on the restricted sample. Outliers are considered sufficiently influential if they change the point estimate by more than $2/\sqrt(N)$ Belsley *et al.* (1980).

Figure VII plots the DFBETA for total investment and its components. This analysis shows that the empirical results of BCE are sensitive to a small number of observations. These observations are the volatility spikes in the sample that the BCE estimator does not predict. These spikes typically occur in and around recessions. Figure VIII plots the standard deviation of residuals against the lagged average of investment. It shows a few influential observations drive the empirical analysis in BCE. When those observations are excluded from the sample, there is no longer a significant relationship between the conditional volatility of aggregate investment and lagged average investment.

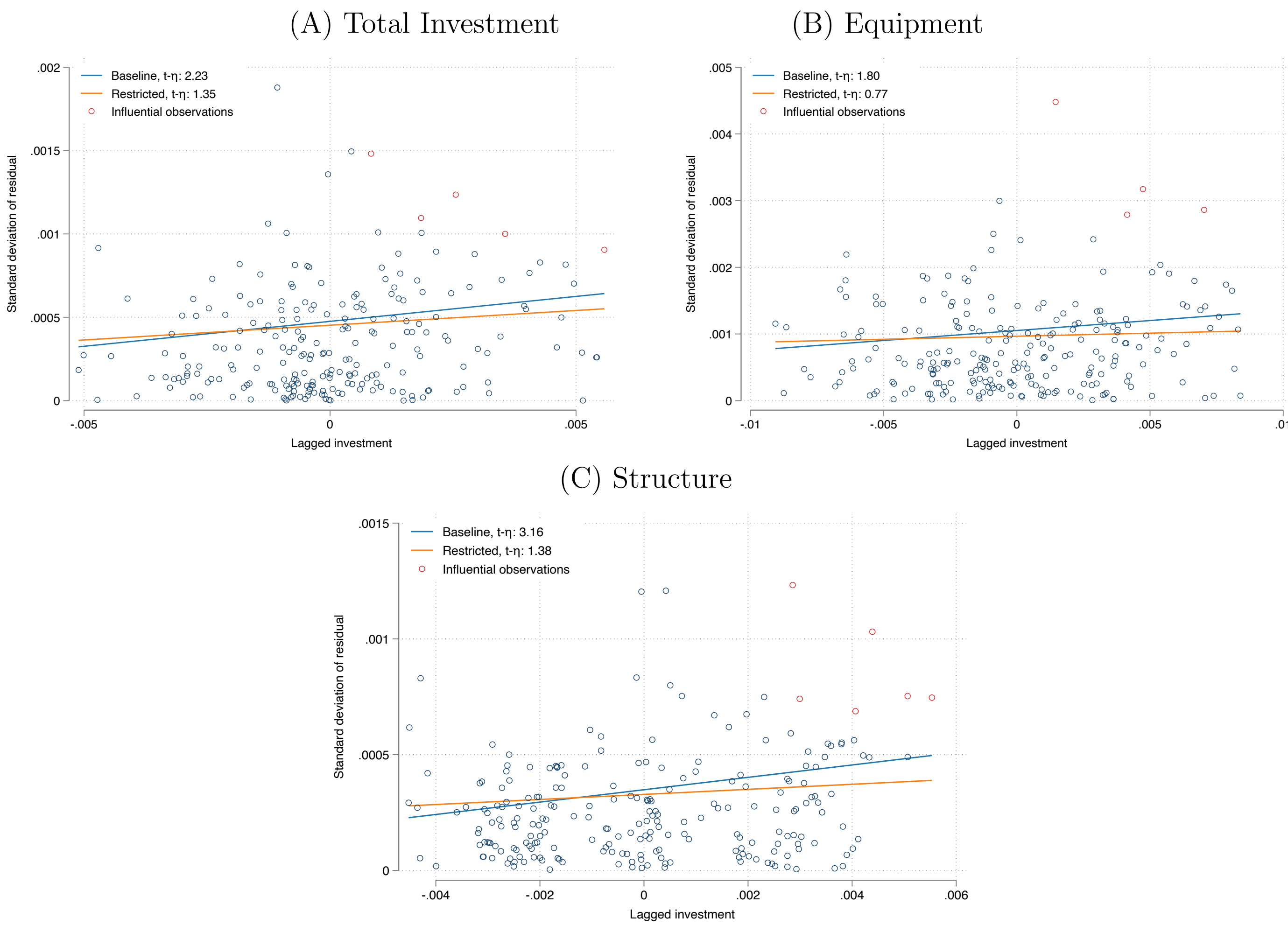
The alternative GARCH specifications are not equivalent to the BCE specification. Reposing the BCE specification as an asymmetric GARCH(∞) model shows that the two families of specifications use the information from residuals in distinct ways. Thus, it is unlikely that the two specifications provide similar estimates of the conditional volatility of aggregate investment. That said, one may be interested in whether the data generating processes implied by these estimators are somehow related. Appendix 9 responds to this question using two Monte Carlo experiments and confirms that the two data generating processes implied by these estimators are not equivalent.

Figure VII – Influential Observations in BCE Specifications



Notes: This plot contains six subplots. Subplots A, C, and E plot the DFBETA statistic for U.S. aggregate total investment rate series, U.S. aggregate equipment investment rate series, U.S. aggregate structure investment rate series, respectively. Panel B plots the normalized residuals, otherwise referred to as the standard deviation of residuals, from a univariate autoregression of lag order 6 against the lagged average of investment for the U.S. aggregate total investment rate series. Panel D plots the normalized residuals, otherwise referred to as the standard deviation of residuals, from a univariate autoregression of lag order 7 against the lagged average of investment for the U.S. aggregate equipment investment rate series. Panel F plots the normalized residuals, otherwise referred to as the standard deviation of residuals, from a univariate autoregression of lag order 6 against the lagged average of investment for the U.S. aggregate structure investment rate series.
Sources: BEA and author calculations.

