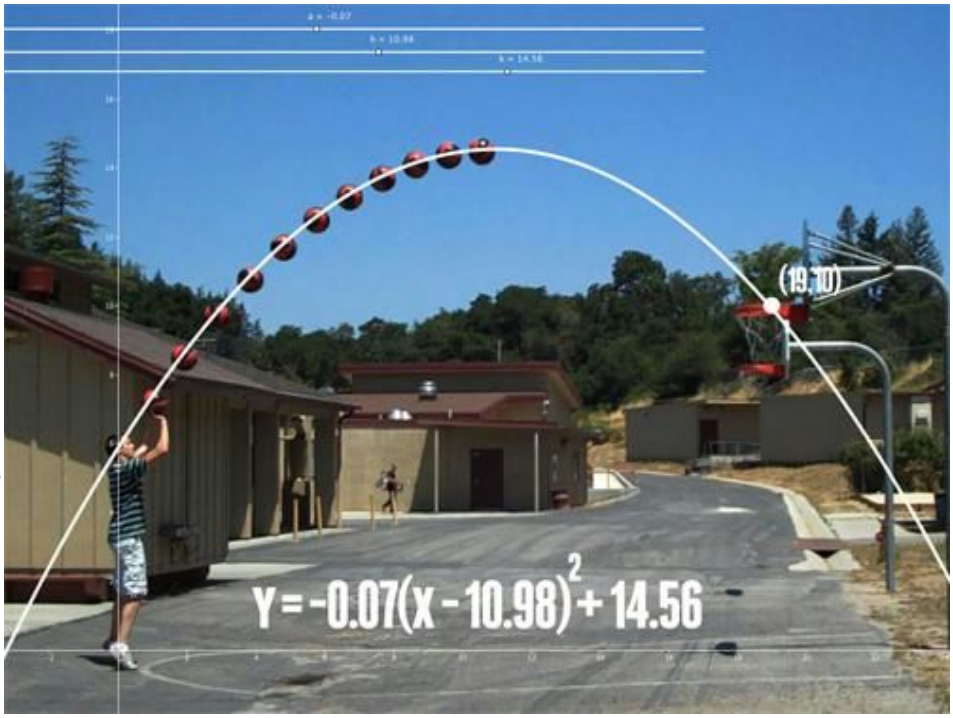
Quadratic Calculator Calendar (90 min block)

Week	Mon	Tues	Wed	Thur	Fri	
1	- Group intro activity - Group contracts - Entry Doc - **K/NTK**/NS	- Do Now: Graphing - K/NTK/**NS** - **py** - Graphing with code - graphing.py - Graphing Exploration - Debrief	- Do Now: Find the zeros - Introduce quadratic formula - Quadratic Formula Worksheet	- Add quadratic formula solver to program - Check your answers from yesterday's worksheet	- Do Now: How many solutions? - Add discriminant solver to program	
2	- Do Now: Axis of Symmetry - Completing the Square Exploration (part 1)	- Do Now: Vertex (pg 2) - Finish Completing the Square Exploration (part 2)	- Use the patterns you noticed about the vertex and the axis of symmetry to add these to your program	- Group Problem Solving Quiz (may use program)	- Try out another group's program - Critical friends feedback about functionality and comments	---------- Possible student solution - py code (w/ functions) - ch code (w/o functions) ----------
3	Review	Test				

Quadratic Calculator:

Dear Students,

Quadratics are all around us, from the path of a basketball, to the area of a garden, to the net centripetal force of a satellite orbiting the earth. Unfortunately, many students have trouble with them. Wouldn't it be nice if there were a Quadratic Calculator where you could just enter the coefficients of the quadratic and get the graph of the quadratic between -10 and 10, zeros, value of the discriminant (and what it means), the vertex, and the equation of the axis of symmetry? You can use the power of coding to make this dream a reality!



Sincerely,
Your Teacher

Know	Need to Know	Next Steps

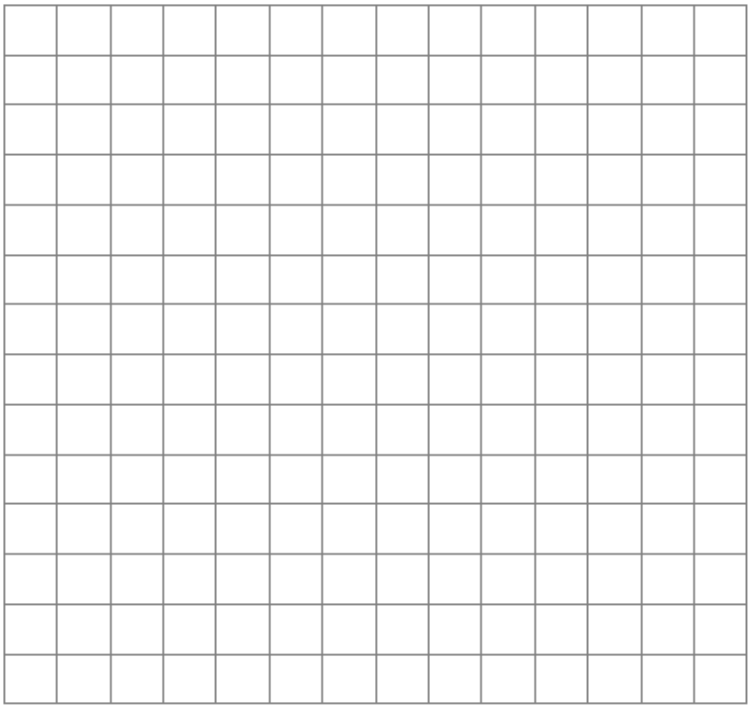
Photo credit: Dan Meyer from http://blog.mrmeyer.com/2010/wcydwt-will-it-hit-the-hoop/

