

# Optimal Trading Across Coexisting Exchanges: Limit-Order Books & Automated Market Makers

Anthony Coache (Imperial)

[anthonycoache.ca](http://anthonycoache.ca)

**IMPERIAL**

Joint work with

Agostino Capponi (Columbia)

Johannes Muhle-Karbe (Imperial)



SIAM Conference on Financial Mathematics and Engineering

July 16, 2025

# LOBs vs AMMs

Many digital assets trade in centralized **and** decentralized exchanges

↳ ETH/USDC can be exchanged on Binance (LOB) and Uniswap (AMM)

Large literature that models price impact and optimal trading in LOBs

↳ Almgren and Chriss [2001]; Alfonsi et al. [2010]; Obizhaeva and Wang [2013]; Cheridito and Sepin [2014]; Cartea and Jaimungal [2016]; Neuman and Voß [2022]; etc.

Can we get corresponding results for AMMs? What if we have access to both venues?

↳ Static game theoretical models in Aoyagi and Ito [2021]; Malinova and Park [2023]; Lehar and Parlour [2025]; dynamic models in Cartea et al. [2023] with temporary price impact and approximations; etc.

# This Paper

We develop a dynamic model, with **minimal reduced-form cuts** and **nonlinear price dynamics**

- ✓ Derive consistent budget equations for trading in LOBs and geometric mean AMMs
- ✓ Solve optimization problem of large liquidity taker
- ✓ Verify the absence of price manipulation
- ✓ For calibrated parameters: optimal trades vary substantially, but optimal impacts do not

We do not take into account:

- ✗ Discrete settlement on AMM
- ✗ Proportional fees
- ✗ Endogenous liquidity provision, or concentrated liquidity (Uniswap v3)

# Trading in the LOB

Consider two assets  $X, Y$  (e.g., ETH and USDC) and a **LOB** with immediate linear impact  $\lambda$

- Order flow of a large trader  $(\mathcal{X}_t)_t$  and other market participants  $(X_t)_t$
- Unaffected price  $(P_t)_t$ , exogenous process such as a Brownian motion for the market price in the absence of large traders
- Exchange rate  $(\mathcal{P}_t)_t$ , with arbitrageurs exploiting mispricings relative to  $P_t$

$$d\mathcal{P}_t = -\beta\lambda(\mathcal{P}_t - P_t)dt + \lambda(d\mathcal{X}_t + dX_t)$$

What is the analogue for AMMs?

# Trading in the AMM

Consider an **AMM** with bonding curve  $f(x, y) = xy = L^2$

For any swap  $d\mathcal{X}_t \leftrightarrow d\mathcal{Y}_t$ , we must have

$$(X_{t-} - d\mathcal{X}_t)(Y_{t-} + d\mathcal{Y}_t) = X_{t-}Y_{t-} = L^2$$

Unaffected price, specified in terms of  $(X_t)_t$

$$P_t = \frac{f_x(X_t, Y_t)}{f_y(X_t, Y_t)} = \frac{L^2}{X_t^2}$$

Price paid for amount  $d\mathcal{X}_t$ :

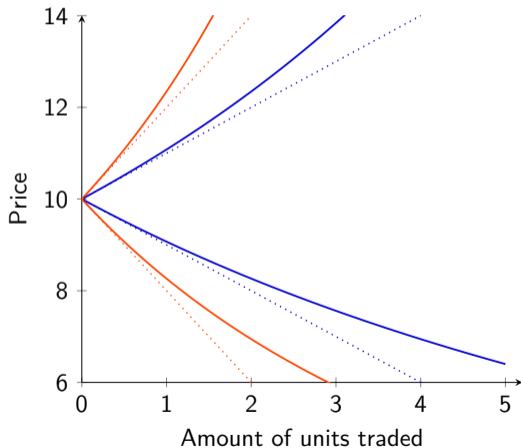
$$d\mathcal{Y}_t = -\frac{L^2 d\mathcal{X}_t}{X_{t-}^2 - X_{t-} d\mathcal{X}_t} = -P_t d\mathcal{X}_t - \frac{2P_t^{3/2}}{L} d\langle \mathcal{X} \rangle_t$$

