Localizing WiFi Access Points Using Signal Strength

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***Abstract*—When estimating the location of WiFi access points (AP) in an anonymous environment, the required parameters of the radio propagation model are not readily obtainable. This letter investigates the use of received signal strength (RSS) for range-based AP localization when no information on the radio propagation model is provided. To achieve this goal, we linearly approximate the exponential relationship between RSS and distance, and apply a multilateration technique. Our simulation results validate that estimation of an AP location is possible given four or more RSS measurements at different locations.**

***Index Terms*—AP localization, multilateration, RSS linearization, WiFi networks.**

I. INTRODUCTION

FINDING the locations of WiFi access points (AP) is one of the important research fields in WiFi-based location systems [1] [2], in which a mobile node estimates its position based on the locations of the APs. In most of such system, AP locations are assumed to be provided a priori or obtained via costly offline training processes. In many situations, however, these assumptions are not applicable; hence, any means of automatic location estimation of AP is required in practice [3] [4].

Given ranging information to an AP at three or more different positions, the location of the AP can be estimated by applying multilateration technique [5]. Due to its low cost and implementation simplicity, the received signal strength (RSS) is commonly used to obtain ranging information between radio devices even though the accuracies are low. RSS-based ranging information is commonly inferred from the pathloss model of radio propagation [6]. In the model, the distance between nodes is exponentially related to the RSS and several radio propagation parameters such as transmission power and a pathloss exponent.

In anonymous environments where radio propagation characteristics were not studied a priori, neither the transmission power nor the pathloss exponent is given. In such situations, RSS cannot be directly converted to a distance [3]. Online calibration of pathloss exponent estimation was studied in [7]. Although the study shows accurate estimation results, the scheme requires quadrilateral measurements and information on transmission powers which are impractical in anonymous environments.

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In this letter, we linearly approximate the relationship between RSS and distance, which is applied to a multilateration scheme. Through the linear approximation, RSSs are usefully used in estimating AP location under the situation where pathloss exponent and transmission power are unknown. The performance of the proposed algorithm is evaluated by simulations.

II. RSS-BASED MULTILATERATION

Consider an AP whose location is being estimated and m different positions where RSSs are measured. The unknown location of the AP is denoted as (x_0, y_0), and the ith measurement position as (x_i, y_i), where $1 < i < m$ and r_i is the measured RSS at the ith position. The distance between the AP and the ith reference position is defined as:

$$d_i = \sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2} \tag{1}$$

Previous studies show that the indoor pathloss model follows the distance power law: $P_r = P_0 - 10n\log_{10}(d/l_0) + X_\sigma$, where P_r is the RSS in dB, P_0 the signal strength at the distance l_0 from transmitter, and n the pathloss exponent. X_σ represents the shadow noise and is modeled as a normal random variable with the standard deviation σdB [6]. Typically, l_0 is set to $1m$. The value of n depends on the surrounding environments. Given the measurement r_i at the ith measurement position, the distance $\hat{d}_i$ from AP can be estimated as:

$$\hat{d}_i = 10^{(P_0 - r_i)/10n} \tag{2}$$

In real environments, the distance estimation is error prone because of various environmental factors such as scattering, shadowing and heights of transmitter and receiver.

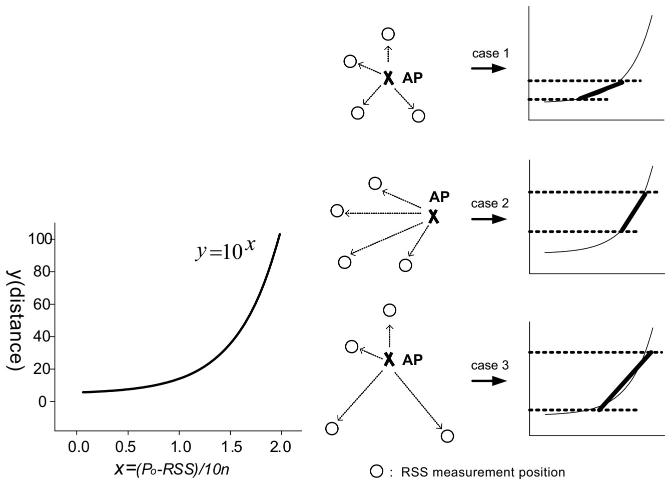
Multilateration is a common technique for estimating node location if distance measurements are available in at least three different, non-collinear positions. From Equation (1) and (2), the relation between two positions and the RSS is represented as:

