Definitions

In[1]:=
```
(*Definition of Kij;
note that β (lapse function) is assumed to be a function of x and t*)
CurvatureComponent[α_, β_, i_, j_, γij_, γxj_, γix_] :=
 (1 / (2 α)) ((-D[γij, t]) + ((β) D[γij, x]) + ((γxj) D[β, x]) + ((γix) D[β, j]))
```

Constraints & Equations of Motion

In[2]:=
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(*Kϕ Transformation*)
Kϕ[t, x] = ArcSin[λ * s[t, x]] / λ
Kϕ^(0,1)[t, x] = D[Kϕ[t, x], x]
```

Out[2]=
$$\frac{\text{ArcSin}[\lambda\, s[t, x]]}{\lambda}$$

Out[3]=
$$\frac{s^{(0,1)}[t, x]}{\sqrt{1-\lambda^2\, s[t, x]^2}}$$

In[4]:=
```
(*Setting up necessary definitions from Spanish paper*)
$Assumptions = x > 0


(*Hamiltonian Constraint*) (*confirmed λ→0 limit matches classical case*)
Hamiltonian = (((-Eϕ[t, x]) / (2 Sqrt[Ex[t, x]] Sqrt[1 + λ^2])) (1 + (Sin[λ * Kϕ[t, x]])^2 / λ^2)) -
 ((Sqrt[Ex[t, x]]) (Kx[t, x]) Sin[2 λ * Kϕ[t, x]] / (λ Sqrt[1 + λ^2])) (1 + ((λ * D[Ex[t, x], x]) / (2 Eϕ[t, x]))^2)) +
 ((Cos[λ * Kϕ[t, x]])^2 / (2 Sqrt[1 + λ^2]))
  (((D[Ex[t, x], x]) / (2 Eϕ[t, x])) (D[Sqrt[Ex[t, x]], x])) + (Sqrt[Ex[t, x]] * D[(Ex^(0,1)[t, x]) / Eϕ[t, x], x])


(*Diffeomorphism Constraint*) (*confirmed λ→0 limit matches classical case*)
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$$\text{Diffeo} = (-Kx[t,x] * D[Ex[t,x],x]) + \left(E\phi[t,x] * K\phi^{(0,1)}[t,x]\right)$$

(*E^x dot*) (*confirmed $\lambda\to 0$ limit matches classical case*)

$$Ex^{(1,0)}[t,x] = (\beta[t,x] * D[Ex[t,x],x]) + \left(\alpha[t,x] * \sqrt{Ex[t,x]} * \frac{\text{Sin}[2\lambda * K\phi[t,x]]}{\lambda\sqrt{1+\lambda^2}} * \left(1 + \left(\frac{\lambda * D[Ex[t,x],x]}{2\,E\phi[t,x]}\right)^2\right)\right)$$

(*E^ϕ dot*) (*confirmed $\lambda\to 0$ limit matches classical case*)

$$E\phi^{(1,0)}[t,x] = (D[\beta[t,x] * E\phi[t,x],x]) + \left(2\,\alpha[t,x] * \sqrt{Ex[t,x]} * Kx[t,x] * \frac{\text{Cos}[2\lambda * K\phi[t,x]]}{\sqrt{1+\lambda^2}} \left(1 + \left(\frac{\lambda * D[Ex[t,x],x]}{2\,E\phi[t,x]}\right)^2\right)\right) + \left(\alpha[t,x]\left(\frac{\text{Sin}[2\lambda * K\phi[t,x]]}{\lambda\sqrt{1+\lambda^2}}\right)\right)\left(\frac{E\phi[t,x]}{2\sqrt{Ex[t,x]}} + \frac{\lambda^2}{2}\left(\left(\frac{D[Ex[t,x],x]}{2\,E\phi[t,x]}\left(D\left[\sqrt{Ex[t,x]},x\right]\right)\right) + \left(\sqrt{Ex[t,x]} * D\left[\left(\frac{Ex^{(0,1)}[t,x]}{E\phi[t,x]}\right),x\right]\right)\right)\right)$$

(*K_x dot*) (*confirmed $\lambda\to 0$ limit matches classical case*)

$$Kx^{(1,0)}[t,x] = (D[\beta[t,x] * Kx[t,x],x]) + \left(\alpha^{(0,2)}[t,x] * \frac{\sqrt{Ex[t,x]}\,(\text{Cos}[\lambda * K\phi[t,x]])^2}{2\,E\phi[t,x]\sqrt{1+\lambda^2}}\right) + \left(\left(\frac{\alpha^{(0,1)}[t,x] * \sqrt{Ex[t,x]}}{2\sqrt{1+\lambda^2}\left(E\phi[t,x]^2\right)}\right)\left(\left(\lambda * \text{Sin}[2\lambda * K\phi[t,x]]\left(\left(Kx[t,x] * Ex^{(0,1)}[t,x]\right) - 2\left(E\phi[t,x] * K\phi^{(0,1)}[t,x]\right)\right)\right) + \left((\text{Cos}[\lambda * K\phi[t,x]])^2\left(\frac{E\phi[t,x] * Ex^{(0,1)}[t,x]}{2\,Ex[t,x]} - D[E\phi[t,x],x]\right)\right)\right)\right) + \frac{\alpha[t,x]}{\sqrt{1+\lambda^2}}\left(\left(\frac{E\phi[t,x]\left((\text{Sin}[\lambda * K\phi[t,x]])^2 + \lambda^2\right)}{4\lambda^2\left(Ex[t,x]^{3/2}\right)}\right) + \frac{(\text{Cos}[\lambda * K\phi[t,x]])^2}{4\,E\phi[t,x] * \sqrt{Ex[t,x]}}\left(Ex^{(0,2)}[t,x] - \frac{\left(Ex^{(0,1)}[t,x]\right)^2}{4\,Ex[t,x]} - \frac{Ex^{(0,1)}[t,x] * E\phi^{(0,1)}[t,x]}{E\phi[t,x]}\right) - \left(\frac{Kx[t,x] * \text{Sin}[2\lambda * K\phi[t,x]]}{2\lambda\sqrt{Ex[t,x]}}\left(1 + \left(\frac{\lambda * D[Ex[t,x],x]}{2\,E\phi[t,x]}\right)^2\right)\right) -$$

$$D\left[\text{Sin}[2\lambda * K\phi[t,x]] * \lambda \frac{\sqrt{Ex[t,x]}}{2\left(E\phi[t,x]^2\right)} * \text{Diffeo}, x\right]\Bigg)$$

(K_ϕ dot*)*

$$K\phi^{(1,0)}[t,x] = \left(\beta[t,x] * K\phi^{(0,1)}[t,x]\right) +$$

$$\left(\frac{\alpha^{(0,1)}[t,x] * D[Ex[t,x],x] * (\text{Cos}[\lambda * K\phi[t,x]])^2 * \sqrt{Ex[t,x]}}{2\left(E\phi[t,x]^2\right)\sqrt{1+\lambda^2}}\right) -$$

$$\left(\alpha[t,x] * \frac{\left((\text{Sin}[\lambda * K\phi[t,x]])^2 + \lambda^2\right)}{2\lambda^2\sqrt{Ex[t,x]} * \sqrt{1+\lambda^2}}\right) +$$

$$\left(\alpha[t,x] * \frac{(D[Ex[t,x],x])^2}{8\sqrt{Ex[t,x]}\left(E\phi[t,x]^2\right)} * \frac{(\text{Cos}[\lambda * K\phi[t,x]])^2}{\sqrt{1+\lambda^2}}\right) -$$

$$\left(\alpha[t,x] * \frac{\text{Sin}[2\lambda * K\phi[t,x]]}{\sqrt{1+\lambda^2}} * \frac{\lambda\sqrt{Ex[t,x]} * D[Ex[t,x],x]}{2\left(E\phi[t,x]^3\right)} * \text{Diffeo}\right)$$

Out[4]= $x > 0$

Out[5]= $$-\frac{E\phi[t,x]\left(1+s[t,x]^2\right)}{2\sqrt{1+\lambda^2}\sqrt{Ex[t,x]}} - \frac{\sqrt{Ex[t,x]}\,Kx[t,x]\,\text{Sin}[2\,\text{ArcSin}[\lambda\,s[t,x]]]\left(1+\frac{\lambda^2 Ex^{(0,1)}[t,x]^2}{4E\phi[t,x]^2}\right)}{\lambda\sqrt{1+\lambda^2}} +$$
$$\frac{\left(1-\lambda^2 s[t,x]^2\right)\left(\frac{Ex^{(0,1)}[t,x]^2}{4\sqrt{Ex[t,x]}\,E\phi[t,x]} + \sqrt{Ex[t,x]}\left(-\frac{Ex^{(0,1)}[t,x]\,E\phi^{(0,1)}[t,x]}{E\phi[t,x]^2} + \frac{Ex^{(0,2)}[t,x]}{E\phi[t,x]}\right)\right)}{2\sqrt{1+\lambda^2}}$$

Out[6]= $$-Kx[t,x]\,Ex^{(0,1)}[t,x] + \frac{E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}$$

Out[7]= $$\beta[t,x]\,Ex^{(0,1)}[t,x] + \frac{\sqrt{Ex[t,x]}\,\text{Sin}[2\,\text{ArcSin}[\lambda\,s[t,x]]]\,\alpha[t,x]\left(1+\frac{\lambda^2 Ex^{(0,1)}[t,x]^2}{4E\phi[t,x]^2}\right)}{\lambda\sqrt{1+\lambda^2}}$$

Out[8]= $$\frac{2\,\text{Cos}[2\,\text{ArcSin}[\lambda\,s[t,x]]]\sqrt{Ex[t,x]}\,Kx[t,x]\,\alpha[t,x]\left(1+\frac{\lambda^2 Ex^{(0,1)}[t,x]^2}{4E\phi[t,x]^2}\right)}{\sqrt{1+\lambda^2}} + \beta[t,x]\,E\phi^{(0,1)}[t,x] +$$
$$E\phi[t,x]\,\beta^{(0,1)}[t,x] + \frac{1}{\lambda\sqrt{1+\lambda^2}}\text{Sin}[2\,\text{ArcSin}[\lambda\,s[t,x]]]\,\alpha[t,x]\left(\frac{E\phi[t,x]}{2\sqrt{Ex[t,x]}} +\right.$$
$$\left.\frac{1}{2}\lambda^2\left(\frac{Ex^{(0,1)}[t,x]^2}{4\sqrt{Ex[t,x]}\,E\phi[t,x]} + \sqrt{Ex[t,x]}\left(-\frac{Ex^{(0,1)}[t,x]\,E\phi^{(0,1)}[t,x]}{E\phi[t,x]^2} + \frac{Ex^{(0,2)}[t,x]}{E\phi[t,x]}\right)\right)\right)$$

Out[9]=

$$\beta[t,x]\,Kx^{(0,1)}[t,x]+\frac{1}{2\sqrt{1+\lambda^2}\,E\phi[t,x]^2}$$

$$\sqrt{Ex[t,x]}\left(\left(1-\lambda^2 s[t,x]^2\right)\left(\frac{E\phi[t,x]\,Ex^{(0,1)}[t,x]}{2\,Ex[t,x]}-E\phi^{(0,1)}[t,x]\right)+\right.$$

$$\left.\lambda\,Sin[2\,ArcSin[\lambda\,s[t,x]]]\left(Kx[t,x]\,Ex^{(0,1)}[t,x]-\frac{2\,E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}\right)\right)$$

$$\alpha^{(0,1)}[t,x]+Kx[t,x]\,\beta^{(0,1)}[t,x]+\frac{1}{\sqrt{1+\lambda^2}}$$

$$\alpha[t,x]\left(\frac{E\phi[t,x]\left(\lambda^2+\lambda^2 s[t,x]^2\right)}{4\lambda^2\,Ex[t,x]^{3/2}}-\frac{Kx[t,x]\,Sin[2\,ArcSin[\lambda\,s[t,x]]]\left(1+\frac{\lambda^2\,Ex^{(0,1)}[t,x]^2}{4\,E\phi[t,x]^2}\right)}{2\lambda\sqrt{Ex[t,x]}}-\right.$$

$$\frac{\lambda\,Sin[2\,ArcSin[\lambda\,s[t,x]]]\,Ex^{(0,1)}[t,x]\left(-Kx[t,x]\,Ex^{(0,1)}[t,x]+\frac{E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}\right)}{4\sqrt{Ex[t,x]}\,E\phi[t,x]^2}+$$

$$\frac{1}{E\phi[t,x]^3}\lambda\sqrt{Ex[t,x]}\,Sin[2\,ArcSin[\lambda\,s[t,x]]]$$

$$E\phi^{(0,1)}[t,x]\left(-Kx[t,x]\,Ex^{(0,1)}[t,x]+\frac{E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}\right)-$$

$$\left(\lambda^2\,Cos[2\,ArcSin[\lambda\,s[t,x]]]\sqrt{Ex[t,x]}\,s^{(0,1)}[t,x]\right.$$

$$\left.\left(-Kx[t,x]\,Ex^{(0,1)}[t,x]+\frac{E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}\right)\right)\Big/\left(E\phi[t,x]^2\sqrt{1-\lambda^2 s[t,x]^2}\right)+$$

$$\frac{\left(1-\lambda^2 s[t,x]^2\right)\left(-\frac{Ex^{(0,1)}[t,x]^2}{4\,Ex[t,x]}-\frac{Ex^{(0,1)}[t,x]\,E\phi^{(0,1)}[t,x]}{E\phi[t,x]}+Ex^{(0,2)}[t,x]\right)}{4\sqrt{Ex[t,x]}\,E\phi[t,x]}-\frac{1}{2\,E\phi[t,x]^2}\lambda\sqrt{Ex[t,x]}$$

$$Sin[2\,ArcSin[\lambda\,s[t,x]]]\left(-Ex^{(0,1)}[t,x]\,Kx^{(0,1)}[t,x]+\frac{E\phi^{(0,1)}[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}+\right.$$

$$\left.\frac{\lambda^2\,E\phi[t,x]\times s[t,x]\,s^{(0,1)}[t,x]^2}{\left(1-\lambda^2 s[t,x]^2\right)^{3/2}}-Kx[t,x]\,Ex^{(0,2)}[t,x]+\frac{E\phi[t,x]\,s^{(0,2)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}\right)+$$

$$\frac{\sqrt{Ex[t,x]}\left(1-\lambda^2 s[t,x]^2\right)\alpha^{(0,2)}[t,x]}{2\sqrt{1+\lambda^2}\,E\phi[t,x]}$$

Out[10]=

$$-\frac{\left(\lambda^2+\lambda^2\,s[t,x]^2\right)\alpha[t,x]}{2\lambda^2\sqrt{1+\lambda^2}\,\sqrt{Ex[t,x]}}+\frac{\left(1-\lambda^2\,s[t,x]^2\right)\alpha[t,x]\,Ex^{(0,1)}[t,x]^2}{8\sqrt{1+\lambda^2}\,\sqrt{Ex[t,x]}\,E\phi[t,x]^2}+$$

$$\frac{\beta[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2\,s[t,x]^2}}-\frac{1}{2\sqrt{1+\lambda^2}\,E\phi[t,x]^3}\lambda\sqrt{Ex[t,x]}\,\text{Sin}[2\,\text{ArcSin}[\lambda\,s[t,x]]]$$

$$\alpha[t,x]\,Ex^{(0,1)}[t,x]\left(-Kx[t,x]\,Ex^{(0,1)}[t,x]+\frac{E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2\,s[t,x]^2}}\right)+$$

$$\frac{\sqrt{Ex[t,x]}\,\left(1-\lambda^2\,s[t,x]^2\right)Ex^{(0,1)}[t,x]\,\alpha^{(0,1)}[t,x]}{2\sqrt{1+\lambda^2}\,E\phi[t,x]^2}$$