Figure

# Price Impact

Price impact is locally linear, but flattens (spikes) when reserves are high (low)

↳ Magnitude of impact depends on price level, and thus previous trades



# Aggregate Market

Without fees, arbitrageurs can realize a true arbitrage if they observe a discrepancy between quotes in the LOB and AMM. **How to route trades across exchanges?**

- ↳ Split so that post-trade prices in both venues are the same, the **aggregate market**
- ↳ Effective price is inf-convolution of local execution prices [see e.g. Biais et al., 2000]
- ↳ Leaves no profits for inter-exchange arbitrageurs

The flow  $(1 - \pi_t)d\mathcal{X}_t$  is executed in the AMM, while  $\pi_td\mathcal{X}_t$  is executed in the LOB:

$$\pi_t := \frac{2\mathcal{P}_t^{3/2}/L}{\lambda + 2\mathcal{P}_t^{3/2}/L}$$

Local impacts  $2\mathcal{P}_t^{3/2}/L$  and  $\lambda$  are replaced by the aggregate impact

$$\Lambda(\mathcal{P}_t) := \frac{2\lambda\mathcal{P}_t^{3/2}/L}{\lambda + 2\mathcal{P}_t^{3/2}/L}$$

# Continuous-Time Scaling Limits

**Order flow** of a large trader  $(\mathcal{X}_t)_t$  and other market participants  $(Q_t)_t$

**Unaffected price**, fully driven by  $(Q_t)_t$

$$dP_t = P_t(\mu_t dt + \sigma_t dW_t)$$

**Exchange rate**, with arbitrageurs exploiting mispricings relative to  $P_t$

$$d\mathcal{P}_t = -\beta\Lambda(\mathcal{P}_t)(\mathcal{P}_t - P_t)dt + \Lambda(\mathcal{P}_t)(d\mathcal{X}_t + dQ_t) + \frac{3L\Lambda(\mathcal{P}_t)^3}{8\mathcal{P}_t^{5/2}}d\langle\mathcal{X} + Q\rangle_t$$

- Optimal trading strategies are generally not smooth
- Immediate reaction to diffusive parameters
- Correction for trades with nontrivial quadratic variation [Milionis et al., 2024]

# Optimal Trading with Alpha Signals

Consider a large risk-neutral trader who

- cannot frontrun market flow
- has a forecast  $\alpha_t := \mathbb{E}_t[P_\tau] - P_t$  at a later time  $\tau \geq T$
- has a cash account  $(\mathcal{Y}_t)_t$  with dynamics

$$d\mathcal{Y}_t = -\mathcal{P}_t d\mathcal{X}_t - \frac{\Lambda(\mathcal{P}_t)}{2} (d\langle \mathcal{X} \rangle_t + 2d\langle \mathcal{X}, Q \rangle_t)$$

Large trader's goal functional: maximize the expected PnL

$$\sup_{(\mathcal{X}_t)_{t \in [0, T]}} \mathbb{E}[\text{PnL}_T] = \sup_{(\mathcal{X}_t)_{t \in [0, T]}} \mathbb{E} \left[ \mathcal{X}_T (\alpha_T + P_T) + \int_0^T d\mathcal{Y}_t \right]$$

# Passage to Impact Space

We redefine the control variable from order flow  $(\mathcal{X}_t)_t$  to aggregate price  $(\mathcal{P}_t)_t$

- ↳ Also leads to explicit solution in models with nonlinear or stochastic impact [see e.g. Fruth et al., 2019; Hey et al., 2025]
- ↳ One-to-one correspondence between order flow and exchange rate

$$d\mathcal{X}_t = \beta (\mathcal{P}_t - P_t) dt + \frac{1}{\Lambda(\mathcal{P}_t)} d\mathcal{P}_t - \frac{3L}{8\mathcal{P}_t^{5/2}} d\langle \mathcal{P} \rangle_t - dQ_t$$

- ↳ Additional terms from integration by parts and Itô's formula
- ↳ Cancellations between variations and final expressions in terms of prices
- ↳ “Passage to impact space” makes the problem tractable, allows pointwise optimization!

