Optimal Trading Across Coexisting Exchanges: Limit-Order Books & Automated Market Makers

Anthony Coache (Imperial)
anthonycoache.ca

IMPERIAL

Joint work with
Agostino Capponi (Columbia)
Johannes Muhle-Karbe (Imperial)

NSERC
CRSNG

9th London-Paris Bachelier Workshop
November 6, 2025

LOBs vs AMMs

Large literature that models price impact and optimal trading in LOBs

↳ Alfonsi, Almgren, Biais, Bouchaud, Carmona, Cartea, Chriss, Cont, Fruth, Gârleanu, Gatheral, Guéant, Hasbrouck, Jaimungal, Lehalle, Lillo, Mastromatteo, Muhle-Karbe, Neuman, Obizhaeva, Pedersen, Rosenbaum, Schied, Voß, Wang, Webster, etc.

Can we get corresponding results for AMMs? What if we have access to both venues?

↳ Static game theoretical models in Aoyagi and Ito [2021]; Malinova and Park [2023]; Lehar and Parlour [2025]; dynamic models in Cartea et al. [2023] with temporary price impact and approximations; etc.

This Paper

We develop a dynamic model, with minimal reduced-form cuts and nonlinear price dynamics

- ✓ Derive consistent budget equations for trading in LOBs and geometric mean AMMs
- ✓ Solve optimization problem of large risk-neutral liquidity taker
- ✓ Verify the absence of price manipulation
- ✓ For calibrated parameters: optimal trades vary substantially, but optimal impacts do not

We do not take into account:

- × Discrete settlement on AMM
- × Proportional fees
- × Endogenous liquidity provision, or concentrated liquidity (Uniswap v3)

Price Dynamics in LOB and AMM

Consider two assets X, Y (e.g., ETH and USDC) and a **LOB** with immediate linear impact λ

- Order flow of a large trader $(\mathcal{X}_t)_t$
- Unaffected or fundamental price $(P_t)_t$, exogenous model such as a Brownian motion
- Market price $(\mathcal{P}_t)_t$, with $d(\mathcal{P}_t - P_t) = \lambda d\mathcal{X}_t - \beta\,(\mathcal{P}_t - P_t)\,dt$

What is the analogue for an **AMM** with bonding curve $f(x, y) = xy = L^2$?

↪ Price impact is locally linear, but flattens (spikes) when reserves are high (low)

↪ Magnitude of impact depends on price level, and thus previous trades

Price Dynamics & Budget Equations

Dynamics of the marginal price:

$$d\mathcal{P}_t = -\lambda(\mathcal{P}_t)\beta_t(\mathcal{P}_t - P_t)dt + \lambda(\mathcal{P}_t)d\mathcal{X}_t + \frac{\lambda(\mathcal{P}_t)}{\lambda(P_t)}dP_t + \xi(\mathcal{P}_t)d\langle \mathcal{X}\rangle_t$$
$$+ \frac{\xi(\mathcal{P}_t)}{\lambda(P_t)^2}\left(1 - \frac{\sqrt{P_t}}{\sqrt{\mathcal{P}_t}}\right)d\langle P\rangle_t + \frac{2\xi(\mathcal{P}_t)}{\lambda(P_t)}d\langle \mathcal{X}, P\rangle_t$$

Price paid for amount $d\mathcal{X}_t$:

$$d\mathcal{Y}_t = -\frac{L^2 d\mathcal{X}_t}{X_{t-}^2 - X_{t-}d\mathcal{X}_t} = -\mathcal{P}_t d\mathcal{X}_t - \frac{\lambda(\mathcal{P}_t)}{\lambda(P_t)}d\langle P, \mathcal{X}\rangle_t - \frac{\lambda(\mathcal{P}_t)}{2}d\langle \mathcal{X}\rangle_t$$

where

$$\lambda(x) := \frac{2}{L}x^{3/2} \quad \text{and} \quad \xi(x) := \frac{3}{L^2}x^2$$

