Radioactivebots Calendar (90 min blocks)

Week	Mon	Tues	Wed	Thur	Fri	
1	- Radioactivebots Entry Doc - K/NTK/NS	- Do now: Fractional Exponent Exploration with Bacteria - Planning Document	- Do now: Sequences - Use planning document to write code.	- Do now: Fill in tables - Finish and test code	- Do now: More tables - Run simulation (>=3 times) - Record data	Example program: - python - ch
2	- Do now: Function from graph - Analyze data	- Do now: Simplify exponents - Twist: Simulation without Robots	- Finish Twist - Record and Analyze Data	- Review	- Test	Example program: - python - ch
3						

Radioactivebots

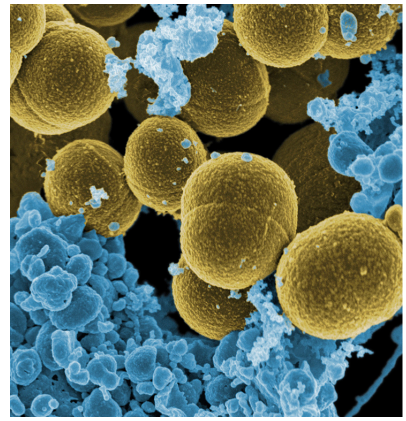
Dear Students,

I found this sign in my storage shed on top of a large and beautiful diamond (it weighs 0.5g). I have reason to believe it is contaminated with carbon-10 which is a radioactive isotope of carbon. I would like to know when it will be safe to give the diamond to my spouse without subjecting them to radiation poisoning.

To figure this out we will be running a simulation with the robots where they decay or not each second in the same way that the carbon isotopes do. Once we take data using the simulation, we will use the graph and equation to predict how long it takes for all of the carbon-10 atoms be gone.

Thanks,
Your Teacher

Warm-up: Fractional Exponent Exploration with Bacteria



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[This task was adopted from the *Illustrative Mathematics Project: http://www.illustrativemathematics.org/illustrations/385*
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Travis and Miriam are studying bacterial growth. They were surprised to find that the population of the bacteria doubled every hour.

1. Complete the following table and plot the data on the graph at the end of this task.

Hours into the study	0	1	2	3	4
Bacteria population (in thousands)	4				

2. Write an equation for P, the population of the bacteria, as a function of time, t, and verify that it produces correct populations for $t = 1, 2, 3$, and 4 hours.

Travis and Miriam want to create a table with more entries; specifically, they want to fill in the population at each half hour. Unfortunately, they forgot to make these measurements so they decide to estimate the values.

Travis makes the following claim:

> "If the population doubles in 1 hour, then half that growth occurs in the first half-hour and the other half in the second half-hour. So for example, we can find the population at $t = \frac{1}{2}$ by finding the average of the populations at $t = 0$ and $t = 1$."

3. Fill in the parts of the table below that you've already computed, and then decide how you might use Travis' strategy to fill in the missing data. Also plot Travis' data on the graph at the end of the task.

Hours into the study	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
Bacteria population (in thousands)	4								

4. Comment on Travis' idea. How does it compare to the table generated in problem 1? For what kind of function would this reasoning work?

Miriam suggests they should fill in the data in the table in the following way:

> "To make the estimates, I noticed that the population increases by the same factor each hour, and I think that this property should hold over each half-hour interval as well."

4. Fill in the parts of the table below that you've already computed in problem 1, and then decide how you might use Miriam's new strategy to fill in the missing data. As in the table in problem 1, each entry should be multiplied by some constant factor in order to produce consistent results. Use this constant multiplier to complete the table. Also plot Miriam's data on the graph at the end of this task.

