

Optimal Trading Across Coexisting Exchanges: Limit-Order Books & Automated Market Makers

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LOBs vs AMMs

Many digital assets trade in centralized **and** decentralized exchanges

- ↳ ETH/USDC can be exchanged on Binance (LOB) and Uniswap (AMM)

Large literature that models price impact and optimal trading in LOBs

- ↳ Almgren and Chriss [2001]; Alfonsi et al. [2010]; Obizhaeva and Wang [2013]; Cheridito and Sepin [2014]; Cartea and Jaimungal [2016]; Neuman and Voß [2022]; etc.

Can we get corresponding results for AMMs? What if we have access to both venues?

- ↳ Static game theoretical models in Aoyagi and Ito [2021]; Malinova and Park [2023]; Lehar and Parlour [2025]; dynamic models in Cartea et al. [2023] with temporary price impact and approximations; etc.

This Paper

We develop a dynamic model, with **minimal reduced-form cuts** and **nonlinear price dynamics**

- ✓ Derive consistent budget equations for trading in LOBs and geometric mean AMMs
- ✓ Solve optimization problem of large liquidity taker
- ✓ Verify the absence of price manipulation
- ✓ For calibrated parameters: optimal trades vary substantially, but optimal impacts do not

We do not take into account:

- ✗ Discrete settlement on AMM
- ✗ Proportional fees
- ✗ Endogenous liquidity provision, or concentrated liquidity (Uniswap v3)

Trading in the LOB

Consider two assets X, Y (e.g., ETH and USDC) and a **LOB** with immediate linear impact λ

- Order flow of a large trader $(\mathcal{X}_t)_t$ and other market participants $(X_t)_t$
- Unaffected price $(P_t)_t$, exogenous process such as a Brownian motion for the market price in the absence of large traders
- Exchange rate $(\mathcal{P}_t)_t$, with arbitrageurs exploiting mispricings relative to P_t

$$d\mathcal{P}_t = -\beta\lambda(\mathcal{P}_t - P_t)dt + \lambda(d\mathcal{X}_t + dX_t)$$

What is the analogue for AMMs?

Trading in the AMM

Consider an **AMM** with bonding curve $f(x, y) = xy = L^2$

For any swap $d\mathcal{X}_t \leftrightarrow d\mathcal{Y}_t$, we must have

$$(X_{t-} - d\mathcal{X}_t)(Y_{t-} + d\mathcal{Y}_t) = X_{t-} Y_{t-} = L^2$$

Unaffected price, specified in terms of $(X_t)_t$

$$P_t = \frac{f_x(X_t, Y_t)}{f_y(X_t, Y_t)} = \frac{L^2}{X_t^2}$$

Price paid for amount $d\mathcal{X}_t$:

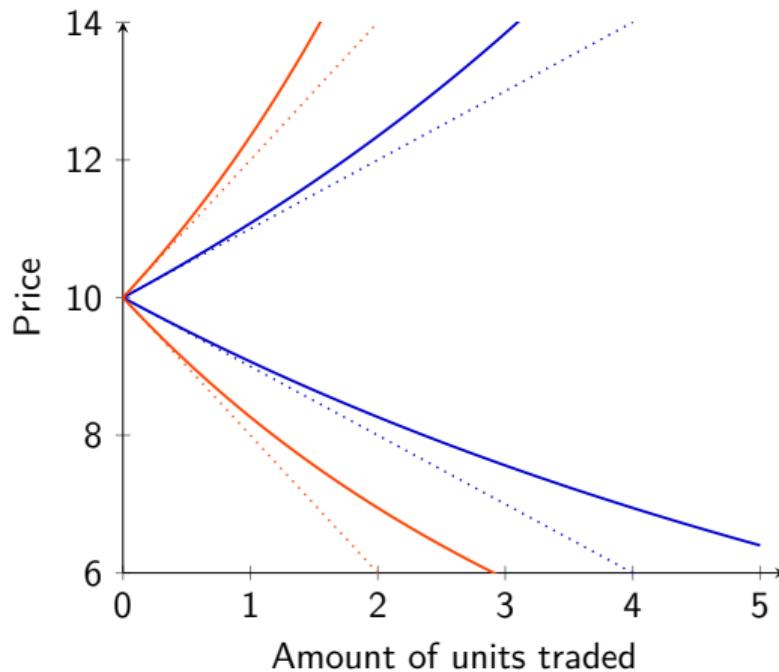
$$d\mathcal{Y}_t = -\frac{L^2 d\mathcal{X}_t}{X_{t-}^2 - X_{t-} d\mathcal{X}_t} = -P_t d\mathcal{X}_t - \frac{2P_t^{3/2}}{L} d\langle \mathcal{X} \rangle_t$$