After Manually Simplifying Arcsins

In[11]:=

```
(*Via Stack Exchange:
   Sin[2ArcSin[x]] = 2x√(1-x^2)
   Cos[2ArcSin[x]] = 1-2x^2
*)
```

In[12]:=

$$\text{Hamiltonian} = -\frac{E\phi[t,x]\,\left(1+s[t,x]^2\right)}{2\sqrt{1+\lambda^2}\,\sqrt{Ex[t,x]}}-\frac{1}{\lambda\sqrt{1+\lambda^2}}\sqrt{Ex[t,x]}\,Kx[t,x]$$

$$\left(2\lambda\,s[t,x]\,\sqrt{1-\lambda^2\,s[t,x]^2}\right)\left(1+\frac{\lambda^2\,Ex^{(0,1)}[t,x]^2}{4\,E\phi[t,x]^2}\right)+\frac{1}{2\sqrt{1+\lambda^2}}\left(1-\lambda^2\,s[t,x]^2\right)$$

$$\left(\frac{Ex^{(0,1)}[t,x]^2}{4\sqrt{Ex[t,x]}\,E\phi[t,x]}+\sqrt{Ex[t,x]}\left(-\frac{Ex^{(0,1)}[t,x]\,E\phi^{(0,1)}[t,x]}{E\phi[t,x]^2}+\frac{Ex^{(0,2)}[t,x]}{E\phi[t,x]}\right)\right)$$

$$Ex^{(1,0)}[t,x] = \beta[t,x]\,Ex^{(0,1)}[t,x]+$$

$$\frac{1}{\lambda\sqrt{1+\lambda^2}}\sqrt{Ex[t,x]}\;2\lambda\,s[t,x]\,\sqrt{1-\lambda^2\,s[t,x]^2}\;\alpha[t,x]\left(1+\frac{\lambda^2\,Ex^{(0,1)}[t,x]^2}{4\,E\phi[t,x]^2}\right)$$

$$E\phi^{(1,0)}[t,x] =$$

$$\frac{1}{\sqrt{1+\lambda^2}}\,2\left(1-2\lambda^2\,s[t,x]^2\right)\sqrt{Ex[t,x]}\,Kx[t,x]\,\alpha[t,x]\left(1+\frac{\lambda^2\,Ex^{(0,1)}[t,x]^2}{4\,E\phi[t,x]^2}\right)+$$

$$\beta[t,x]\,E\phi^{(0,1)}[t,x]+E\phi[t,x]\,\beta^{(0,1)}[t,x]+\frac{1}{\lambda\sqrt{1+\lambda^2}}\left(2\lambda\,s[t,x]\,\sqrt{1-\lambda^2\,s[t,x]^2}\right)\alpha[t,x]\left(\frac{E\phi[t,x]}{2\sqrt{Ex[t,x]}}+\frac{1}{2}\lambda^2\left(\frac{Ex^{(0,1)}[t,x]^2}{4\sqrt{Ex[t,x]}\,E\phi[t,x]}+\sqrt{Ex[t,x]}\left(-\frac{Ex^{(0,1)}[t,x]\,E\phi^{(0,1)}[t,x]}{E\phi[t,x]^2}+\frac{Ex^{(0,2)}[t,x]}{E\phi[t,x]}\right)\right)\right)$$

$$Kx^{(1,0)}[t,x]=\beta[t,x]\,Kx^{(0,1)}[t,x]+\frac{1}{2\sqrt{1+\lambda^2}\,E\phi[t,x]^2}\sqrt{Ex[t,x]}\left(\left(1-\lambda^2\,s[t,x]^2\right)\left(\frac{E\phi[t,x]\,Ex^{(0,1)}[t,x]}{2\,Ex[t,x]}-E\phi^{(0,1)}[t,x]\right)+\left(2\lambda\,s[t,x]\,\sqrt{1-\lambda^2\,s[t,x]^2}\right)\left(Kx[t,x]\,Ex^{(0,1)}[t,x]-\frac{2\,E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2\,s[t,x]^2}}\right)\right)$$

$$\alpha^{(0,1)}[t,x]+Kx[t,x]\,\beta^{(0,1)}[t,x]+\frac{1}{\sqrt{1+\lambda^2}}\alpha[t,x]\left(\frac{E\phi[t,x]\left(\lambda^2+\lambda^2\,s[t,x]^2\right)}{4\lambda^2\,Ex[t,x]^{3/2}}-\left(Kx[t,x]\left(2\lambda\,s[t,x]\,\sqrt{1-\lambda^2\,s[t,x]^2}\right)\left(1+\frac{\lambda^2\,Ex^{(0,1)}[t,x]^2}{4\,E\phi[t,x]^2}\right)\right)\Big/\left(2\lambda\,\sqrt{Ex[t,x]}\right)-\left(\lambda\left(2\lambda\,s[t,x]\,\sqrt{1-\lambda^2\,s[t,x]^2}\right)Ex^{(0,1)}[t,x]\left(-Kx[t,x]\,Ex^{(0,1)}[t,x]+\frac{E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2\,s[t,x]^2}}\right)\right)\Big/\left(4\sqrt{Ex[t,x]}\,E\phi[t,x]^2\right)+\frac{1}{E\phi[t,x]^3}\lambda\sqrt{Ex[t,x]}\left(2\lambda\,s[t,x]\,\sqrt{1-\lambda^2\,s[t,x]^2}\right)E\phi^{(0,1)}[t,x]\left(-Kx[t,x]\,Ex^{(0,1)}[t,x]+\frac{E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2\,s[t,x]^2}}\right)-\left(\lambda^2\left(1-2\lambda^2\,s[t,x]^2\right)\sqrt{Ex[t,x]}\,s^{(0,1)}[t,x]\left(-Kx[t,x]\,Ex^{(0,1)}[t,x]+\frac{E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2\,s[t,x]^2}}\right)\right)\Big/\left(E\phi[t,x]^2\sqrt{1-\lambda^2\,s[t,x]^2}\right)+\left(\left(1-\lambda^2\,s[t,x]^2\right)\left(-\frac{Ex^{(0,1)}[t,x]^2}{4\,Ex[t,x]}-\frac{Ex^{(0,1)}[t,x]\,E\phi^{(0,1)}[t,x]}{E\phi[t,x]}+Ex^{(0,2)}[t,x]\right)\right)\Big/\left(4\sqrt{Ex[t,x]}\,E\phi[t,x]\right)-\frac{1}{2\,E\phi[t,x]^2}\lambda\sqrt{Ex[t,x]}\left(2\lambda\,s[t,x]\,\sqrt{1-\lambda^2\,s[t,x]^2}\right)$$

$$\left(-\text{Ex}^{(0,1)}[t,x]\,\text{Kx}^{(0,1)}[t,x]+\frac{\text{E}\phi^{(0,1)}[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}+\right.$$

$$\left.\frac{\lambda^2\,\text{E}\phi[t,x]\times s[t,x]\,s^{(0,1)}[t,x]^2}{\left(1-\lambda^2 s[t,x]^2\right)^{3/2}}-\text{Kx}[t,x]\,\text{Ex}^{(0,2)}[t,x]+\frac{\text{E}\phi[t,x]\,s^{(0,2)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}\right)+$$

$$\frac{\sqrt{\text{Ex}[t,x]}\,\left(1-\lambda^2 s[t,x]^2\right)\alpha^{(0,2)}[t,x]}{2\sqrt{1+\lambda^2}\,\text{E}\phi[t,x]}$$

$$\text{K}\phi^{(1,0)}[t,x]=-\frac{\left(\lambda^2+\lambda^2 s[t,x]^2\right)\alpha[t,x]}{2\lambda^2\sqrt{1+\lambda^2}\,\sqrt{\text{Ex}[t,x]}}+\frac{\left(1-\lambda^2 s[t,x]^2\right)\alpha[t,x]\,\text{Ex}^{(0,1)}[t,x]^2}{8\sqrt{1+\lambda^2}\,\sqrt{\text{Ex}[t,x]}\,\text{E}\phi[t,x]^2}+$$

$$\frac{\beta[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}-\frac{1}{2\sqrt{1+\lambda^2}\,\text{E}\phi[t,x]^3}\lambda\sqrt{\text{Ex}[t,x]}\,\left(2\lambda\,s[t,x]\,\sqrt{1-\lambda^2 s[t,x]^2}\right)$$

$$\alpha[t,x]\,\text{Ex}^{(0,1)}[t,x]\left(-\text{Kx}[t,x]\,\text{Ex}^{(0,1)}[t,x]+\frac{\text{E}\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}\right)+$$

$$\frac{\sqrt{\text{Ex}[t,x]}\,\left(1-\lambda^2 s[t,x]^2\right)\text{Ex}^{(0,1)}[t,x]\,\alpha^{(0,1)}[t,x]}{2\sqrt{1+\lambda^2}\,\text{E}\phi[t,x]^2}$$

Out[12]=
$$-\frac{\text{E}\phi[t,x]\,\left(1+s[t,x]^2\right)}{2\sqrt{1+\lambda^2}\,\sqrt{\text{Ex}[t,x]}}-\frac{2\sqrt{\text{Ex}[t,x]}\,\text{Kx}[t,x]\times s[t,x]\,\sqrt{1-\lambda^2 s[t,x]^2}\,\left(1+\frac{\lambda^2\,\text{Ex}^{(0,1)}[t,x]^2}{4\,\text{E}\phi[t,x]^2}\right)}{\sqrt{1+\lambda^2}}+$$

$$\frac{\left(1-\lambda^2 s[t,x]^2\right)\left(\frac{\text{Ex}^{(0,1)}[t,x]^2}{4\sqrt{\text{Ex}[t,x]}\,\text{E}\phi[t,x]}+\sqrt{\text{Ex}[t,x]}\,\left(-\frac{\text{Ex}^{(0,1)}[t,x]\,\text{E}\phi^{(0,1)}[t,x]}{\text{E}\phi[t,x]^2}+\frac{\text{Ex}^{(0,2)}[t,x]}{\text{E}\phi[t,x]}\right)\right)}{2\sqrt{1+\lambda^2}}$$

Out[13]=
$$\beta[t,x]\,\text{Ex}^{(0,1)}[t,x]+\frac{2\sqrt{\text{Ex}[t,x]}\,s[t,x]\,\sqrt{1-\lambda^2 s[t,x]^2}\,\alpha[t,x]\,\left(1+\frac{\lambda^2\,\text{Ex}^{(0,1)}[t,x]^2}{4\,\text{E}\phi[t,x]^2}\right)}{\sqrt{1+\lambda^2}}$$

Out[14]=
$$\frac{2\sqrt{\text{Ex}[t,x]}\,\text{Kx}[t,x]\,\left(1-2\lambda^2 s[t,x]^2\right)\alpha[t,x]\,\left(1+\frac{\lambda^2\,\text{Ex}^{(0,1)}[t,x]^2}{4\,\text{E}\phi[t,x]^2}\right)}{\sqrt{1+\lambda^2}}+\beta[t,x]\,\text{E}\phi^{(0,1)}[t,x]+$$

$$\text{E}\phi[t,x]\,\beta^{(0,1)}[t,x]+\frac{1}{\sqrt{1+\lambda^2}}2\,s[t,x]\,\sqrt{1-\lambda^2 s[t,x]^2}\,\alpha[t,x]\left(\frac{\text{E}\phi[t,x]}{2\sqrt{\text{Ex}[t,x]}}+\right.$$

$$\left.\frac{1}{2}\lambda^2\left(\frac{\text{Ex}^{(0,1)}[t,x]^2}{4\sqrt{\text{Ex}[t,x]}\,\text{E}\phi[t,x]}+\sqrt{\text{Ex}[t,x]}\,\left(-\frac{\text{Ex}^{(0,1)}[t,x]\,\text{E}\phi^{(0,1)}[t,x]}{\text{E}\phi[t,x]^2}+\frac{\text{Ex}^{(0,2)}[t,x]}{\text{E}\phi[t,x]}\right)\right)\right)$$

Out[15]=

$$\beta[t,x]\,Kx^{(0,1)}[t,x]+\frac{1}{2\sqrt{1+\lambda^2}\,E\phi[t,x]^2}$$

$$\sqrt{Ex[t,x]}\left(\left(1-\lambda^2 s[t,x]^2\right)\left(\frac{E\phi[t,x]\,Ex^{(0,1)}[t,x]}{2\,Ex[t,x]}-E\phi^{(0,1)}[t,x]\right)+\right.$$

$$\left.2\lambda\,s[t,x]\,\sqrt{1-\lambda^2 s[t,x]^2}\left(Kx[t,x]\,Ex^{(0,1)}[t,x]-\frac{2\,E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}\right)\right)$$

$$\alpha^{(0,1)}[t,x]+Kx[t,x]\,\beta^{(0,1)}[t,x]+\frac{1}{\sqrt{1+\lambda^2}}$$

$$\alpha[t,x]\left(\frac{E\phi[t,x]\left(\lambda^2+\lambda^2 s[t,x]^2\right)}{4\lambda^2\,Ex[t,x]^{3/2}}-\frac{Kx[t,x]\times s[t,x]\,\sqrt{1-\lambda^2 s[t,x]^2}\left(1+\frac{\lambda^2\,Ex^{(0,1)}[t,x]^2}{4\,E\phi[t,x]^2}\right)}{\sqrt{Ex[t,x]}}-\right.$$

$$\frac{\lambda^2 s[t,x]\,\sqrt{1-\lambda^2 s[t,x]^2}\,Ex^{(0,1)}[t,x]\left(-Kx[t,x]\,Ex^{(0,1)}[t,x]+\frac{E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}\right)}{2\sqrt{Ex[t,x]}\,E\phi[t,x]^2}+$$

$$\frac{1}{E\phi[t,x]^3}2\lambda^2\sqrt{Ex[t,x]}\,s[t,x]\,\sqrt{1-\lambda^2 s[t,x]^2}$$

$$E\phi^{(0,1)}[t,x]\left(-Kx[t,x]\,Ex^{(0,1)}[t,x]+\frac{E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}\right)-$$

$$\frac{\lambda^2\sqrt{Ex[t,x]}\left(1-2\lambda^2 s[t,x]^2\right)s^{(0,1)}[t,x]\left(-Kx[t,x]\,Ex^{(0,1)}[t,x]+\frac{E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}\right)}{E\phi[t,x]^2\sqrt{1-\lambda^2 s[t,x]^2}}+$$

$$\frac{\left(1-\lambda^2 s[t,x]^2\right)\left(-\frac{Ex^{(0,1)}[t,x]^2}{4\,Ex[t,x]}-\frac{Ex^{(0,1)}[t,x]\,E\phi^{(0,1)}[t,x]}{E\phi[t,x]}+Ex^{(0,2)}[t,x]\right)}{4\sqrt{Ex[t,x]}\,E\phi[t,x]}-\frac{1}{E\phi[t,x]^2}\lambda^2\sqrt{Ex[t,x]}$$

$$s[t,x]\,\sqrt{1-\lambda^2 s[t,x]^2}\left(-Ex^{(0,1)}[t,x]\,Kx^{(0,1)}[t,x]+\frac{E\phi^{(0,1)}[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}+\right.$$

$$\left.\left.\frac{\lambda^2\,E\phi[t,x]\times s[t,x]\,s^{(0,1)}[t,x]^2}{\left(1-\lambda^2 s[t,x]^2\right)^{3/2}}-Kx[t,x]\,Ex^{(0,2)}[t,x]+\frac{E\phi[t,x]\,s^{(0,2)}[t,x]}{\sqrt{1-\lambda^2 s[t,x]^2}}\right)\right)+$$

$$\frac{\sqrt{Ex[t,x]}\left(1-\lambda^2 s[t,x]^2\right)\alpha^{(0,2)}[t,x]}{2\sqrt{1+\lambda^2}\,E\phi[t,x]}$$