Hours into the study	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
Bacteria population (in thousands)	4								

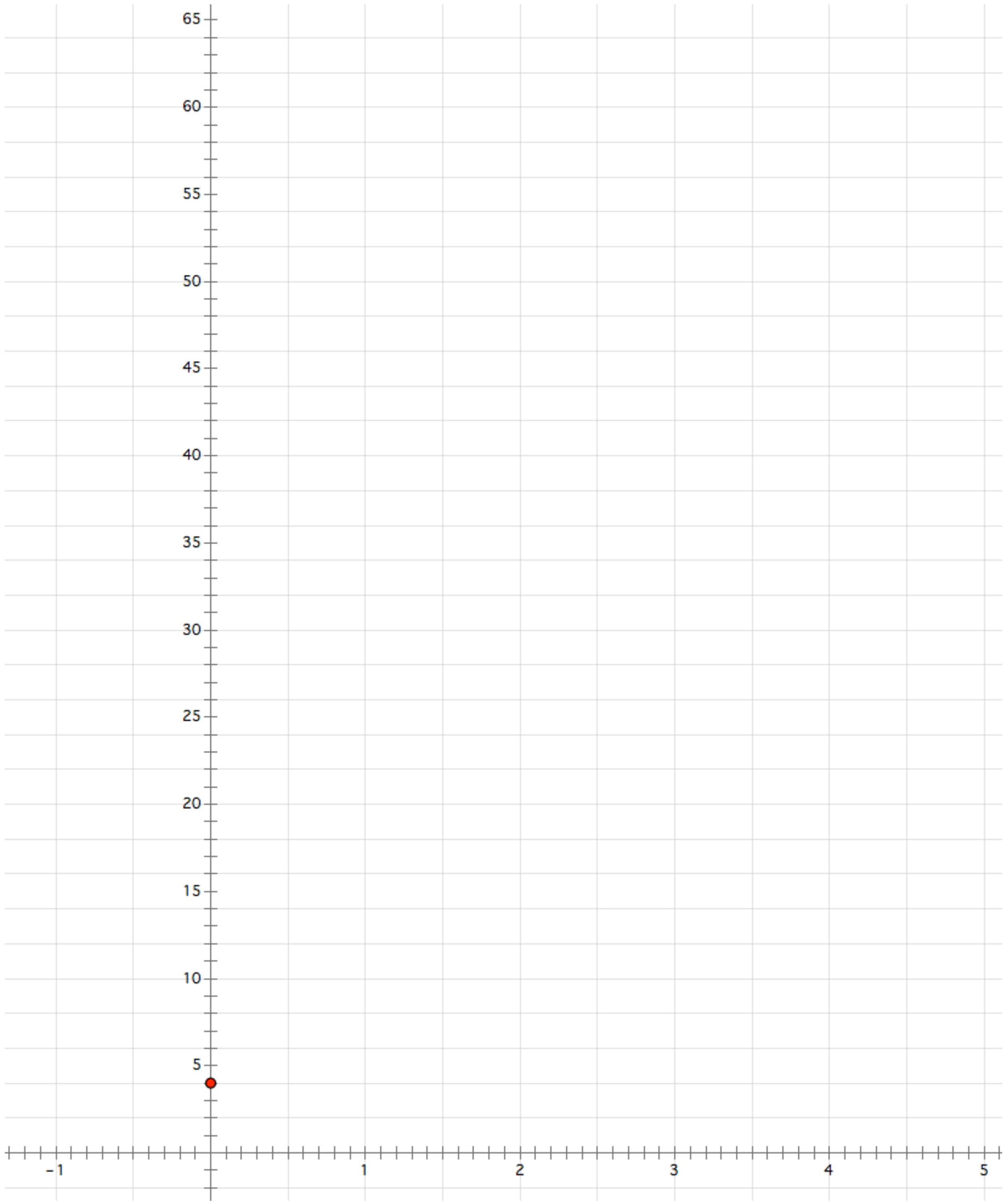
5. What if Miriam wanted to estimate the population every 20 minutes instead of every 30 minutes? What multiplier would she use for every third of an hour to be consistent with the population doubling every hour? Use this multiplier to complete the following table.

Hours into the study	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$	3
Bacteria population (in thousands)	4									

6. What number did you use as a multiplier to complete the table in problem 4?

7. What number did you use as a multiplier to complete the table in problem 5?

8. Give a detailed description of how you would estimate the population, *P*, at $t = \frac{5}{3}$ hours.


65
60
55
50
45
40
35
30
25
20
15
10
5
-1
1
2
3
4
5

Planning Document

Chance of decay:

The half life of carbon-10 is _______ seconds. That means after that many seconds, fifty percent of the atoms will still be carbon-10 and the other half will have decayed into boron.

What percent of atoms remain after one second?

What are the chances your bot will remain carbon each second?