Figure

Price Impact

Price impact is locally linear, but flattens (spikes) when reserves are high (low)

↳ Magnitude of impact depends on price level, and thus previous trades



Aggregate Market

Without fees, arbitrageurs can realize a true arbitrage if they observe a discrepancy between quotes in the LOB and AMM. **How to route trades across exchanges?**

- ↳ Split so that post-trade prices in both venues are the same, the **aggregate market**
- ↳ Effective price is inf-convolution of local execution prices [see e.g. Biais et al., 2000]
- ↳ Leaves no profits for inter-exchange arbitrageurs

The flow $(1 - \pi_t)d\mathcal{X}_t$ is executed in the AMM, while $\pi_t d\mathcal{X}_t$ is executed in the LOB:

$$\pi_t := \frac{2\mathcal{P}_t^{3/2}/L}{\lambda + 2\mathcal{P}_t^{3/2}/L}$$

Local impacts $2\mathcal{P}_t^{3/2}/L$ and λ are replaced by the aggregate impact

$$\Lambda(\mathcal{P}_t) := \frac{2\lambda\mathcal{P}_t^{3/2}/L}{\lambda + 2\mathcal{P}_t^{3/2}/L}$$

Continuous-Time Scaling Limits

Order flow of a large trader $(\mathcal{X}_t)_t$ and other market participants $(Q_t)_t$

Unaffected price, fully driven by $(Q_t)_t$

$$dP_t = P_t (\mu_t dt + \sigma_t dW_t)$$

Exchange rate, with arbitrageurs exploiting mispricings relative to P_t

$$d\mathcal{P}_t = -\beta \Lambda(\mathcal{P}_t) (\mathcal{P}_t - P_t) dt + \Lambda(\mathcal{P}_t) (d\mathcal{X}_t + dQ_t) + \frac{3L\Lambda(\mathcal{P}_t)^3}{8\mathcal{P}_t^{5/2}} d\langle \mathcal{X} + Q \rangle_t$$

- Optimal trading strategies are generally not smooth
- Immediate reaction to diffusive parameters
- Correction for trades with nontrivial quadratic variation [Milionis et al., 2024]

Optimal Trading with Alpha Signals

Consider a large risk-neutral trader who

- cannot frontrun market flow
- has a forecast $\alpha_t := \mathbb{E}_t[P_\tau] - P_t$ at a later time $\tau \geq T$
- has a cash account $(\mathcal{Y}_t)_t$ with dynamics

$$d\mathcal{Y}_t = -\mathcal{P}_t d\mathcal{X}_t - \frac{\Lambda(\mathcal{P}_t)}{2} (d\langle \mathcal{X} \rangle_t + 2d\langle \mathcal{X}, Q \rangle_t)$$

Large trader's goal functional: maximize the expected PnL

$$\sup_{(\mathcal{X}_t)_{t \in [0, T]}} \mathbb{E}[\text{PnL}_T] = \sup_{(\mathcal{X}_t)_{t \in [0, T]}} \mathbb{E} \left[\mathcal{X}_T (\alpha_T + P_T) + \int_0^T d\mathcal{Y}_t \right]$$

Passage to Impact Space

We redefine the control variable from order flow $(\mathcal{X}_t)_t$ to aggregate price $(\mathcal{P}_t)_t$

- ↳ Also leads to explicit solution in models with nonlinear or stochastic impact [see e.g. Fruth et al., 2019; Hey et al., 2025]
- ↳ One-to-one correspondence between order flow and exchange rate

$$d\mathcal{X}_t = \beta (\mathcal{P}_t - P_t) dt + \frac{1}{\Lambda(\mathcal{P}_t)} d\mathcal{P}_t - \frac{3L}{8\mathcal{P}_t^{5/2}} d\langle \mathcal{P} \rangle_t - dQ_t$$

- ↳ Additional terms from integration by parts and Itô's formula
- ↳ Cancellations between variations and final expressions in terms of prices
- ↳ “Passage to impact space” makes the problem tractable, allows pointwise optimization!