Figure VIII – Effect of Influential Observations on BCE Coefficient

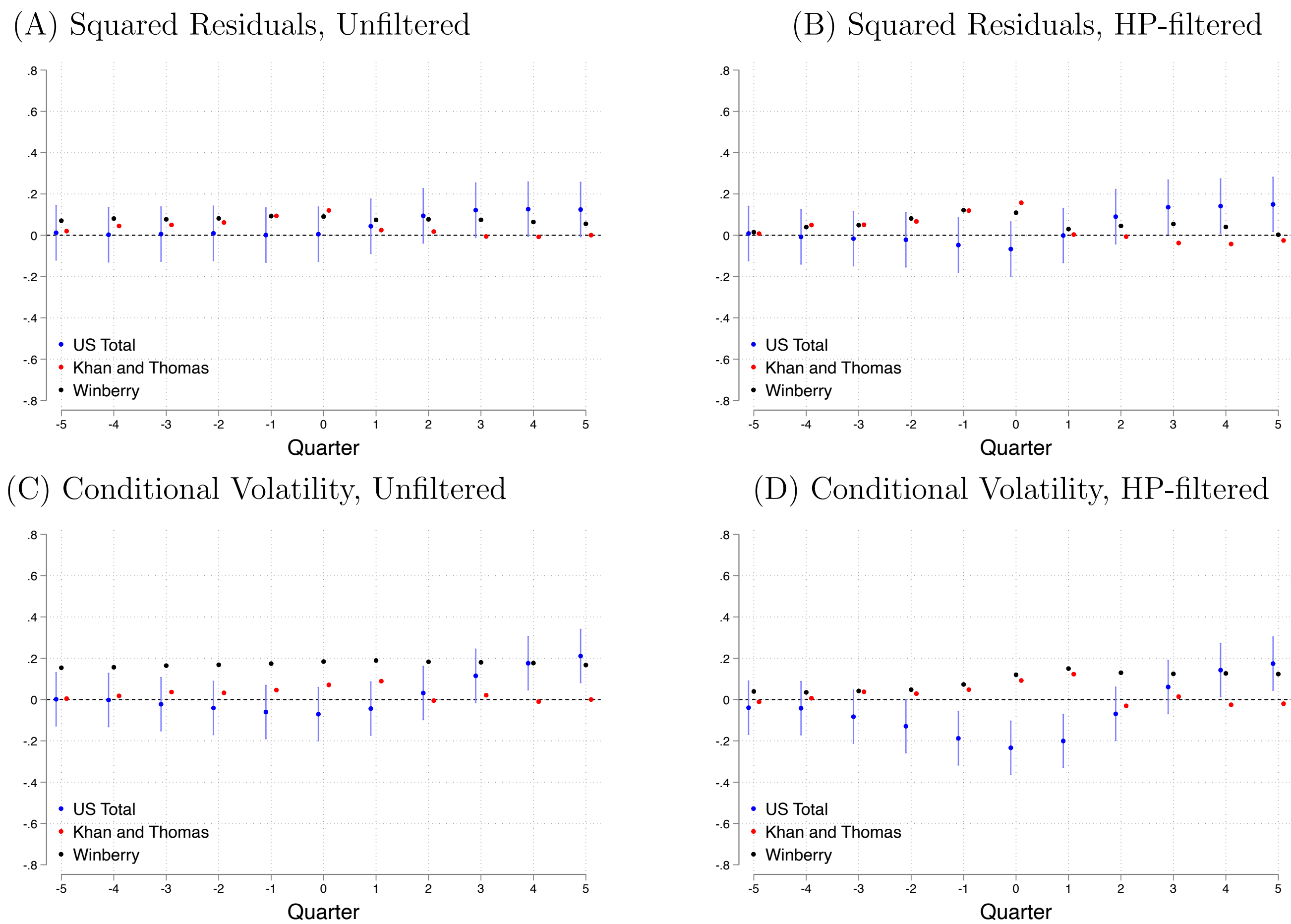


Notes: This plot contains three subplots. Panel A plots the estimation results of the BCE specification against a scatter plot of the standard deviation of residuals from a univariate autoregression of lag order 6 against the lagged average of investment for the U.S. aggregate total investment rate series. Data points with a DFBETA above $2/\sqrt(N)$ are highlighted in red, and the legend reports the t-statistic of η for the baseline sample and the restricted sample, which excludes the influential observations. Panel B plots the estimation results of the BCE specification against a scatter plot of the standard deviation of residuals from a univariate autoregression of lag order 7 against the lagged average of investment for the U.S. aggregate equipment investment rate series. Panel C plots the estimation results of the BCE specification against a scatter plot of the standard deviation of residuals from a univariate autoregression of lag order 6 against the lagged average of investment for the U.S. aggregate structure investment rate series.
Sources: BEA and author calculations.

6 Conditional Volatility of Simulated Environments

In this section, I estimate the two families of GARCH specifications on simulated data from two benchmark lumpy investment models with varying levels of state dependence: Khan and Thomas (2008) and Winberry (2021).[1] First, I recover the point estimates of the two simulated samples. Next, I compare the point estimates of the simulated data to the confidence intervals of the US data to test against the null that a given quantitative model could have produced the observed US data. Table I and II report the regression results. Figure IX plots the cyclicality of the simulated squared residuals and conditional variances.

Figure IX – Cyclicality of Simulated Squared Residuals and Conditional Volatilities



Notes: Panels A and B report correlograms of different output lags and the squared residuals recovered from autoregressive processes with a lag order of 6 estimated on U.S. aggregate total investment data and data from simulated Khan and Thomas (2008) and Winberry (2021) models. Panels C and D report correlograms of different output lags and the conditional variances estimated using a GARCH(1,1) specification.The unfiltered correlation is estimated on output detrended using a second order deterministic filter. The hp-filtered correlation is estimated on output detrended using an HP-filter with a smoothing parameter of 1600.
Sources: BEA and author calculations.

I fail to reject the null that cyclicality of squared residuals from both Khan and Thomas (2008) and Winberry (2021) models are statistically different from the residuals recovered from aggregate US investment rates. This is relevant because the two classes of models produce significantly different

[1] I use the codes provided by Winberry (2021) and Winberry (2018) to run the simulations for consistency with other studies.