Pseudo-Random numbers:

You will need to generate a random number each second to check if your radioactivebot has decayed or not. After you “import random” you have a choice of “decimal=random.random()” which returns 0 <= decimal < 1 or “integer=random.randint(a,b)” which returns a <= integer <= b.

Which will you use?

How will you use the number that is generated along with the percent of atoms that remain after each second to check to see if your bot has decayed or not?

Program considerations:

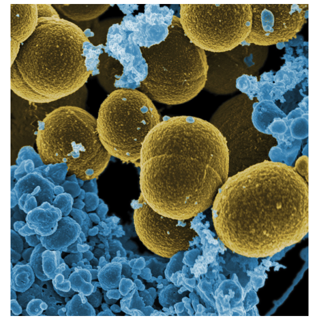
What could you add to your program so that it waits until the class is ready (and you hit enter)?

How will you know how many seconds have passed before it decays?

How will the bot show you it still is carbon? How will it show you it has decayed into boron?

How will your radioactivebot behave while it is still radioactive?

Warm-up: Sequences



Ready

Topic: Comparing additive and multiplicative patterns.

The sequences below exemplify either an additive (arithmetic) or a multiplicative (geometric) pattern. Identify the type of sequence, fill in the missing values on the table and write an equation.

1.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	2	4	8	16	32			

Type of Sequence: **Equation:**

2.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	66	50	34	18				

Type of Sequence: **Equation:**

3.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-3	9	-27	81				

Type of Sequence: **Equation:**

4.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	160	80	40	20				

Type of Sequence: **Equation:**

5.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-9	-2	5	12				

Type of Sequence: **Equation:**

Set

Topic: Evaluate the expressions with rational exponents.

Fill in the missing values of the table based on the growth that is described.

12.

The growth in the table is triple at each whole year.

Years	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
bacteria	2		6						

13.

The growth in the table is triple at each whole year.

Years	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$
bacteria	2			6					

14.

The values in the table grow by a factor of four at each whole year.

Years	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
bacteria	2		8						

Go

Topic: Simplifying exponents

Simplify the following expressions using exponent rules and relationships, write your answers in exponential form. (For example: $2^2 \cdot 2^5 = 2^7$)

15. $3^2 \cdot 3^5$

16. $\dfrac{5^3}{5^2}$

17. 2^{-5}

18. 17^0

19. $\dfrac{7^5}{7^2} \cdot \dfrac{7^3}{7^4}$

20. $\dfrac{3^{-2} \cdot 3^5}{3^7}$

NAME

Warm-up: Fill in Tables

Set

Topic: Finding arithmetic and geometric means and making meaning of rational exponents.

You may have found arithmetic and geometric means in your prior work. Finding arithmetic and geometric means requires finding values of a sequence between given values from non-consecutive terms. In each of the sequences below determine the means and show how you found them.

Find the *arithmetic* means for the following, show your work.

10.

x	1	2	3
y	5		11

11.

x	1	2	3	4	5
y	18				-10

12.

x	1	2	3	4	5	6	7
y	12						-6

Find the *geometric* means for the following, show your work.

13.

x	1	2	3
y	3		12

14.

x	1	2	3	4
y	7			875

15.

x	1	2	3	4	5	6
y	4					972

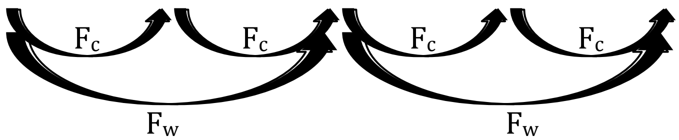
NAME

Warm-up: More Tables

16.
Fill in the table of values and find the factor used to move between whole number values, F_w, as well as the factor, F_c, used to move between each column of the table.

x	0	½	1	3⁄2	2
y	4 €		16€		

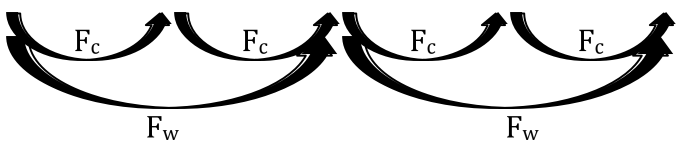
$F_w =$
$F_c =$



17.
Fill in the table of values and find the factor used to move between whole number values, F_w, as well as the factor, F_c, used to move between each column of the table.