Name:_____ Date:_____

Graph the equation: $y=2x^2+3x-4$

Table:

Graph:



1. What do you notice about the graph?

2. How did you graph it? (First I did..., then I did..., etc)

Remember how you did the graph of the parabola for the warm-up?

Graphing with code is similar, this command:

```
xList=numpy.linspace(-10,10,100)
```

makes a list of 100 evenly spaced x values between -10 and 10.

Then we calculate the y values for each x value:

```
yList=a*xList**2+b*xList+c
```

Finally, this block of code puts the points we just created on a graph:

```
pyplot.plot(xList,yList)      #plots all points
pyplot.xlabel('x value')      #labels x axis
pyplot.ylabel('y value')      #labels y axis
pyplot.title('Parabola')      #labels graph
pyplot.grid(True)             #shows grid
pyplot.show()                       #tells computer to show the graph on the screen
```

What does the graph look like if you only do 10 evenly spaced points between -10 and 10?

What would you change if you wanted to graph between 0 and 1 instead of -10 to 10?

What would you change if you wanted to graph a straight line instead of a quadratic?

Algebra I Name _____

Date _____ Period ___

Graphing Exploration for Quadratics

Standard form of a Quadratic Equation $y = ax^2 + bx + c$	Standard form of a Quadratic Function $f(x) = ax^2 + bx + c$

1. To graph the function $f(x) = x^2 + x + 1$ you need to enter a=1, b=1, and c=1
   What does the graph look like?

2. Before changing the values, try to predict what will happen (Hint: try evaluating the value of the function for the same value of x but slightly changing the value of the number that is represented by the slider bar.

   a) What happens when you change a to a positive number, 0, a negative number?
   Try the following values $a = 2$, $a = 0$, $a = -2$ (don't forget b=1 and c=1 still).

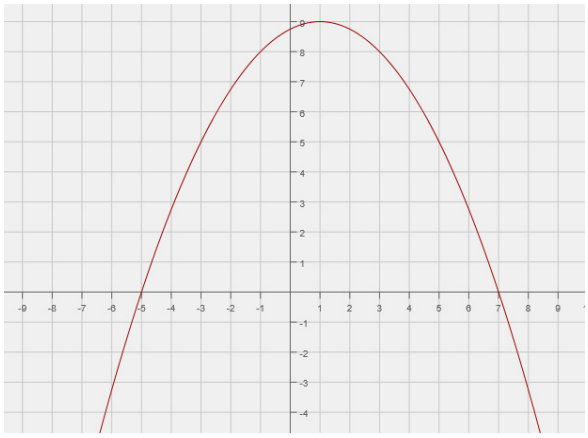
   b) What happens when you change b to a positive number, 0, a negative number?
   Try the following values $b = 2$, $b = 0$, $b = -2$ (don't forget a=1 and c=1 still).

   c) What happens when you change c to a positive number, 0, a negative number?
   Try the following values $c = 2$, $c = 0$, $c = -2$ (don't forget a=1 and b=1 still).