Table I – Regression Results, BCE Specification, Simulated Data

Series	Filter	$\hat{\eta}$	SE
Khan and Thomas	HP	-.00012	.00021
Winberry	HP	.00008	.00004
Khan and Thomas	NF	.00006	.00013
Winberry	NF	.00007	.00002

Notes: This table reports conditional variance regression results using the BCE specification on simulated data from Khan and Thomas (2008) and Winberry (2021), using OLS. The first column denotes the model that generates the underlying data. The second column denotes the filter used prior to estimation. NF implies that the data is not filtered prior to estimation. HP denotes that the data is filtered using an HP-filter with a smoothing parameter of 1600 prior to estimation. The third column denotes the point estimate of η, which measures the relationship between the conditional volatility and the lagged average of aggregate investment.
Sources: Winberry (2021), Winberry (2018), and author calculations.

Table II – GARCH Results, Simulated Data

Series	Filter	Specification	β_a	SE_a	β_g	SE_g
K & T	NF	GARCH(0,1)	.0331634	.0338382		
K & T	NF	GARCH(1,1)	.032849	.0335836	-.3376483	.4932293
K & T	HP	GARCH(0,1)	.065429	.0374582		
K & T	HP	GARCH(1,1)	.0688493	.0383658	-.299037	.3152482
Winberry	NF	GARCH(0,1)	.0110846	.0357373		
Winberry	NF	GARCH(1,1)	.0305299	.0276434	.7427631	.2682626
Winberry	HP	GARCH(0,1)	.0098734	.0360306		
Winberry	HP	GARCH(1,1)	.0368468	.0307327	.7355672	.2557767

Notes: This table reports conditional variance regression results using the GARCH specifications on simulated data from Khan and Thomas (2008) and Winberry (2021). The first column denotes the model that generates the underlying data. The second column denotes the filter used prior to estimation. NF implies that the data is not filtered prior to estimation. The third column reports the point estimate of β_a, the coefficient on the moving average component. The fourth column reports the standard error of β_a. The fifth column reports the point estimate of β_g, the coefficient on the autoregressive component. The sixth column reports the standard error of β_g. HP denotes that the data is filtered using an HP-filter with a smoothing parameter of 1600 prior to estimation.
Sources: Winberry (2021), Winberry (2018), and author calculations.

aggregate dynamics, most notably their interest rate elasticity discussed in House (2014) and Winberry (2021), among others. Second, both models exhibit procyclical conditional variances. Third, the simulated series both fail to capture the relative size of the moving average component relative to the autoregressive component. In models with high state dependence akin to Winberry (2021), the conditional variance exhibits significant mean reversion and persistence, as the β_g coefficient is large, while undershooting the importance of recent innovations, as the β_a coefficient is statistically insignificant. Unsurprisingly, models with low state dependence like Khan and Thomas (2008) exhibit insignificant persistence, as β_g and β_a are statistically insignificant at the 5% level for all specifications.

7 Conclusion

The presence of conditional heteroskedasticity neither implies cyclical nor persistent heteroskedasticity. The conditional volatility of aggregate total and equipment investment is acyclical and impersistent, while the sensitivity of structure investment is countercyclical and persistent. The cyclicality of the investment heteroskedasticity documented in BCE mechanically follows from the cyclicality of the lagged investment.

Since the cyclicality of aggregate investment varies across capital types, future quantitative work should consider possible explanations for why the conditional volatility of aggregate structure investment is more persistent than that of aggregate equipment investment. Possible work should consider the interaction of maintenance investment with fixed costs and other nonlinear investment frictions. For example, partial irreversibility, whether due to adverse selection or adjustment costs, creates a wedge between the buying price and selling price of different capital goods. As equipment tends to have a thicker resale market, the importance of maintenance investment may vary across capital types. Alternatively, equipment is also more tradable than structures, so the price elasticity of equipment supply is more elastic. This feature, outlined in numerous studies, could explain why the conditional volatility of investment varies across capital goods.

References

Bachmann, R., Caballero, R. J. and Engel, E. M. R. A. (2013). Aggregate Implications of Lumpy Investment: New Evidence and a DSGE Model. *American Economic Journal: Macroeconomics*, **5** (4), 29–67.

Baley, I. and Blanco, A. (2021). Aggregate Dynamics in Lumpy Economies. *Econometrica*, **89** (3), 1235–1264.

Belsley, D. A., Kuh, E. and Welsch, R. E. (1980). *Regression Diagnostics: Identifying Influential Data and Sources of Collinearity*. John Wiley & Sons.

Berger, D. and Vavra, J. (2015). Consumption Dynamics during Recessions. *Econometrica*, **83** (1), 101–154.

Caballero, R. J., Engel, E. M. R. A., Haltiwanger, J. C., Woodford, M. and Hall, R. E. (1995). Plant-Level Adjustment and Aggregate Investment Dynamics. *Brookings Papers on Economic Activity*, **1995** (2), 1–54.

Carnero, M. A. and Prez, A. (2021). Outliers and Misleading Leverage Effect in Asymmetric GARCH-type Models. *Studies in Nonlinear Dynamics & Econometrics*, **25** (1), 20180073.

Engle, R. F. (1990). Stock Volatility and the Crash of '87: Discussion. *The Review of financial studies*, **3** (1), 103?106.

House, C. L. (2014). Fixed Costs and Long-Lived Investments. *Journal of Monetary Economics*, **68**, 86–100.

Khan, A. and Thomas, J. (2008). Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics. *Econometrica*, **76** (2), 395–436.

Kim, J. and Meddahi, N. (2020). Volatility Regressions with Fat Tails. *Journal of Econometrics*, **218** (2), 690–713.

Lamoureux, C. G. and Lastrapes, W. D. (1990). Persistence in Variance, Structural Change, and the GARCH Model. *Journal of Business & Economic Statistics*, **8** (2), 225–234.

Winberry, T. (2018). A Method for Solving and Estimating Heterogeneous Agent Macro Models. *Quantitative Economics*, **9** (3), 1123–1151.

— (2021). Lumpy Investment, Business Cycles, and Stimulus Policy. *American Economic Review*, **111** (1), 364–96.

8 Data and BCE Replication

In this appendix, I discuss my replication of time series exercise featured in BCE and how I construct quarterly aggregate investment rate series using data from the Bureau of Economic Analysis (BEA). I follow the procedure outlined in BCE. This procedure incorporates data on total, equipment, and structure investment and capital from the following national account and fixed asset tables: Table 1.1.5 Gross Domestic Product lines 9-11 provides nominal annual private fixed nonresidential investment; Table 1.1. Fixed Assets and Consumer Durable Goods lines 4-6 provides annual capital stock at year-end prices; Tables 1.3 Fixed Assets and Consumer Durable Goods lines 4-6 provides nominal annual private nonresidential depreciation; Table 1.1.5 Gross Domestic Product lines 9-11 provides quarterly nominal fixed nonresidential investment; And, Table 1.1.9 Gross Domestic Product lines 9-11 quarterly implicit price deflators. BCE explains their data-cleaning procedure in detail in their appendix.