x	0	½	1	3⁄2	2
y	4 €		8 €		

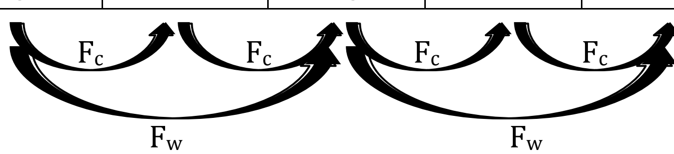
$F_w =$
$F_c =$



18.
Fill in the table of values and find the factor used to move between whole number values, F_w, as well as the factor, F_c, used to move between each column of the table.

x	0	½	1	3⁄2	2
y	5 €		15€		

$F_w =$
$F_c =$



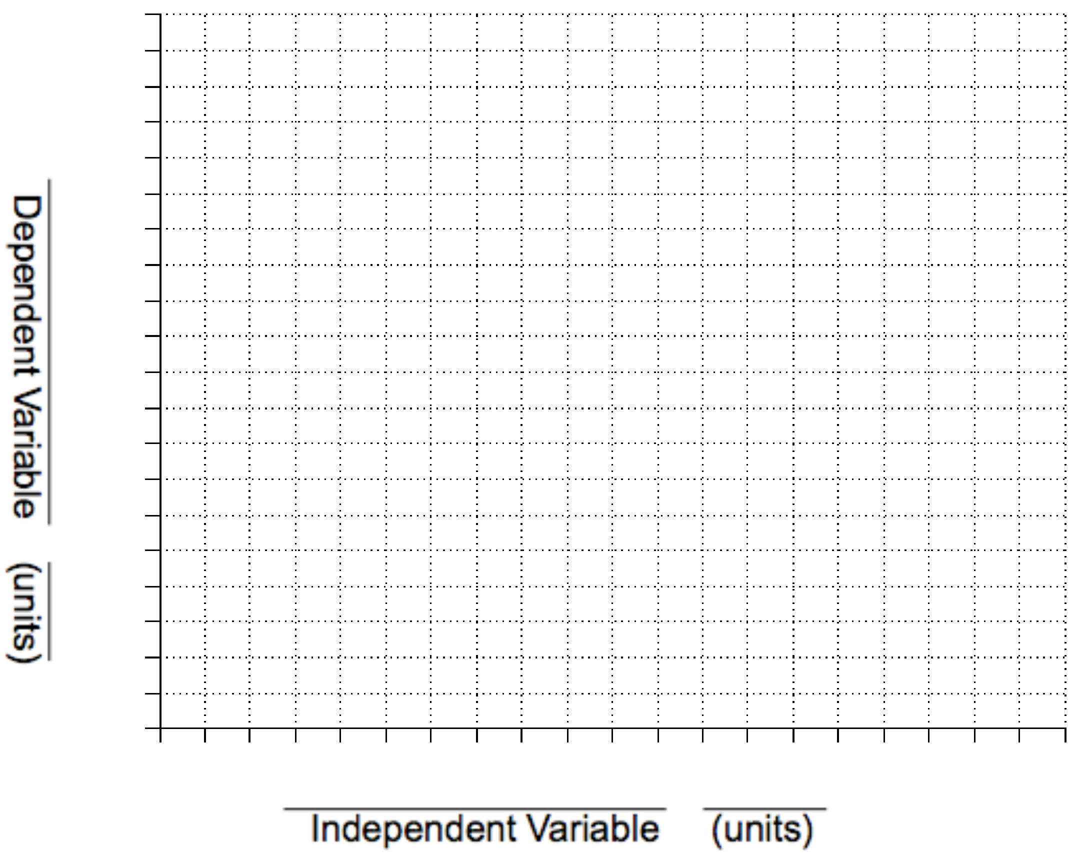
Radioactivebots Data

1. Data Collection: Collect data from your experiment. Be sure to include title, labels, and units.

Table 1: ___________________________________

Independent Variable:	**Dependent Variable:**			
	Trial #1 ( )	Trial #2 ( )	Trial #3 ( )	Average ( )

2. Graph: Make a graph of the data below. Be sure to include all 5 parts of a graph (<u>title, IV and DV, units, scales, and data</u>)