Out[16]=
$$-\frac{\left(\lambda^2+\lambda^2\,s[t,x]^2\right)\alpha[t,x]}{2\,\lambda^2\,\sqrt{1+\lambda^2}\,\sqrt{Ex[t,x]}}+\frac{\left(1-\lambda^2\,s[t,x]^2\right)\alpha[t,x]\,Ex^{(0,1)}[t,x]^2}{8\,\sqrt{1+\lambda^2}\,\sqrt{Ex[t,x]}\,E\phi[t,x]^2}+$$
$$\frac{\beta[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2\,s[t,x]^2}}-\frac{1}{\sqrt{1+\lambda^2}\,E\phi[t,x]^3}\lambda^2\,\sqrt{Ex[t,x]}\,s[t,x]\,\sqrt{1-\lambda^2\,s[t,x]^2}$$
$$\alpha[t,x]\,Ex^{(0,1)}[t,x]\left(-Kx[t,x]\,Ex^{(0,1)}[t,x]+\frac{E\phi[t,x]\,s^{(0,1)}[t,x]}{\sqrt{1-\lambda^2\,s[t,x]^2}}\right)+$$
$$\frac{\sqrt{Ex[t,x]}\,\left(1-\lambda^2\,s[t,x]^2\right)Ex^{(0,1)}[t,x]\,\alpha^{(0,1)}[t,x]}{2\,\sqrt{1+\lambda^2}\,E\phi[t,x]^2}$$

Line Element

In[17]:=
```
(*Defining E^x = x^2*)
Ex[t, x] = x^2
Ex^(0,1)[t, x] = 2 x
Ex^(0,2)[t, x] = 2



(*Spacial Metric Components*)
n = ((1 - (k / Sqrt[Ex[t, x]]))^-1) * ((Eϕ[t, x])^2 / Ex[t, x])  (*γxx*)
l = Ex[t, x] (*γΘΘ*)

q = Ex[t, x] * ((Sin[Θ])^2) (*γϕϕ*)
```

Out[17]= x^2

Out[18]= $2\,x$

Out[19]= 2

Out[20]=
$$\frac{E\phi[t,x]^2}{x^2\left(1-\frac{k}{\sqrt{x^2}}\right)}$$

Out[21]= x^2

Out[22]= $x^2\,\text{Sin}[\Theta]^2$

Calculations

Curvature & Trace

In[23]:=

```
(*Curvature Components*)
Kxx = CurvatureComponent[α[t, x], β[t, x], x, x, n, n, n]
Kθθ = CurvatureComponent[α[t, x], β[t, x], θ, θ, 1, 0, 0]
Kϕϕ = CurvatureComponent[α[t, x], β[t, x], ϕ, ϕ, q, 0, 0]



(*γ^ij K_ij = 0, the trace condition/maximality constraint*)
(*confirmed λ→0 limit matches classical case*)
TraceCondn = (Kxx * 1/n) + (Kθθ * 1/1) + (Kϕϕ * 1/q)
```

Out[23]=

$$\frac{1}{2\,\alpha[t,x]}\left(\beta[t,x]\left(-\frac{k\,E\phi[t,x]^2}{x\left(x^2\right)^{3/2}\left(1-\frac{k}{\sqrt{x^2}}\right)^2}-\frac{2\,E\phi[t,x]^2}{x^3\left(1-\frac{k}{\sqrt{x^2}}\right)}+\frac{2\,E\phi[t,x]\,E\phi^{(0,1)}[t,x]}{x^2\left(1-\frac{k}{\sqrt{x^2}}\right)}\right)+\right.$$

$$\frac{2\,E\phi[t,x]^2\,\beta^{(0,1)}[t,x]}{x^2\left(1-\frac{k}{\sqrt{x^2}}\right)}-\frac{1}{x^2\left(1-\frac{k}{\sqrt{x^2}}\right)}$$

$$2\,E\phi[t,x]\left(\frac{2\sqrt{x^2}\left(1+\frac{x^2\lambda^2}{E\phi[t,x]^2}\right)Kx[t,x]\left(1-2\lambda^2 s[t,x]^2\right)\alpha[t,x]}{\sqrt{1+\lambda^2}}+\beta[t,x]\,E\phi^{(0,1)}[t,x]+\right.$$

$$\frac{2\,s[t,x]\sqrt{1-\lambda^2 s[t,x]^2}\,\alpha[t,x]\left(\frac{E\phi[t,x]}{2\sqrt{x^2}}+\frac{1}{2}\lambda^2\left(\frac{\sqrt{x^2}}{E\phi[t,x]}+\sqrt{x^2}\left(\frac{2}{E\phi[t,x]}-\frac{2\,x\,E\phi^{(0,1)}[t,x]}{E\phi[t,x]^2}\right)\right)\right)}{\sqrt{1+\lambda^2}}+$$

$$\left.\left.E\phi[t,x]\,\beta^{(0,1)}[t,x]\right)\right)$$

Out[24]=

$$\frac{x\,\beta[t,x]}{\alpha[t,x]}$$

Out[25]=

$$\frac{x\,\text{Sin}[\theta]^2\,\beta[t,x]}{\alpha[t,x]}$$

Out[26]=

$$\frac{2\,\beta[t,x]}{x\,\alpha[t,x]}+\frac{1}{2\,E\phi[t,x]^2\,\alpha[t,x]}$$

$$x^2\left(1-\frac{k}{\sqrt{x^2}}\right)\left(\beta[t,x]\left(-\frac{k\,E\phi[t,x]^2}{x\left(x^2\right)^{3/2}\left(1-\frac{k}{\sqrt{x^2}}\right)^2}-\frac{2\,E\phi[t,x]^2}{x^3\left(1-\frac{k}{\sqrt{x^2}}\right)}+\frac{2\,E\phi[t,x]\,E\phi^{(0,1)}[t,x]}{x^2\left(1-\frac{k}{\sqrt{x^2}}\right)}\right)+\right.$$

$$\frac{2\,E\phi[t,x]^2\,\beta^{(0,1)}[t,x]}{x^2\left(1-\frac{k}{\sqrt{x^2}}\right)}-\frac{1}{x^2\left(1-\frac{k}{\sqrt{x^2}}\right)}$$

$$2\,E\phi[t,x]\left(\frac{2\sqrt{x^2}\left(1+\frac{x^2\lambda^2}{E\phi[t,x]^2}\right)Kx[t,x]\left(1-2\lambda^2\,s[t,x]^2\right)\alpha[t,x]}{\sqrt{1+\lambda^2}}+\beta[t,x]\,E\phi^{(0,1)}[t,x]+\right.$$

$$\frac{2\,s[t,x]\sqrt{1-\lambda^2\,s[t,x]^2}\,\alpha[t,x]\left(\frac{E\phi[t,x]}{2\sqrt{x^2}}+\frac{1}{2}\lambda^2\left(\frac{\sqrt{x^2}}{E\phi[t,x]}+\sqrt{x^2}\left(\frac{2}{E\phi[t,x]}-\frac{2\,x\,E\phi^{(0,1)}[t,x]}{E\phi[t,x]^2}\right)\right)\right)}{\sqrt{1+\lambda^2}}+$$

$$\left.\left.E\phi[t,x]\,\beta^{(0,1)}[t,x]\right)\right)$$

Solving Trace Condition for K_x

In[27]:=

```
(*Solving γ^ij K_ij = 0 for K_x in terms of Eϕ, K_ϕ*)

Solve[TraceCondn == 0, Kx[t, x]]
```

Out[27]=

$$\left\{\left\{Kx[t,x] \to -\left(\left(\sqrt{1+\lambda^2}\ E\phi[t,x]\left(-\frac{2\beta[t,x]}{x\,\alpha[t,x]}+\frac{\beta[t,x]\ E\phi^{(0,1)}[t,x]}{E\phi[t,x]\,\alpha[t,x]}-\right.\right.\right.\right.$$

$$\left.\frac{\left(1-\frac{k}{x}\right)x^2\beta[t,x]\left(-\frac{k\,E\phi[t,x]^2}{\left(1-\frac{k}{x}\right)^2x^4}-\frac{2\,E\phi[t,x]^2}{\left(1-\frac{k}{x}\right)x^3}+\frac{2\,E\phi[t,x]\,E\phi^{(0,1)}[t,x]}{\left(1-\frac{k}{x}\right)x^2}\right)}{2\,E\phi[t,x]^2\,\alpha[t,x]}+\right.$$

$$\left.\left.\left.\left.\frac{2\,s[t,x]\,\sqrt{1-\lambda^2\,s[t,x]^2}\left(\frac{E\phi[t,x]}{2x}+\frac{1}{2}\lambda^2\left(\frac{x}{E\phi[t,x]}+x\left(\frac{2}{E\phi[t,x]}-\frac{2\,x\,E\phi^{(0,1)}[t,x]}{E\phi[t,x]^2}\right)\right)\right)}{\sqrt{1+\lambda^2}\ E\phi[t,x]}\right)\right)\right/$$

$$\left.\left(2x\left(1+\frac{x^2\lambda^2}{E\phi[t,x]^2}\right)\left(1-2\lambda^2\,s[t,x]^2\right)\right)\right\}\Bigg\}$$

In[28]:=

Solving $\dot{E}^x$ for β

In[29]:=

```
(*Solving E^x dot = 0 for shift N^x*)
Solve[Ex^(1,0)[t, x] == 0, β[t, x]]
```

Out[29]=

$$\left\{\left\{\beta[t,x] \to -\frac{\left(x^2\lambda^2+E\phi[t,x]^2\right)s[t,x]\ \sqrt{1-\lambda^2\,s[t,x]^2}\ \alpha[t,x]}{\sqrt{1+\lambda^2}\ E\phi[t,x]^2}\right\}\right\}$$

In[30]:=

```
(*Shift / N^x given from E^x dot = 0, β given by β = γN^x *)
(*confirmed λ→0 limit matches classical case*)

(*β[t,x] = -((x^2 λ^2+Eϕ[t,x]^2) Sin[2 λ Kϕ[t,x]] α[t,x])/(2λ Sqrt[1+λ^2] Eϕ[t,x]^2) *)


(*Note for λ=0 case: In the classical case,
we changed the sign of β through time reversal. However,
to recover the classical case of K_x from the nonzero λ case,
the sign of β cannot be changed, or else K_x goes to 0.*)
```

In[31]:=

```
(*Manually simplified
 (Mathematica will sometimes give this result and sometimes give it in terms of Kϕ,
  so this is just in case)*)

β[t, x] = -((x^2 λ^2 + Eϕ[t, x]^2) s[t, x] Sqrt[1 - λ^2 s[t, x]^2] α[t, x])/(Sqrt[1 + λ^2] Eϕ[t, x]^2)
```

Out[31]=

$$-\frac{\left(x^2 \lambda^2 + E\phi[t, x]^2\right) s[t, x] \sqrt{1 - \lambda^2 s[t, x]^2}\, \alpha[t, x]}{\sqrt{1 + \lambda^2}\, E\phi[t, x]^2}$$

K_x After Plugging in β

In[32]:=

$$\mathrm{Kx}[t,x] = -\frac{1}{2x\left(1+\frac{x^2\lambda^2}{E\phi[t,x]^2}\right)\left(1-2\lambda^2 s[t,x]^2\right)}\sqrt{1+\lambda^2}\,E\phi[t,x]\left(-\frac{2\beta[t,x]}{x\,\alpha[t,x]}+\frac{\beta[t,x]\,E\phi^{(0,1)}[t,x]}{E\phi[t,x]\,\alpha[t,x]}-\frac{\left(1-\frac{k}{x}\right)x^2\beta[t,x]\left(-\frac{k\,E\phi[t,x]^2}{\left(1-\frac{k}{x}\right)^2x^4}-\frac{2\,E\phi[t,x]^2}{\left(1-\frac{k}{x}\right)x^3}+\frac{2\,E\phi[t,x]\,E\phi^{(0,1)}[t,x]}{\left(1-\frac{k}{x}\right)x^2}\right)}{2\,E\phi[t,x]^2\alpha[t,x]}+\frac{2\,s[t,x]\sqrt{1-\lambda^2 s[t,x]^2}\left(\frac{E\phi[t,x]}{2x}+\frac{1}{2}\lambda^2\left(\frac{x}{E\phi[t,x]}+x\left(\frac{2}{E\phi[t,x]}-\frac{2x\,E\phi^{(0,1)}[t,x]}{E\phi[t,x]^2}\right)\right)\right)}{\sqrt{1+\lambda^2}\,E\phi[t,x]}\right)$$

Out[32]=

$$-\left(\left(\sqrt{1+\lambda^2}\,E\phi[t,x]\left(\frac{2\left(x^2\lambda^2+E\phi[t,x]^2\right)s[t,x]\sqrt{1-\lambda^2 s[t,x]^2}}{x\sqrt{1+\lambda^2}\,E\phi[t,x]^2}-\frac{\left(x^2\lambda^2+E\phi[t,x]^2\right)s[t,x]\sqrt{1-\lambda^2 s[t,x]^2}\,E\phi^{(0,1)}[t,x]}{\sqrt{1+\lambda^2}\,E\phi[t,x]^3}+\frac{1}{2\sqrt{1+\lambda^2}\,E\phi[t,x]^4}\left(1-\frac{k}{x}\right)x^2\left(x^2\lambda^2+E\phi[t,x]^2\right)s[t,x]\sqrt{1-\lambda^2 s[t,x]^2}\left(-\frac{k\,E\phi[t,x]^2}{\left(1-\frac{k}{x}\right)^2x^4}-\frac{2\,E\phi[t,x]^2}{\left(1-\frac{k}{x}\right)x^3}+\frac{2\,E\phi[t,x]\,E\phi^{(0,1)}[t,x]}{\left(1-\frac{k}{x}\right)x^2}\right)+\frac{2\,s[t,x]\sqrt{1-\lambda^2 s[t,x]^2}\left(\frac{E\phi[t,x]}{2x}+\frac{1}{2}\lambda^2\left(\frac{x}{E\phi[t,x]}+x\left(\frac{2}{E\phi[t,x]}-\frac{2x\,E\phi^{(0,1)}[t,x]}{E\phi[t,x]^2}\right)\right)\right)}{\sqrt{1+\lambda^2}\,E\phi[t,x]}\right)\right)\Big/\left(2x\left(1+\frac{x^2\lambda^2}{E\phi[t,x]^2}\right)\left(1-2\lambda^2 s[t,x]^2\right)\right)$$