$$\sqrt{(x_0 - x_i)^2 + (y_0 - y_i)^2} \cong 10^{(P_0 - r_i)/10n} \tag{3}$$

By squaring, rearranging terms, and subtracting the kth equation from the ith equation, a linear function of x_0 and y_0 is obtained as below [5]:

$$\begin{aligned} -x_i^2 - y_i^2 + x_k^2 + y_k^2 + 10^{(P_0 - r_i)/5n} - 10^{(P_0 - r_k)/5n} \\ = x_0(-2x_i + 2x_k) + y_0(-2y_i + 2y_k) \end{aligned} \tag{4}$$

Considering multiple positions, the final equation of multilat-



(a) Exponential relationship between RSS and distance (b) Linear approximation examples

Fig. 1. Relationship between distance and RSS. The accuracy of linear approximation is affected by the geometry of measurement positions.

eration has the form of $y = Xb$ as shown in Equation (5).

$$\begin{bmatrix} -x_1^2 - y_1^2 + x_k^2 + y_k^2 + 10^{\frac{(P_0-r_1)}{5n}} - 10^{\frac{P_0-r_k}{5n}} \\ -x_2^2 - y_2^2 + x_k^2 + y_k^2 + 10^{\frac{P_0-r_2}{5n}} - 10^{\frac{P_0-r_k}{5n}} \\ \cdots \\ -x_{k-1}^2 - y_{k-1}^2 + x_k^2 + y_k^2 + 10^{\frac{P_0-r_{k-1}}{5n}} - 10^{\frac{P_0-r_k}{5n}} \end{bmatrix} = \begin{bmatrix} -2x_1+2x_k & -2y_1+2y_k \\ -2x_2+2x_k & -2y_2+2y_k \\ \cdots & \\ -2x_{k-1}+2x_k & -2y_{k-1}+2y_k \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \quad (5)$$

Due to the measurement errors contained in vector y, a least-square solution is used, resulting in the least-square estimate for b as

$$\hat{b} = (X^T X)^{-1} X^T y \quad (6)$$

The solution requires the values of P_0 and n, which may not be obtained in anonymous environments. Consequently, the RSS-based multilateration is not solvable.

III. Multilateration with RSS Linearization

Distances between nodes are exponentially related to the RSS as illustrated in Figure 1(a). However, the exponential relationship leaves the multilateration unsolvable in case P_0 and n are unknown. To make the problem simple, the relationship is linearly approximated. The approximated section is decided by the geometric relationship between the AP and the RSS measurement positions. Figure 1(b) shows examples of the linearization. The red lines show approximations. In case 1, the measurement positions are closely located to the AP. Hence, linearization section is limited to the lower narrow area. In case 2, the geometric relationships among measurement positions are the same as in case 1. However, the measurement positions are far from the APs position, which shifts the linearization section to the upper area. In case 3, the distances between measurement positions and the AP are diverse. This requires larger area to be linearly approximated. To handle all the situations, the linearization range should be set to the entire area.

Linearization of the exponential relationship in Equation (2) is formulized as below:

$$10^{(P_0-r_i)/10n} \approx a_0 + a_1((P_0 - r_i)/10n) \quad (7)$$

Here, a_0 and a_1 are the coefficients of linearization. In case the range of r_i as well as the values of P_0 and n are given, the coefficients can be estimated by using linear regression or other linear approximation techniques. With the linearization, Equation (4) is reformed as:

$$-x_i^2 - y_i^2 + x_k^2 + y_k^2 = x_0(-2x_i + 2x_k) + y_0(-2y_i + 2y_k) + a_1(r_i - r_k)/5n \quad (8)$$

All terms are linearly related. The final solution is obtained in the form of $y = Xb$:

$$\begin{bmatrix} -x_1^2 - y_1^2 + x_k^2 + y_k^2 \\ -x_2^2 - y_2^2 + x_k^2 + y_k^2 \\ \cdots \\ -x_{k-1}^2 - y_{k-1}^2 + x_k^2 + y_k^2 \end{bmatrix} = \begin{bmatrix} -2x_1+2x_k & -2y_1+2y_k & \frac{a_1(r_1-r_k)}{5} \\ -2x_2+2x_k & -2y_2+2y_k & \frac{a_1(r_2-r_k)}{5} \\ \cdots & \cdots & \cdots \\ -2x_{k-1}+2x_k & -2y_{k-1}+2y_k & \frac{a_1(r_{k-1}-r_k)}{5} \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ \frac{1}{n} \end{bmatrix} \quad (9)$$