Adapted from Jordan Pasqualin's *Helicopter Lab* by Jim Town

3. Claim: What cause and effect relationship have you discovered?

As the _____________________ increases the ______________________ ________________________

(Independent Variable) (Dependent Variable) (Increases, Decreases, or Stays Constant)

4. Evidence: Explain what data you based your claim on.

5. Reasoning: Explain how your *evidence* proves your *claim*?

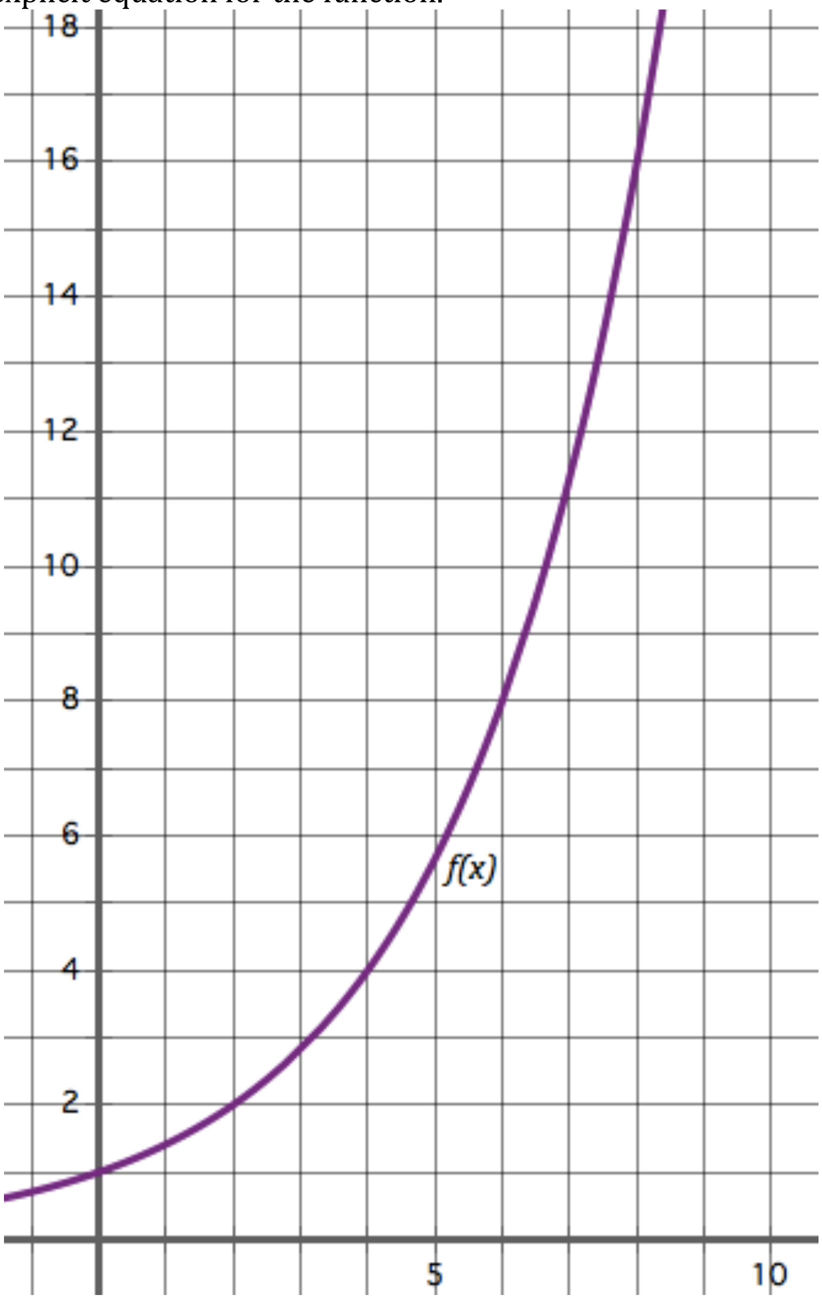
6. Extend: Find a function that fits your data.

Why does the type of function you chose fit the situation we are modeling?

What do the parameters in the function mean in the situation we are modeling? Are they the values you expected them to be? What might have caused differences in the expected value vs the measured value?

Warm-up: Function from Graph

Use the graph of the function to find the desired values of the function. Also create an explicit equation for the function.



