Optimal Trading Across Coexisting Exchanges: Limit-Order Books & Automated Market Makers

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LOBs vs AMMs

Many digital assets trade in centralized **and** decentralized exchanges

↳ ETH/USDC can be exchanged on Binance (LOB) and Uniswap (AMM)

Large literature that models price impact and optimal trading in LOBs

↳ Almgren and Chriss [2001]; Alfonsi et al. [2010]; Obizhaeva and Wang [2013]; Cheridito and Sepin [2014]; Cartea and Jaimungal [2016]; Neuman and Voß [2022]; etc.

Can we get corresponding results for AMMs? What if we have access to both venues?

↳ Static game theoretical models in Aoyagi and Ito [2021]; Malinova and Park [2023]; Lehar and Parlour [2025]; dynamic models in Cartea et al. [2023] with temporary price impact and approximations; etc.

This Paper

We develop a dynamic model, with minimal reduced-form cuts and nonlinear price dynamics

- ✓ Derive consistent budget equations for trading in LOBs and geometric mean AMMs
- ✓ Solve optimization problem of large liquidity taker
- ✓ Verify the absence of price manipulation
- ✓ For calibrated parameters: optimal trades vary substantially, but optimal impacts do not

We do not take into account:

- × Discrete settlement on AMM
- × Proportional fees
- × Endogenous liquidity provision, or concentrated liquidity (Uniswap v3)

Trading in the LOB

Consider two assets X, Y (e.g., ETH and USDC) and a **LOB** with immediate linear impact λ

- Order flow of a large trader $(Q_t)_t$ and other market participants $(\overline{Q}_t)_t$
- Unaffected martingale price $(\overline{P}_t)_t$, exogenous process such as a Brownian motion for the market price in the absence of large traders
- Exchange rate $(P_t)_t$, with arbitrageurs exploiting mispricings relative to $\overline{P}_t$

$$dP_t = -\beta\lambda\left(P_t - \overline{P}_t\right)dt + \lambda\left(d\overline{Q}_t + dQ_t\right)$$

What is the analogue for AMMs?

Trading in the AMM

Consider an **AMM** with bonding curve $f(x, y) = xy = L^2$

For any swap $dQ_t \leftrightarrow dY_t$, we must have

$$(X_{t-} - dQ_t)(Y_{t-} + dY_t) = X_{t-}Y_{t-} = L^2$$

Unaffected price, specified in terms of $(\overline{Q}_t)_t$

$$\overline{P}_t = \frac{f_x(\overline{Q}_t, \overline{Y}_t)}{f_y(\overline{Q}_t, \overline{Y}_t)} = \frac{L^2}{\overline{Q}_t^2}$$

Price paid for amount dQ_t:

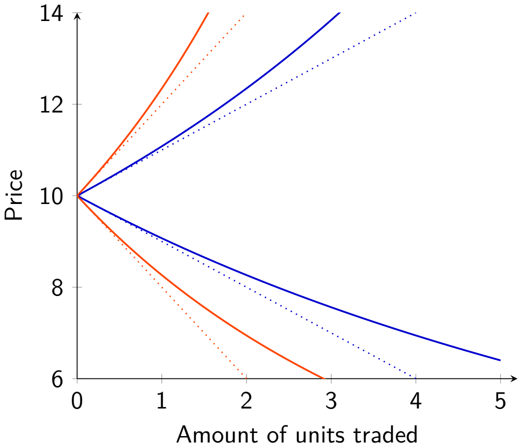
$$dY_t = \frac{L^2 dQ_t}{\overline{Q}_{t-}^2 - \overline{Q}_{t-} dQ_t} = \overline{P}_t dQ_t + \frac{\overline{P}_t^{3/2}}{L} d\langle Q \rangle_t$$

Figure

Price Impact

Price impact is locally linear, but flattens (spikes) when reserves are high (low)

↳ Magnitude of impact depends on price level, and thus previous trades



Aggregate Market

Without fees, arbitrageurs can realize a true arbitrage if they observe a discrepancy between quotes in the LOB and AMM. **How to route trades across exchanges?**

↳ Split so that post-trade prices in both venues are the same, the aggregate market

↳ Effective price is inf-convolution of local execution prices [see e.g. Biais et al., 2000]