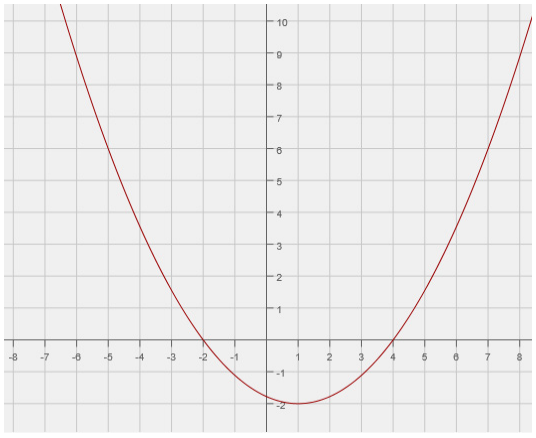
Warm-up: Find the Zeros

1. What are the x-intercepts of the four graphs below?

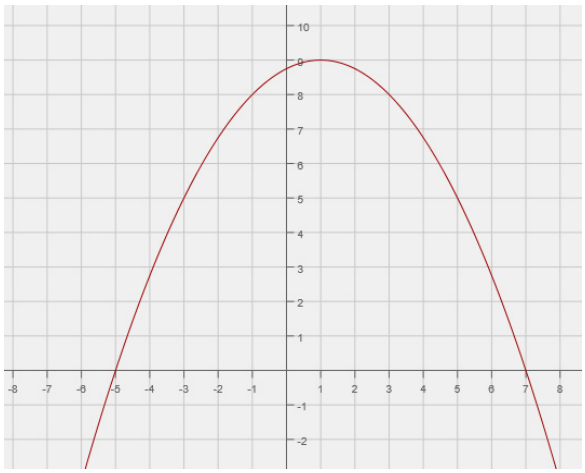
Graph A



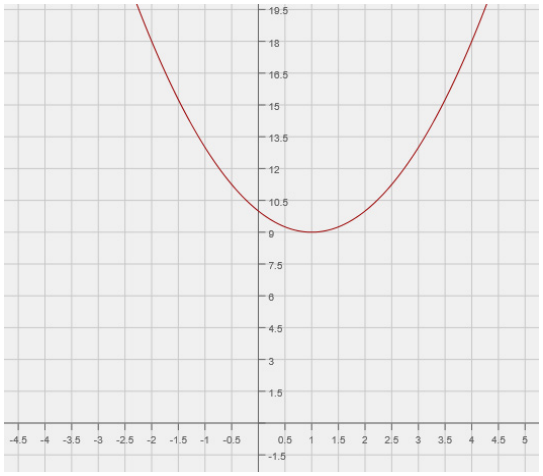
Graph B



Graph C



Graph D



a.

b.

c.

d.

Workshop: Quadratic Formula

Lesson Summary

The quadratic formula, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, is derived by completing the square on the general form of a quadratic equation: $ax^2 + bx + c = 0$, where $a \neq 0$. The formula can be used to solve any quadratic equation, and is especially useful for those that are not easily solved using any other method (i.e., factoring or completing the square).

Problem Set

Use the quadratic formula to solve each equation.

1. Solve for z: $z^2 - 3z - 8 = 0$.

2. Solve for q: $2q^2 - 8 = 3q$

3. Solve for m: $\frac{1}{3}m^2 + 2m + 8 = 5$.

Worksheet: Quadratic Formula

Lesson 15: Using the Quadratic Formula

Classwork

Opening Exercise

Solve:

1. $4x^2 + 5x + 3 = 2x^2 - 3x$

2. $c^2 - 14 = 5c$

Exercises 1–5

Solve the following equations using the quadratic formula.

1. $x^2 - 2x + 1 = 0$

2. $3b^2 + 4b + 8 = 0$

3. $2t^2 + 7t - 4 = 0$

4. $q^2 - 2q - 1 = 0$

5. $m^2 - 4 = 3$

Warm-up: How Many Solutions?

Lesson Summary

You can use the sign of the discriminant, $b^2 - 4ac$, to determine the number of real solutions to a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$. If the equation has a positive discriminant, there are two real solutions. A negative discriminant yields no real solutions and a discriminant equal to zero yields only one real solution.

Problem Set

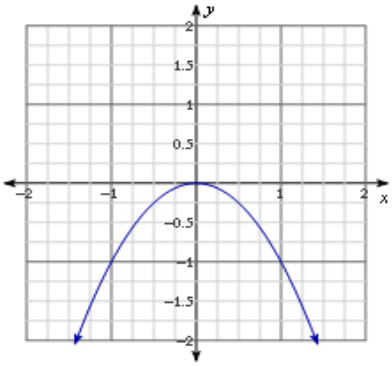
Without solving, determine the number of real solutions for each quadratic equation.

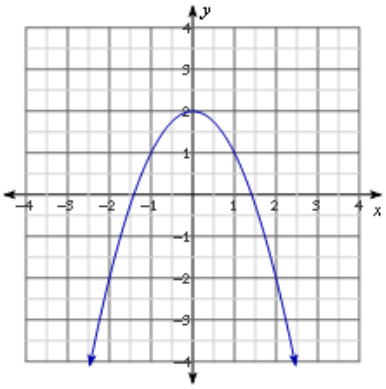
1. $b^2 - 4b + 3 = 0$

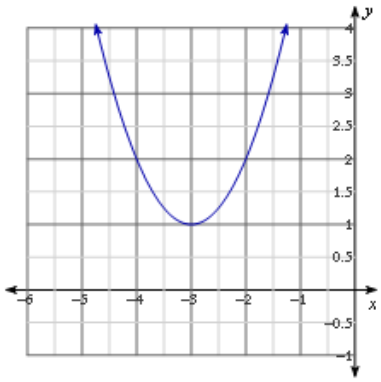
2. $2n^2 + 7 = -4n + 5$

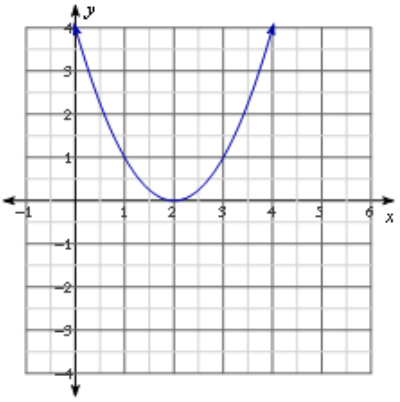
3. $x - 3x^2 = 5 + 2x - x^2$

4. $4q + 7 = q^2 - 5q + 1$

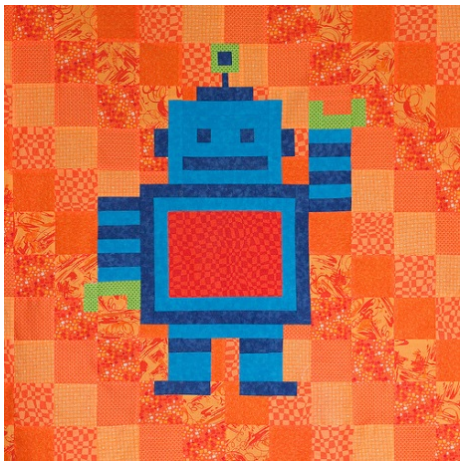
5. 