6. Find the value of $f(2)$

7. Find where $f(x) = 4$

8. Find the value of $f(6)$

9. Find where $f(x) = 16$

10. What do you notice about the way that inputs and outputs for this function relate? (Create an in-out table if you need to.)

11. What is the explicit equation for this function?

Warm-up: Simplify Exponents

Set

Topic: Finding equivalent expressions and functions.

Determine whether the expressions or functions in each problem below are equivalent. Justify why or why not they are not equivalent.

7. $5(3^{x-1})$ $\quad 15(3^{x-2})$ $\quad \frac{3}{5}(3^x)$

8. $64\ (2^{-x})$ $\quad \frac{64}{2^x}$ $\quad 64\left(\frac{1}{2}\right)^x$

9. 3(x-1)+4 $\quad$ 3x - 1 $\quad$ 3(x-2) +7

10. $50(2^{x+2})$ $\quad 25(2^{2x+1})$ $\quad 50(4^x)$

11. $30(1.05^x)$ $\quad 30\left(1.05^{\frac{1}{7}}\right)^{7x}$ $\quad 30\left(1.05^{\frac{x}{2}}\right)^2$

12. $20\ (1.1^x)$ $\quad 20\ (1.1^{-1})^{-1x}$ $\quad 20\left(1.1^{\frac{1}{5}}\right)^{5x}$

NAME

Go

Topic: Using rules of exponents

Simplify each expression.

13. $7^3 \cdot 7^5 \cdot 7^2$

14. $(3^4)^5$

15. $(5^3)^4 \cdot 5^7$

16. $x^3 \cdot x^5$

17. x^{-b}

18. $x^a \cdot x^b$

19. $(x^a)^b$

20. $\dfrac{y^a}{y^b}$

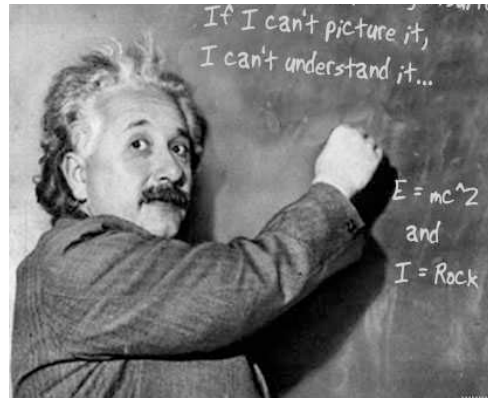
21. $\dfrac{(y^a)^c}{y^b}$

22. $\dfrac{\left(3^4\right)^6}{3^7} =$

23. $\dfrac{r^5 s^3}{rs^2} =$

24. $\dfrac{x^5 y^{12} z^0}{x^8 y^9} =$

3.4 Radical Ideas
A Practice Understanding Task



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Now that Tia and Tehani know that $a^{m/n} = \left(\sqrt[n]{a}\right)^m$ they are wondering which form, radical form or exponential form, is best to use when working with numerical and algebraic expressions.

Tia says she prefers radicals since she understands the following properties for radicals (and there are not too many properties to remember):

If *n* is a positive integer greater than 1 and both *a* and *b* are positive real numbers then,

1. $\sqrt[n]{a^n} = a$
2. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
3. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Tehania says she prefers exponents since she understands the following properties for exponents (and there are more properities to work with):

1. $a^m \cdot a^n = a^{m+n}$
2. $\left(a^m\right)^n = a^{mn}$
3. $(ab)^n = a^n \cdot b^n$
4. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
5. $\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$
6. $a^{-n} = \frac{1}{a^n}$

DO THIS: Illustrate with examples and explain, using the properties of radicals and exponents, why $a^{1/n} = \sqrt[n]{a}$ and $a^{m/n} = \left(\sqrt[n]{a}\right)^m$ are true identities.

Using their preferred notation, Tia might simplify $\sqrt[3]{x^8}$ as follows:

$$\sqrt[3]{x^8} = \sqrt[3]{x^3 \cdot x^3 \cdot x^2} = \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^2} = x \cdot x \cdot \sqrt[3]{x^2} = x^2 \cdot \sqrt[3]{x^2}$$

(Tehani points out that Tia also used some exponent rules in her work.)