Equivalence Results

Theorem. The large trader's optimization problem in “**order flow space**”

$$\sup_{(\mathcal{X}_t)_{t \in [0, T]}} \mathbb{E} \left[\mathcal{X}_T (\alpha_T + P_T) + \int_0^T d\mathcal{Y}_t \right]$$

is equivalent to the following problem in “**impact space**”

$$\begin{aligned} \sup_{(\mathcal{P}_t)_{t \in (0, T]}} \mathbb{E} \left[& \int_{0+}^{T-} \left((\alpha_t + P_t - \mathcal{P}_t) \left(\beta(\mathcal{P}_t - P_t) - \frac{\mu_t P_t}{\Lambda(P_t)} + \frac{3L\sigma_t^2}{8P_t^{1/2}} \right) + \frac{\sigma_t^2 P_t^2 \Lambda(\mathcal{P}_t)}{2\Lambda(P_t)^2} \right) dt \right. \\ & \left. + \frac{4P_t^{3/2}/L - \lambda}{2\lambda P_t^{3/2}/L} d\langle P, \alpha + P \rangle_t + f(\alpha_T, P_T, \mathcal{P}_T) - f(\alpha_0, P_0, \mathcal{P}_0) \right]. \end{aligned}$$

Solutions

Theorem. Under regularity assumptions, the problem in “impact space” admits a unique solution. The **optimal price** is $\mathcal{P}_T^* = \alpha_T + P_T$ and $\mathcal{P}_t^*, t \in (0, T)$ such that

$$\underbrace{\beta(\alpha_t - 2(\mathcal{P}_t - P_t)) + \frac{\mu_t P_t}{\Lambda(P_t)}}_{\text{strength of alpha signal}} + \underbrace{\frac{3L\sigma_t^2}{8P_t^{1/2}} \left(\frac{P_t^{5/2}\Lambda(\mathcal{P}_t)^2}{\mathcal{P}_t^{5/2}\Lambda(P_t)^2} - 1 \right)}_{\text{“LVR” profit from mispricings}} = 0$$

We recover the corresponding **optimal order flow** via the mapping

$$\mathcal{X}_t = \int_0^t \beta (\mathcal{P}_s - P_s) ds + \int_0^t \frac{1}{\Lambda(\mathcal{P}_s)} d\mathcal{P}_s - \int_0^t \frac{1}{\Lambda(P_s)} dP_s - \int_0^t \frac{3L}{8\mathcal{P}_s^{5/2}} d\langle \mathcal{P} \rangle_s + \int_0^t \frac{3L\sigma_s^2}{8P_s^{1/2}} ds$$

Solutions and Limiting Cases

Pointwise optimization remains well-posed, but no longer has an explicit solution

- ↳ Optimal price of $\mathcal{P}_t^* = P_t$ without alpha signal, **no price manipulation**
- ↳ Explicit solution when volatility of P_t is zero
- ↳ Approximation for small σ_t^2 via implicit function theorem