7. 

6. 

7. 

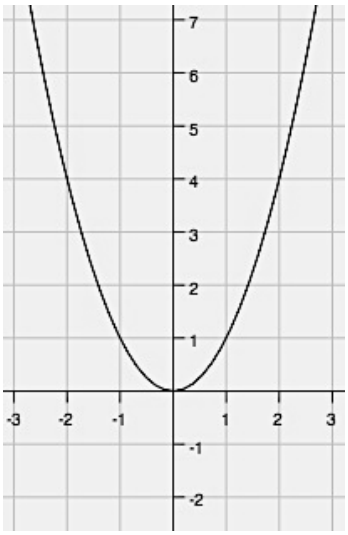
Warm-up: Axis of Symmetry



Ready Topic: Finding key features in the graph of a quadratic equation

Make a point on the **vertex** and draw a dotted line for the **axis of symmetry**.
Label the coordinates of the vertex and state whether it's a **maximum** or a **minimum.**
Write the **equation for the axis of symmetry**.

1.



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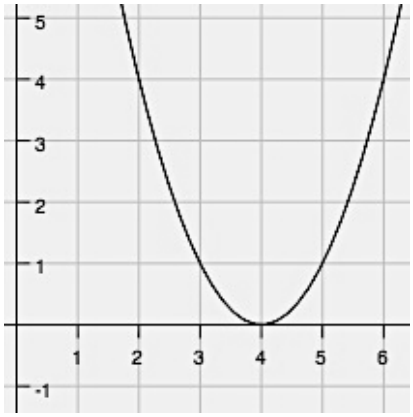

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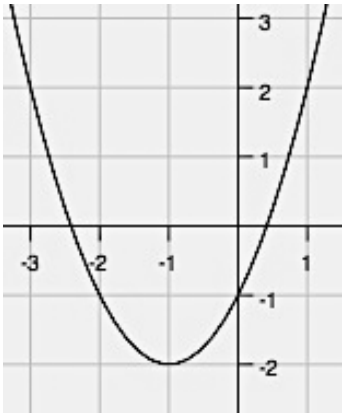

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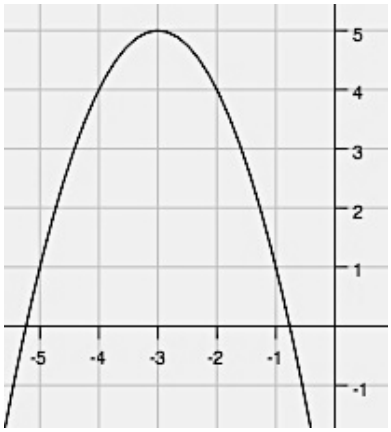

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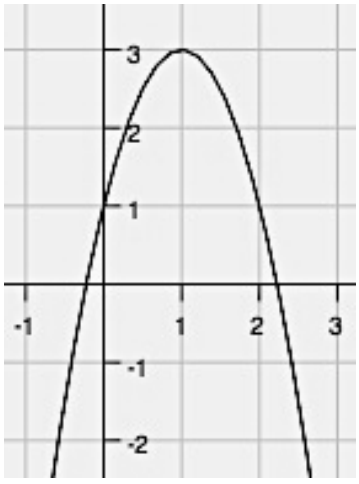

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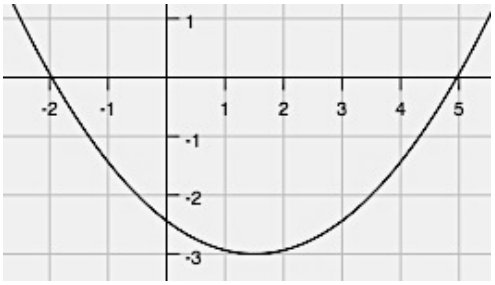
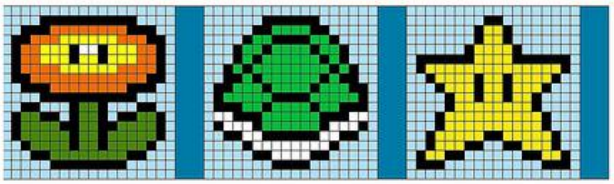