On the other hand, Tehani might simplify $\sqrt[3]{x^8}$ as follows:

$$\sqrt[3]{x^8} = x^{8/3} = x^{2+2/3} = x^2 \cdot x^{2/3} \text{ or } x^2 \cdot \sqrt[3]{x^2}$$

For each of the following problems, simplify the expression in the ways you think Tia and Tehani might do it.

Original expression	**What Tia and Tehani might do to simplify the expression:**
$\sqrt{27}$	Tia's method
	Tehani's method
$\sqrt[3]{32}$	Tia's method
	Tehani's method
$\sqrt{20x^7}$	Tia's method
	Tehani's method
$\sqrt[3]{\dfrac{16xy^5}{x^7y^2}}$	Tia's method
	Tehani's method

The Twist

Well, it turns out it is actually carbon-11 which has a much longer half life so using the robots to simulate it is not feasible. Your new task is to update your program to run the simulation without the robots. You should run at least 10 trials with a sample size of at least 100 atoms.

Chance of decay:

The half life of carbon-11 is _______ seconds. That means after that many seconds, fifty percent of the atoms will still be carbon-11 and the other half will have decayed into boron.

What percent of atoms remain after one second?

What are the chances each atom will remain carbon each second?

Program considerations:

How will you keep track of time?

How will you keep track of the trials?

How will you show your results?

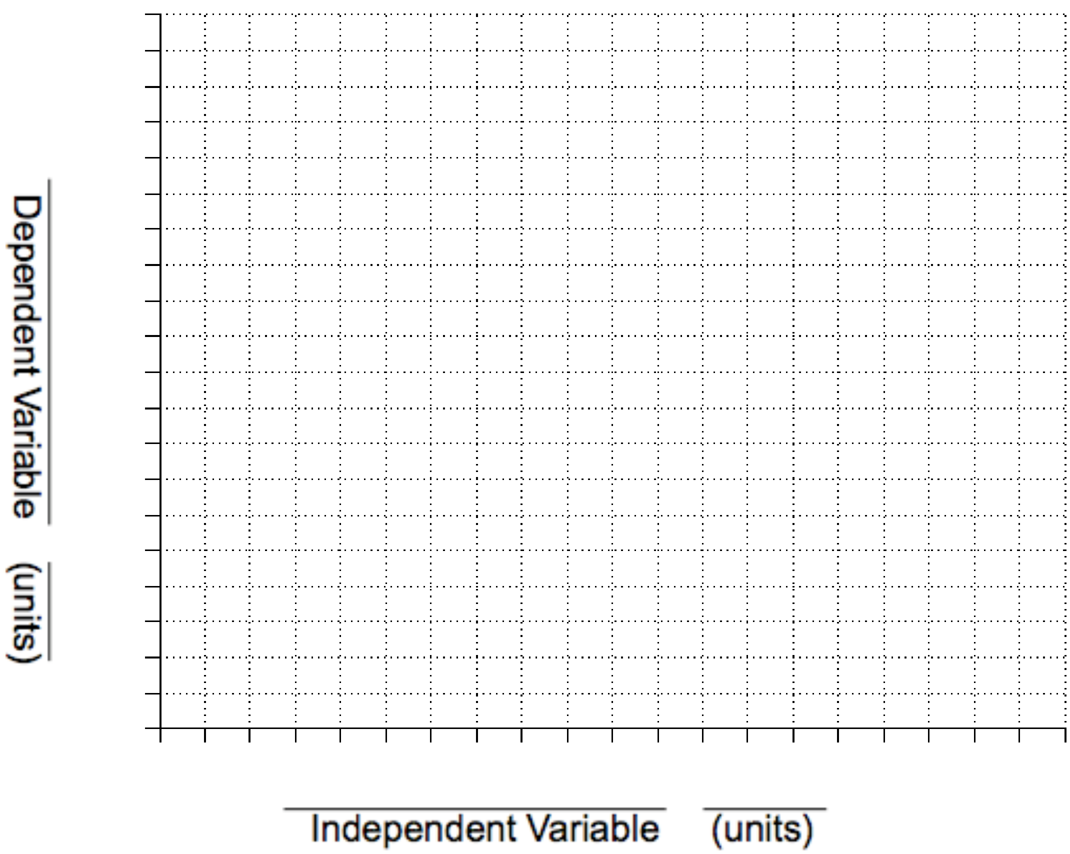
Radioactivebots Twist Data

1. Data Collection: Collect data from your experiment. Be sure to include title, labels, and units.

Table 1: ____________________________________

Independent Variable:	**Dependent Variable:**			
	Trial #1 ( )	Trial #2 ( )	Trial #3 ( )	Average ( )