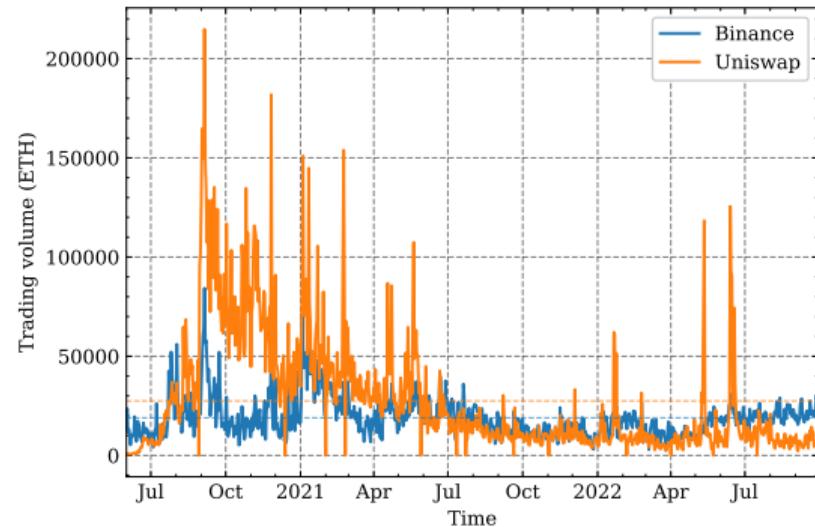
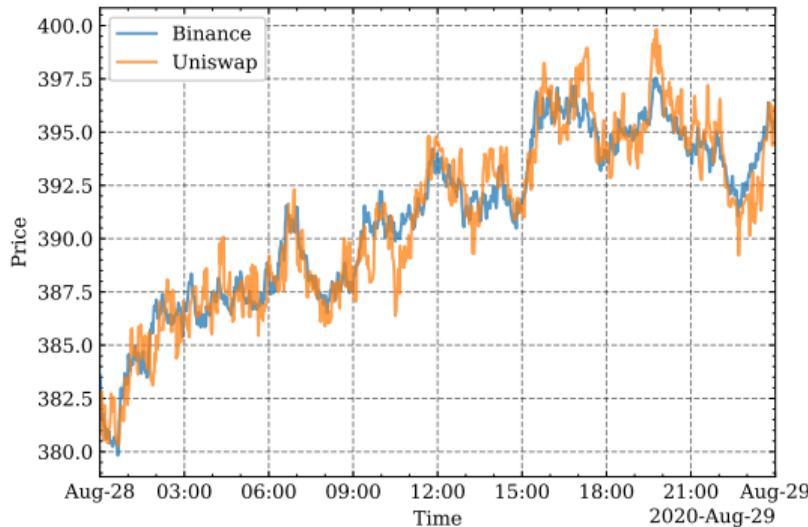
- ✓ **LOB limit** ($L \rightarrow 0$): Obizhaeva-Wang model with $\Lambda(\cdot) \rightarrow \lambda$ and

$$\mathcal{P}_t^* = P_t + \frac{1}{2} \left(\alpha_t + \frac{\mu_t P_t}{\beta \lambda} \right)$$

- ✓ **AMM limit** ($\lambda \rightarrow \infty$): Price-dependent $\Lambda(\mathcal{P}_t) \rightarrow 2\mathcal{P}_t^{3/2}/L$ and FOC is quadratic in $\sqrt{\mathcal{P}_t}$

Empirical Study

We use 10-second bins for price and trade data from **Binance** and **Uniswap v2** between Aug. 2020 and Sept. 2022 for the pair ETH/USDC