- Optimal trades are generally not smooth; immediate reaction to diffusive parameters
- Correction for trades with nontrivial quadratic variation [see e.g. Milionis et al., 2024]

Optimal Trading with Alpha Signals

Consider a large risk-neutral trader who has a forecast $\alpha_t := \mathbb{E}_t[P_\tau - P_t]$, $\tau \geq T$, and

$$\sup_{(\mathcal{X}_t)_{t\in[0,T]}} \mathbb{E}\Big[\mathrm{PnL}_T\Big] = \sup_{(\mathcal{X}_t)_{t\in[0,T]}} \mathbb{E}\Big[\mathcal{X}_T(\alpha_T + P_T) + \mathcal{Y}_T\Big]$$

At first glance, it seems hopeless to solve this optimization problem in closed-form...

↳ One-to-one correspondence between holdings and exchange rate

↳ **Changing the control variable** from order flow $(\mathcal{X}_t)_t$ to market price $(\mathcal{P}_t)_t$

↳ Also leads to explicit solution in models with nonlinear or stochastic impact [see e.g. Fruth et al., 2019; Hey et al., 2025]

↳ Substantially more involved algebra due to convexity corrections, trader's actions affect $\lambda(\mathcal{P}_t)$... but miraculously **makes the problem tractable**, allows pointwise optimization!

Equivalence Result for AMM

Theorem. For any trading strategy $(\mathcal{X}_t)_{t\in[0,T]}$, the value of the risk-neutral goal functional can be rewritten in terms of the corresponding market price $(\mathcal{P}_t)_{t\in[0,T]}$ as

$$\mathbb{E}\Bigg[\int_{0+}^{T-}\left((\alpha_t+P_t-\mathcal{P}_t)\left(\beta_t(\mathcal{P}_t-P_t)-\frac{P_t}{\lambda(P_t)}\left(\mu_t-\frac{3\sigma_t^2}{4}\right)\right)+\frac{P_t^2\sigma_t^2\lambda(\mathcal{P}_t)}{2\lambda(P_t)^2}\right)dt \\ -\frac{1}{\lambda(P_t)}d\langle P,\alpha+P\rangle_t-\frac{L(\alpha_T+P_T+\mathcal{P}_T)}{\sqrt{\mathcal{P}_T}}+\frac{L(\alpha_0+P_0+\mathcal{P}_0)}{\sqrt{\mathcal{P}_0}}\Bigg].$$

Conversely, for strictly positive $(\mathcal{P}_t)_t$, the corresponding trades can be recovered via

$$d\mathcal{X}_t=\frac{1}{\lambda(\mathcal{P}_t)}d\mathcal{P}_t-\frac{1}{\lambda(P_t)}dP_t-\frac{\xi(\mathcal{P}_t)}{\lambda(\mathcal{P}_t)^3}d\langle\mathcal{P}\rangle_t+\frac{\xi(P_t)}{\lambda(P_t)^3}d\langle P\rangle_t+\beta_t(\mathcal{P}_t-P_t)dt,$$

$$\Delta\mathcal{X}_0=\frac{L}{\sqrt{\mathcal{P}_0}}-\frac{L}{\sqrt{\mathcal{P}_{0+}}}\quad\text{and}\quad\Delta\mathcal{X}_T=\frac{L}{\sqrt{\mathcal{P}_{T-}}}-\frac{L}{\sqrt{\mathcal{P}_T}}.$$

Solution for AMM

Theorem. Under regularity assumptions, the problem in "price space" admits a unique positive maximizer. The optimal price at the terminal time T is $\mathcal{P}_T^* = \alpha_T + P_T$, while at the intermediate times $t \in (0, T)$ it is the unique positive root of the first-order condition

$$\beta_t(\alpha_t + 2(P_t - \mathcal{P}_t)) + \frac{P_t \mu_t}{\lambda(P_t)} + \frac{3P_t\sigma_t^2}{4\lambda(P_t)}\left(\frac{\sqrt{\mathcal{P}_t}}{\sqrt{P_t}} - 1\right) = 0.$$