2.3 Building the Perfect Square

A Solidify Understanding Task

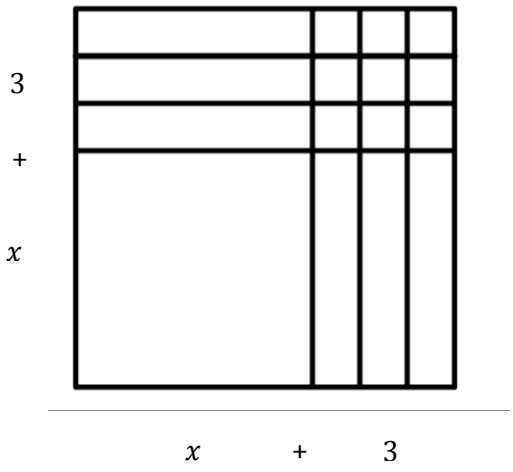


Part 1: Quadratic Quilts

Optima has a quilt shop where she sells many colorful quilt blocks for people who want to make their own quilts. She has quilt designs that are made so that the can be sized to fit any bed. She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square x, and the area of the basic square is the function $A(x) = x^2$. In this way, she can customize the designs by making bigger squares or smaller squares.

1. If Optima adds 3 inches to the side of the square, what is the area of the square?

When Optima draws a pattern for the square in problem #1, it looks like this:



2. Use both the diagram and the equation, $A(x) = (x + 3)^2$ to explain why the area of the quilt block square, $A(x)$, is also equal to the $x^2 + 6x + 9$.

The customer service representatives at Optima's shop work with customer orders and write up the orders based on the area of the fabric needed for the order. As you can see from problem #2 there are two ways that customers can call in and describe the area of the quilt block. One way describes the length of the sides of the block and the other way describes the areas of each of the four sections of the block.

For each of the following quilt blocks, draw the diagram of the block and write two equivalent equations for the area of the block.

3. Block with side length: $x + 2$.

4. Block with side length: $x + 1$.

5. What patterns do you notice when you relate the diagrams to the two expressions for the area?

6. Optima likes to have her little dog, Clementine, around the shop. One day the dog got a little hungry and started to chew up the orders. When Optima found the orders, one of them was so chewed up that there were only partial expressions for the area remaining. Help Optima by completing each of the following expressions for the area so that they describe a perfect square. Then, write the two equivalent equations for the area of the square.

   a. $x^2 + 4x$

   b. $x^2 + 6x$

   c. $x^2 + 8x$

   d. $x^2 + 12x$

7. If $x^2 + bx + c$ is a perfect square, what is the relationship between b and c? How do you use b to find c, like in problem 6?

   Will this strategy work if b is negative? Why or why not?

   Will the strategy work if b is an odd number? What happens to c if b is odd?

Sometimes a customer orders more than one quilt block of a given size. For instance, when a customer orders 4 blocks of the basic size, the customer service representatives write up an order for $A(x) = 4x^2$.

8. What would they write if the order was for 2 blocks that are 1 inch longer than the basic block? Write the area function in two equivalent forms. Verify your algebra using a diagram.

Part 2: Quilts and Quadratic Graphs

Optima's niece, Jenny works in the shop, taking orders and drawing quilt diagrams. When the shop isn't too busy, Jenny pulls out her math homework and works on it. One day, she is working on graphing parabolas and notices that the equations she is working with look a lot like an order for a quilt block. For instance, Jenny is supposed to graph the equation: $y = (x - 3)^2 + 4$. She thinks, "That's funny. This would be an order where the length of the standard square is reduced by 3 and then we add a little piece of fabric that has as area of 4. We don't usually get orders like that, but it still makes sense. I better get back to thinking about parabolas. Hmmm…"

9. Fully describe the parabola that Jenny has been assigned to graph.

10. Jenny returns to her homework, which is about graphing quadratic functions. Much to her dismay, she finds that she has been given: $y = x^2 - 6x + 9$. "Oh dear", thinks Jenny. "I can't tell where the vertex is or any of the transformations of the parabola in this form. Now what am I supposed to do?"

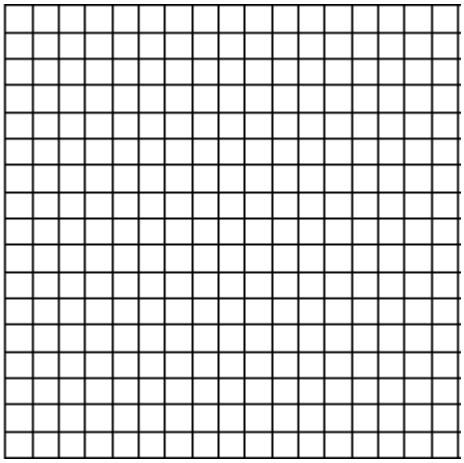
    "Wait a minute—is this the area of a perfect square?" Use your work from Part 1of this task to answer Jenny's question and justify your answer.