Equation (9) depends only on measurement positions, the RSS values at each position, and the linear approximation coefficient a_1. In the given solution, the location of AP, (x_0, y_0), is estimated without any information on the radio propagation parameters. Only four or more RSS measurements at different positions are required.

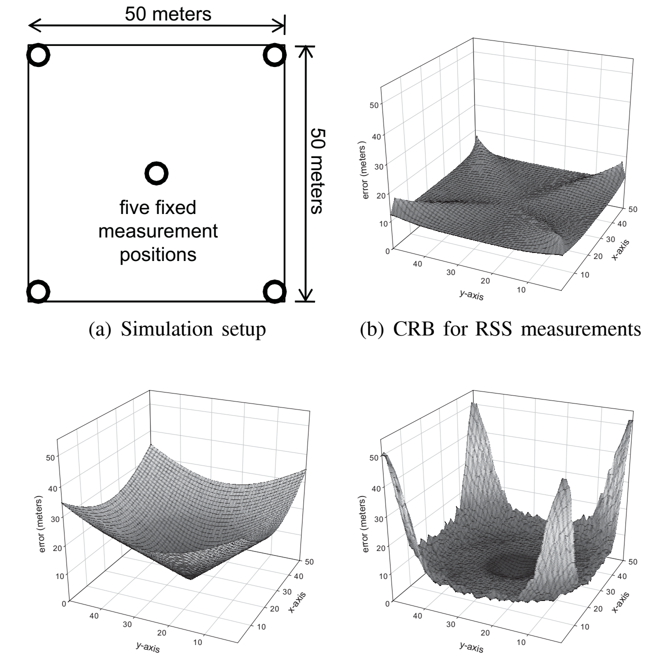
The accuracy of linearization is affected by the range of r_i and the values of P_0 and n. In the proposed algorithm, however, the accuracy of linearization does not affect the performance of location estimation. That is, the localization performance is independent to the values of a_0 and a_1. Assuming that X is a 3x3 matrix in Equation (9), b is estimated as $X^{-1}y$. For x_0 and y_0, X^{-1} is obtained as below:

$$X^{-1} = \frac{1}{|X|} adj(X) = \frac{1}{|X|}[m_{i,j}] = \frac{g(x,y,r)}{f(x,y,r)} \quad \text{for } i,j = 0,1$$
$$where,\ |X| = a_1 f(x,y,r),$$
$$m_{i,j} = a_1 g(x,y,r) \quad (10)$$

Here, $f(\cdot)$ and $g(\cdot)$ are the linear functions of x, y and r, and are independent of a_1. Therefore, the first and second rows of the inverse matrix of X are independent of a_1. As a result, the estimated location (x_0, y_0) is not affected by a_1, which means the accuracy of linearization does not affect the performance of the multilateration system. This fact eliminates the linearization issues as stated previously. However, the pathloss exponent n is severely influenced by the value a_1. Higher-order matrix cases are validated with simulation experiments.

IV. Evaluation

To evaluate the feasibility of the proposed scheme, simulations are conducted. The simulated area is 50 m X 50 m



(a) Simulation setup (b) CRB for RSS measurements

(c) Centroid (d) RSS Linearization

Fig. 2. Performance comparison between RSS-CRB, Centroid, and Multilateration with RSS linearization (σ = 2).

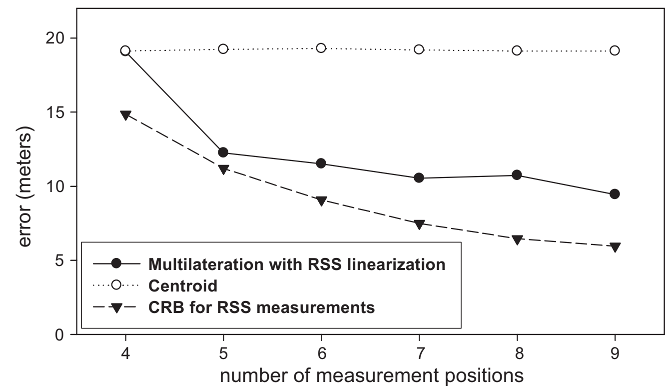


Fig. 3. Localization errors versus the number of measurement positions (σ = 2). RSSs are measured at (1,1), (1,50), (50,1), (50,50), (13, 25), (25, 13), (38, 25), (25, 38), and (25, 25).

