Straightening à la Joyal

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The development of ∞-category theory in Lurie's book *Higher Topos Theory* is founded on a series of rectification theorems, the first of which is the (unmarked) Straightening Theorem. This theorem states that, for each simplicial set A, the straightening–unstraightening adjunction is a Quillen equivalence between the simplicial presheaf category $[\mathfrak{C}(A)^{\mathrm{op}}, \mathbf{sSet}]$ equipped with the projective Kan model structure and the slice category $\mathbf{sSet}/A$ equipped with the contravariant model structure (whose fibrant objects are the right fibrations over A). Lurie's proof of this theorem is notoriously difficult; alternative proofs – substantially different from Lurie's proof and from each other – have since been given by Stevenson and by Heuts and Moerdijk.

In this talk I will present a new proof of the Straightening Theorem, which I contend is much simpler than all preceding proofs. This proof is based on an idea which may be found in §51 of Joyal's *Notes on quasi-categories*: we factorise the straightening–unstraightening adjunction as the composite of three adjunctions (in fact, two adjunctions and one equivalence), each of which we show to be a Quillen equivalence. One of these adjunctions (the equivalence) is easily seen to be a Quillen equivalence (in fact, an equivalence of model categories). To prove that the remaining two adjunctions are Quillen equivalences, I will use my recent proof of Joyal's Cylinder Conjecture.