Model Calibration

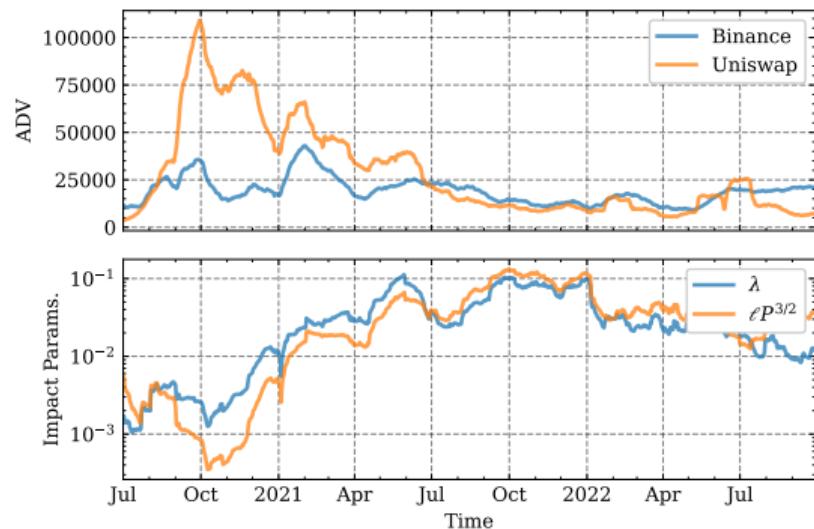
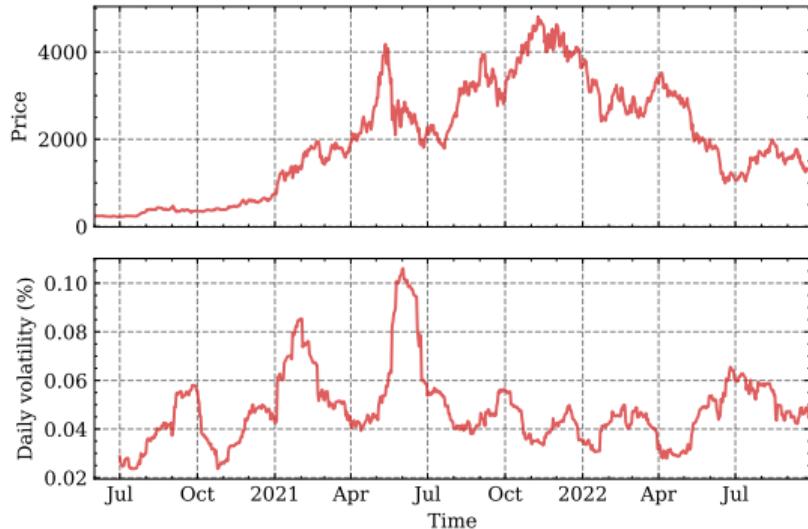
We calibrate the model to price and trade data:

- Analysis repeated daily, with a rolling window of 30 days
- Estimation of aggregate market activity by an exponentially weighted moving average
- Regression of price change against aggregate flow [Muhle-Karbe et al., 2024]
- Grid search for the decay parameter

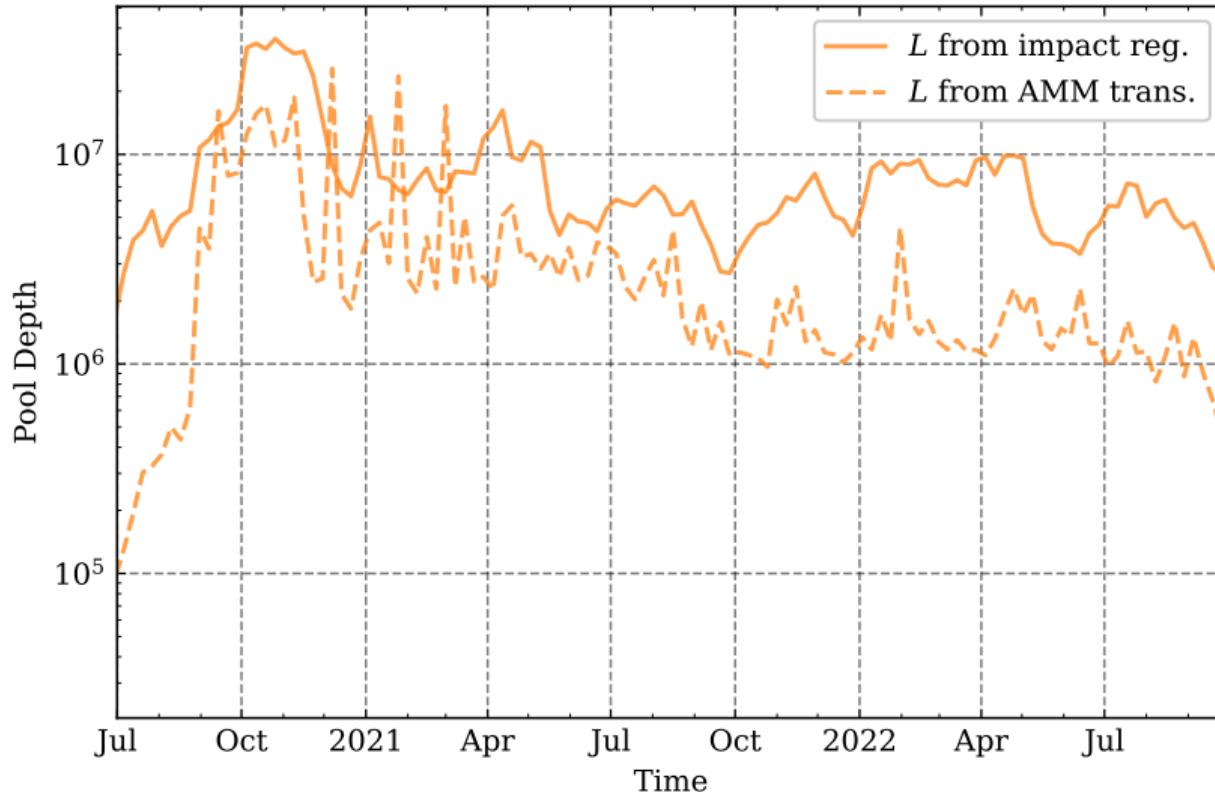
We choose the local depths to match the observed split of trading volume

