

## Greater New York Programming Contest

Stony Brook University Stony Brook, NY



# **E • Faulhaber's Triangle**

The sum of the  $m^{th}$  powers of the first n integers

$$S(n,m) = SUM (j = 1 to n) (j^m)$$

Can be written as a polynomial of degree m+1 in n:

$$S(n,m) = SUM (k = 1 to m+1) (F(m,k) * n^k)$$

For example:

$$S(n,1) = (1 + ... + n) = (1/2)*n^{2} + (1/2)*n$$

$$S(n,2) = (1 + ... + n^{2}) = (1/3)*n^{3} + (1/2)*n^{2} + (1/6)*n$$

$$S(n,3) = (1 + ... + n^{3}) = (1/4)*n^{4} + (1/2)*n^{3}) + (1/4)*n^{2}$$

$$S(n,4) = (1 + ... + n^{4}) = (1/5)*n^{5} + (1/2)*n^{4}) + (1/3)*n^{3} - (1/30)*n$$

The coefficients F(m,k) of these formulas form Faulhaber's Triangle:

1						
1/2	1/2					
1/6	1/2	1/3				
0	1/4	1/2	1/4			
-1/30	0	1/3	1/2	1/5		
0	-1/12	0	5/12	1/2	1/6	
1/42	0	-1/6	0	1/2	1/2	1/7

where rows m start with 0 (at the top) and columns k go from 1 to m+1

Each row of Faulhaber's Triangle can be computed from the previous row by:

- a) The element in row i and column j (j > 1) is (i/j) \* (the element above left); that is: F(i,j) = (i/j) \* F(i-1,j-1)
- b) The first element in each row F(i,1) is chosen so the sum of the elements in the row is 1.

Write a program to find entries in Faulhaber's Triangle as decimal fractions in lowest terms.



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### Input

The first line of input contains a single integer P, (1  $\leq P \leq 1000$ ), which is the number of data sets that follow. Each data set should be processed identically and independently.

Each data set consists of a single line of input consisting of three space separated decimal integers. The first integer is the data set number. The second integer is row number m, and the third integer is the index k within the row of the entry for which you are to find F(m, k), the Faulhaber's Triangle entry (0 <= m <= 400, 1 <= k <= n+1).

### Output

For each data set there is a single line of output. It contains the data set number, followed by a single space which is then followed by either the value if it is an integer OR by the numerator of the entry, a forward slash and the denominator of the entry.

Sample Input	Sample Output
4	1 -1/30
1 4 1	2 1/3
2 4 3	3 -22388337
3 86 79	4 1/401
4 400 401	