# Equivalence Results

**Theorem.** The large trader's optimization problem in “**order flow space**”

$$\sup_{(\mathcal{X}_t)_{t \in [0, T]}} \mathbb{E} \left[ \mathcal{X}_T(\alpha_T + P_T) + \int_0^T d\mathcal{Y}_t \right]$$

is equivalent to the following problem in “**impact space**”

$$\begin{aligned} \sup_{(\mathcal{P}_t)_{t \in (0, T]}} \mathbb{E} \left[ \int_{0+}^{T-} \left( (\alpha_t + P_t - \mathcal{P}_t) \left( \beta(\mathcal{P}_t - P_t) - \frac{\mu_t P_t}{\Lambda(P_t)} + \frac{3L\sigma_t^2}{8P_t^{1/2}} \right) + \frac{\sigma_t^2 P_t^2 \Lambda(\mathcal{P}_t)}{2\Lambda(P_t)^2} \right) dt \right. \\ \left. + \frac{4P_t^{3/2}/L - \lambda}{2\lambda P_t^{3/2}/L} d\langle P, \alpha + P \rangle_t + f(\alpha_T, P_T, \mathcal{P}_T) - f(\alpha_0, P_0, \mathcal{P}_0) \right]. \end{aligned}$$

# Solutions

**Theorem.** Under regularity assumptions, the problem in “impact space” admits a unique solution. The **optimal price** is  $\mathcal{P}_T^* = \alpha_T + P_T$  and  $\mathcal{P}_t^*, t \in (0, T)$  such that

$$\underbrace{\beta(\alpha_t - 2(\mathcal{P}_t - P_t)) + \frac{\mu_t P_t}{\Lambda(P_t)}}_{\text{strength of alpha signal}} + \underbrace{\frac{3L\sigma_t^2}{8P_t^{1/2}} \left( \frac{P_t^{5/2} \Lambda(\mathcal{P}_t)^2}{\mathcal{P}_t^{5/2} \Lambda(P_t)^2} - 1 \right)}_{\text{“LVR” profit from mispricings}} = 0$$

We recover the corresponding **optimal order flow** via the mapping

$$\mathcal{X}_t = \int_0^t \beta(\mathcal{P}_s - P_s) ds + \int_0^t \frac{1}{\Lambda(\mathcal{P}_s)} d\mathcal{P}_s - \int_0^t \frac{1}{\Lambda(P_s)} dP_s - \int_0^t \frac{3L}{8\mathcal{P}_s^{5/2}} d\langle \mathcal{P} \rangle_s + \int_0^t \frac{3L\sigma_s^2}{8P_s^{1/2}} ds$$

# Solutions and Limiting Cases

Pointwise optimization remains well-posed, but no longer has an explicit solution

- ↳ Optimal price of  $\mathcal{P}_t^* = P_t$  without alpha signal, **no price manipulation**
- ↳ Explicit solution when volatility of  $P_t$  is zero
- ↳ Approximation for small  $\sigma_t^2$  via implicit function theorem