↳ Leaves no profits for inter-exchange arbitrageurs

The flow $(1-\pi_t)dQ_t$ is executed in the AMM, while $\pi_t dQ_t$ is executed in the LOB:

$$\pi_t := \frac{\ell P_t^{3/2}}{\lambda + \ell P_t^{3/2}}, \quad \text{where} \quad \ell := 2/L$$

Local impacts $\ell P_t^{3/2}$ and λ are replaced by the aggregate impact

$$\kappa(P_t) := \frac{\lambda \ell P_t^{3/2}}{\lambda + \ell P_t^{3/2}}$$

Continuous-Time Scaling Limits

Order flow of a large trader $(Q_t)_t$ and other market participants $(\overline{Q}_t)_t$

Unaffected price, fully driven by $(\overline{Q}_t)_t$

$$d\overline{P}_t = \overline{P}_t\Big(\nu_t dt + \varsigma_t dW_t\Big)$$

Exchange rate, with arbitrageurs exploiting mispricings relative to $\overline{P}_t$

$$dP_t = -\beta\kappa(P_t)\left(P_t - \overline{P}_t\right)dt + \kappa(P_t)\Big(dQ_t + d\overline{Q}_t\Big) + \frac{3\kappa(P_t)^3}{4\ell P_t^{5/2}} d\langle Q + \overline{Q}\rangle_t$$

- Optimal trading strategies are generally not smooth
- Immediate reaction to diffusive parameters
- Correction for trades with nontrivial quadratic variation [Milionis et al., 2024]

Optimal Trading with Alpha Signals

Consider a large risk-neutral trader who

- cannot frontrun market flow
- has a forecast $\alpha_t := \mathbb{E}_t[\overline{P}_\tau] - \overline{P}_t$ at a later time $\tau \geq T$
- has a cash account $(C_t)_t$ with dynamics

$$dC_t = -P_t dQ_t - \frac{\kappa(P_t)}{2}\Big(d\langle Q\rangle_t + 2d\langle Q, \overline{Q}\rangle_t\Big)$$

Large trader's goal functional: maximize the expected PnL

$$\sup_{(Q_t)_{t\in[0,T]}} \mathbb{E}[\mathrm{PnL}_T] = \sup_{(Q_t)_{t\in[0,T]}} \mathbb{E}\left[Q_T \mathbb{E}_T[\overline{P}_\tau] + \int_{0+}^{T-} dC_t + \sum_{s\in\{0,T\}} \Delta Q_s \mathbb{E}_s[\overline{P}_\tau] + \Delta C_s\right]$$

Passage to Impact Space

We redefine the control variable from order flow $(Q_t)_t$ to aggregate price $(P_t)_t$

↳ Also leads to explicit solution in models with nonlinear or stochastic impact [see e.g. Fruth et al., 2019; Hey et al., 2025]

↳ One-to-one correspondence between order flow and exchange rate

$$dQ_t = \frac{1}{\kappa(P_t)} dP_t - \frac{3}{4\ell P_t^{5/2}} d\langle P \rangle_t + \beta \left(P_t - \overline{P}_t \right) dt - d\overline{Q}_t$$

↳ Additional terms from integration by parts and Itô's formula

↳ Cancellations between variations and final expressions in terms of prices

↳ "Passage to impact space" makes the problem tractable, allows pointwise optimization!

Equivalence Results

Theorem. The large trader's optimization problem in **"order flow space"**

$$\sup_{(Q_t)_{t\in[0,T]}} \mathbb{E}\left[Q_T \mathbb{E}_T[\overline{P}_\tau] + \int_{0+}^{T-} dC_t + \sum_{s\in\{0,T\}} \Delta Q_s \mathbb{E}_s[\overline{P}_\tau] + \Delta C_s \right]$$

is equivalent to the following problem in **"impact space"**

$$\sup_{(P_t)_{t\in(0,T]}} \mathbb{E}\left[\int_{0+}^{T-} \left((\alpha_t + \overline{P}_t - P_t)\left(\beta(P_t - \overline{P}_t) - \frac{\nu_t \overline{P}_t}{\kappa(\overline{P}_t)} + \frac{3\varsigma_t^2}{4\ell \overline{P}_t^{1/2}} \right) + \frac{\varsigma_t^2 \overline{P}_t^2 \kappa(P_t)}{2\kappa(\overline{P}_t)^2} \right) dt \right.$$
$$\left. + \frac{2\ell \overline{P}_t^{3/2} - \lambda}{\lambda \ell \overline{P}_t^{3/2}} d\langle \overline{P}, \alpha + \overline{P} \rangle_t \right] + f(\alpha_T, \overline{P}_T, P_T) - f(\alpha_0, \overline{P}_0, P_0).$$