Solving Diffeomorphism for s (K_ϕ)

```
(*Solving K_ϕ from diffeomorphism, to plug into K_ϕ dot later*)
(*This may or may not be commented out just
 to make running entire notebook faster; it does work.*)

DSolve[Diffeo == 0, s[t, x], x]
```

Out[33]=

$$\left\{\left\{s[t, x] \to -\frac{\sqrt{\frac{1}{\lambda^2} - \frac{\sqrt{x^9 \lambda^4 + 2 x^7 \lambda^2 E\phi[t,x]^2 + x^5 E\phi[t,x]^4 + 4 e^{2 c_1} k \lambda^2 E\phi[t,x]^4 - 4 e^{2 c_1} x \lambda^2 E\phi[t,x]^4}}{x^{5/2} \lambda^2 \sqrt{\left(x^2 \lambda^2 + E\phi[t,x]^2\right)^2}}}}{\sqrt{2}}\right\},\right.$$

$$\left\{s[t, x] \to \frac{\sqrt{\frac{1}{\lambda^2} - \frac{\sqrt{x^9 \lambda^4 + 2 x^7 \lambda^2 E\phi[t,x]^2 + x^5 E\phi[t,x]^4 + 4 e^{2 c_1} k \lambda^2 E\phi[t,x]^4 - 4 e^{2 c_1} x \lambda^2 E\phi[t,x]^4}}{x^{5/2} \lambda^2 \sqrt{\left(x^2 \lambda^2 + E\phi[t,x]^2\right)^2}}}}{\sqrt{2}}\right\},$$

$$\left\{s[t, x] \to -\frac{\sqrt{\frac{1}{\lambda^2} + \frac{\sqrt{x^9 \lambda^4 + 2 x^7 \lambda^2 E\phi[t,x]^2 + x^5 E\phi[t,x]^4 + 4 e^{2 c_1} k \lambda^2 E\phi[t,x]^4 - 4 e^{2 c_1} x \lambda^2 E\phi[t,x]^4}}{x^{5/2} \lambda^2 \sqrt{\left(x^2 \lambda^2 + E\phi[t,x]^2\right)^2}}}}{\sqrt{2}}\right\},$$

$$\left.\left\{s[t, x] \to \frac{\sqrt{\frac{1}{\lambda^2} + \frac{\sqrt{x^9 \lambda^4 + 2 x^7 \lambda^2 E\phi[t,x]^2 + x^5 E\phi[t,x]^4 + 4 e^{2 c_1} k \lambda^2 E\phi[t,x]^4 - 4 e^{2 c_1} x \lambda^2 E\phi[t,x]^4}}{x^{5/2} \lambda^2 \sqrt{\left(x^2 \lambda^2 + E\phi[t,x]^2\right)^2}}}}{\sqrt{2}}\right\}\right\}$$

In[34]:=
```
(*Chosen Kϕ to match λ→0 limit*)

s[t, x] = (Sqrt[1/λ^2 - 1/λ^2 Sqrt[(x^9 λ^4+2 x^7 λ^2 Eϕ[t,x]^2+x^5 Eϕ[t,x]^4+4 ℯ^(2 c1) k λ^2 Eϕ[t,x]^4-4 ℯ^(2 c1) x λ^2 Eϕ[t,x]^4)/(x^5 (x^2 λ^2+Eϕ[t,x]^2)^2)]])/Sqrt[2]

s^(0,1)[t, x] = D[s[t, x], x]

(*We can set c1 = ln(c2) *)
```

Out[34]=
$$\frac{\sqrt{\frac{1}{\lambda^2}-\frac{\sqrt{\frac{x^9\lambda^4+2x^7\lambda^2 E\phi[t,x]^2+x^5 E\phi[t,x]^4+4e^{2c1}k\lambda^2 E\phi[t,x]^4-4e^{2c1}x\lambda^2 E\phi[t,x]^4}{x^5\left(x^2\lambda^2+E\phi[t,x]^2\right)^2}}{\lambda^2}}}{\sqrt{2}}$$

Out[35]=
$$-\left(\left(-\frac{5\left(x^9\lambda^4+2x^7\lambda^2 E\phi[t,x]^2+x^5 E\phi[t,x]^4+4e^{2c1}k\lambda^2 E\phi[t,x]^4-4e^{2c1}x\lambda^2 E\phi[t,x]^4\right)}{x^6\left(x^2\lambda^2+E\phi[t,x]^2\right)^2}-\frac{1}{x^5\left(x^2\lambda^2+E\phi[t,x]^2\right)^3}\right.\right.$$
$$2\left(x^9\lambda^4+2x^7\lambda^2 E\phi[t,x]^2+x^5 E\phi[t,x]^4+4e^{2c1}k\lambda^2 E\phi[t,x]^4-4e^{2c1}x\lambda^2 E\phi[t,x]^4\right)$$
$$\left(2x\lambda^2+2E\phi[t,x]\,E\phi^{(0,1)}[t,x]\right)+\frac{1}{x^5\left(x^2\lambda^2+E\phi[t,x]^2\right)^2}$$
$$\left(9x^8\lambda^4+14x^6\lambda^2 E\phi[t,x]^2+5x^4 E\phi[t,x]^4-4e^{2c1}\lambda^2 E\phi[t,x]^4+\right.$$
$$4x^7\lambda^2 E\phi[t,x]\,E\phi^{(0,1)}[t,x]+4x^5 E\phi[t,x]^3 E\phi^{(0,1)}[t,x]+$$
$$\left.\left.16e^{2c1}k\lambda^2 E\phi[t,x]^3 E\phi^{(0,1)}[t,x]-16e^{2c1}x\lambda^2 E\phi[t,x]^3 E\phi^{(0,1)}[t,x]\right)\right)\Big/$$
$$\left(4\sqrt{2}\,\lambda^2\sqrt{\frac{x^9\lambda^4+2x^7\lambda^2 E\phi[t,x]^2+x^5 E\phi[t,x]^4+4e^{2c1}k\lambda^2 E\phi[t,x]^4-4e^{2c1}x\lambda^2 E\phi[t,x]^4}{x^5\left(x^2\lambda^2+E\phi[t,x]^2\right)^2}}\right.$$
$$\left.\left.\sqrt{\frac{1}{\lambda^2}-\frac{\sqrt{\frac{x^9\lambda^4+2x^7\lambda^2 E\phi[t,x]^2+x^5 E\phi[t,x]^4+4e^{2c1}k\lambda^2 E\phi[t,x]^4-4e^{2c1}x\lambda^2 E\phi[t,x]^4}{x^5\left(x^2\lambda^2+E\phi[t,x]^2\right)^2}}{\lambda^2}}\right)\right)$$

In[36]:=

```
Kϕ[t, x]
```

Out[36]=

$$\frac{\text{ArcSin}\left[\frac{\lambda\sqrt{\frac{1}{\lambda^2}-\frac{\sqrt{\frac{x^9\lambda^4+2x^7\lambda^2 E\phi[t,x]^2+x^5 E\phi[t,x]^4+4e^{2c1}k\lambda^2 E\phi[t,x]^4-4e^{2c1}x\lambda^2 E\phi[t,x]^4}{x^5\left(x^2\lambda^2+E\phi[t,x]^2\right)^2}}{\lambda^2}}}{\sqrt{2}}\right]}{\lambda}$$

In[37]:=

```
Limit[Kϕ[t, x], λ → 0]
```

Out[37]=

$$\frac{\sqrt{e^{2c1}\ (-k+x)}}{x^{5/2}}$$

In[38]:=

```
Kϕ^(0,1)[t, x] = D[Kϕ[t, x], x]
```

Out[38]=

$$-\left(\left(-\frac{5\left(x^9\lambda^4+2x^7\lambda^2 E\phi[t,x]^2+x^5 E\phi[t,x]^4+4e^{2c1}k\lambda^2 E\phi[t,x]^4-4e^{2c1}x\lambda^2 E\phi[t,x]^4\right)}{x^6\left(x^2\lambda^2+E\phi[t,x]^2\right)^2}-\right.\right.$$

$$\frac{1}{x^5\left(x^2\lambda^2+E\phi[t,x]^2\right)^3}$$

$$2\left(x^9\lambda^4+2x^7\lambda^2 E\phi[t,x]^2+x^5 E\phi[t,x]^4+4e^{2c1}k\lambda^2 E\phi[t,x]^4-4e^{2c1}x\lambda^2 E\phi[t,x]^4\right)$$

$$\left(2x\lambda^2+2E\phi[t,x]\ E\phi^{(0,1)}[t,x]\right)+\frac{1}{x^5\left(x^2\lambda^2+E\phi[t,x]^2\right)^2}$$

$$\left(9x^8\lambda^4+14x^6\lambda^2 E\phi[t,x]^2+5x^4 E\phi[t,x]^4-4e^{2c1}\lambda^2 E\phi[t,x]^4+\right.$$

$$4x^7\lambda^2 E\phi[t,x]\ E\phi^{(0,1)}[t,x]+4x^5 E\phi[t,x]^3 E\phi^{(0,1)}[t,x]+$$

$$\left.\left.16e^{2c1}k\lambda^2 E\phi[t,x]^3 E\phi^{(0,1)}[t,x]-16e^{2c1}x\lambda^2 E\phi[t,x]^3 E\phi^{(0,1)}[t,x]\right)\right)\Big/$$

$$\left(4\sqrt{2}\ \lambda^2\sqrt{\frac{x^9\lambda^4+2x^7\lambda^2 E\phi[t,x]^2+x^5 E\phi[t,x]^4+4e^{2c1}k\lambda^2 E\phi[t,x]^4-4e^{2c1}x\lambda^2 E\phi[t,x]^4}{x^5\left(x^2\lambda^2+E\phi[t,x]^2\right)^2}}\right.$$

$$\sqrt{\frac{1}{\lambda^2}-\frac{\sqrt{\frac{x^9\lambda^4+2x^7\lambda^2 E\phi[t,x]^2+x^5 E\phi[t,x]^4+4e^{2c1}k\lambda^2 E\phi[t,x]^4-4e^{2c1}x\lambda^2 E\phi[t,x]^4}{x^5\left(x^2\lambda^2+E\phi[t,x]^2\right)^2}}{\lambda^2}}$$

$$\left.\left.\sqrt{1-\frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2}-\frac{\sqrt{\frac{x^9\lambda^4+2x^7\lambda^2 E\phi[t,x]^2+x^5 E\phi[t,x]^4+4e^{2c1}k\lambda^2 E\phi[t,x]^4-4e^{2c1}x\lambda^2 E\phi[t,x]^4}{x^5\left(x^2\lambda^2+E\phi[t,x]^2\right)^2}}{\lambda^2}\right)}\right)\right)$$

Asymptotic Limit

E^{ϕ} assumption + necessary derivatives

In[39]:=
```
Eϕ[t, x] = x * d1[t] + d2[t] + d3[t]/x
Eϕ^(0,1)[t, x] = D[Eϕ[t, x], x]
β^(0,1)[t, x] = D[β[t, x], x]
c1 = Log[c2[t]]
```

Out[39]= $x\,d1[t] + d2[t] + \frac{d3[t]}{x}$

Out[40]= $d1[t] - \frac{d3[t]}{x^2}$

Out[41]=
$$-\left(\left(\left(x^2\lambda^2 + \left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right)^2\right)\right.\right.$$
$$\left(\frac{1}{x^5\left(x^2\lambda^2 + \left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right)^2\right)^2}\left(9x^8\lambda^4 + 4x^7\lambda^2\left(d1[t] - \frac{d3[t]}{x^2}\right)\right.\right.$$
$$\left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right) + 14x^6\lambda^2\left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right)^2 + 4x^5\left(d1[t] - \frac{d3[t]}{x^2}\right)$$
$$\left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right)^3 + 16e^{2c1}k\lambda^2\left(d1[t] - \frac{d3[t]}{x^2}\right)\left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right)^3 -$$
$$16e^{2c1}x\lambda^2\left(d1[t] - \frac{d3[t]}{x^2}\right)\left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right)^3 +$$
$$\left.5x^4\left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right)^4 - 4e^{2c1}\lambda^2\left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right)^4\right) -$$
$$\left(2\left(2x\lambda^2 + 2\left(d1[t] - \frac{d3[t]}{x^2}\right)\left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right)\right)\right.$$
$$\left(x^9\lambda^4 + 2x^7\lambda^2\left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right)^2 + x^5\left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right)^4 +\right.$$
$$\left.\left.4e^{2c1}k\lambda^2\left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right)^4 - 4e^{2c1}x\lambda^2\left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right)^4\right)\right)\Big/$$
$$\left.\left.\left(x^5\left(x^2\lambda^2 + \left(x\,d1[t] + d2[t] + \frac{d3[t]}{x}\right)^2\right)^3\right) -\right.$$

$$\left(5\left(x^9\lambda^4+2x^7\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+x^5\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+4e^{2c1}k\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4e^{2c1}x\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\right)\Big/\left(x^6\left(x^2\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)$$

$$\sqrt{\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(x^9\lambda^4+2x^7\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+x^5\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+4e^{2c1}k\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4e^{2c1}x\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\Big/\left(x^5\left(x^2\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)\right)\right)\alpha[t,x]\right)\Big/\left(8\sqrt{2}\sqrt{1+\lambda^2}\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\sqrt{\left(\left(x^9\lambda^4+2x^7\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+x^5\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+4e^{2c1}k\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4e^{2c1}x\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\Big/\left(x^5\left(x^2\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)\right)}\right)$$

$$\sqrt{\left(1-\frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(x^9\lambda^4+2x^7\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+x^5\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+4e^{2c1}k\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4e^{2c1}x\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\Big/\left(x^5\left(x^2\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)\right)\right)\right)\Big/\left(x^5\left(x^2\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)\right)+\left(\left(x^2\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)\left(\frac{1}{x^5\left(x^2\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2}\left(9x^8\lambda^4+4x^7\lambda^2\left(d1[t]-\frac{d3[t]}{x^2}\right)\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)+14x^6\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+\right.\right.\right.$$

$$4\,x^5\left(d1[t]-\frac{d3[t]}{x^2}\right)\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^3+$$
$$16\,e^{2\,c1}\,k\,\lambda^2\left(d1[t]-\frac{d3[t]}{x^2}\right)\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^3-$$
$$16\,e^{2\,c1}\,x\,\lambda^2\left(d1[t]-\frac{d3[t]}{x^2}\right)\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^3+$$
$$5\,x^4\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4\,e^{2\,c1}\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\Bigg)-$$
$$\left(2\left(2\,x\,\lambda^2+2\left(d1[t]-\frac{d3[t]}{x^2}\right)\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)\right)\right.$$
$$\left(x^9\,\lambda^4+2\,x^7\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+x^5\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+\right.$$
$$\left.\left.4\,e^{2\,c1}\,k\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4\,e^{2\,c1}\,x\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\right)\Big/$$
$$\left(x^5\left(x^2\,\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^3\right)-$$
$$\left(5\left(x^9\,\lambda^4+2\,x^7\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+x^5\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+\right.\right.$$
$$\left.\left.4\,e^{2\,c1}\,k\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4\,e^{2\,c1}\,x\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\right)\Big/$$
$$\left.\left(x^6\left(x^2\,\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)\right)$$
$$\sqrt{\left(1-\frac{1}{2}\,\lambda^2\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(x^9\,\lambda^4+2\,x^7\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+x^5\right.\right.\right.\right.\right.}\right.$$
$$\left.\left.\left.\left.\left.\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+4\,e^{2\,c1}\,k\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4\,e^{2\,c1}\,x\,\lambda^2\right.\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left.\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\Big/\left(x^5\left(x^2\,\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)\right)\right)\right)\right)$$
$$\alpha[t,x]\Bigg)\Big/\left(4\,\sqrt{2}\,\lambda^2\,\sqrt{1+\lambda^2}\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right.$$
$$\sqrt{\left(\left(x^9\,\lambda^4+2\,x^7\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+x^5\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+\right.\right.}$$
$$\left.\left.4\,e^{2\,c1}\,k\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4\,e^{2\,c1}\,x\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\Big/\right.$$

