A study for profitability of Alchemix multiplier strategy delegating a vault to itself

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Summary

The Alchemix multiplier strategy could be applied to a vault such that, it takes the allocated amount, multiplies it, and reinvests the multiplied amount again to the same vault. Although, in another document, it has been shown that the fees will always increase, in this document, I will show that according to function of generated yield to invested amount for a vault, applying the strategy to the same vault may produce less yield for the investors.

Assumptions

1. It will be assumed, the Dai Vault is being studied here, this could easily be adopted to other vaults.
2. Price per share is 1 Dai.
3. Initially the vault has N Dai, and from this, a Dai is allocated to the Alchemix multiplier strategy.
4. After the allocated amount is doubled and returned to the strategy, the whole amount will be allocated to other strategies, generating actual yield.
5. The actual generated yield (by other strategies) for N Dai will be N Dai, and for $N+a$ Dai will be $N+ka$ Dai, with $0 \leqslant k \leqslant 1$.
6. Applying Alchemix multiplier strategy is profitable to be applied, if the generated yield for the investors after applying the strategy is not lower than what it has been before applying the strategy (N).
7. The multiplication factor is 2, the Alchemix fee is 10%, the Yearn fee is 20%.

Analysis

Before applying the Alchemix multiplier strategy, the yield for N shares is N Dai.

Since the Alchemix multipler strategy transfers the $2\mathrm{a}$ Dai to the Alchemix smart contracts and they in return, invest that in Yearn Dai vault, the number of shares of Yearn Dai vault will become $N+2\mathrm{a}$.

When $N+ka$ Dai yield is generated by other strategies (assumptions 4 and 5), it is divided between N shares which belong to actual investors and $2\mathrm{a}$ shares which belong to Alchemix and thus Alchemix multiplier strategy. From what goes to Alchemix, it takes its fees and gives 0.9 of it to Alchemix multiplier strategy and the strategy reports 0.8 of that 0.9 of the yield for $2\mathrm{a}$ shares as the new yield. The same process happens for the new yield, and this process continues infinitely. This requires harvests from Yearn and Alchemix.

So the yield for the investors, is the sum of what is earned in multiple steps as below

$$S=(\frac{N}{N+2\mathrm{a}})(N+ka)+(\frac{N}{N+2\mathrm{a}})*0.72*(\frac{2\mathrm{a}}{N+2\mathrm{a}})(N+ka)+(\frac{N}{N+2\mathrm{a}})*0.72*(\frac{2\mathrm{a}}{N+2\mathrm{a}})^2(N+ka)+...$$
$$\Rightarrow S=\frac{N}{N+2\mathrm{a}}(N+ka)(1+\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+...),\ \ r=0.72(\frac{2\mathrm{a}}{N+2\mathrm{a}})\ \ (r<1,\ \textit{thus the series converge})$$
$$\Rightarrow S=\frac{N}{N+2\mathrm{a}}(N+ka)(\frac{1}{1-r}),\ \ r=0.72(\frac{2\mathrm{a}}{N+2\mathrm{a}})$$

It is desired that $S\geq N$, thus

$$S=(\frac{N^2+kaN}{N+2\mathrm{a}})(\frac{N+2\mathrm{a}}{N+0.56\mathrm{a}})\geq N$$
$$\Rightarrow\frac{N^2+kaN}{N+0.56\mathrm{a}}\geq N$$
$$\Rightarrow N^2+kaN\geq N^2+0.56\mathrm{aN}$$
$$\Rightarrow k\geq 0.56$$

thus, there can be situations that although the actual generated yield is larger than before applying Alchemix multiplier strategy, what is returned back to investors is even less than when the strategy is not applied (The remaining being consumed as Yearn and Alchemix fees).

Now we briefly look at how many rounds of harvests is required (for two different values of k returning more yield), in order for a yield equal to the amount before applying the Alchemix multiplier is returned to the investors. It should be found out that for studied values of k , for what s the term $D=(\sum_{i=0}^{s}\frac{N}{N+2\mathrm{a}}(N+ka)(\frac{1.44\mathrm{a}}{N+2\mathrm{a}})^i)-N$ is not negative.

Case 1 ($k=1$):

This is the case when increasing the amount put into actual yield generating strategies, does not affect their performance. It turns out:

$$D=\frac{a\,N(0.11\,N-0.14\,a)}{(a+0.5\,N)^2},\ \ s=1,\textit{which can be negative for some values of } a\,,N>0$$
$$D=\frac{a\,N(0.1192\,a^2+0.2992\,a\,N+0.055\,N^2)}{(a+0.5\,N)^3},\ \ s=2,\textit{which is always positive for } a\,,N>0$$

so for $k=1$, generally three rounds of Yearn and Alchemix harvests are required, in order for the investors to receive yields equal to what they got before Alchemix multiplier strategy is applied and start receiving Alchemix multiplier strategy generated yields (three rounds will cover all $a\,,N$, depending on their values, two rounds maybe sufficient).

Case 2 ($k=\frac{2}{3}$):

It turns out:

$$D=\frac{(a\,N(-0.0398711\,a^4+0.201868\,a^3\,N+0.115712\,a^2\,N^2+0.0314667\,a\,N^3))}{(a+0.5\,N)^5}$$

$$+\frac{0.00333333\,N^4}{(a+0.5\,N)^5},\quad s=4, \textit{which can be negative for some values of } a\,,N>0$$

$$D=\frac{(a\,N(0.0246261\,a^5+0.278678\,a^4\,N+0.216646\,a^3\,N^2+0.0893227\,a^2\,N^3+0.0190667\,a\,N^4))}{(a+0.5\,N)^6}$$

$$+\frac{0.00166667\,N^5}{(a+0.5\,N)^6},\quad s=5, \textit{which is always positive for } a\,,N>0$$

so for $k=\frac{2}{3}$, generally 6 rounds of Yearn and Alchemix harvests are required, in order for the investors to receive yields equal to what they got before Alchemix multiplier strategy is applied and start receiving Alchemix multiplier strategy generated yields (six rounds will cover all $a\,,N$, depending on their values, less rounds maybe sufficient).

Conclusion

From my analysis, it is shown that depending on the characteristics of the vault (how the yield increase s when the deposited amount increases), applying the Alchemix multiplier strategy as a self delegation strategy may or may not generate more yield for depositors, and also some rounds of Yearn and Alchemix harvests may be required for the strategy to show its generated profitability.