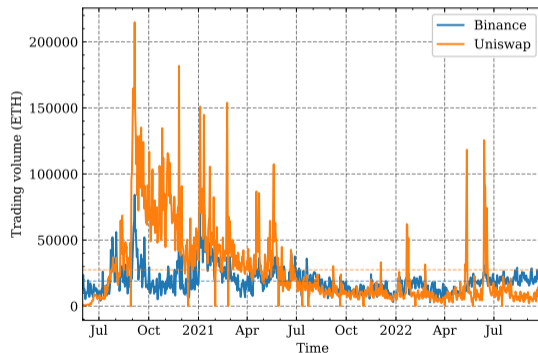
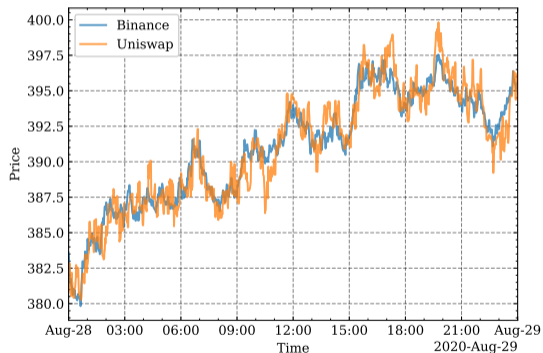
- ✓ **LOB limit** ( $L \rightarrow 0$ ): Obizhaeva-Wang model with  $\Lambda(\cdot) \rightarrow \lambda$  and

$$\mathcal{P}_t^* = P_t + \frac{1}{2} \left( \alpha_t + \frac{\mu_t P_t}{\beta \lambda} \right)$$

- ✓ **AMM limit** ( $\lambda \rightarrow \infty$ ): Price-dependent  $\Lambda(\mathcal{P}_t) \rightarrow 2\mathcal{P}_t^{3/2}/L$  and FOC is quadratic in  $\sqrt{\mathcal{P}_t}$

# Empirical Study

We use 10-second bins for price and trade data from **Binance** and **Uniswap v2** between Aug. 2020 and Sept. 2022 for the pair ETH/USDC



# Model Calibration

We calibrate the model to price and trade data:

- Analysis repeated daily, with a rolling window of 30 days
- Estimation of aggregate market activity by an exponentially weighted moving average
- Regression of price change against aggregate flow [Muhle-Karbe et al., 2024]
- Grid search for the decay parameter

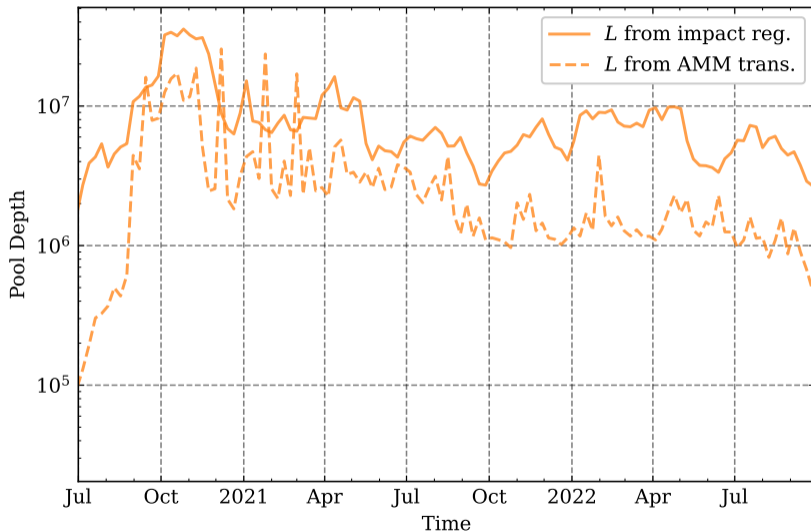
We choose the local depths to match the observed split of trading volume

- Alternative: directly estimate depth of AMM from pre- and post-trade prices
- Both approaches turn out to be (roughly) consistent