In 2013, the BEA announced a comprehensive revision of their national account and fixed asset data tables. As part of this revision, the BEA changed their treatment of intellectual property, equipment, and private nonresidential investment. Specifically, the BEA changed the name of Table 1.1.5 Gross Domestic Product line 11 from "equipment" to "equipment and software," separating equipment and software investment. Accompanying this change, they included intellectual property investment as a component of private nonresidential fixed investment. After 2013, Table 1.1.5 Gross Domestic Product line 12 reports annual private nonresidential fixed investment, defined as "investment in software, in research and development (R&D), and in entertainment, literary, and artistic originals by private business." While this revision also included minor changes in the accounting of transactions costs and depreciation, the two revisions discussed above in detail are the revisions most pertinent to this analysis. Figure X plots the extended sample against the replication data provided by BCE. The structure investment rate series closely tracks the series used in BCE, as the 2013 revision did not significantly affect the accounting of this variable. The equipment and total investment rate series are highly correlated with the series used in BCE, but diverge towards the end of the original sample, which tracks the recent upward trend in intellectual property investment in the US economy.

The time horizon in BCE is 1960Q1-2005Q4. The time horizon for my baseline specification is 1960Q1-2019Q4. To understand the relationship between the original study and this project, I replicate their baseline empirical specifications below using the data provided in their replication file. Their baseline empirical specifications take the follow form:

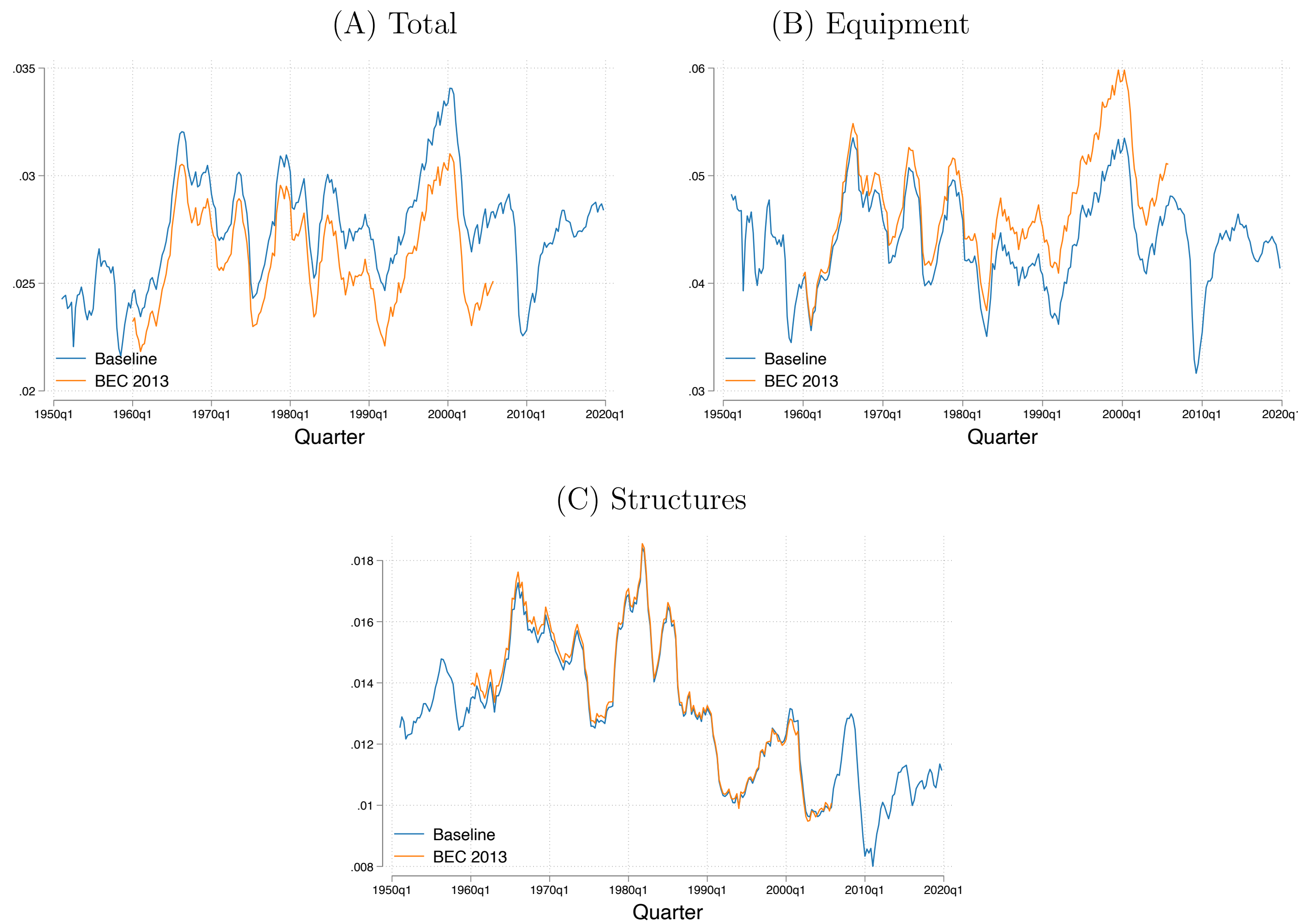
$$\left(\sigma_t^{BCE}\right)^2 = \alpha + \eta \bar{x}_{t-1}^k \tag{12}$$

$$|\sigma_t^{BCE}| = \alpha + \eta \bar{x}_{t-1}^k \tag{13}$$

I am able to replicate their baseline results in Stata 14. Table XI reports the original empirical results of BCE and correspond to Table 3 in their original text. Table XII presents the replication of their baseline results. In general the estimates are very close, varying only marginally. As an exercise in data validation, I reproduce their baseline specification for my extended sample. Table XIII presents the results of this exercise. In general, the estimates are close.

