Poisson Regression and Applications

Adam Dawson, Sarah Onstad-Hawes, and Asare Buahin

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Introduction: Main Project Questions

- What is the Poisson Regression and what are it's applications?
- Using the Poisson Regression, can we find significant relationships within our data set.

Introduction: Our Data Set

- “Contact with Medical Doctors”
  - Cross-Sectional data collected in North Carolina between 1977-78
  - Examines 20186 observations of individuals
    - Randomly selected subset of 2000 observations for ease of computation
  - Collected from RAND Health Insurance Experiment (RHIE), which is the longest and largest socially controlled experiment regarding medical care (Price, D. 2002)
  - Our Response Variable of interest is mdu, which captures the number of times an individual visited a medical health profession during the study
  - Data Set also contains 14 other variables of interest

Introduction: Our Research Question

- How is the number of doctor visits impacted by various factors in an individual's life?
  - Specifically, we want see how one's age, sex, income, physical limitations, and present diseases influence their ability to seek out medical care.
- Use the Poisson Regression to model the data and see what relationships, if any, exist in between our variables.

Methodology: The Poisson Distribution

- A probability function which is especially useful for count data
    - Y is a variable with discrete outcomes $(0, 1, 2, ...)$ where high counts for Y are rare
    - $f(Y) = \frac{\mu^{Y} * e^{-\mu}}{Y!}$
    - $E[Y] = \mu$
    - $P(Y = y) = \frac{\mu^{y} * e^{-\mu}}{y!}$
    - μ also sometimes noted as λ

Methodology: Necessary Conditions for the Poisson Distribution

- The Mean and Variance of Y are the equal
  - $E[Y] = V[Y] = \mu$
- Independence
  - An event A occuring does not impact event B from occuring
- Probabilities are porportional to time
  - A is twice as likely to occur in two-hour window than in one hour.
- Each observation is recorder over the same fixed period of time.
- Data Set is not overloaded with zero counts.

Methodology: The Poisson Regression

- Poisson Regression finds estimates for the linear equation relating the log of a response variable to predictors.
  - $log(\hat{\mu}) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_p X_p$
  - $\hat{\mu} = e^{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_p X_p}$
- Allows us to see how a marginal change in our predictor variable impacts the estimated count of our variable.
- Coefficients found using the Maximum Likelihood Estimate process
- Coefficients evalued for statisical significance using z-statistic and corresponding p-value
- We can use R glm function where we indicate the family to be "poisson" in order to obtain our fitted equation and coefficient estimates.

Methodology: The Poisson vs. Other Regressions

- As mentioned, Poisson deals with discrete quantitative variables, as opposed to continuous quantitative variables (Linear Regression) or Yes/No outcomes (Logistical Regression)
- Similar to the Logistical Regression in that the coefficients are interpreted in the context of the log
- Poisson is particularly helpful when looking at specific count issues, such as traffic accidents at a particular intersection, prime staffing numbers during peak hours at a business, or number of soliders killed by a mule's kick

Results and Conclusion: An Introduction to our Data

- Observational Units: Individuals in North Carolina
- Overall, we have chosen five predictor variables to estimate our response variable
- Response Variable:
  - mdu measures the number of doctors visits one person attends in a year: Count
- Predictor Variables used:
  - Linc denotes a person's yearly log(income): Quantitative
  - Age denotes a person's age at the time of the study: Quantitative
  - Physlim denotes if a person has any sort of physical limitation: Categorical, Binary
  - ndiseases denotes the number of diagnosed diseases a person has at the time of the study:Quantitative
  - Sex denotes the sex of the patient: Categorical, Binary (Male == 1, Female == 0)

Results and Conclusions: An Introduction to our Data continued

count	st.dev	sample_mean	med	min	max
2000	4.56	2.87	1	0	65

- $\mu = 2.87$, $\sigma^2 = 4.56$
  - Over-disperion present

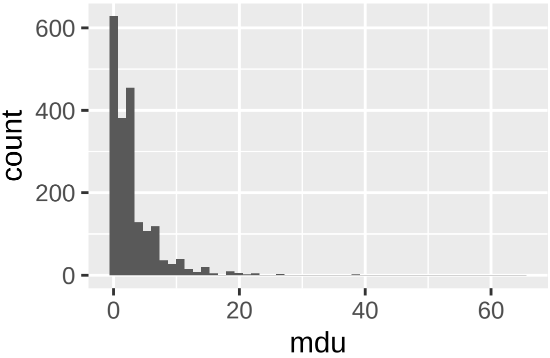

Figure 1: Count of an Individual's Doctors Visits in a Year

- Histogram shows a distribution skewed to the right
    - A high number of 0 counts in our data set