$$\left(x^5\left(x^2\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)$$

$$\sqrt{\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(x^9\lambda^4+2x^7\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+x^5\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.4e^{2c1}k\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4e^{2c1}x\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\Big/\right.\right.\right.$$

$$\left.\left.\left.\left(x^5\left(x^2\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)\right)\right)\right)-$$

$$\frac{1}{\sqrt{2}\ \sqrt{1+\lambda^2}\ \left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2}$$

$$\left(2\right.$$

$$x$$

$$\lambda^2+2$$

$$\left(d1[t]-\frac{d3[t]}{x^2}\right)$$

$$\left.\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)\right)$$

$$\sqrt{\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(x^9\lambda^4+2x^7\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+x^5\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.4e^{2c1}k\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4e^{2c1}x\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\Big/\right.\right.\right.$$

$$\left.\left.\left.\left(x^5\left(x^2\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)\right)\right)\right)$$

$$\sqrt{\left(1-\frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(x^9\lambda^4+2x^7\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.x^5\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+4e^{2c1}k\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4e^{2c1}x\lambda^2\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\Big/\left(x^5\left(x^2\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)\right)\right)\right)\right)$$

$$\alpha[t,x]+\frac{1}{\sqrt{1+\lambda^2}\ \left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^3}$$

$$\sqrt{2}$$

$$\left(d1[$$

$$t]-\frac{d3[t]}{x^2}\right)$$

$$\left(x^2\,\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)\sqrt{\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(x^9\,\lambda^4+2\,x^7\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+x^5\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+4\,e^{2\,c1}\,k\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4\,e^{2\,c1}\,x\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\Big/\left(x^5\left(x^2\,\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)\right)\right)\right)\sqrt{\left(1-\frac{1}{2}\,\lambda^2\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(x^9\,\lambda^4+2\,x^7\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+x^5\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+4\,e^{2\,c1}\,k\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4\,e^{2\,c1}\,x\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\Big/\left(x^5\left(x^2\,\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)\right)\right)\right)\right)\alpha[t,x]-\frac{1}{\sqrt{2}\,\sqrt{1+\lambda^2}\,\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2}\left(x^2\,\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)\sqrt{\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(x^9\,\lambda^4+2\,x^7\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+x^5\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+4\,e^{2\,c1}\,k\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4\,e^{2\,c1}\,x\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\Big/\left(x^5\left(x^2\,\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)\right)\right)\right)\sqrt{\left(1-\frac{1}{2}\,\lambda^2\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(x^9\,\lambda^4+2\,x^7\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2+x^5\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4+4\,e^{2\,c1}\,k\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4-4\,e^{2\,c1}\,x\,\lambda^2\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^4\right)\Big/$$

$$\left(x^5\left(x^2\lambda^2+\left(x\,d1[t]+d2[t]+\frac{d3[t]}{x}\right)^2\right)^2\right)\alpha^{(0,1)}[t,x]$$

Out[42]= Log[c2[t]]

In[43]:=

In[44]:=

Replacing x -> 1/y in Hamiltonian

In[•]:=
```
(*Replacing x with 1/y to allow a series expansion in 1/x*)

$Assumptions = y > 0
Hamiltonian /. {x → 1/y, x^2 → 1/y^2, x^3 → 1/y^3, x^4 → 1/y^4, x^(5/2) → 1/y^(5/2), x^5 → 1/y^5, x^7 → 1/y^7, x^9 → 1/y^9}
```

Out[•]= $y > 0$

Out[•]=

$$-\frac{1}{2\sqrt{\frac{1}{y^2}}\sqrt{1+\lambda^2}}\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)$$

$$\left(1+\frac{1}{2}\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}{y^7}+\frac{\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y^5}+4k\lambda^2\right.\right.\right.\right.$$

$$\left.\left.\left.\left.c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4-\frac{4\lambda^2 c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y}\right)\right)\Big/\right.\right.$$

$$\left.\left.\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\right)+\frac{1}{2\sqrt{1+\lambda^2}}$$

$$\left(\frac{\sqrt{\frac{1}{y^2}}}{\frac{d1[t]}{y}+d2[t]+y\,d3[t]}+\sqrt{\frac{1}{y^2}}\left(\frac{2}{\frac{d1[t]}{y}+d2[t]+y\,d3[t]}-\frac{2\left(d1[t]-y^2\,d3[t]\right)}{y\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}\right)\right)$$

$$\left(1-\frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}{y^7}+\frac{\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y^5}+4k\lambda^2\right.\right.\right.\right.$$

$$c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4-\frac{4\lambda^2 c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\Bigg)\Bigg)+\Bigg(\sqrt{\frac{1}{y^2}}\;y\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)$$

$$\sqrt{\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}{y^7}+\frac{\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y^5}+4k\right.\right.\right.$$

$$\left.\left.\left.\lambda^2 c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4-\frac{4\lambda^2 c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y}\right)\right)\Bigg/$$

$$\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\Bigg)\sqrt{\left(1-\frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2}-\right.\right.}$$

$$\left.\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}{y^7}+\frac{\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y^5}+4k\lambda^2 c2[t]^2\right.\right.\right.$$

$$\left.\left.\left.\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4-\frac{4\lambda^2 c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y}\right)\right)\Bigg/$$

$$\left.\left.\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\right)\right)$$

$$\left(\frac{1}{\sqrt{1+\lambda^2}\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}\sqrt{2}\;y\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\sqrt{\left(\frac{1}{\lambda^2}-\right.\right.}$$

$$\left.\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}{y^7}+\frac{\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y^5}+4k\lambda^2 c2[t]^2\right.\right.\right.$$

$$\left.\left.\left.\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4-\frac{4\lambda^2 c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y}\right)\right)\Bigg/$$

$$\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\sqrt{\left(1-\frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}{y^7}+\frac{\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y^5}+4k\lambda^2c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4-\frac{4\lambda^2c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y}\right)}\right)\Big/\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\right)\right)}-\left(\left(d1[t]-y^2\,d3[t]\right)\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\sqrt{\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}{y^7}+\frac{\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y^5}+4k\lambda^2c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4-\frac{4\lambda^2c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y}\right)}\right)\Big/\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\right)\right)}\right)\Big/\left(\sqrt{\left(1-\frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}{y^7}+\frac{\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y^5}+4k\lambda^2c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4-\frac{4\lambda^2c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y}\right)}\right)\Big/\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\right)\right)}\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\right)$$

$$\left(\sqrt{2}\ \sqrt{1+\lambda^2}\ \left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^3\right)+\frac{1}{2\sqrt{2}\ y^2\ \sqrt{1+\lambda^2}\ \left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}$$

$$(1-k\,y)\left(\frac{\lambda^2}{y^2}+\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2\right)$$

$$\left(-\frac{k\,y^4\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2}{(1-k\,y)^2}-\frac{2\,y^3\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2}{1-k\,y}+\right.$$

$$\left.\frac{2\,y^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)\left(\text{d1}[t]-y^2\,\text{d3}[t]\right)}{1-k\,y}\right)\sqrt{\left(\frac{1}{\lambda^2}-\right.}$$

$$\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2}{y^7}+\frac{\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y^5}+4\,k\,\lambda^2\,\text{c2}[t]^2\right.}\right.$$

$$\left.\left.\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4-\frac{4\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y}\right)\right)\Big/$$

$$\left.\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2\right)\right)\right)$$

$$\sqrt{\left(1-\frac{1}{2}\,\lambda^2\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2}{y^7}+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y^5}+4\,k\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4-\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{4\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y}\right)\right)\Big/\right.\right.$$

$$\left.\left.\left.\left.\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2\right)\right)\right)\right)\right)+$$

$$\frac{1}{\sqrt{1+\lambda^2}\ \left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)}\ \sqrt{2}\ \sqrt{\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2}{y^7}+\right.\right.\right.}\right.$$

$$\frac{\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y^5}+4\,k\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4-\frac{4\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2\right)\right)\Bigg)$$

$$\sqrt{\left(1-\frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2}{y^7}+\frac{\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y^5}+4\,k\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4-\frac{4\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y}\right)}\right)\Bigg)\Bigg)\Bigg/\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2\right)\right)\Bigg)\Bigg)\Bigg)\left(\frac{1}{2}\,y\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)+\frac{1}{2}\lambda^2\left(\frac{1}{y\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)}+\frac{\frac{2}{\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]}-\frac{2\left(\text{d1}[t]-y^2\,\text{d3}[t]\right)}{y\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2}}{y}\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(1-\lambda^2\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2}{y^7}+\frac{\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y^5}+4\,k\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4-\frac{4\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y}\right)}\right)\Bigg)\Bigg)\Bigg/\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2\right)\right)\Bigg)\Bigg)\Bigg)\Bigg)$$

Hamiltonian Series Expansion

In[ ]:=

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(*Series expansion of Hamiltonian after manual square root simplification*)

Series[-1/(2 Sqrt[1/y^2] Sqrt[1+λ^2]) (d1[t]/y + d2[t] + y d3[t])
 (1 + 1/2 (1/λ^2 - (Sqrt[λ^4/y^9 + (2 λ^2 (d1[t]/y + d2[t] + y d3[t])^2)/y^7 + (d1[t]/y + d2[t] + y d3[t])^4/y^5 + 4 k λ^2
     c2[t]^2 (d1[t]/y + d2[t] + y d3[t])^4 - (4 λ^2 c2[t]^2 (d1[t]/y + d2[t] + y d3[t])^4)/y])/
  ((1/y)^(5/2) λ^2 (λ^2/y^2 + (d1[t]/y + d2[t] + y d3[t])^2)))) + 1/(2 Sqrt[1+λ^2])
 (Sqrt[1/y^2]/(d1[t]/y + d2[t] + y d3[t]) + Sqrt[1/y^2] (2/(d1[t]/y + d2[t] + y d3[t]) - (2 (d1[t] - y^2 d3[t]))/(y (d1[t]/y + d2[t] + y d3[t])^2)))
 (1 - 1/2 λ^2 (1/λ^2 - (Sqrt[λ^4/y^9 + (2 λ^2 (d1[t]/y + d2[t] + y d3[t])^2)/y^7 + (d1[t]/y + d2[t] + y d3[t])^4/y^5 + 4 k λ^2
     c2[t]^2 (d1[t]/y + d2[t] + y d3[t])^4 - (4 λ^2 c2[t]^2 (d1[t]/y + d2[t] + y d3[t])^4)/y])/
  ((1/y)^(5/2) λ^2 (λ^2/y^2 + (d1[t]/y + d2[t] + y d3[t])^2)))) +
 (Sqrt[1/y^2] y (d1[t]/y + d2[t] + y d3[t]) Sqrt[1/λ^2 - (Sqrt[λ^4/y^9 + (2 λ^2 (d1[t]/y + d2[t] + y d3[t])^2)/y^7 +
```

$$\frac{\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y^5}+4\,k\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4-\frac{4\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2\right)\right)\Bigg)\sqrt{\Bigg(1-\frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2}{y^7}+\frac{\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y^5}+4\,k\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4-\frac{4\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y}\right)\Bigg)\Bigg/$$

$$\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2\right)\right)\Bigg)\Bigg)\Bigg)$$

$$\left(\frac{1}{\sqrt{1+\lambda^2}\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2}\sqrt{2}\,y\left(\frac{\lambda^2}{y^2}+\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2\right)\right.$$

$$\sqrt{\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2}{y^7}+\frac{\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y^5}+4\,k\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4-\frac{4\,\lambda^2\,\text{c2}[t]^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^4}{y}\right)\Bigg)\Bigg/$$

$$\left.\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2\right)\right)\Bigg)\right)$$

$$\sqrt{\left(1-\frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{\text{d1}[t]}{y}+\text{d2}[t]+y\,\text{d3}[t]\right)^2}{y^7}+\right.\right.\right.}\right.$$

$$\frac{\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y^5}+4k\lambda^2 c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4-\frac{4\lambda^2 c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y}\Bigg)\Bigg/\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\Bigg)\Bigg)-$$

$$\left(\left(d1[t]-y^2\,d3[t]\right)\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\sqrt{\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}{y^7}+\frac{\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y^5}+4k\lambda^2 c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4-\frac{4\lambda^2 c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y}\right)}\Bigg)\Bigg/\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\right)\right)$$

$$\sqrt{\left(1-\frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}{y^7}+\frac{\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y^5}+4k\lambda^2 c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4-\frac{4\lambda^2 c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y}\right)}\Bigg)\Bigg/\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\right)\right)\Bigg/\left(\sqrt{2}\,\sqrt{1+\lambda^2}\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^3\right)+\frac{1}{2\sqrt{2}\,y^2\sqrt{1+\lambda^2}\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}$$

$$(1 - k\,y)\left(\frac{\lambda^2}{y^2} + \left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^2\right)$$

$$\left(-\frac{k\,y^4\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^2}{(1 - k\,y)^2} - \frac{2\,y^3\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^2}{1 - k\,y} + \right.$$

$$\left.\frac{2\,y^2\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)\left(d1[t] - y^2\,d3[t]\right)}{1 - k\,y}\right)\sqrt{\left(\frac{1}{\lambda^2} - \right.}$$

$$\left(\sqrt{\left(\frac{\lambda^4}{y^9} + \frac{2\,\lambda^2\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^2}{y^7} + \frac{\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^4}{y^5} + 4\,k\,\lambda^2\,c2[t]^2\right.\right.$$

$$\left.\left.\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^4 - \frac{4\,\lambda^2\,c2[t]^2\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^4}{y}\right)\right)\Big/$$

$$\left.\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2} + \left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^2\right)\right)\right)$$

$$\sqrt{\left(1 - \frac{1}{2}\,\lambda^2\left(\frac{1}{\lambda^2} - \left(\sqrt{\left(\frac{\lambda^4}{y^9} + \frac{2\,\lambda^2\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^2}{y^7} + \right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^4}{y^5} + 4\,k\,\lambda^2\,c2[t]^2\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^4 - \right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{4\,\lambda^2\,c2[t]^2\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^4}{y}\right)\right)\Big/\right.\right.$$

$$\left.\left.\left.\left.\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2} + \left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^2\right)\right)\right)\right)\right) +$$

$$\frac{1}{\sqrt{1 + \lambda^2}\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)}\sqrt{2}\sqrt{\left(\frac{1}{\lambda^2} - \left(\sqrt{\left(\frac{\lambda^4}{y^9} + \frac{2\,\lambda^2\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^2}{y^7} + \right.\right.\right.}\right.$$

$$\left.\left.\left.\frac{\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^4}{y^5} + 4\,k\,\lambda^2\,c2[t]^2\left(\frac{d1[t]}{y} + d2[t] + y\,d3[t]\right)^4 - \right.\right.\right.$$

$$\frac{4\,\lambda^2\,c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y}\Bigg)\Bigg)\Bigg/$$

$$\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\Bigg)$$

$$\sqrt{\left(1-\frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}{y^7}+\right.\right.\right.\right.}$$

$$\left.\left.\left.\left.\frac{\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y^5}+4\,k\,\lambda^2\,c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4-\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{4\,\lambda^2\,c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y}\right)\right)\Bigg/\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\right)\right)\left(\frac{1}{2}\,y\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)+\right.$$

$$\left.\frac{1}{2}\,\lambda^2\left(\frac{1}{y\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)}+\frac{\frac{2}{\frac{d1[t]}{y}+d2[t]+y\,d3[t]}-\frac{2\left(d1[t]-y^2\,d3[t]\right)}{y\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}}{y}\right)\right)\Bigg)\Bigg)\Bigg/$$

$$\left(\sqrt{2}\left(1-\lambda^2\left(\frac{1}{\lambda^2}-\left(\sqrt{\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2}{y^7}+\frac{\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y^5}+\right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\left.4\,k\,\lambda^2\,c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4-\right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\left.\frac{4\,\lambda^2\,c2[t]^2\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^4}{y}\right)\right)\Bigg/\right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\left.\left(\left(\frac{1}{y}\right)^{5/2}\lambda^2\left(\frac{\lambda^2}{y^2}+\left(\frac{d1[t]}{y}+d2[t]+y\,d3[t]\right)^2\right)\right)\right)\right)\right)\right),\ \{y,\ 0,\ 2\}\Bigg]$$

In[ ]:=