This first-order condition is quadratic in $\sqrt{\mathcal{P}_t}$ and the solution is given explicitly by

$$\mathcal{P}_t^* = P_t\left(\frac{3\sigma_t^2}{16\beta_t\lambda(P_t)} + \sqrt{\left(\frac{3\sigma_t^2}{16\beta_t\lambda(P_t)}\right)^2 + \frac{\alpha_t}{2P_t} + 1 + \frac{1}{2\beta_t\lambda(P_t)}\left(\mu_t - \frac{3\sigma_t^2}{4}\right)}\right)^2.$$

Remarks

Obtaining a wellposed optimization problem requires sufficient impact decay

↳ Impact decay has to grow sufficiently fast as the affected price becomes large or small i.e., $\mathcal{P}_t^{3/2}/\beta(\mathcal{P}_t, P_t) \to 0$ as $\mathcal{P}_t \to \infty$ or $\mathcal{P}_t/\beta(\mathcal{P}_t, P_t) \to \infty$ as $\mathcal{P}_t \to 0$

↳ For $\beta(\mathcal{P}_t, P_t) = \beta_t(\mathcal{P}_t - P_t)$, such regularity assumptions are satisfied with parameters where P_t moves at most by a fixed percentage over the trading horizon

Trade-off between exploiting the alpha signal, impact decay and curvature of bonding curve

↳ Optimal price of $\mathcal{P}_t^* = P_t$ without alpha signal, no price manipulation

↳ Last trade exhausts all remaining alpha, like in the Obizhaeva-Wang model

↳ With no volatility for P_t, solution has the same form as in Obizhaeva-Wang model

Result for Aggregate Market

Without fees, arbitrageurs can realize a true arbitrage if they observe a discrepancy between quotes in the LOB and AMM. **How to route trades across exchanges?**

↳ Split so that post-trade prices in both venues are the same, the aggregate market

↳ Effective price is inf-convolution of local execution prices [see e.g. Biais et al., 2000]

Approach for AMM can be adapted, same key insights continue to apply somewhat

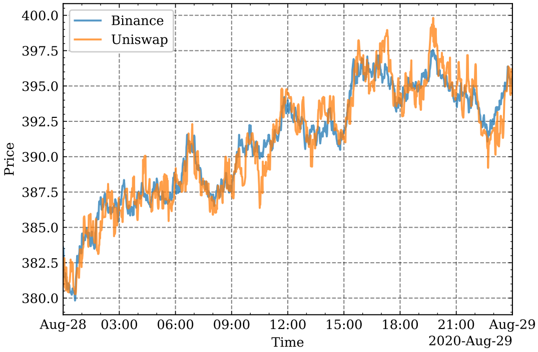
↳ Local impacts $2x^{3/2}/L$ and λ are replaced by the aggregate impact $\Lambda(x) := \frac{2\lambda x^{3/2}/L}{\lambda + 2x^{3/2}/L}$

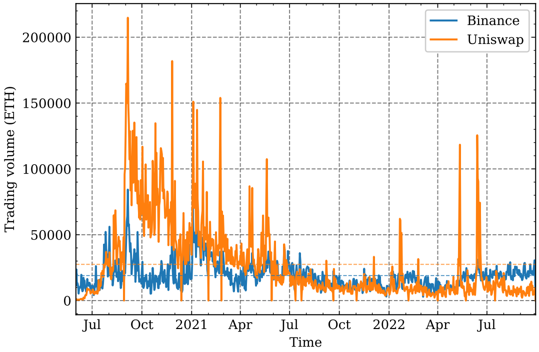
↳ Pointwise optimization remains well-posed, but no longer has an explicit solution

✓ **LOB limit** ($L \to 0$): $\Lambda(\cdot) \to \lambda$ and FOC is linear in $\mathcal{P}_t$, optimal price as in OW model

✓ **AMM limit** ($\lambda \to \infty$): Price-dependent $\Lambda(x) \to 2x^{3/2}/L$ and FOC is quadratic in $\sqrt{\mathcal{P}_t}$