TABLE I
ESTIMATION ERROR (M) FOR DIFFERENT PARAMETERS (σ=5)

Number of measurements	n=3 P_0=-30dB a_1=26	n=3 P_0=-30dB a_1=10	n=3 P_0=-50dB a_1=26	n=5 P_0=-30dB a_1=26
5	19.02	19.02	19.02	16.57
7	17.44	17.44	17.44	16.17
9	17.06	17.06	17.06	15.99

square. RSSs are generated by the indoor pathloss model. P_0 is set to -30 dBm and n is set to 3. We fixed five RSS measurement positions as shown in Figure 2(a). The location estimation was conducted in every 1 meter grids. Figure 2(b) shows the RMS error of Cramer-Rao Bound (CRB) [8] for RSS measurements when P_0 and n are known, which represents the accuracy limit of location estimator using RSS measurements. Centroid, the representative algorithm of range-free method [3], does not need any information about radio propagation, but performance is directly related to the locations of AP and measurement positions, which are shown in Figure 2(c). The performance of the proposed algorithm is shown in Figure 2(d). In most of the area, the localization error is close to the CRB. However, the error is relatively large at the corners where the geometric condition is poor. The geometric condition is represented as the sum of areas of all triangles composed by an AP and any set of two measurement positions [8].

We also investigated the effects of the number of measurement positions. Figure 3 shows the results. In case of Centroid, the improvement of performance is trivial as the number of measurement positions is increased. However, the localization error of the proposed algorithm is shown to decrease as the number of measurement positions increases.

The performance of our algorithm is related to several parameters which include the linearization coefficient a_1, the transmitting power of an AP, and the path loss factor in an environment. To make the experiment result more reliable, we increased the noise level. Table I summarizes the effects of the parameters. P_0 does not affect the performance, whereas the bigger n has the lower estimation error because fast-decaying of RSS has more information according to the Fisher Information [8]. As proved in Equation (10), a_1 does not affect the performance.

V. CONCLUSION

We proposed a WiFi AP localization algorithm that is applicable in anonymous environments. The algorithm does not require that the pathloss exponent and transmission power are known a priori; hence no offline training is necessary in the area of AP. The only constraint is that RSSs are measured at four or more different positions. Simulation results show that RSS is are properly used in location estimation without information on radio propagation characteristics.

REFERENCES

[1] P. Bahl and V. N. Padmanabhan, “RADAR: an in-building RF-based user location and tracking system,” *IEEE INFOCOM*, 2000.

[2] H. Lim, L. Kung, J. C. Hou, and H. Luo, “Zero-configuration, robust indoor localization: theory and experimentation,” *IEEE INFOCOM*, 2006.

[3] Y.-C. Cheng, Y. Chawathe, A. LaMarca, and J. Krumm, “Accuracy characterization for metropolitan-scale Wi-Fi localization,” *ACM MOBISYS*, 2005.

[4] A. Subramanian, P. Deshpande, J. Gaojgao, and S. Das, “Drive-by localization of roadside WiFi networks,” *IEEE INFOCOM*, 2008.

[5] A. Savvides, C.-C. Han, and M. B. Srivastava, “Dynamic fine-grained localization in ad-hoc networks of sensors,” *ACM MOBICOM*, 2001.

[6] T. Rappaport, *Wireless Communications–Principles and Practice*. Prentice-Hall, 1996.

[7] G. Mao, B. D. O. Anderson, and B, Fidan, “Path loss exponent estimation for wireless sensor network localization,” *Computer Networks*, vol. 51, no. 10, pp. 2467-2483, 2007.

[8] N. Patwari, A. Hero III, M. Perkins, N. S. Correal, and R. Dea, “Relative location estimation in wireless sensor networks,” *IEEE Trans. Signal Process.*, 2003.