- Alternative: directly estimate depth of AMM from pre- and post-trade prices
- Both approaches turn out to be (roughly) consistent

Daily Estimates of Calibrated Model

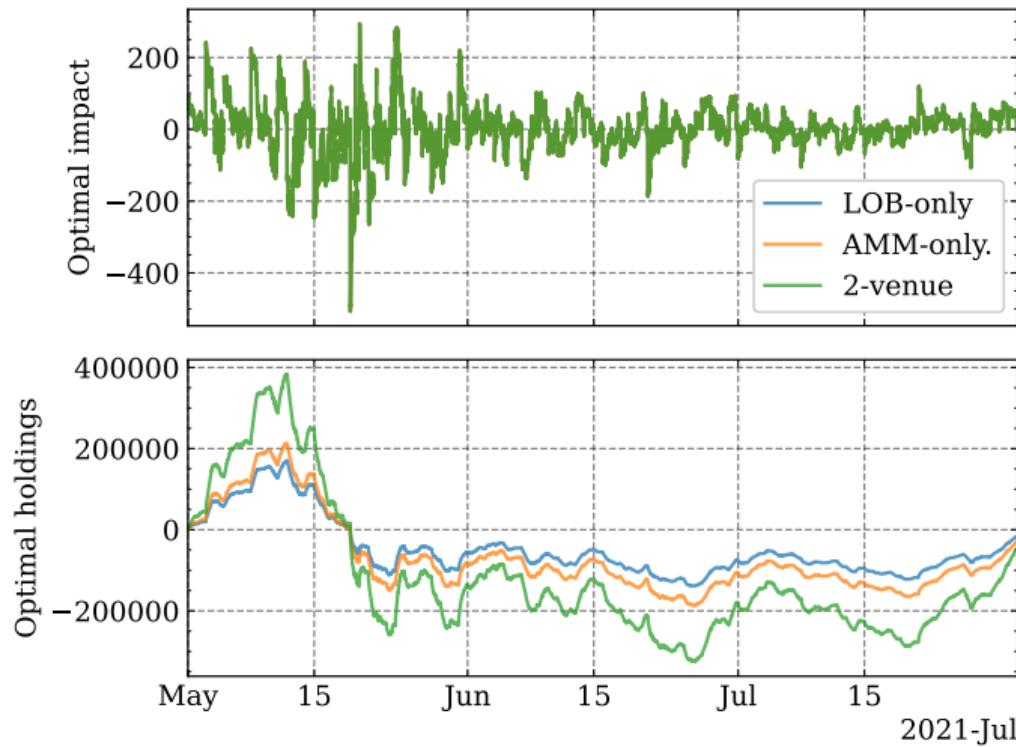


Comparison of AMM's Pool Depth



Optimal Impacts & Order Flows

Optimal solutions are **nearly identical** in impact space, but **differs** in order flow space



Contributions

We study **optimal trading across a LOB and AMM**:

- ↳ **Nonlinear dynamics** of the aggregate exchange with **diffusive trading strategies**
- ↳ **Tractable solution** in “impact space”; results can be generalized to G3Ms
- ↳ Empirical case study on Binance & Uniswap data

Future directions:

- ↳ Inclusion of fees, multi-dimensional model for all prices
- ↳ Interactions between liquidity takers and liquidity providers
- ↳ Competition between many LOBs and AMMs

Thank you!

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