11. Jenny says, "I think I've figured out how to change the form of my quadratic equation so that I can graph the parabola. I'll check to see if I can make my equation a perfect square." Jenny's equation is: $y = x^2 - 6x + 9$. Change the form of the equation, find the vertex, and graph the parabola.

    a. $y = x^2 - 6x + 9$ New form of the equation: _____

    b. Vertex of the parabola: _____

    c. Graph (with at least 3 accurate points on each side of the line of symmetry):

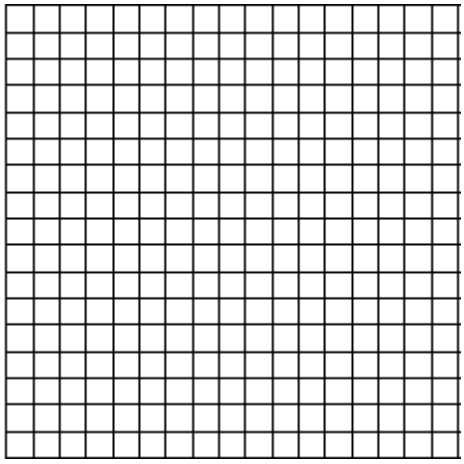


12. The next quadratic to graph on Jenny's homework is $y = x^2 + 4x + 2$. Does this expression fit the pattern for a perfect square? Why or why not?

a. Use an area model to figure out how to complete the square so that the equation can be written in vertex form, $y = a(x - h)^2 + k$.

b. Is the equation you have written equivalent to the original equation? If not, what adjustments need to be made? Why?

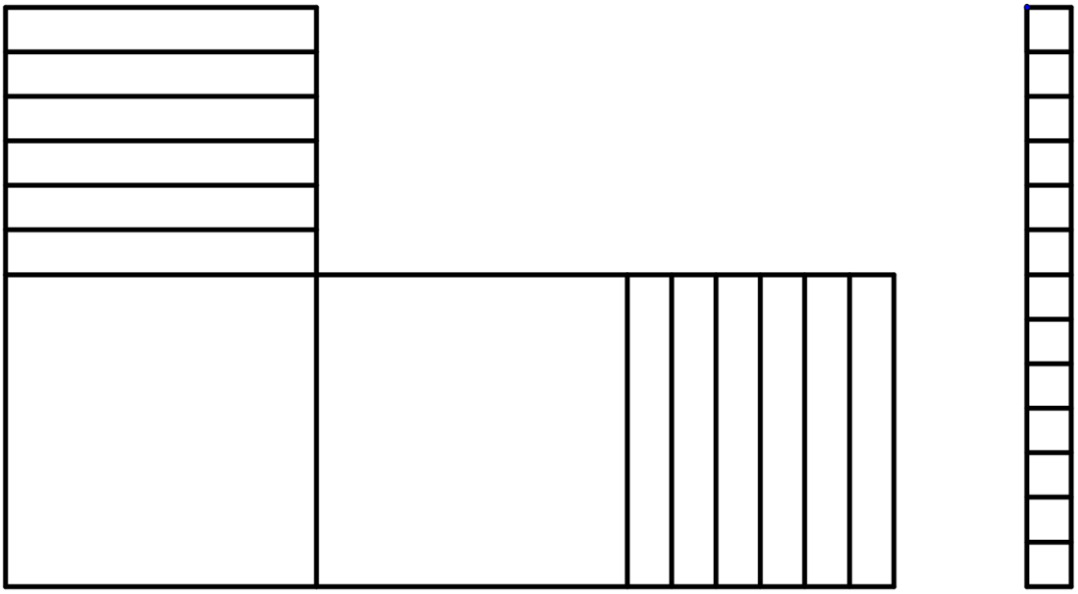
c. Identify the vertex and graph the parabola with three accurate points on both sides of the line of symmetry.



13. Jenny hoped that she wasn't going to need to figure out how to complete the square on an equation where b is an odd number. Of course, that was the next problem. Help Jenny to find the vertex of the parabola for this quadratic function:

$g(x) = x^2 + 7x + 10$

14. Jenny's last quadratic function to graph is $f(x) = 2x^2 + 12x + 13$. She draws the following diagram and says, "I'm not sure how this helps me. I don't see how to make this a square." Help Jenny to complete the square and find the vertex of the parabola using either the diagram or the equation.



Warm-up: Vertex (Qs 7-8)

Ready Topic: Finding key features in the graph of a quadratic equation

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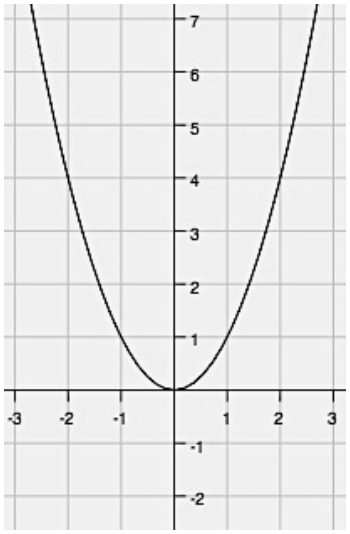

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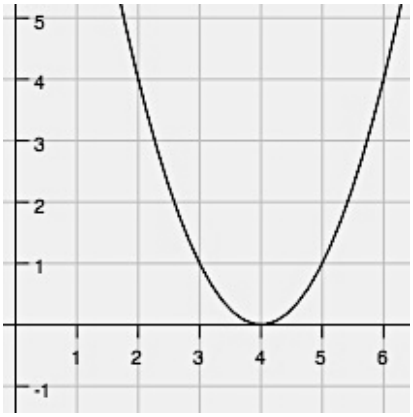