Departing from these results, I use their second model, which regresses squared residuals onto lagged investment, for the rest of the replication exercises, as the regressor matches the regressor used in the standard GARCH(p,q) specification. Since I employ a Quasi-Maximum-Likelihood estimator (QMLE) to estimate the GARCH(1,1) specification, I replicate the BCE specification using QMLE and recover comparable estimates for the original and extended sample. Table XIV presents the estimates recovered from the BCE replication data. Table XVI presents the results of this exercise for the full sample. The coefficient of interest in the BCE estimated using QMLE is well within the 95% confidence interval of the original estimate recovered through OLS. As discussed in the main text of the paper, given the impersistence of conditional heteroskedasticity in the total and equipment investment series, the GARCH(1,1) series does not converge for total and equipment investment in the replication sample. In the extended sample, the autoregressive coefficient in the GARCH equation is statistically insignificant and negative, suggesting that those specifications are misspecified for total and equipment investment.

Figure X – Comparison of Baseline Sample and BCE



Notes: This plot contains three subplots, which compare the baseline sample used for empirical analysis in this paper against the replication data provided by Bachmann *et al.* (2013). Panel A plots the U.S. aggregate total investment rate series used in the baseline analysis against U.S. total aggregate investment rate series used in Bachmann *et al.* (2013). Panel A plots the U.S. aggregate equipment investment rate series used in the baseline analysis against U.S. equipment aggregate investment rate series used in Bachmann *et al.* (2013). Panel A plots the U.S. aggregate structure investment rate series used in the baseline analysis against U.S. structure aggregate investment rate series used in Bachmann *et al.* (2013).
Sources: BEA, Bachmann *et al.* (2013), and author's calculations.

Figure XI – Table 3 from BCE

	TOT - model 1	TOT - model 2	EQ - model 1	EQ - model 2	ST - model 1	ST - model 2
p	6	6	7	7	6	6
k	6	6	8	8	2	2
$1000 * \eta$	45.93	.03731	30.62	.053780	39.96	.02581
t - η	3.121	2.496	2.089	1.724	4.097	3.246
p-value ($\eta > 0$) - bootstrap	.0088	.0236	.0375	.0742	.0033	.0094
$\log(\sigma_{max}/\sigma_{min})$	.7367	.5933	.5521	.4395	1.1167	1.1169
$\log(\sigma_{95}/\sigma_5)$	.6118	.4816	.4520	.3547	.9194	.8994
$\log(\sigma_{90}/\sigma_{10})$	.51203	.4082	.3355	.2646	.8003	.7403
Skewness	.1574	.1574	.3759	.3759	-.1051	-.1051
Excess Kurtosis	-.9803	-.9803	-.1401	-.1401	-.9864	-.9864
Autocorr. e_t	-.0452	-.0412	-.0151	-.0131	-.0823	-.0826
N	172	172	172	172	172	172

*Notes:*This table reports a replication Table 3 in Bachmann *et al.* (2013). The table features 6 GARCH-X regression results and summary statistics using aggregate investment rate data from 1960-2005. The first two columns report summary statistics and regression results for two specifications estimated on total US aggregate investment. The third and fourth columns report summary statistics and regression results for two specifications estimated on US aggregate equipment investment. The fifth and sixth columns report summary statistics and regression results for two specifications estimated on US aggregate structure investment. Model 1 estimates the volatility as a function of the absolute value of the lagged average of aggregate investment. Model 2 estimates squared volatility as a function of the lagged average of aggregate investment. The first row reports the lag order of the univariate autoregression that estimates the residuals. The second row reports the lag order of the lagged average of aggregate investment used in the second stage of the estimation. The third row presents the rescaled regression coefficient for the lagged average of aggregate investment in the second stage. The fourth reports the t-statistic of η. The fifth row reports the p-value of a bootstrap of 20,000 simulations. The sixth through ninth rows report different measures of quantile distance for the predicted volatility of aggregate investment. The tenth and eleventh rows report the skewness and excess kurtosis of the underlying series, respectively. The twelfth row reports the first order autocorrelation of the latent structural shock. The last row reports the sample size.

Sources: BEA, Bachmann *et al.* (2013).

Figure XII – Replication of BCE, Replication Sample

	TOT - model 1	TOT - model 2	EQ - model 1	EQ - model 2	ST - model 1	ST - model 2
p	6	6	7	7	6	6
k	6	6	8	8	2	2
$1000 * \eta$	45.94	.037315	30.63	.053796	39.96	.025806
t - η	3.122	2.496	2.089	1.724	4.097	3.246
p-value ($\eta > 0$) - bootstrap	.0021	.0211	.0182	.0686	.0008	.0061
$\log(\sigma_{max}/\sigma_{min})$	.7368	.5933	.5521	.4396	1.1168	1.117
$\log(\sigma_{95}/\sigma_5)$	.6121	.4819	.4534	.3558	.9198	.89
$\log(\sigma_{90}/\sigma_{10})$	.5186	.407	.3355	.2646	.7944	.7329
Skewness	.1574	.1574	.376	.376	-.1051	-.1051
Excess Kurtosis	-.9803	-.9803	-.14	-.14	-.9863	-.9863
Autocorr. e_t	-.0452	-.0411	-.0151	-.0131	-.0822	-.0826
N	172	172	172	172	172	172

Notes: This table reports a replication of the original empirical analysis in Table 3 of Bachmann *et al.* (2013), using the replication data provided by the authors. The sample spans 1960-2005. This replication uses OLS to estimate the first and second stage of the GARCH process. The table features 6 GARCH-X regression results and summary statistics using aggregate investment rate data. The first two columns report summary statistics and regression results for two specifications estimated on total US aggregate investment. The third and fourth columns report summary statistics and regression results for two specifications estimated on US aggregate equipment investment. The fifth and sixth columns report summary statistics and regression results for two specifications estimated on US aggregate structure investment. Model 1 estimates the volatility as a function of the absolute value of the lagged average of aggregate investment. Model 2 estimates squared volatility as a function of the lagged average of aggregate investment. The first row reports the lag order of the univariate autoregression that estimates the residuals. The second row reports the lag order of the lagged average of aggregate investment used in the second stage of the estimation. The third row presents the rescaled regression coefficient for the lagged average of aggregate investment in the second stage. The fourth reports the t-statistic of η. The fifth row reports the p-value of a bootstrap of 20,000 simulations. The sixth through ninth rows report different measures of quantile distance for the predicted volatility of aggregate investment. The tenth and eleventh rows report the skewness and excess kurtosis of the underlying series, respectively. The twelfth row reports the first order autocorrelation of the latent structural shock. The last row reports the sample size.