# Daily Estimates of Calibrated Model

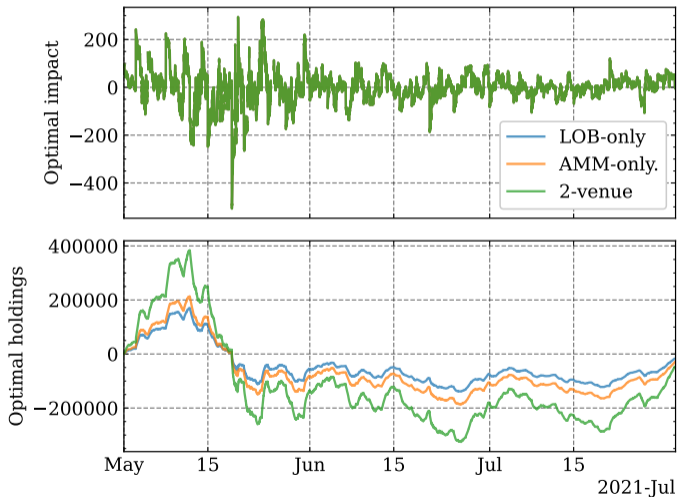


# Comparison of AMM's Pool Depth



# Optimal Impacts & Order Flows

Optimal solutions are **nearly identical** in impact space, but **differs** in order flow space



# Contributions

We study **optimal trading across a LOB and AMM**:

- ↳ **Nonlinear dynamics** of the aggregate exchange with **diffusive trading strategies**
- ↳ **Tractable solution** in “impact space”; results can be generalized to G3Ms
- ↳ Empirical case study on Binance & Uniswap data

Future directions:

- ↳ Inclusion of fees, multi-dimensional model for all prices
- ↳ Interactions between liquidity takers and liquidity providers
- ↳ Competition between many LOBs and AMMs

## Thank you!

Slides: [anthonycoache.ca](http://anthonycoache.ca)

*Travel support for this presentation was provided by G-Research.*

# References I

- Alfonsi, A., Fruth, A., and Schied, A. (2010). Optimal execution strategies in limit order books with general shape functions. *Quantitative Finance*, 10(2):143–157.
- Almgren, R. and Chriss, N. (2001). Optimal execution of portfolio transactions. *Journal of Risk*, 3:5–40.
- Aoyagi, J. and Ito, Y. (2021). Coexisting exchange platforms: Limit order books and automated market makers. Available at SSRN 3808755.
- Biais, B., Martimort, D., and Rochet, J.-C. (2000). Competing mechanisms in a common value environment. *Econometrica*, 68(4):799–837.
- Cartea, Á., Drissi, F., and Monga, M. (2023). Decentralised finance and automated market making: Execution and speculation. *arXiv preprint arXiv:2307.03499*.
- Cartea, Á. and Jaimungal, S. (2016). Incorporating order-flow into optimal execution. *Mathematics and Financial Economics*, 10:339–364.
- Cheridito, P. and Sepin, T. (2014). Optimal trade execution under stochastic volatility and liquidity. *Applied Mathematical Finance*, 21(4):342–362.
- Fruth, A., Schöneborn, T., and Urusov, M. (2019). Optimal trade execution in order books with stochastic liquidity. *Mathematical Finance*, 29(2):507–541.
- Hey, N., Mastromatteo, I., Muhle-Karbe, J., and Webster, K. (2025). Trading with concave price impact and impact decay—theory and evidence. *Operations Research*.

# References II

- Lehar, A. and Parlour, C. (2025). Decentralized exchange: The Uniswap automated market maker. *The Journal of Finance*, 80(1):321–374.
- Malinova, K. and Park, A. (2023). Learning from DeFi: Would automated market makers improve equity trading? *Available at SSRN*, 4531670.
- Milionis, J., Moallemi, C. C., and Roughgarden, T. (2024). Automated market making and arbitrage profits in the presence of fees. In *International Conference on Financial Cryptography and Data Security*, pages 159–171. Springer.
- Muhle-Karbe, J., Wang, Z., and Webster, K. (2024). Stochastic liquidity as a proxy for nonlinear price impact. *Operations Research*, 72(2):444–458.
- Neuman, E. and Voß, M. (2022). Optimal signal-adaptive trading with temporary and transient price impact. *SIAM Journal on Financial Mathematics*, 13(2):551–575.
- Obizhaeva, A. A. and Wang, J. (2013). Optimal trading strategy and supply/demand dynamics. *Journal of Financial Markets*, 16(1):1–32.