Derivatives / Integrals

Exponentials

e form	$f(x)$	$f'(x)$	$\int f(x)dx$
	e^x	e^x	$e^x + C$
$e^{\ln(b)\cdot x}$	b^x	$\ln(b)\cdot b^x$	$\frac{1}{\ln(b)}\cdot b^x + C$
	$\ln\lvert x\rvert$*	$\frac{1}{x}$	???
$\frac{\ln(x)}{\ln(b)}$	$\log_b(x)$	$\frac{1}{\ln(b)\cdot x}$	???

*: $\ln(x)$ has the same derivative; the absolute value is used to give a larger domain for the integral of $\frac{1}{x}$.

Trig

$f(x)$	$f'(x)$	$\int f(x)dx$
$\tan(x)$	$\sec^2(x)$	$-\ln\lvert\cos(x)\rvert + C$
$\cot(x)$	$-\csc^2(x)$	$\ln\lvert\sin(x)\rvert + C$
$\sec(x)$	$\sec(x)\cdot\tan(x)$	???
$\csc(x)$	$-\cot(x)\cdot\csc(x)$	???

Some Rules

- $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$

Trig Identities

Pythagorean

- $\sin^2(x) + \cos^2(x) = 1$
- $1 + \cot^2(x) = \csc^2(x)$
- $\tan^2(x) + 1 = \sec^2(x)$

Euler's Formula (just derive them lol)

$e^{i\theta} = \cos(\theta) + i\sin(\theta)$

Double-Angle

- $\cos(2x) = \cos^2(x) - \sin^2(x)$

  $= 2\cdot\cos^2(x) + 1$

  $= 1 - 2\cdot\sin^2(x)$
- $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Power Reduction

- $\sin^2(x) = \frac{1-\cos(2x)}{2}$
- $\cos^2(x) = \frac{1+\cos(2x)}{2}$
- $\tan^2(x) = \frac{1-\cos(2x)}{1+\cos(2x)}$

Angle-Sum

- $\cos(x \pm y) = ???$ $\qquad \sin(x \pm y) = ???$

Hint: $e^{x+y} = e^x e^y$

Even/Odd Functions

$f(x)$	$g(x)$	$f(x)+g(x)$	$f(x)\cdot g(x)$
E	E	E	E
O	O	O	O
O	E	N	O

Difference of Powers

- $a^2 - b^2 = (a-b)(a+b)$
- $a^3 - b^3 = (a-b)(a^3 + a^2b + ab^2 + b^3)$
- $a^4 - b^4 = (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

Exponents/Logarithms

Log Rules

1. $\log_b(b^x) = x$ $\qquad b^{\log_b(x)} = x$
2. $\log_b(x\cdot y) = \log_{b(x)} + \log_{b(y)}$
3. $\log_b(x^n) = n\cdot\log_b(x)$
5. $\frac{\log_a(x)}{\log_a(x)} = \log_b(x)$

Hyperbolic Trig

Definitions

- $\sinh(x) = \frac{e^x - e^{-x}}{2}$ $\qquad \sinh^{-1}(x) = \ln\left(x + \sqrt{x^2+1}\right)$
- $\cosh(x) = \frac{e^x + e^{-x}}{2}$ $\qquad \cosh^{-1}(x) = \ln\left(x + \sqrt{x^2-1}\right)$

Identities

- Pythagorean: $\cosh^2(t) - \sinh^2(t) = 1$

  $1 - \tanh^2(t) = \operatorname{sech}^2(t)$
- Double-angle: $\sinh(2t) = 2\sinh(t)\cosh(t)$

  $\cosh(2t) = \cosh^2(t) + \sinh^2(t)$

Derivatives

$f(x)$	$f'(x)$
$\operatorname{sech}(t)$	$-\operatorname{sech}(t)\tanh(t)$
$\operatorname{csch}(t)$	$-\operatorname{csch}(t)\coth(t)$

Note on deriving formulas:

When encountering a $\pm$ inside of the logarithm, it is the case for the two inverse hyp. trig functions this happens to that the two forms are equivalent. Multiplying by the conjugate and recognizing the exponent -1 can be put into the expression using log rules will show this.

Various tools that may be needed:

- quadratic equation
- triangle method + implicit differentiation
- log rules, exponent rules
- go slow on derivatives, ***especially*** those with quotients!
- logarithmic differentiation might be better for the exponential function
- can you differentiate any inverse trig function?
- the tanh inverse and derivative