Sources: BEA, Bachmann *et al.* (2013), and author's calculations.

Figure XIII – Replication of BCE with Extended Sample

	TOT - model 1	TOT - model 2	EQ - model 1	EQ - model 2	ST - model 1	ST - model 2
p	6	6	7	7	6	6
k	6	6	8	8	2	2
$1000 * \eta$	37.95	.028454	28.87	.062561	24.87	.01867
t - η	3.031	2.23	1.944	1.796	3.218	3.156
p-value ($\eta > 0$) - bootstrap	.0052	.0517	.0286	.0556	.0098	.0193
$\log(\sigma_{max}/\sigma_{min})$	.9515	.7612	.5145	.5453	.7398	.858
$\log(\sigma_{95}/\sigma_5)$	.6075	.4535	.3821	.3849	.5078	.5415
$\log(\sigma_{90}/\sigma_{10})$	.4676	.3494	.2991	.2999	.4555	.4843
Skewness	.1475	.1475	-.0678	-.0678	.1261	.1261
Excess Kurtosis	-.198	-.198	-.1155	-.1155	-.7475	-.7475
Autocorr. e_t	-.0344	-.0308	-.0017	-.0022	-.0238	-.0346
N	228	228	228	228	228	228

Notes: This table reports the results of a replication of the original empirical analysis in Table 3 of Bachmann *et al.* (2013), using an extended sample from 1960-2019. The aggregate investment rate series is constructed following the process applied in Bachmann *et al.* (2013). This replication uses OLS to estimate the first and second stage of the GARCH process. The table features 6 GARCH-X regression results and summary statistics using aggregate investment rate data. The first two columns report summary statistics and regression results for two specifications estimated on total US aggregate investment. The third and fourth columns report summary statistics and regression results for two specifications estimated on US aggregate equipment investment. The fifth and sixth columns report summary statistics and regression results for two specifications estimated on US aggregate structure investment. Model 1 estimates the volatility as a function of the absolute value of the lagged average of aggregate investment. Model 2 estimates squared volatility as a function of the lagged average of aggregate investment. The first row reports the lag order of the univariate autoregression that estimates the residuals. The second row reports the lag order of the lagged average of aggregate investment used in the second stage of the estimation. The third row presents the rescaled regression coefficient for the lagged average of aggregate investment in the second stage. The fourth reports the t-statistic of η. The fifth row reports the p-value of a bootstrap of 20,000 simulations. The sixth through ninth rows report different measures of quantile distance for the predicted volatility of aggregate investment. The tenth and eleventh rows report the skewness and excess kurtosis of the underlying series, respectively. The twelfth row reports the first order autocorrelation of the latent structural shock. The last row reports the sample size.
Sources: BEA, Bachmann *et al.* (2013), and author's calculations.

Figure XIV – Comparison of Baseline Specifications and BCE, Replication Sample

	T - BCE	T - G0,1)	T - G(1,1)	E - BCE	E - G(0,1)	E - G(1,1)	S - BCE	S - G(0,1)	S - G(1,1)
$1000 * \eta$	.039297	.	.	.042259	.	.	.024307	.	.
t - η	2.977	.	.	1.604	.	.	4.772	.	.
β^a	.	-.083	.014	.	-.061	.	.	.042	.039
t - β^a	.	-6.166	.274	.	-.685	.	.	.509	.912
β^g	.	.	-.915	.	.	.	.	.	.888
t - β^g	.	.	-1.895	.	.	.	.	.	7.524
$\log(\sigma_{max}/\sigma_{min})$	.634	1.788	.132	.344	.365	.	1.003	.209	.355
$\log(\sigma_{95}/\sigma_5)$	.512	.146	.066	.281	.139	.	.809	.093	.267
$\log(\sigma_{90}/\sigma_{10})$	.432	.118	.055	.209	.083	.	.674	.051	.215
Autocorr e_t	-.042	-.006	-.015	-.011	-.013	.	-.075	-.037	-.023
p-value ($e_t > 0$)	.579	.935	.836	.884	.862	.	.324	.628	.756
N	172	172	172	172	172	172	172	172	172

Notes: This table reports a replication of the original empirical analysis in Table 3 of Bachmann *et al.* (2013), using the replication data provided by the authors. The sample spans 1960-2005. The aggregate investment rate series is constructed following the process applied in Bachmann *et al.* (2013). This replication uses OLS to estimate the first stage and MLE to estimate the second stage of the GARCH process. The table features 9 GARCH regression results and summary statistics using aggregate investment rate data. The first three columns report summary statistics and regression results for three specifications estimated on total US aggregate investment. The fourth, fifth, and sixth columns report summary statistics and regression results for two specifications estimated on US aggregate equipment investment. The seventh, eighth, and ninth columns report summary statistics and regression results for two specifications estimated on US aggregate structure investment. Columns labeled BCE estimates squared volatility as a function of the lagged average of aggregate investment. Columns labeled G(.,.) estimates standard GARCH(a, b) estimations, where a is the autoregressive lag order and b is the moving average lag order. The first row presents the rescaled regression coefficient for the lagged average of aggregate investment in the second stage for BCE specifications. The second row reports the t-statistic of η for BCE specifications. The third row presents the regression coefficient for the moving average component of GARCH specifications. The fourth row reports the t-statistic of the moving average component of GARCH specifications. The fifth row presents the regression coefficient for the autoregressive component of GARCH(1,1) specifications. The sixth row reports the t-statistic of the autoregressive component of GARCH(1,1) specifications. The seventh through tenth rows report different measures of quantile distance for the predicted volatility of aggregate investment. The eleventh and twelfth rows report the skewness and excess kurtosis of the underlying series, respectively. The thirteenth row reports the first order autocorrelation of the latent structural shock. The last row reports the sample size.
Sources: BEA, Bachmann *et al.* (2013), and author's calculations.