2. Graph: Make a graph of the data below. Be sure to include all 5 parts of a graph (<u>title, IV and DV, units, scales, and data</u>)



3. Claim: What cause and effect relationship have you discovered?

As the ____________________ increases the ____________________ ____________________
(Independent Variable) (Dependent Variable) (Increases, Decreases, or Stays Constant)

4. Evidence: Explain what data you based your claim on.

5. Reasoning: Explain how your *evidence* proves your *claim*?

6. Extend: Find a function that fits your data.

Why does the type of function you chose fit the situation we are modeling?

What do the parameters in the function mean in the situation we are modeling? Are they the values you expected them to be? What might have caused differences in the expected value vs the measured value?

Exponents Test

Name ____ Date ____________________

1. Use properties of exponents to explain why it makes sense to define $16^{\frac{1}{4}}$ as $\sqrt[4]{16}$.

2. Use properties of exponents to rewrite each expression as either an integer or as a quotient of integers $\frac{p}{q}$ to show the expression is a rational number.

   a. $\sqrt[4]{2}\,\sqrt[4]{8}$

   b. $\dfrac{\sqrt[3]{54}}{\sqrt[3]{2}}$

c. $16^{\frac{3}{2}} \cdot \left(\frac{1}{27}\right)^{\frac{2}{3}}$

5. A scientist is studying the growth of a population of bacteria. At the beginning of her study, she has 800 bacteria. She notices that the population is quadrupling every hour.

   a. What quantities, including units, need to be identified to further investigate the growth of this bacteria population?

   b. The scientist recorded the following information in her notebook, but she forgot to label each row. Label each row to show what quantities, including appropriate units, are represented by the numbers in the table, and then complete the table.

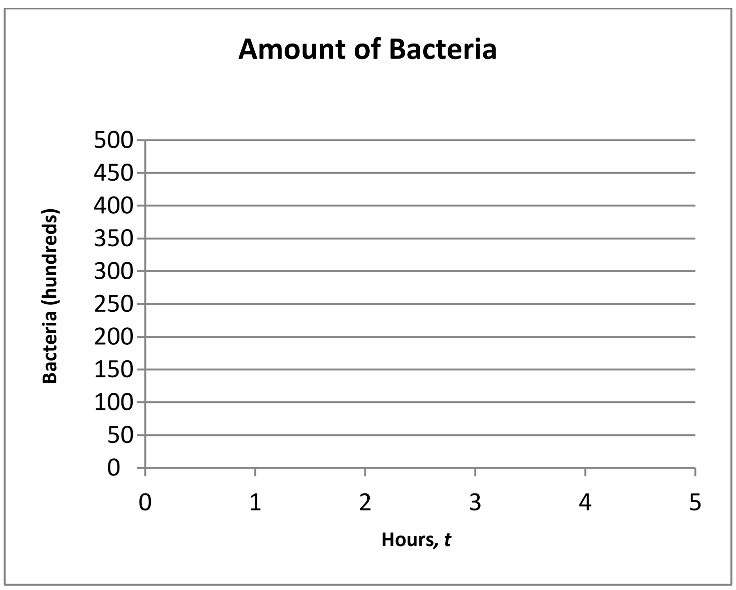
	0	1	2	3	4
	8	32	128		

c. Write an explicit formula for the number of bacteria present after t hours.

d. Another scientist studying the same population notices that the population is doubling every half an hour. Complete the table, and write an explicit formula for the number of bacteria present after x half hours.

Time, t (hours)	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
Time, x (half hours)	0	1	2	3	4	5	6
Bacteria (hundreds)	8	16	32				

e.



f. A scientist calculated the average rate of change for the bacteria in the first three hours to be 168. Which units should the scientist use when reporting this number? Explain how you know.

Find the time, in hours, when there will be $5{,}120{,}000$ bacteria. Express your answer as a logarithmic expression.