Solutions

Theorem. The **optimal price** is $P_T^* = \alpha_T + \overline{P}_T$ and $P_t^*, t \in (0, T)$ such that

$$\underbrace{\beta(\alpha_t - 2(P_t - \overline{P}_t)) + \frac{\nu_t \overline{P}_t}{\kappa(\overline{P}_t)}}_{\text{strength of alpha signal}} + \underbrace{\frac{3\varsigma_t^2}{4\ell \overline{P}_t^{1/2}} \left(\frac{\overline{P}_t^{5/2} \kappa(P_t)^2}{P_t^{5/2} \kappa(\overline{P}_t)^2} - 1 \right)}_{\text{“LVR” profit from mispricings}} = 0$$

We recover the corresponding **optimal order flow** via the mapping

$$Q_t = \int_0^t \frac{1}{\kappa(P_s)} dP_s - \int_0^t \frac{1}{\kappa(\overline{P}_s)} d\overline{P}_s - \int_0^t \frac{3}{4\ell P_s^{5/2}} d\langle P \rangle_s + \int_0^t \frac{3\varsigma_s^2}{4\ell \overline{P}_s^{1/2}} ds + \int_0^t \beta \left(P_s - \overline{P}_s \right) ds$$

Solutions and Limiting Cases

Pointwise optimization remains well-posed, but no longer has an explicit solution

↳ Optimal price of $P_t^* = \overline{P}_t$ without alpha signal, no price manipulation

↳ Explicit solution when volatility of $\overline{P}_t$ is zero

↳ Approximation for small ς_t^2 via implicit function theorem

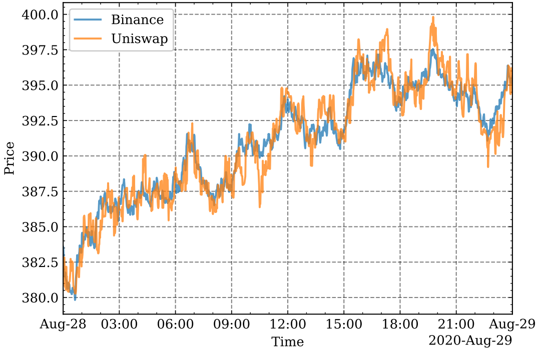
✓ **LOB limit** $(\ell \to \infty)$: Obizhaeva-Wang model with $\kappa(P) \to \lambda$ and

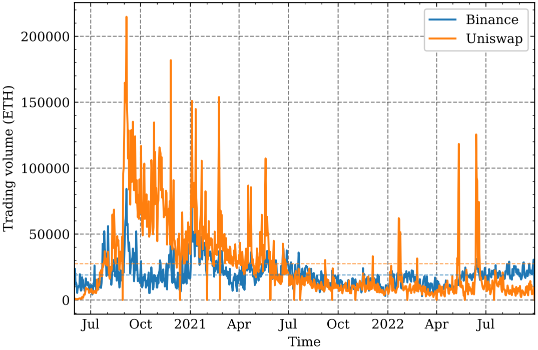
$$P_t^* = \overline{P}_t + \frac{1}{2}\left(\alpha_t + \frac{\nu_t \overline{P}_t}{\beta\lambda}\right)$$

✓ **AMM limit** $(\lambda \to \infty)$: Price-dependent $\kappa(P) \to \ell P^{3/2}$ and FOC is quadratic in $\sqrt{P_t}$

Empirical Study

We use 10-second bins for price and trade data from **Binance** and **Uniswap v2** between Aug. 2020 and Sept. 2022 for the pair ETH/USDC