Figure XV – Comparison of Baseline Specifications and BCE, Full Sample

	T - BCE	T - G(0,1)	T - G(1,1)	E - BCE	E - G(0,1)	E - G(1,1)	S - BCE	S - G(0,1)	S - G(1,1)
$1000 * \eta$	.024554	.	.	.046526	.	.	.012395	.	.
t - η	2.9721	.	.	2.0265	.	.	3.3554	.	.
β^a	.	.156	.1687	.	.0802	.1032	.	.0192	.027
t - β^a	.	1.7982	1.9923	.	.9969	1.1597	.	.27	.6635
β^g	.	.	-.2994	.	.	-.4573	.	.	.8855
t - β^g	.	.	-1.1143	.	.	-.8819	.	.	5.1541
$\log(\sigma_{max}/\sigma_{min})$	.6323	.668	1.634	.3882	.4706	.7775	.5227	.1061	.2403
$\log(\sigma_{95}/\sigma_5)$	.3899	.2552	.3358	.2825	.1337	.2175	.3524	.041	.1746
$\log(\sigma_{90}/\sigma_{10})$	.3005	.2044	.2415	.2209	.1055	.1662	.3162	.024	.1449
Autocorr. e_t	-.0301	-.0336	-.0292	-.0008	-.0052	-.0057	-.0213	-.0183	-.0033
p-value ($e_t > 0$)	.6519	.6144	.6606	.9904	.9371	.9308	.7489	.7839	.96
N	228	228	228	228	228	228	228	228	228

Notes: This table reports a replication of the original empirical analysis in Table 3 of Bachmann *et al.* (2013), using an extended sample from 1960-2019. This replication uses OLS to estimate the first stage and MLE to estimate the second stage of the GARCH process. The table features 9 GARCH regression results and summary statistics using aggregate investment rate data. The first three columns report summary statistics and regression results for three specifications estimated on total US aggregate investment. The fourth, fifth, and sixth columns report summary statistics and regression results for two specifications estimated on US aggregate equipment investment. The seventh, eighth, and ninth columns report summary statistics and regression results for two specifications estimated on US aggregate structure investment. Columns labeled BCE estimates squared volatility as a function of the lagged average of aggregate investment. Columns labeled G(.,.) estimates standard GARCH(a,b) estimations, where a is the autoregressive lag order and b is the moving average lag order. The first row presents the rescaled regression coefficient for the lagged average of aggregate investment in the second stage for BCE specifications. The second row reports the t-statistic of η for BCE specifications. The third row presents the regression coefficient for the moving average component of GARCH specifications. The fourth row reports the t-statistic of the moving average component of GARCH specifications. The fifth row presents the regression coefficient for the autoregressive component of GARCH(1,1) specifications. The sixth row reports the t-statistic of the autoregressive component of GARCH(1,1) specifications. The seventh through tenth rows report different measures of quantile distance for the predicted volatility of aggregate investment. The eleventh and twelfth rows report the skewness and excess kurtosis of the underlying series, respectively. The thirteenth row reports the first order autocorrelation of the latent structural shock. The last row reports the sample size.
Sources: BEA and author's calculations.

Figure XVI – Comparison of Baseline Specifications and BCE, Full Sample

	T - BCE	T - G(0,1)	T - G(1,1)	E - BCE	E - G(0,1)	E - G(1,1)	S - BCE	S - G(0,1)	S - G(1,1)
$1000 * \eta$	.024554	.	.	.046526	.	.	.012395	.	.
t - η	2.9721	.	.	2.0265	.	.	3.3554	.	.
β^a	.	.156	.1687	.	.0802	.1032	.	.0192	.027
t - β^a	.	1.7982	1.9923	.	.9969	1.1597	.	.27	.6635
β^g	.	.	-.2994	.	.	-.4573	.	.	.8855
t - β^g	.	.	-1.1143	.	.	-.8819	.	.	5.1541
$\log(\sigma_{max}/\sigma_{min})$	.6323	.668	1.634	.3882	.4706	.7775	.5227	.1061	.2403
$\log(\sigma_{95}/\sigma_5)$	.3899	.2552	.3358	.2825	.1337	.2175	.3524	.041	.1746
$\log(\sigma_{90}/\sigma_{10})$	.3005	.2044	.2415	.2209	.1055	.1662	.3162	.024	.1449
Autocorr. e_t	-.0301	-.0336	-.0292	-.0008	-.0052	-.0057	-.0213	-.0183	-.0033
p-value ($e_t > 0$)	.6519	.6144	.6606	.9904	.9371	.9308	.7489	.7839	.96
N	228	228	228	228	228	228	228	228	228

Notes: This table reports a replication of the original empirical analysis in Table 3 of Bachmann *et al.* (2013), using an extended sample from 1960-2019. This replication uses OLS to estimate the first stage and MLE to estimate the second stage of the GARCH process. The table features 9 GARCH regression results and summary statistics using aggregate investment rate data. The first three columns report summary statistics and regression results for three specifications estimated on total US aggregate investment. The fourth, fifth, and sixth columns report summary statistics and regression results for two specifications estimated on US aggregate equipment investment. The seventh, eighth, and ninth columns report summary statistics and regression results for two specifications estimated on US aggregate structure investment. Columns labeled BCE estimates squared volatility as a function of the lagged average of aggregate investment. Columns labeled G(.,.) estimates standard GARCH(a, b) estimations, where a is the autoregressive lag order and b is the moving average lag order. The first row presents the rescaled regression coefficient for the lagged average of aggregate investment in the second stage for BCE specifications. The second row reports the t-statistic of η for BCE specifications. The third row presents the regression coefficient for the moving average component of GARCH specifications. The fourth row reports the t-statistic of the moving average component of GARCH specifications. The fifth row presents the regression coefficient for the autoregressive component of GARCH(1,1) specifications. The sixth row reports the t-statistic of the autoregressive component of GARCH(1,1) specifications. The seventh through tenth rows report different measures of quantile distance for the predicted volatility of aggregate investment. The eleventh and twelfth rows report the skewness and excess kurtosis of the underlying series, respectively. The thirteenth row reports the first order autocorrelation of the latent structural shock. The last row reports the sample size.
Sources: BEA and author's calculations.