Results and Conclusions: Forward Selction:Step 1

```r
regmod1<-glm(mdu~physlim, Doc_sample, family=poisson)
regmod2<-glm(mdu~ndisease, Doc_sample, family=poisson)
regmod3<-glm(mdu~linc, Doc_sample, family=poisson)
regmod4<-glm(mdu~age, Doc_sample, family=poisson)
regmod5<-glm(mdu~sex, Doc_sample, family=poisson)
p.physlim <- summary(regmod1)$coefficients[2,4]
p.disease <- summary(regmod2)$coefficients[2,4]
p.linc <- summary(regmod3)$coefficients[2,4]
p.age <- summary(regmod4)$coefficients[2,4]
p.sex <- summary(regmod5)$coefficients[2,4]
```

Results and Conclusions: Forward Selction:Step 1

```
##                      [,1]
## p.physlim  9.880857e-95
## p.disease 2.247778e-151
## p.linc     1.925348e-25
## p.age      5.061433e-39
## p.sex      1.898671e-38
```

Results and Conclusions: Forward Selction:Step 2

```r
regmod11<-glm(mdu~ndisease+physlim, Doc_sample,
              family=poisson)
regmod12<-glm(mdu~ndisease+linc, Doc_sample,
              family=poisson)
regmod13<-glm(mdu~ndisease+age, Doc_sample,
              family=poisson)
regmod14<-glm(mdu~ndisease+sex, Doc_sample,
              family=poisson)
p.dp <- summary(regmod11)$coefficients[3,4]
p.dli <- summary(regmod12)$coefficients[3,4]
p.da <- summary(regmod13)$coefficients[3,4]
p.ds <- summary(regmod14)$coefficients[3,4]
```

```
##               [,1]
## p.dp  1.037162e-39
## p.dli 1.220258e-25
## p.da  5.939466e-10
## p.ds  9.587483e-18
```

Results and Conclusions: Forward Selction:Step 3

```r
regmod21<-glm(mdu~ndisease+physlim+age,
              Doc_sample, family=poisson)
regmod22<-glm(mdu~ndisease+physlim+sex,
              Doc_sample, family=poisson)
regmod23<-glm(mdu~ndisease+physlim+linc,
              Doc_sample, family=poisson)
p.dpa <- summary(regmod21)$coefficients[4,4]
p.dps <- summary(regmod22)$coefficients[4,4]
p.dpl <- summary(regmod23)$coefficients[4,4]
```

```
##                [,1]
## p.dpa 1.367631e-09
## p.dps 2.372625e-16
## p.dpl 4.288775e-29
```

Results and Conclusions: Forward Selction:Step 4

```
regmod31<-glm(mdu~ndisease+physlim+linc+sex,
              Doc_sample, family=poisson)
regmod32<-glm(mdu~ndisease+physlim+linc+age,
              Doc_sample, family=poisson)
p.dpls <- summary(regmod31)$coefficients[5,4]
p.dpla <- summary(regmod32)$coefficients[5,4]
```

```
##                 [,1]
## p.dpls 2.339247e-19
## p.dpla 7.851804e-09
```

Results and Conclusions: Forward Selction:Step 5

```r
regmod41<-glm(mdu~ndisease+physlim+linc+sex+age,
              Doc_sample, family=poisson)
p.dlspa <- summary(regmod41)$coefficients[6,4]
```

```
##                 [,1]
## p.dlspa 6.771765e-08
```

Results and Conclusions: Summary Output of model

From our forward selection test, we found that the model considering disease, physical limitation, log of income and age as explanatory variables was the best model to explain the number of times a person visits the doctor in a year

```
##
## Call:
## glm(formula = mdu ~ ndisease + physlim + linc + sex + ag
##     data = Doc_sample)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -4.1501  -1.9434  -0.8458   0.5508  16.8987
##
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -1.0282135  0.1450822  -7.087 1.37e-12 ***
## ndisease     0.0287146  0.0018776  15.293  < 2e-16 ***
## physlimTRUE  0.4349585  0.0313656  13.867  < 2e-16 ***
```

Results and Conclusions: Final Model

From our knowledge on Poisson Distribution, we can write out the regression model like so

$log(MDU) = -1.03 + 0.029 NDisease + 0.435 PhysLim + 0.184 linc - 0.245 Ma$

Which is equivalent to

$MDU = e^{-1.03 + 0.029 NDisease + 0.435 PhysLim + 0.184 linc - 0.245 Male + 0.004 Age}$