We use 10-second bins for price and trade data from **Binance** and **Uniswap v2** between Aug. 2020 and Sept. 2022 for the pair ETH/USDC





Model Calibration

We calibrate the model to price and trade data:

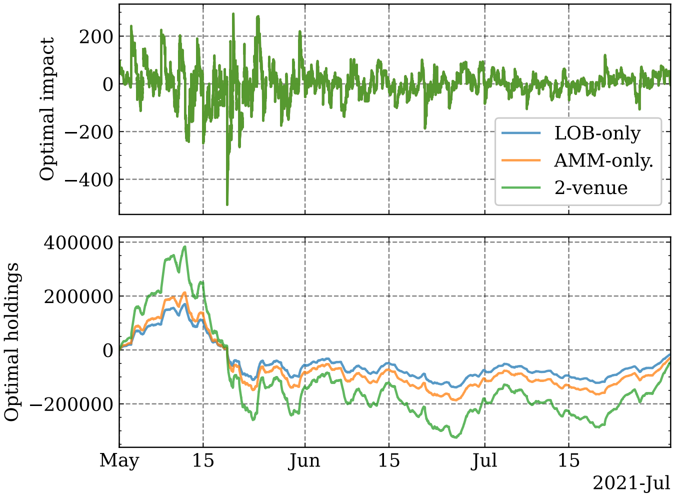
- Analysis repeated daily, with a rolling window of 30 days
- Estimation of aggregate market activity by an exponentially weighted moving average
- Regression of price change against aggregate flow [Muhle-Karbe et al., 2024]
- Grid search for the decay parameter

We choose the local depths to match the observed split of trading volume

- Alternative: directly estimate depth of AMM from pre- and post-trade prices
- Both approaches turn out to be roughly consistent

Optimal Impacts & Order Flows

Optimal solutions are **nearly identical** in impact space, but **differs** in order flow space



Contributions

We study optimal trading across a LOB and AMM:

↳ **Nonlinear dynamics** of the aggregate exchange with **diffusive trading strategies**

↳ **Tractable solution** in "price space"; results can be generalized to G3Ms

↳ Empirical case study on Binance & Uniswap data

Future directions:

↳ Inclusion of fees, multi-dimensional model for all prices

↳ Interactions between liquidity takers and liquidity providers

↳ Competition between many LOBs and AMMs

Thank you!

Paper in preparation; slides on: `anthonycoache.ca`

References I

Aoyagi, J. and Ito, Y. (2021). Coexisting exchange platforms: Limit order books and automated market makers. *Available at SSRN 3808755*.

Biais, B., Martimort, D., and Rochet, J.-C. (2000). Competing mechanisms in a common value environment. *Econometrica*, 68(4):799–837.

Cartea, Á., Drissi, F., and Monga, M. (2023). Decentralised finance and automated market making: Execution and speculation. *arXiv preprint arXiv:2307.03499*.

Fruth, A., Schöneborn, T., and Urusov, M. (2019). Optimal trade execution in order books with stochastic liquidity. *Mathematical Finance*, 29(2):507–541.

Hey, N., Mastromatteo, I., Muhle-Karbe, J., and Webster, K. (2025). Trading with concave price impact and impact decay—theory and evidence. *Operations Research*.

Lehar, A. and Parlour, C. (2025). Decentralized exchange: The Uniswap automated market maker. *The Journal of Finance*, 80(1):321–374.

Malinova, K. and Park, A. (2023). Learning from DeFi: Would automated market makers improve equity trading? *Available at SSRN*, 4531670.

Milionis, J., Moallemi, C. C., and Roughgarden, T. (2024). Automated market making and arbitrage profits in the presence of fees. In *International Conference on Financial Cryptography and Data Security*, pages 159–171. Springer.

Muhle-Karbe, J., Wang, Z., and Webster, K. (2024). Stochastic liquidity as a proxy for nonlinear price impact. *Operations Research*, 72(2):444–458.