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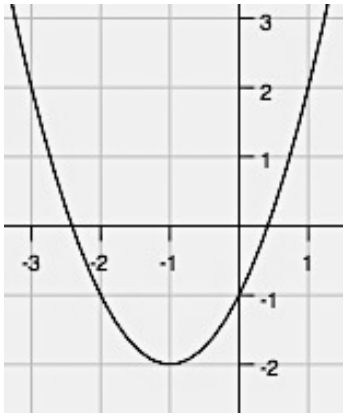

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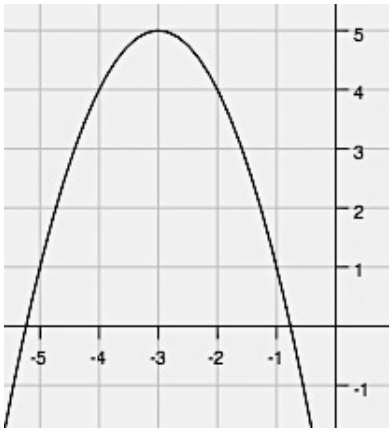

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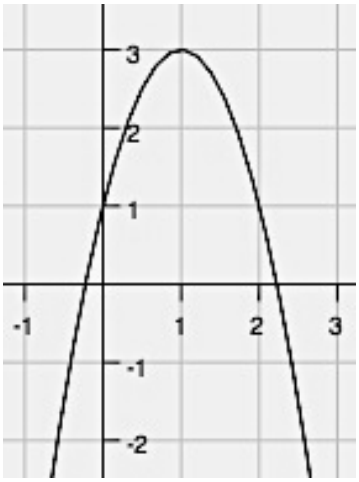

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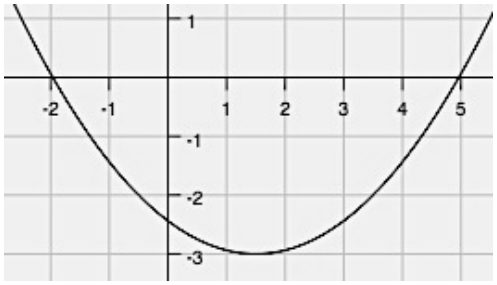


7. What connection exists between the coordinates of the vertex and the equation of the axis of symmetry?

8. Look back at #6. Try to find a way to find the ***exact*** value of the coordinates of the vertex. Test your method with each vertex in 1 - 5. Explain your conjecture.

Will it Hit the Hoop?

Dan Meyer, 3-Act Task
Group Quiz

Act 1:

1. **<u>On your own</u>**:

a.

Is he going to make it? Can you draw me the path of a shot that will make it? That will miss it?

b.

How about now? Can you draw me the path of a shot that will make it? That will miss it?

c.

How about now? Can you draw me the path of a shot that will make it? That will miss it?

2. **Work with a Partner or Team**:
   Watch the video. Which shots will go in the hoop? If your answer is "no", predict why the ball won't go in. Will the ball be short, long, or something else entirely?
   A.

   B.

   C.

   D.

   E.

   F.

   G.

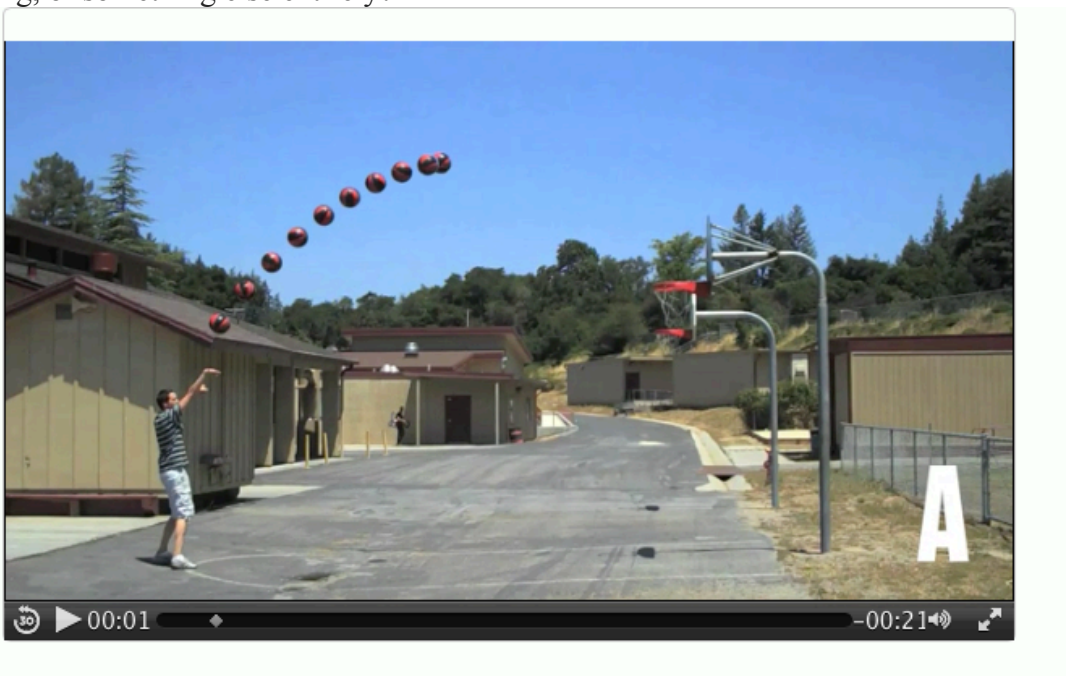
3. **Discuss**: What information/action would help you be surer of your answer?