Results and Conclusions: Another Application

- In addition to regression, we can use the poisson distribution to find probabilities of specific count occurences or ranges of count occurences in the data.

```
Probof4ormore = ppois(4,lambda=2.86, lower.tail=FALSE)
Probof4ormore
```

```
## [1] 0.1617866
```

- This gives us the probability that a person will go to the doctor's office 4 or more times in a year. We can edit the lower.tail component of this code to give us the probablity that a person will go to the doctor's office 4 or less times in a year

Discussion and Critiques: Model Assumptions

- We have good reason to question a few necessary assumptions of the Poisson Distribution in regards to our data set
- Mean and Variance are not equal $E[Y] = 2.8$ and $V[Y] = 4.5$
  - Over-dispersion, which could indicate that we should use a different model
- We have a large number of observations for which the count is 0
- Consider using a different model to fit to our data
  - Zero-Inflated Poisson Regression
    - accounts for large number of 0 counts in data set
  - Zero-Inflated Negative Binomial Regression
    - helps with both larger number of 0s and over-dispersion

Discussion and Critiques: Inference

- With our data being so significant (perhaps abnormally), we were unsure of our ability to use the fitted equation with any confidence

- With a high number of possible predictors, we could have ommitted some very significant variables from the regression, which could be vital in understanding what impacts utilization of healthcare.

- Data set is from the 1970s
  - Comprehensive, but outdated
  - If using data to enact policy shifts or structural changes, our data analysis might not be "in touch" enough with the current climate of health care.

- Data is collected from N.C. only, which limits the scope in which we can make inferences.

- Perhaps some cultural/societal impacts in N.C. region impacted our data

Conclusion

- Poisson Regression is useful for analyzing count data, but it is crucial to check the data set and see that conditions are met prior to analysis and inference
- Having a large sample size is helpful, but also must be aware of how that potentially impacts analysis.
- Working on real world data is tough and messy. Takes a lot more time and careful thought
  - No exact, clear path for analysis
- "One step forward, two steps back"

References

https://stats.idre.ucla.edu/r/dae/poisson-regression/
https://stats.idre.ucla.edu/stata/output/poisson-regression/
https://stats.idre.ucla.edu/r/dae/zip/ https:
//www.sciencedirect.com/science/article/pii/S0167629602000085
https://www.dataquest.io/blog/tutorial-poisson-regression-in-r/
#:~:targetText=Poisson%20Regression%20models%20are%20best,where%

R Markdown

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When you click the **Knit** button a document will be generated that includes both content as well as the output of any embedded R code chunks within the document.

Slide with Bullets

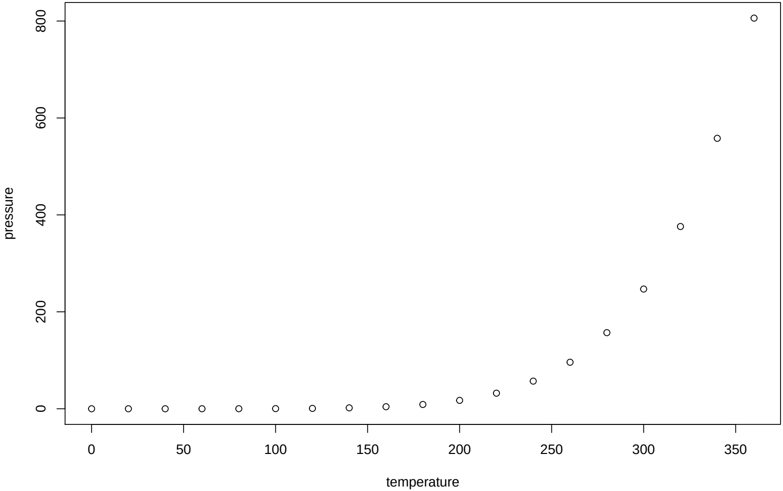
- Bullet 1
- Bullet 2
- Bullet 3

Slide with R Output

```r
summary(cars)
```

```
##      speed           dist
##  Min.   : 4.0   Min.   :  2.00
##  1st Qu.:12.0   1st Qu.: 26.00
##  Median :15.0   Median : 36.00
##  Mean   :15.4   Mean   : 42.98
##  3rd Qu.:19.0   3rd Qu.: 56.00
##  Max.   :25.0   Max.   :120.00
```

Slide with Plot


pressure
temperature