```
(*System of equations for d1, d2, d3*)

Solve[{{(λ^4 + 2 λ^2 d1[t]^2 + d1[t]^4 + λ^2 Sqrt[(λ^2 + d1[t]^2)^2] - d1[t]^2 Sqrt[(λ^2 + d1[t]^2)^2] -
        2 λ^2 d1[t]^2 Sqrt[(λ^2 + d1[t]^2)^2]) / (4 λ^2 Sqrt[1 + λ^2] d1[t] Sqrt[(λ^2 + d1[t]^2)^2]) == 0,
  ((λ^4 d2[t] + 2 λ^2 d1[t]^2 d2[t] + d1[t]^4 d2[t] + λ^2 Sqrt[(λ^2 + d1[t]^2)^2] d2[t] -
      d1[t]^2 Sqrt[(λ^2 + d1[t]^2)^2] d2[t] - 2 λ^2 d1[t]^2 Sqrt[(λ^2 + d1[t]^2)^2] d2[t]) y) /
   (4 λ^2 Sqrt[1 + λ^2] d1[t]^2 Sqrt[(λ^2 + d1[t]^2)^2]) == 0,
  (3 (λ^2 + d1[t]^2 + Sqrt[(λ^2 + d1[t]^2)^2]) (-d2[t]^2 + d1[t] × d3[t])) /
   (4 Sqrt[1 + λ^2] d1[t]^3 (λ^2 + d1[t]^2)) +
  (λ^2 d3[t] + d1[t]^2 d3[t] - Sqrt[(λ^2 + d1[t]^2)^2] d3[t] - 2 λ^2 Sqrt[(λ^2 + d1[t]^2)^2] d3[t]) /
   (4 λ^2 Sqrt[1 + λ^2] Sqrt[(λ^2 + d1[t]^2)^2]) +
  1/Sqrt[2] (-(((λ^2 + d1[t]^2 - Sqrt[(λ^2 + d1[t]^2)^2]) (λ^2 + d1[t]^2 + Sqrt[(λ^2 + d1[t]^2)^2])
        d2[t] (k λ^2 d1[t] + k d1[t]^3 + 4 λ^2 d2[t])) /
      (4 Sqrt[2] λ^2 Sqrt[1 + λ^2] d1[t]^3 (λ^2 + d1[t]^2) Sqrt[(λ^2 + d1[t]^2)^2])) +
    1/(Sqrt[1 + λ^2] d1[t]^2) (λ^2 + d1[t]^2) Sqrt[(λ^2 + d1[t]^2 - Sqrt[(λ^2 + d1[t]^2)^2]) /
      (λ^2 (λ^2 + d1[t]^2))]
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$$\sqrt{\frac{\lambda^2+\text{d1}[t]^2+\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2+\text{d1}[t]^2}}\left(\left(\text{d1}[t]\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}\right.\right.$$

$$\sqrt{\frac{\lambda^2+\text{d1}[t]^2-\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\text{d1}[t]^2\right)}}\sqrt{\frac{\lambda^2+\text{d1}[t]^2+\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2+\text{d1}[t]^2}}$$

$$\left(-\frac{4\,\text{d1}[t]\times\text{d2}[t]\left(\lambda^2\,\text{d1}[t]\times\text{d2}[t]+\text{d1}[t]^3\,\text{d2}[t]\right)}{\left(\left(\lambda^2+\text{d1}[t]^2\right)^2\right)^{3/2}}+\frac{1}{\left(\lambda^2+\text{d1}[t]^2\right)^3}\right.$$

$$\sqrt{\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4}\left(-\lambda^2\,\text{d2}[t]^2+3\,\text{d1}[t]^2\,\text{d2}[t]^2-\right.$$

$$\left.2\lambda^2\,\text{d1}[t]\times\text{d3}[t]-2\,\text{d1}[t]^3\,\text{d3}[t]\right)+\frac{1}{2\left(\lambda^2+\text{d1}[t]^2\right)}$$

$$\sqrt{\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4}\left(-\frac{\left(4\lambda^2\,\text{d1}[t]\times\text{d2}[t]+4\,\text{d1}[t]^3\,\text{d2}[t]\right)^2}{4\left(\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4\right)^2}+\right.$$

$$\left.\left.\left.\frac{2\lambda^2\left(\text{d2}[t]^2+2\,\text{d1}[t]\times\text{d3}[t]\right)+2\,\text{d1}[t]^2\left(3\,\text{d2}[t]^2+2\,\text{d1}[t]\times\text{d3}[t]\right)}{\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4}\right)\right)\right)\Big/$$

$$\left(2\sqrt{2}\left(\lambda^2+\text{d1}[t]^2+\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}\right)\right)+\frac{1}{\sqrt{2}}\sqrt{\frac{\lambda^2+\text{d1}[t]^2+\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2+\text{d1}[t]^2}}$$

$$\left(\text{d1}[t]\sqrt{\frac{\lambda^2+\text{d1}[t]^2-\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\text{d1}[t]^2\right)}}\right.$$

$$\left(\frac{4\,\text{d1}[t]\times\text{d2}[t]\left(\lambda^2\,\text{d1}[t]\times\text{d2}[t]+\text{d1}[t]^3\,\text{d2}[t]\right)}{\left(\left(\lambda^2+\text{d1}[t]^2\right)^2\right)^{3/2}}-\frac{1}{\left(\lambda^2+\text{d1}[t]^2\right)^3}\right.$$

$$\sqrt{\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4}\left(-\lambda^2\,\text{d2}[t]^2+3\,\text{d1}[t]^2\,\text{d2}[t]^2-\right.$$

$$\left.2\lambda^2\,\text{d1}[t]\times\text{d3}[t]-2\,\text{d1}[t]^3\,\text{d3}[t]\right)-\frac{1}{2\left(\lambda^2+\text{d1}[t]^2\right)}$$

$$\sqrt{\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4}\left(-\frac{\left(4\lambda^2\,\text{d1}[t]\times\text{d2}[t]+4\,\text{d1}[t]^3\,\text{d2}[t]\right)^2}{4\left(\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4\right)^2}+\right.$$

$$\frac{2\lambda^2\left(\mathrm{d2}[t]^2+2\,\mathrm{d1}[t]\times\mathrm{d3}[t]\right)+2\,\mathrm{d1}[t]^2\left(3\,\mathrm{d2}[t]^2+2\,\mathrm{d1}[t]\times\mathrm{d3}[t]\right)}{\lambda^4+2\lambda^2\,\mathrm{d1}[t]^2+\mathrm{d1}[t]^4}\Bigg)\Bigg)+$$

$$\frac{1}{\sqrt{\left(\lambda^2+\mathrm{d1}[t]^2\right)^2}}\left(\lambda^2+\mathrm{d1}[t]^2\right)\left(\sqrt{\frac{\lambda^2+\mathrm{d1}[t]^2-\sqrt{\left(\lambda^2+\mathrm{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\mathrm{d1}[t]^2\right)}}\,\mathrm{d3}[t]+\right.$$

$$\frac{1}{2\lambda^2\left(\frac{1}{\lambda^2}-\frac{\sqrt{\left(\lambda^2+\mathrm{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\mathrm{d1}[t]^2\right)}\right)}\,\mathrm{d1}[t]\,\sqrt{\frac{\lambda^2+\mathrm{d1}[t]^2-\sqrt{\left(\lambda^2+\mathrm{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\mathrm{d1}[t]^2\right)}}$$

$$\left(\frac{4\,\mathrm{d1}[t]\times\mathrm{d2}[t]\left(\lambda^2\,\mathrm{d1}[t]\times\mathrm{d2}[t]+\mathrm{d1}[t]^3\,\mathrm{d2}[t]\right)}{\left(\left(\lambda^2+\mathrm{d1}[t]^2\right)^2\right)^{3/2}}-\frac{1}{\left(\lambda^2+\mathrm{d1}[t]^2\right)^3}\right.$$

$$\sqrt{\lambda^4+2\lambda^2\,\mathrm{d1}[t]^2+\mathrm{d1}[t]^4}\left(-\lambda^2\,\mathrm{d2}[t]^2+3\,\mathrm{d1}[t]^2\,\mathrm{d2}[t]^2-\right.$$

$$\left.2\lambda^2\,\mathrm{d1}[t]\times\mathrm{d3}[t]-2\,\mathrm{d1}[t]^3\,\mathrm{d3}[t]\right)-\frac{1}{2\left(\lambda^2+\mathrm{d1}[t]^2\right)}$$

$$\sqrt{\lambda^4+2\lambda^2\,\mathrm{d1}[t]^2+\mathrm{d1}[t]^4}\left(-\frac{\left(4\lambda^2\,\mathrm{d1}[t]\times\mathrm{d2}[t]+4\,\mathrm{d1}[t]^3\,\mathrm{d2}[t]\right)^2}{4\left(\lambda^4+2\lambda^2\,\mathrm{d1}[t]^2+\mathrm{d1}[t]^4\right)^2}+\right.$$

$$\left(2\lambda^2\left(\mathrm{d2}[t]^2+2\,\mathrm{d1}[t]\times\mathrm{d3}[t]\right)+2\,\mathrm{d1}[t]^2\left(3\,\mathrm{d2}[t]^2+\right.\right.$$

$$\left.\left.\left.\left.\left.\left.2\,\mathrm{d1}[t]\times\mathrm{d3}[t]\right)\right)/\left(\lambda^4+2\lambda^2\,\mathrm{d1}[t]^2+\mathrm{d1}[t]^4\right)\right)\right)\right)\right)+$$

$$\frac{1}{\sqrt{2}\,\sqrt{\left(\lambda^2+\mathrm{d1}[t]^2\right)^2}}\,\mathrm{d1}[t]\left(\lambda^2+\mathrm{d1}[t]^2\right)\sqrt{\frac{\lambda^2+\mathrm{d1}[t]^2-\sqrt{\left(\lambda^2+\mathrm{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\mathrm{d1}[t]^2\right)}}$$

$$\sqrt{\frac{\lambda^2+\mathrm{d1}[t]^2+\sqrt{\left(\lambda^2+\mathrm{d1}[t]^2\right)^2}}{\lambda^2+\mathrm{d1}[t]^2}}$$

$$\left(-\frac{1}{4\sqrt{1+\lambda^2}\,\mathrm{d1}[t]^4}\sqrt{\frac{\lambda^2+\mathrm{d1}[t]^2-\sqrt{\left(\lambda^2+\mathrm{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\mathrm{d1}[t]^2\right)}}\right.$$

$$\sqrt{\frac{\lambda^2+\text{d1}[t]^2+\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2+\text{d1}[t]^2}}\ \left(\text{k}^2\,\lambda^2\,\text{d1}[t]^2+\text{k}^2\,\text{d1}[t]^4-2\,\text{k}\,\lambda^2\,\text{d1}[t]\times\right.$$

$$\left.\text{d2}[t]-6\,\lambda^2\,\text{d2}[t]^2-2\,\text{d1}[t]^2\,\text{d2}[t]^2+4\,\lambda^2\,\text{d1}[t]\times\text{d3}[t]+4\,\text{d1}[t]^3\,\text{d3}[t]\right)+$$

$$\frac{1}{\sqrt{1+\lambda^2}}\ \sqrt{2}\ \left(\left(\left(\lambda^2+\text{d1}[t]^2\right)^2\sqrt{\frac{\lambda^2+\text{d1}[t]^2-\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\text{d1}[t]^2\right)}}\right.\right.$$

$$\sqrt{\frac{\lambda^2+\text{d1}[t]^2+\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2+\text{d1}[t]^2}}$$

$$\left(-\frac{4\,\text{d1}[t]\times\text{d2}[t]\ \left(\lambda^2\,\text{d1}[t]\times\text{d2}[t]+\text{d1}[t]^3\,\text{d2}[t]\right)}{\left(\left(\lambda^2+\text{d1}[t]^2\right)^2\right)^{3/2}}+\frac{1}{\left(\lambda^2+\text{d1}[t]^2\right)^3}\right.$$

$$\sqrt{\lambda^4+2\,\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4}\ \left(-\lambda^2\,\text{d2}[t]^2+3\,\text{d1}[t]^2\,\text{d2}[t]^2-\right.$$

$$\left.2\,\lambda^2\,\text{d1}[t]\times\text{d3}[t]-2\,\text{d1}[t]^3\,\text{d3}[t]\right)+\frac{1}{2\left(\lambda^2+\text{d1}[t]^2\right)}$$

$$\sqrt{\lambda^4+2\,\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4}\ \left(-\frac{\left(4\,\lambda^2\,\text{d1}[t]\times\text{d2}[t]+4\,\text{d1}[t]^3\,\text{d2}[t]\right)^2}{4\left(\lambda^4+2\,\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4\right)^2}+\right.$$

$$\left.\left.\left.\frac{2\,\lambda^2\left(\text{d2}[t]^2+2\,\text{d1}[t]\times\text{d3}[t]\right)+2\,\text{d1}[t]^2\left(3\,\text{d2}[t]^2+2\,\text{d1}[t]\times\text{d3}[t]\right)}{\lambda^4+2\,\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4}\right)\right)\right)$$

$$\left(2\,\sqrt{2}\ \text{d1}[t]^2\left(\lambda^2+\text{d1}[t]^2+\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}\right)\right)+$$

$$\frac{1}{\sqrt{2}}\ \sqrt{\frac{\lambda^2+\text{d1}[t]^2+\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2+\text{d1}[t]^2}}\ \left(\sqrt{\frac{\lambda^2+\text{d1}[t]^2-\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\text{d1}[t]^2\right)}}\right.$$

$$\left(-\frac{4\,\text{d2}[t]^2}{\text{d1}[t]^2}+\frac{\left(\lambda^2+\text{d1}[t]^2\right)\left(3\,\text{d2}[t]^2-2\,\text{d1}[t]\times\text{d3}[t]\right)}{\text{d1}[t]^4}+\right.$$