Act 2: Analyzing the Data and Mathematical Modeling

4. **Frozen Strobes**: Which shots A. – G. will go in the hoop?

For each of the following still photographs, create a quadratic function model for each situation and use it to help you answer each question.

A. **Yes or No?** If your answer is "no", predict why the ball won't go in. Will the ball be short, long, or something else entirely?



B. **Yes or No?** If your answer is "no", predict why the ball won't go in. Will the ball be short, long, or something else entirely?

C. **Yes or No?** If your answer is “no”, predict why the ball won't go in. Will the ball be short, long, or something else entirely?

D. **Yes or No?** If your answer is “no”, predict why the ball won't go in. Will the ball be short, long, or something else entirely?

E. **Yes or No?** If your answer is “no”, predict why the ball won't go in. Will the ball be short, long, or something else entirely?

F. **Yes or No?** If your answer is “no”, predict why the ball won't go in. Will the ball be short, long, or something else entirely?

G. **Yes or No?** If your answer is “no”, predict why the ball won't go in. Will the ball be short, long, or something else entirely?

Act 3: **Conclusion**

Once everyone submits their quiz we'll watch the “answers” to shots A. – G.

Which ones were you right about?

For the ones you got wrong, how did you get them wrong?

CRITICAL FRIENDS PEER REVIEW

This peer evaluation activity can be used as either a midway feedback opportunity for longer projects or as a final assessment for shorter projects. The process forces students to practice their listening skills and provides a safe means for peer evaluation. Each phase can take from 5-10 minutes so plan accordingly. Form teams of 3 or 4 groups who will present to each other.

PHASE ONE: Presentation

Presenting Group: Describe their product, standards and phases of their project.

Critical Friends: Friends remain silent. They are not allowed to ask clarifying or follow up questions. They should be taking notes to evaluate the product as it is presented.

PHASE TWO: Critique

Presenting Group: Presenter(s) remain silent and are not allowed to respond to the comments of the "Friends."

Critical Friends: Friends talk amongst themselves about the project as if the presenters were not in the room and use the phrases below to start each topic. Start by focusing on the strengths, then on suggestions for improvement, and lastly, ideas for "next steps."

I like the fact that...

I wonder if...

A next step might be...

PHASE THREE: Response

Open discussion period for presenter(s) to respond to the comments of the "Friends" and to follow up on ideas or suggestions.

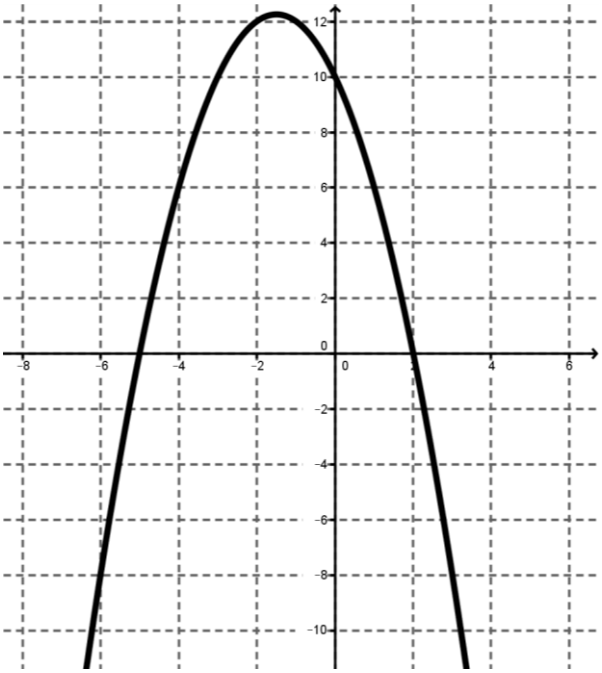
Quadratics Test

For each problem below, you are given a piece of information that tells you a lot. Use what you know about that information to fill in the rest.

1.	You get this:	Fill in this:
	$y = x^2 - x - 12$	Vertex form on the equation: Graph of the equation:

2.	You get this:	Fill in this:
	$y = x^2 - 6x + 3$	Vertex form of the equation:
		Graph of the equation:

3.	You get this:	Fill in this:
		Vertex form of the equation:
		Standard form of the equation:

4.	You get this:	Fill in this:
		Vertex form of the equation:
		Standard form of the equation:

5.	You get this:	Fill in this:
	$y = -x^2 - 6x + 16$	Either form of the equation other than standard form.
		Vertex of the parabola
		x-intercepts and *y*-intercept

6.	You get this:	Fill in this:
	$y = 2x^2 + 12x + 13$	Either form of the equation other than standard form.
		Vertex of the parabola
		x-intercepts and *y*-intercept

7.	You get this:	Fill in this:
	$y = -2x^2 + 14x + 60$	Either form of the equation other than standard form.
		Vertex of the parabola
		x-intercepts and *y*-intercept