Econ 675: HW 1

Erin Markiewitz

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Problem 1.1.

By LLN,

$$\begin{aligned} \hat{\beta_{LS}} &= \frac{\tilde{\mathbf{x}}'\mathbf{x}}{\tilde{\mathbf{x}}'\tilde{\mathbf{x}}}\beta + \frac{\tilde{\mathbf{x}}'\epsilon}{\tilde{\mathbf{x}}'\mathbf{x}} = \frac{\tilde{\mathbf{x}}'\mathbf{x}/n}{\tilde{\mathbf{x}}'\tilde{\mathbf{x}}/n}\beta + \frac{\tilde{\mathbf{x}}'\epsilon/n}{\tilde{\mathbf{x}}'\mathbf{x}/n} \\ &\to_p \frac{\mathbb{E}[(x_i+u_i)x_i]}{\mathbb{E}[(x_i+u_i)^2]}\beta = \frac{\sigma_x^2}{\sigma_x^2+\sigma_u^2}\beta = \lambda\beta \end{aligned} \tag{1}$$

As $\sigma_x^2, \sigma_u^2 > 0 \implies \frac{\sigma_x^2}{\sigma_x^2+\sigma_u^2} < 1, \lambda\beta < \beta$. $\hat{\beta}_{LS}$ is biased downward.

Problem 1.2.

By LLN,

$$\begin{aligned} \hat{\sigma}_\epsilon^2 &= (\mathbf{y} - (\mathbf{x}+\mathbf{u})(\hat{\beta}_{LS})'(\mathbf{y} - (\mathbf{x}+\mathbf{u})(\hat{\beta}_{LS})/n \\ &= (\epsilon - (\hat{\beta}_{LS}-\beta)\mathbf{x} - \mathbf{u}\hat{\beta}_{LS})'(\epsilon - (\hat{\beta}_{LS}-\beta)\mathbf{x} - \mathbf{u}\hat{\beta}_{LS})/n \\ &= \epsilon'\epsilon/n - (\hat{\beta}_{LS}-\beta)\epsilon'\mathbf{x}/n - \epsilon'\mathbf{u}/n \\ &+ (\hat{\beta}_{LS}-\beta)\mathbf{x}'\epsilon/n + (\hat{\beta}_{LS}-\beta)^2\mathbf{x}'\mathbf{x}/n \\ &+ (\hat{\beta}_{LS}-\beta)\hat{\beta}_{LS}\mathbf{x}'\mathbf{u}/n + \hat{\beta}_{LS}\mathbf{x}'\epsilon/n + \beta)\hat{\beta}_{LS}\mathbf{u}'\mathbf{x}/n + \hat{\beta}_{LS}^2\mathbf{u}'\mathbf{u}/n \\ &\to_p \sigma_\epsilon^2 + o_p(1) + o_p(1) + o_p(1) + (1-\lambda)^2\beta^2\sigma_x^2 + o_p(1) + o_p(1) + o_p(1) + \lambda^2\beta^2\sigma_u^2 \\ &= \sigma_\epsilon^2 + (1-\lambda)^2\beta^2\sigma_x^2 + \lambda^2\beta^2\sigma_u^2 \end{aligned} \tag{2}$$

which has an upward bias.

Now considering $\sigma_\epsilon^2(\tilde{\mathbf{x}}'\tilde{\mathbf{x}}/n)^{-1}$ using our previous results,

$$\sigma_\epsilon^2(\tilde{\mathbf{x}}'\tilde{\mathbf{x}}/n)^{-1} \to_p \frac{\sigma_\epsilon^2 + (1-\lambda)^2\beta^2\sigma_x^2 + \lambda^2\beta^2\sigma_u^2}{\sigma_x^2+\sigma_u^2} = \lambda\frac{\sigma_\epsilon^2}{\sigma_x^2} + \lambda(1-\lambda)\beta^2 \tag{3}$$

We cannot sign the bias with the information provide.

Problem 1.3.

By Slutsky and previous results,

$$\frac{\hat{\beta}_{LS}}{\sqrt{\sigma_\epsilon^2(\tilde{\mathbf{x}}'\tilde{\mathbf{x}}/n)^{-1}}} \to_p \frac{\sqrt{\lambda}\beta}{\sqrt{\frac{\sigma_\epsilon^2}{\sigma_x^2} + (1-\lambda)\beta^2}} \tag{4}$$

$$\frac{\sqrt{\lambda}\beta}{\sqrt{\frac{\sigma_\epsilon^2}{\sigma_x^2} + (1-\lambda)\beta^2}} < \frac{\beta}{\sqrt{\frac{\sigma_\epsilon^2}{\sigma_x^2}}} \tag{5}$$

So the estimate is biased downward.
$\mathbb{E} \quad = \text{'x}\frac{}{\tilde{\mathbf{x}}'\tilde{\mathbf{x}}\beta+\frac{\tilde{\mathbf{x}}'\epsilon}{\tilde{\mathbf{x}}'\mathbf{x}}=\frac{\tilde{\mathbf{x}}'\mathbf{x}/n}{\tilde{\mathbf{x}}'\tilde{\mathbf{x}}/n}\beta+\frac{\tilde{\mathbf{x}}'\epsilon/n}{\tilde{\mathbf{x}}'\mathbf{x}/n}}$

Problem 1.4.