$$\left.\left.\frac{\text{d2}[t]^2+2\,\text{d1}[t]\times\text{d3}[t]}{\text{d1}[t]^2}\right)+\frac{1}{2\,\lambda^2\,\text{d1}[t]^2\left(\frac{1}{\lambda^2}-\frac{\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\text{d1}[t]^2\right)}\right)}\right.$$

$$\left(\lambda^2+\text{d1}[t]^2\right)\sqrt{\frac{\lambda^2+\text{d1}[t]^2-\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\text{d1}[t]^2\right)}}$$

$$\left(\frac{4\,\text{d1}[t]\times\text{d2}[t]\left(\lambda^2\,\text{d1}[t]\times\text{d2}[t]+\text{d1}[t]^3\,\text{d2}[t]\right)}{\left(\left(\lambda^2+\text{d1}[t]^2\right)^2\right)^{3/2}}-\frac{1}{\left(\lambda^2+\text{d1}[t]^2\right)^3}\right.$$

$$\sqrt{\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4}\left(-\lambda^2\,\text{d2}[t]^2+3\,\text{d1}[t]^2\,\text{d2}[t]^2-\right.$$

$$\left.2\lambda^2\,\text{d1}[t]\times\text{d3}[t]-2\,\text{d1}[t]^3\,\text{d3}[t]\right)-\frac{1}{2\left(\lambda^2+\text{d1}[t]^2\right)}$$

$$\sqrt{\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4}\left(-\frac{\left(4\lambda^2\,\text{d1}[t]\times\text{d2}[t]+4\,\text{d1}[t]^3\,\text{d2}[t]\right)^2}{4\left(\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4\right)^2}+\right.$$

$$\left(2\lambda^2\left(\text{d2}[t]^2+2\,\text{d1}[t]\times\text{d3}[t]\right)+2\,\text{d1}[t]^2\left(3\,\text{d2}[t]^2+\right.\right.$$

$$\left.\left.\left.\left.2\,\text{d1}[t]\times\text{d3}[t]\right)\right)\Big/\left(\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4\right)\right)\Big)\Big)\Big)+$$

$$\frac{1}{\sqrt{2}\ \sqrt{1+\lambda^2}}\left(-\left(\left(\left(\lambda^2+\text{d1}[t]^2\right)^2\sqrt{\frac{\lambda^2+\text{d1}[t]^2-\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\text{d1}[t]^2\right)}}\right.\right.\right.$$

$$\left.\left.\sqrt{\frac{\lambda^2+\text{d1}[t]^2+\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2+\text{d1}[t]^2}}\right.$$

$$\left(-\frac{4\,\text{d1}[t]\times\text{d2}[t]\left(\lambda^2\,\text{d1}[t]\times\text{d2}[t]+\text{d1}[t]^3\,\text{d2}[t]\right)}{\left(\left(\lambda^2+\text{d1}[t]^2\right)^2\right)^{3/2}}+\frac{1}{\left(\lambda^2+\text{d1}[t]^2\right)^3}\right.$$

$$\sqrt{\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4}\left(-\lambda^2\,\text{d2}[t]^2+3\,\text{d1}[t]^2\,\text{d2}[t]^2-\right.$$

$$\left.2\lambda^2\,\text{d1}[t]\times\text{d3}[t]-2\,\text{d1}[t]^3\,\text{d3}[t]\right)+\frac{1}{2\left(\lambda^2+\text{d1}[t]^2\right)}$$

$$\sqrt{\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4}\left(-\frac{\left(4\lambda^2\,\text{d1}[t]\times\text{d2}[t]+4\,\text{d1}[t]^3\,\text{d2}[t]\right)^2}{4\left(\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4\right)^2}+\right.$$

$$\left(2\lambda^2\left(\text{d2}[t]^2+2\,\text{d1}[t]\times\text{d3}[t]\right)+2\,\text{d1}[t]^2\left(3\,\text{d2}[t]^2+\right.\right.$$

$$2\,d1[t]\times d3[t]\Big)\Big)\Big/\left(\lambda^4+2\,\lambda^2\,d1[t]^2+d1[t]^4\right)\Big)\Big)\Big)\Big/$$

$$\left(2\sqrt{2}\;d1[t]^2\left(\lambda^2+d1[t]^2+\sqrt{\left(\lambda^2+d1[t]^2\right)^2}\right)\right)-\frac{1}{\sqrt{2}}$$

$$\sqrt{\frac{\lambda^2+d1[t]^2+\sqrt{\left(\lambda^2+d1[t]^2\right)^2}}{\lambda^2+d1[t]^2}}\left(\sqrt{\frac{\lambda^2+d1[t]^2-\sqrt{\left(\lambda^2+d1[t]^2\right)^2}}{\lambda^2\left(\lambda^2+d1[t]^2\right)}}\right.$$

$$\left(-\frac{6\,d2[t]^2}{d1[t]^2}+\frac{d2[t]^2+2\,d1[t]\times d3[t]}{d1[t]^2}+\left(\lambda^2+d1[t]^2\right)\left(-\frac{d3[t]}{d1[t]^3}-\right.\right.$$

$$\left.\left.\frac{3\left(-2\,d2[t]^2+d1[t]\times d3[t]\right)}{d1[t]^4}\right)\right)+\frac{1}{2\,\lambda^2\,d1[t]^2\left(\frac{1}{\lambda^2}-\frac{\sqrt{\left(\lambda^2+d1[t]^2\right)^2}}{\lambda^2\left(\lambda^2+d1[t]^2\right)}\right)}$$

$$\left(\lambda^2+d1[t]^2\right)\sqrt{\frac{\lambda^2+d1[t]^2-\sqrt{\left(\lambda^2+d1[t]^2\right)^2}}{\lambda^2\left(\lambda^2+d1[t]^2\right)}}$$

$$\left(\frac{4\,d1[t]\times d2[t]\left(\lambda^2\,d1[t]\times d2[t]+d1[t]^3\,d2[t]\right)}{\left(\left(\lambda^2+d1[t]^2\right)^2\right)^{3/2}}-\frac{1}{\left(\lambda^2+d1[t]^2\right)^3}\right.$$

$$\sqrt{\lambda^4+2\,\lambda^2\,d1[t]^2+d1[t]^4}\left(-\lambda^2\,d2[t]^2+3\,d1[t]^2\,d2[t]^2-\right.$$

$$\left.2\,\lambda^2\,d1[t]\times d3[t]-2\,d1[t]^3\,d3[t]\right)-\frac{1}{2\left(\lambda^2+d1[t]^2\right)}$$

$$\sqrt{\lambda^4+2\,\lambda^2\,d1[t]^2+d1[t]^4}\left(-\frac{\left(4\,\lambda^2\,d1[t]\times d2[t]+4\,d1[t]^3\,d2[t]\right)^2}{4\left(\lambda^4+2\,\lambda^2\,d1[t]^2+d1[t]^4\right)^2}+\right.$$

$$\left(2\,\lambda^2\left(d2[t]^2+2\,d1[t]\times d3[t]\right)+2\,d1[t]^2\left(3\,d2[t]^2+2\,d1[t]\times\right.\right.$$

$$\left.\left.\left.\left.d3[t]\right)\right)\Big/\left(\lambda^4+2\,\lambda^2\,d1[t]^2+d1[t]^4\right)\Big)\Big)\Big)\Big)+\frac{1}{\sqrt{1+\lambda^2}}\;\sqrt{2}$$

$$\left(-\frac{\left(\lambda^2+\text{d1}[t]^2\right)\sqrt{\frac{\lambda^2+\text{d1}[t]^2-\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\text{d1}[t]^2\right)}}\sqrt{\frac{\lambda^2+\text{d1}[t]^2+\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2+\text{d1}[t]^2}}\,\text{d2}[t]^2}{2\sqrt{2}\,\text{d1}[t]^4}+\right.$$

$$\frac{\sqrt{\frac{\lambda^2+\text{d1}[t]^2-\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\text{d1}[t]^2\right)}}\sqrt{\frac{\lambda^2+\text{d1}[t]^2+\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2+\text{d1}[t]^2}}\left(\frac{\text{d3}[t]}{2}+\frac{3\lambda^2\left(-\text{d2}[t]^2+\text{d1}[t]\times\text{d3}[t]\right)}{2\,\text{d1}[t]^3}\right)}{\sqrt{2}\,\text{d1}[t]}+$$

$$\frac{1}{2\,\text{d1}[t]}\left(\lambda^2+\text{d1}[t]^2\right)\left(\left(\left(\lambda^2+\text{d1}[t]^2\right)\sqrt{\frac{\lambda^2+\text{d1}[t]^2-\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2\left(\lambda^2+\text{d1}[t]^2\right)}}\right.\right.$$

$$\sqrt{\frac{\lambda^2+\text{d1}[t]^2+\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2+\text{d1}[t]^2}}$$

$$\left(-\frac{4\,\text{d1}[t]\times\text{d2}[t]\left(\lambda^2\,\text{d1}[t]\times\text{d2}[t]+\text{d1}[t]^3\,\text{d2}[t]\right)}{\left(\left(\lambda^2+\text{d1}[t]^2\right)^2\right)^{3/2}}+\frac{1}{\left(\lambda^2+\text{d1}[t]^2\right)^3}\right.$$

$$\sqrt{\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4}\left(-\lambda^2\,\text{d2}[t]^2+3\,\text{d1}[t]^2\,\text{d2}[t]^2-\right.$$

$$\left.2\lambda^2\,\text{d1}[t]\times\text{d3}[t]-2\,\text{d1}[t]^3\,\text{d3}[t]\right)+\frac{1}{2\left(\lambda^2+\text{d1}[t]^2\right)}$$

$$\sqrt{\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4}\left(-\frac{\left(4\lambda^2\,\text{d1}[t]\times\text{d2}[t]+4\,\text{d1}[t]^3\,\text{d2}[t]\right)^2}{4\left(\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4\right)^2}+\right.$$

$$\left(2\lambda^2\left(\text{d2}[t]^2+2\,\text{d1}[t]\times\text{d3}[t]\right)+2\,\text{d1}[t]^2\left(3\,\text{d2}[t]^2+\right.\right.$$

$$\left.\left.\left.\left.2\,\text{d1}[t]\times\text{d3}[t]\right)\right)/\left(\lambda^4+2\lambda^2\,\text{d1}[t]^2+\text{d1}[t]^4\right)\right)\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{2}\,\text{d1}[t]\left(\lambda^2+\text{d1}[t]^2+\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}\right)\right)+\frac{1}{\sqrt{2}}$$

$$\sqrt{\frac{\lambda^2+\text{d1}[t]^2+\sqrt{\left(\lambda^2+\text{d1}[t]^2\right)^2}}{\lambda^2+\text{d1}[t]^2}}$$

$$\left(\frac{\sqrt{\frac{\lambda^2+d1[t]^2-\sqrt{(\lambda^2+d1[t]^2)^2}}{\lambda^2(\lambda^2+d1[t]^2)}}\left(d2[t]^2-d1[t]\times d3[t]\right)}{d1[t]^3}+\right.$$

$$\frac{1}{2\lambda^2 d1[t]\left(\frac{1}{\lambda^2}-\frac{\sqrt{(\lambda^2+d1[t]^2)^2}}{\lambda^2(\lambda^2+d1[t]^2)}\right)}\sqrt{\frac{\lambda^2+d1[t]^2-\sqrt{(\lambda^2+d1[t]^2)^2}}{\lambda^2(\lambda^2+d1[t]^2)}}$$

$$\left(\frac{4\,d1[t]\times d2[t]\left(\lambda^2 d1[t]\times d2[t]+d1[t]^3 d2[t]\right)}{\left((\lambda^2+d1[t]^2)^2\right)^{3/2}}-\right.$$

$$\frac{1}{(\lambda^2+d1[t]^2)^3}\sqrt{\lambda^4+2\lambda^2 d1[t]^2+d1[t]^4}\,\left(-\lambda^2 d2[t]^2+\right.$$

$$\left.3\,d1[t]^2 d2[t]^2-2\lambda^2 d1[t]\times d3[t]-2\,d1[t]^3 d3[t]\right)-$$

$$\frac{1}{2(\lambda^2+d1[t]^2)}\sqrt{\lambda^4+2\lambda^2 d1[t]^2+d1[t]^4}$$

$$\left(-\frac{\left(4\lambda^2 d1[t]\times d2[t]+4\,d1[t]^3 d2[t]\right)^2}{4\left(\lambda^4+2\lambda^2 d1[t]^2+d1[t]^4\right)^2}+\right.$$

$$\left(2\lambda^2\left(d2[t]^2+2\,d1[t]\times d3[t]\right)+2\,d1[t]^2\right.$$

$$\left.\left(3\,d2[t]^2+2\,d1[t]\times d3[t]\right)\right)/\left(\lambda^4+2\lambda^2 d1[t]^2+\right.$$

$$\left.\left.\left.\left.d1[t]^4\right)\right)\right)\right)\right)\right)\right)\right) == 0\}, \{d1[t], d2[t], d3[t]\}]$$

Out[ ]=

$$\left\{\left\{d1[t]\to -1 \text{ if } -\lambda^2+\sqrt{(1+\lambda^2)^2}==1,\ d3[t]\to -\frac{3}{2}d2[t]^2 \text{ if } -\lambda^2+\sqrt{(1+\lambda^2)^2}==1\right\},\right.$$

$$\left.\left\{d1[t]\to 1 \text{ if } -\lambda^2+\sqrt{(1+\lambda^2)^2}==1,\ d3[t]\to \frac{3\,d2[t]^2}{2} \text{ if } -\lambda^2+\sqrt{(1+\lambda^2)^2}==1\right\}\right\}$$

Asymptotic α assumption and d constants from Hamiltonian

In[50]:=
```
(*Constants from Hamiltonian expansion*)

d1[t] = 1
d3[t] = 3 d2[t]^2 / 2

(*Setting d2 to be time independent bc of staticity;
this may be wrong!!!*)

d2'[t] = 0
```

Out[50]= 1

Out[51]= $\frac{3\,d2[t]^2}{2}$

Out[52]= 0

In[ ]:=
```
D[d2[t], t]
```
Out[ ]= 0

In[ ]:=
```
d2'[t]
```
Out[ ]= 0

In[53]:=
```
(*Asymptotic α assumption*)

α[t, x] = z1 + z2/x + z3/x^2
α^(0,1)[t, x] = D[α[t, x], x]
```

Out[53]= $z1 + \frac{z2}{x} + \frac{z3}{x^2}$

Out[54]= $-\frac{z2}{x^2} - \frac{2\,z3}{x^3}$

Solve for α?