Figure XVII – Comparison of Baseline Specifications and BCE, Full Sample, HP-Filtered

	T - BCE	T - G(0,1)	T - G(1,1)	E - BCE	E - G(0,1)	E - G(1,1)	S - BCE	S - G(0,1)	S - G(1,1)
$1000 * \eta$	.023094	.	.	.066422	.	.	.019054	.	.
t - η	1.6877	.	.	2.0839	.	.	2.0276	.	.
β^a	.	.1163	.128	.	.0729	.1227	.	-.0648	.0408
t - β^a	.	1.5597	1.6302	.	1.0308	1.2436	.	-1.4834	.8824
β^g	.	.	-.3111	.	.	-.5063	.	.	.8847
t - β^g	.	.	-.6755	.	.	-1.0023	.	.	7.8317
$\log(\sigma_{max}/\sigma_{min})$	.2863	.5288	.9712	.3541	.4185	.9994	.3862	.778	.35
$\log(\sigma_{95}/\sigma_5)$	.2348	.1952	.2604	.2445	.1228	.2495	.235	.1122	.2475
$\log(\sigma_{90}/\sigma_{10})$	.1791	.1442	.1865	.1928	.088	.1871	.1947	.081	.1919
Autocorr. e_t	-.0192	-.0263	-.0181	-.0085	-.015	-.0134	-.0227	-.0322	.011
p-value ($e_t > 0$)	.7732	.6934	.7857	.8985	.8214	.8403	.7336	.6285	.8691
N	228	228	228	228	228	228	228	228	228

Notes: This table reports a replication of the original empirical analysis in Table 3 of Bachmann *et al.* (2013), using an extended sample from 1960-2019. All investment series are HP-filtered prior to estimation using a smoothing parameter of 1600. This replication uses OLS to estimate the first stage and MLE to estimate the second stage of the GARCH process. The table features 9 GARCH regression results and summary statistics using aggregate investment rate data. The first three columns report summary statistics and regression results for three specifications estimated on total US aggregate investment. The fourth, fifth, and sixth columns report summary statistics and regression results for two specifications estimated on US aggregate equipment investment. The seventh, eighth, and ninth columns report summary statistics and regression results for two specifications estimated on US aggregate structure investment. Columns labeled BCE estimates squared volatility as a function of the lagged average of aggregate investment. Columns labeled G(.,.) estimates standard GARCH(a, b) estimations, where a is the autoregressive lag order and b is the moving average lag order. The first row presents the rescaled regression coefficient for the lagged average of aggregate investment in the second stage for BCE specifications. The second row reports the t-statistic of η for BCE specifications. The third row presents the regression coefficient for the moving average component of GARCH specifications. The fourth row reports the t-statistic of the moving average component of GARCH specifications. The fifth row presents the regression coefficient for the autoregressive component of GARCH(1,1) specifications. The sixth row reports the t-statistic of the autoregressive component of GARCH(1,1) specifications. The seventh through tenth rows report different measures of quantile distance for the predicted volatility of aggregate investment. The eleventh and twelfth rows report the skewness and excess kurtosis of the underlying series, respectively. The thirteenth row reports the first order autocorrelation of the latent structural shock. The last row reports the sample size.
Sources: BEA and author's calculations.

9 Monte Carlo Experiments

In this appendix, I run two Monte Carlo experiments to document the performance of BCE specifications on data generated using a GARCH process. This experiment seeks to understand how the BCE would perform when the underlying data generating process is symmetric. It is not intended to assume that the underlying data generating process of aggregate investment is symmetric. The data generating process in this experiment takes the following form

$$x_{1,t} = \rho x_{t-j} + \varepsilon_{1,t}, \tag{14}$$

$$\varepsilon_{1,t} = e_t \sigma_{1,t},\ e_t \sim N(0,1), \tag{15}$$

$$\sigma^2_{1,t} = \alpha + \beta_g \sigma^2_{1,t-1} + \beta_a \varepsilon^2_{1,t-1}, \tag{16}$$

where $\rho = 0.95$, $\alpha = 6e^{-4}$ coincide with the unconditional volatility and autoregressive consistent for aggregate investment. I report the average η and corresponding t-statistic for different values of β_g and β_a. Each Monte Carlo experiment consists of 10,000 simulations with 172 observations each, which is the sample length of the original BCE empirical analysis. Table III reports the average η of the simulations, and Table III reports probability that η is significant at a 5% level. There is no standard systematic between the size of the GARCH coefficinets and the average value of η. That said, the probability that the null hypothesis of $\eta = 0$ is rejected increases as the autoregressive and moving average coefficients increase. For US aggregate investment, the moving average component of the baseline specification is below 0.2. This analysis provides two insights. First, the BCE specification may provide imprecise intuition for the evolution of the conditional volatility of aggregate investment when the underlying data is generated by GARCH(1,1) specification, when the moving average and autoregressive components are high. Second, it is unlikely that an econometrician would observe a relationship between the conditional volatility of aggregate investment and the lagged average of aggregate investment if the underlying data is generated by GARCH(1,1) specification.

Table III – Average η, Monte Carlo Simulation

$\beta_a \setminus \beta_g$	0.2	0.4	0.6	0.8
0.2	-0.0005	0.0008	-0.0011	-0.0018
0.4	-3.420E-05	-0.0012	-0.0004	0.0023
0.6	-4.820E-05	-0.0001	-0.0026	0.0038
0.8	-0.0005	0.0055	0.0041	-0.0016

Notes: This table reports the average η in a Monte Carlo experiment consists of 10,000 simulations with 172 observations each, where the underlying data generating process is a GARCH(1,1). The rows in the table report the coefficient on the moving average component.
Source: Author's calculations.

Table IV – Probability $t_\eta > 1.96$, Monte Carlo Simulation

$\beta_a \setminus \beta_g$	0.2	0.4	0.6	0.8
0.2	0.0484	0.102	0.1509	0.2117
0.4	0.0661	0.1372	0.2378	0.3156
0.6	0.0992	0.2364	0.3317	0.4092
0.8	0.1899	0.4103	0.4349	0.4525

Notes: This table reports the probability that the t_η statistic is greater than 1.96 in a Monte Carlo experiment consists of 10,000 simulations with 172 observations each, where the underlying data generating process is a GARCH(1,1). The rows in the table report the coefficient on the moving average component.
Source: Author's calculations.