Model Calibration

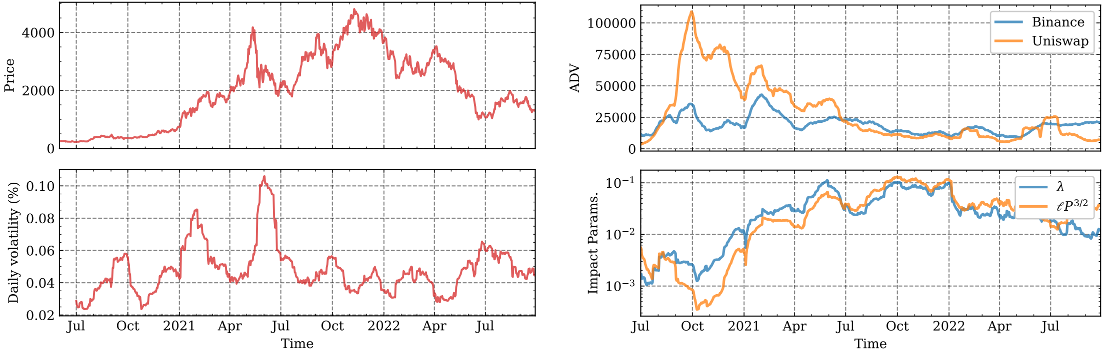
We calibrate the model to price and trade data:

- Analysis repeated daily, with a rolling window of 30 days
- Estimation of aggregate market activity by an exponentially weighted moving average
- Regression of price change against aggregate flow [Muhle-Karbe et al., 2024]
- Grid search for the decay parameter

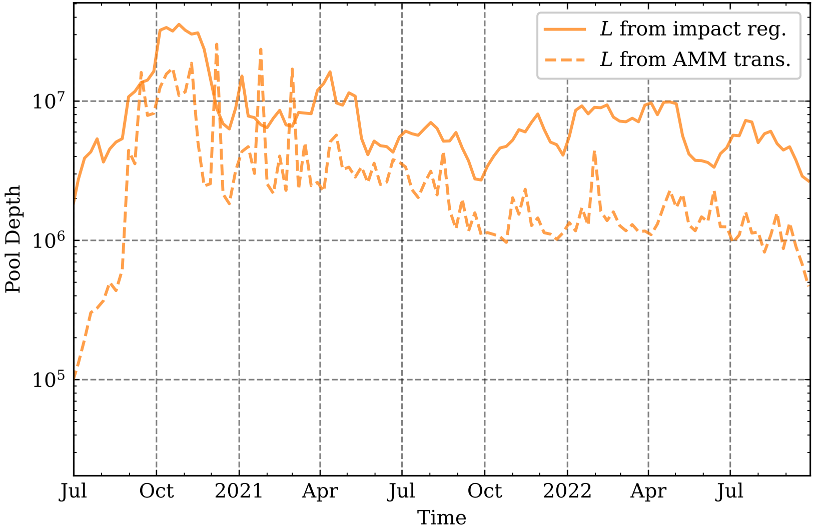
We choose the local depths to match the observed split of trading volume

- Alternative: directly estimate depth of AMM from pre- and post-trade prices
- Both approaches turn out to be (roughly) consistent

Daily Estimates of Calibrated Model

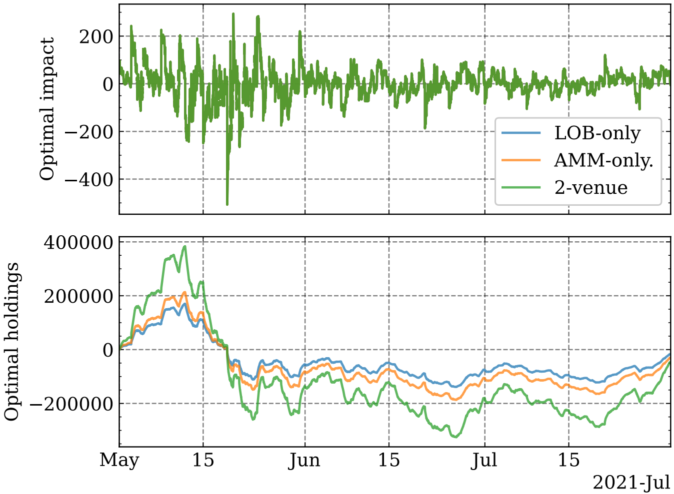


Comparison of AMM's Pool Depth



Optimal Impacts & Order Flows

Optimal solutions are **nearly identical** in impact space, but **differs** in order flow space



Contributions

We study optimal trading across a LOB and AMM:

↳ **Nonlinear dynamics** of the aggregate exchange with **diffusive trading strategies**

↳ **Tractable solution** in "impact space"; results can be generalized to G3Ms

↳ Empirical case study on Binance & Uniswap data

Future directions:

↳ Inclusion of fees, multi-dimensional model for all prices

↳ Interactions between liquidity takers and liquidity providers

↳ Competition between many LOBs and AMMs

↳ Etc.

Thank you!

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