$\dot{K_\phi}$ Expansion

In[55]:=

```
Kϕ^(1,0)[t, x]
```

Replacing x->1/y

In[56]:=

```
(*Kϕ dot in terms of x = 1/y*)

KϕDot[t, y] = Kϕ^(1,0)[t, x] /. {x → 1/y, x^2 → 1/y^2, x^3 → 1/y^3,
   x^4 → 1/y^4, x^(5/2) → 1/y^(5/2), x^5 → 1/y^5, x^6 → 1/y^6, x^7 → 1/y^7, x^8 → 1/y^8, x^9 → 1/y^9}
```

Out[56]=

$$\left(z1 + y\,z2 + y^2\,z3\right)\left(\frac{\lambda^2}{y^2} + \left(\frac{1}{y} + d2[t] + \frac{3}{2}\,y\,d2[t]^2\right)^2\right)$$

$$\left(-\left(\left(\left(5\,y^6\left(\frac{\lambda^4}{y^9} + \frac{2\,\lambda^2\left(\frac{1}{y} + d2[t] + \frac{3}{2}\,y\,d2[t]^2\right)^2}{y^7} + \right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{\left(\frac{1}{y} + d2[t] + \frac{3}{2}\,y\,d2[t]^2\right)^4}{y^5} + 4\,k\,\lambda^2\,c2[t]^2\left(\frac{1}{y} + d2[t] + \frac{3}{2}\,y\,d2[t]^2\right)^4 - \right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{4\,\lambda^2\,c2[t]^2\left(\frac{1}{y} + d2[t] + \frac{3}{2}\,y\,d2[t]^2\right)^4}{y}\right)\right) \Big/ \left(\frac{\lambda^2}{y^2} + \left(\frac{1}{y} + d2[t] + \frac{3}{2}\,y\,d2[t]^2\right)^2\right)^2\right) -$$

$$\left(2\,y^5\left(\frac{\lambda^4}{y^9} + \frac{2\,\lambda^2\left(\frac{1}{y} + d2[t] + \frac{3}{2}\,y\,d2[t]^2\right)^2}{y^7} + \frac{\left(\frac{1}{y} + d2[t] + \frac{3}{2}\,y\,d2[t]^2\right)^4}{y^5} + \right.\right.$$

$$\left.\left.4\,k\,\lambda^2\,c2[t]^2\left(\frac{1}{y} + d2[t] + \frac{3}{2}\,y\,d2[t]^2\right)^4 - \frac{4\,\lambda^2\,c2[t]^2\left(\frac{1}{y} + d2[t] + \frac{3}{2}\,y\,d2[t]^2\right)^4}{y}\right)\right.$$

$$\left(\frac{2\,\lambda^2}{y}+2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)\left(1-\frac{3}{2}\,y^2\,\text{d2}[t]^2\right)\right)\Bigg/$$

$$\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2\right)^3+$$

$$\left(y^5\left(\frac{9\,\lambda^4}{y^8}+\frac{14\,\lambda^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2}{y^6}+\frac{5\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y^4}-4\,\lambda^2\,\text{c2}[t]^2\right.\right.$$

$$\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4+\frac{4\,\lambda^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)\left(1-\frac{3}{2}\,y^2\,\text{d2}[t]^2\right)}{y^7}+$$

$$\frac{4\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^3\left(1-\frac{3}{2}\,y^2\,\text{d2}[t]^2\right)}{y^5}+$$

$$16\,k\,\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^3\left(1-\frac{3}{2}\,y^2\,\text{d2}[t]^2\right)-$$

$$\left.\left.\frac{16\,\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^3\left(1-\frac{3}{2}\,y^2\,\text{d2}[t]^2\right)}{y}\right)\right)\Bigg/$$

$$\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2\right)^2\Bigg)\Bigg)\Bigg/\left(8\,\lambda^2\,\sqrt{1+\lambda^2}\,\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2\right.$$

$$\sqrt{\left(\left(y^5\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y^5}+\right.\right.\right.}$$

$$\left.\left.\left.4\,k\,\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4-\frac{4\,\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y}\right)\right)\Bigg/$$

$$\left.\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2\right)^2\right)\Bigg)+\left(\sqrt{\frac{1}{y^2}}\,\left(\text{z1}+y\,\text{z2}+y^2\,\text{z3}\right)\left(1-\frac{1}{2}\,\lambda^2\left(\frac{1}{\lambda^2}-\right.\right.\right.$$

$$\left(\sqrt{\frac{y^5\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y^5}+4\,k\,\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4-\frac{4\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y}\right)}{\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2\right)^2}}{\lambda^2}\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg/$$

$$\left(2\sqrt{1+\lambda^2}\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2\right)+\left(\sqrt{\frac{1}{y^2}}\left(-y^2\,z2-2\,y^3\,z3\right)\left(1-\frac{1}{2}\,\lambda^2\left(\frac{1}{\lambda^2}-\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\left.\left.\left.\sqrt{\frac{y^5\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y^5}+4\,k\,\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4-\frac{4\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y}\right)}{\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2\right)^2}}{\lambda^2}\right)\right)\right)\right)\right)\right)\right)\Bigg/$$

$$\left(y\sqrt{1+\lambda^2}\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2\right)-\frac{1}{2\sqrt{\frac{1}{y^2}}\,\lambda^2\sqrt{1+\lambda^2}}\left(z1+y\,z2+y^2\,z3\right)\left(\lambda^2+\frac{1}{2}\,\lambda^2\left(\frac{1}{\lambda^2}-\right.\right.$$

$$\left.\left.\left.\left.\left.\left.\left.\left.\sqrt{\frac{y^5\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y^5}+4\,k\,\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4-\frac{4\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y}\right)}{\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2\right)^2}}{\lambda^2}\right)\right)\right)\right)\right)\right)\right)-$$

$$\frac{1}{y\sqrt{1+\lambda^2}\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^3}\sqrt{2}\sqrt{\frac{1}{y^2}}\left(z1+y\,z2+y^2\,z3\right)\lambda^2$$

$$\sqrt{\frac{1}{\lambda^2}-\frac{\sqrt{\frac{y^5\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^4}{y^5}+4k\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^4-\frac{4\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^4}{y}\right)}{\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^2\right)^2}}}{\lambda^2}}$$

$$\sqrt{\left(1-\frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2}-\frac{\sqrt{\frac{y^5\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^4}{y^5}+4k\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^4-\frac{4\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^4}{y}\right)}{\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^2\right)^2}}}{\lambda^2}\right)\right)}$$

$$\left(-\left(\left(\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)\left(-\left(\left(\left(5y^6\left(\frac{\lambda^4}{y^9}+\frac{2\lambda^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^4}{y^5}+4k\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^4-\frac{4\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^4}{y}\right)\right)\Big/\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}y\,\text{d2}[t]^2\right)^2\right)^2\right)-\right.\right.\right.\right.$$

$$\left(2\,y^5\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y^5}+4\,k\right.\right.$$

$$\left.\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4-\frac{4\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y}\right)$$

$$\left.\left(\frac{2\,\lambda^2}{y}+2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)\left(1-\frac{3}{2}\,y^2\,d2[t]^2\right)\right)\right)\Big/$$

$$\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2\right)^3+\frac{1}{\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2\right)^2}$$

$$y^5\left(\frac{9\,\lambda^4}{y^8}+\frac{14\,\lambda^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2}{y^6}+\frac{5\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y^4}-4\,\lambda^2\,c2[t]^2\right.$$

$$\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4+\frac{4\,\lambda^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)\left(1-\frac{3}{2}\,y^2\,d2[t]^2\right)}{y^7}+$$

$$\frac{4\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^3\left(1-\frac{3}{2}\,y^2\,d2[t]^2\right)}{y^5}+$$

$$16\,k\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^3\left(1-\frac{3}{2}\,y^2\,d2[t]^2\right)-$$

$$\left.\left.\left.\frac{16\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^3\left(1-\frac{3}{2}\,y^2\,d2[t]^2\right)}{y}\right)\right)\right)\Big/$$

$$\left(4\,\sqrt{2}\,\lambda^2\sqrt{\left(\left(\left(y^5\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y^5}+\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.4\,k\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4-\frac{4\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y}\right)\right)\right)\Big/\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2\right)^2\right)$$

$$\sqrt{\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(\left(y^5\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y^5}+\right.\right.\right.\right.\right.\right.$$

$$4\,k\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4-\frac{4\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y}\Bigg)\Bigg)\Bigg/\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2\right)^2\Bigg)\Bigg)\Bigg)$$

$$\sqrt{\left(1-\frac{1}{2}\,\lambda^2\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(y^5\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y^5}+4\,k\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4-\frac{4\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y}\right)\right)\Bigg/\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2\right)^2\right)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)+$$

$$\left(\sqrt{1+\lambda^2}\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)\left(\frac{1}{\sqrt{1+\lambda^2}\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2}\right.\right.$$

$$\sqrt{2}\,y\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2\right)$$

$$\sqrt{\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(y^5\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y^5}+4\,k\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4-\frac{4\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y}\right)\right)\Bigg/\left(\frac{\lambda^2}{y^2}+\right.\right.\right.$$

$$\left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^2\Bigg)^2\Bigg)\Bigg)\Bigg)$$

$$\sqrt{\left(1 - \frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2} - \frac{1}{\lambda^2}\left(\sqrt{\left(\left(y^5\left(\frac{\lambda^4}{y^9} + \frac{2\lambda^2\left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^2}{y^7} + \right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\left.\frac{\left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^4}{y^5} + 4k\lambda^2 c2[t]^2\left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^4 - \right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\left.\frac{4\lambda^2 c2[t]^2\left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^4}{y}\right)\right)\right/\left(\frac{\lambda^2}{y^2} + \right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\left.\left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^2\right)^2\right)\right)\right)\right)\Bigg) -$$

$$\frac{1}{\sqrt{2}\ \sqrt{1+\lambda^2}\ \left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^3}\left(1 - \frac{3}{2} y^2\, d2[t]^2\right)$$

$$\left(\frac{\lambda^2}{y^2} + \left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^2\right)$$

$$\sqrt{\left(\frac{1}{\lambda^2} - \frac{1}{\lambda^2}\left(\sqrt{\left(\left(y^5\left(\frac{\lambda^4}{y^9} + \frac{2\lambda^2\left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^2}{y^7} + \right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\left.\frac{\left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^4}{y^5} + 4k\lambda^2 c2[t]^2\left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^4 - \right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\left.\frac{4\lambda^2 c2[t]^2\left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^4}{y}\right)\right)\right/\left(\frac{\lambda^2}{y^2} + \right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\left.\left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^2\right)^2\right)\right)\right)\right)\Bigg)$$

$$\sqrt{\left(1 - \frac{1}{2}\lambda^2\left(\frac{1}{\lambda^2} - \frac{1}{\lambda^2}\left(\sqrt{\left(\left(y^5\left(\frac{\lambda^4}{y^9} + \frac{2\lambda^2\left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^2}{y^7} + \right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\left.\frac{\left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^4}{y^5} + 4k\lambda^2 c2[t]^2\left(\frac{1}{y} + d2[t] + \frac{3}{2} y\, d2[t]^2\right)^4 - \right.\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.\frac{4\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y}\right)\right)\right/\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2\right)^2\right)\right)\right)\right)+$$

$$\frac{1}{2\sqrt{2}\,y^2\sqrt{1+\lambda^2}\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}(1-k\,y)\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2\right)$$

$$\left(-\frac{k\,y^4\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2}{(1-k\,y)^2}-\frac{2\,y^3\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2}{1-k\,y}+\frac{2\,y^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)\left(1-\frac{3}{2}\,y^2\,d2[t]^2\right)}{1-k\,y}\right)$$

$$\sqrt{\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(y^5\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y^5}+4\,k\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4-\frac{4\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y}\right)\right)\right/\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2\right)^2\right)}\right)\right)}$$

$$\sqrt{\left(1-\frac{1}{2}\,\lambda^2\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(y^5\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y^5}+4\,k\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4-\frac{4\,\lambda^2\,c2[t]^2\left(\frac{1}{y}+d2[t]+\frac{3}{2}\,y\,d2[t]^2\right)^4}{y}\right)\right)\right/\left(\frac{\lambda^2}{y^2}+\right.}\right.\right.\right.}$$

$$\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2\Bigg)^2\Bigg)\Bigg)\Bigg)\Bigg)+\frac{1}{\sqrt{1+\lambda^2}\,\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)}$$

$$\sqrt{2}\,\sqrt{\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(y^5\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2}{y^7}+\right.\right.\right.\right.$$

$$\left.\frac{\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y^5}+4\,k\,\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4-\right.$$

$$\left.\left.\left.\left.\frac{4\,\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y}\right)\right)\Big/\left(\frac{\lambda^2}{y^2}+\right.\right.\right.$$

$$\left.\left.\left.\left.\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2\right)^2\right)\right)\right)$$

$$\sqrt{\left(1-\frac{1}{2}\,\lambda^2\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(y^5\left(\frac{\lambda^4}{y^9}+\frac{2\,\lambda^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2}{y^7}+\right.\right.\right.\right.\right.$$

$$\left.\frac{\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y^5}+4\,k\,\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4-\right.$$

$$\left.\left.\left.\left.\frac{4\,\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y}\right)\right)\Big/\right.\right.$$

$$\left.\left.\left.\left.\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2\right)^2\right)\right)\right)\left(\frac{1}{2}\,y\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)+\right.$$

$$\left.\frac{1}{2}\,\lambda^2\left(\frac{1}{y\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)}+\frac{\frac{2}{\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2}-\frac{2\left(1-\frac{3}{2}\,y^2\,\text{d2}[t]^2\right)}{y\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2}}{y}\right)\right)\Big/$$

$$\left(\left(\left(1+\frac{\lambda^2}{y^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2}\right)\left(1-\lambda^2\left(\frac{1}{\lambda^2}-\frac{1}{\lambda^2}\left(\sqrt{\left(\left(y^5\left(\frac{\lambda^4}{y^9}+\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{2\lambda^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2}{y^7}+\frac{\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y^5}+4\,k\,\lambda^2\,\text{c2}[t]^2$$

$$\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4-\frac{4\lambda^2\,\text{c2}[t]^2\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^4}{y}\Bigg)\Bigg/$$

$$\left(\frac{\lambda^2}{y^2}+\left(\frac{1}{y}+\text{d2}[t]+\frac{3}{2}\,y\,\text{d2}[t]^2\right)^2\right)^2\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)$$

Series Expansion

```
(*2nd order Series Expansion of Kϕ dot*)

Series[KϕDot[t, y], {y, 0, 2}]
```

Out[63]=
$$\frac{(-\text{z2}-\text{z1}\,\text{d2}[t])\,y^2}{\sqrt{\frac{1}{y^2}}\;y\;\sqrt{1+\lambda^2}}+O[y]^3$$

In[65]:=
```
Solve[(-z2 - z1 d2[t]) == 0, {z1, z2}]
```

Out[65]=
$$\{\{\text{z2}\to-\text{z1}\,\text{d2}[t]\}\}$$

In[64]:=
```
(*3rd order Series Expansion of Kϕ dot*)

Series[KϕDot[t, y], {y, 0, 3}]
```

Out[64]=
$$\frac{(-\text{z2}-\text{z1}\,\text{d2}[t])\,y^2}{\sqrt{\frac{1}{y^2}}\;y\;\sqrt{1+\lambda^2}}+\frac{(-2\,\text{z3}+\text{z2}\,\text{d2}[t])\,y^3}{\sqrt{\frac{1}{y^2}}\;y\;\sqrt{1+\lambda^2}}+O[y]^4$$

```
Solve[{(-z2 - z1 d2[t]) == 0, (-2 z3 + z2 d2[t]) == 0}, {z1, z2, z3}]
```

Out[66]= $\left\{\left\{z2 \to -z1\, d2[t],\ z3 \to -\frac{1}{2} z1\, d2[t]^2\right\}\right\}$

$\dot{E^{\phi}}$ Expansion

H Simplify

Misc