Derivatives / Integrals

Trig

$f(x)$	$f'(x)$	$\int f(x)dx$
$\tan(x)$	$\sec^2(x)$	$-\ln\|\cos(x)\| + C$
$\cot(x)$	$-\csc^2(x)$	$\ln\|\sin(x)\| + C$
$\sec(x)$	$\sec(x) \cdot \tan(x)$	???
$\csc(x)$	$-\cot(x) \cdot \csc(x)$	???

Polar Integration

$$A = \int dA = \frac{1}{2} \int r^2 d\theta$$

Some Rules

- $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Trig Identities

Pythagorean

- $\sin^2(x) + \cos^2(x) = 1$
- $1 + \cot^2(x) = \csc^2(x)$
- $\tan^2(x) + 1 = \sec^2(x)$

Euler's Formula (just derive them lol)

$e^{i\theta} = \cos(\theta) + i\sin(\theta)$

Double-Angle

- $\cos(2x) = \cos^2(x) - \sin^2(x)$

$$= 2 \cdot \cos^2(x) + 1$$
$$= 1 - 2 \cdot \sin^2(x)$$

- $\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

Power Reduction

- $\sin^2(x) = \frac{1 - \cos(2x)}{2}$
- $\cos^2(x) = \frac{1 + \cos(2x)}{2}$
- $\tan^2(x) = \frac{1 - \cos(2x)}{1 + \cos(2x)}$

Angle-Sum

- $\cos(x \pm y) = ???$ $\quad \sin(x \pm y) = ???$

Hint: $e^{x+y} = e^x e^y$

Differential Equations

Separable Equations

Form:

$$y' = f(x) \cdot g(y) \text{ ; alternatively, } \frac{dy}{dx} = f(x) \cdot g(y)$$

Solve:

$$\int \frac{dy}{g(y)} = \int f(x)dx$$

First-order Linear

Form:

$$y' + a(x) \cdot y = f(x)$$

Solve: Multiply the whole equation by the integrating factor,

$$u(x) = e^{\int a(x)dx}$$

$$u \cdot y' + u' \cdot y = f(x) \cdot u(x)$$

$$(u \cdot y)' = f(x) \cdot u(x)$$

$$u \cdot y = \int f(x) \cdot u(x)dx$$

$$y = \frac{\int f(x) \cdot u(x)dx}{u}$$

Power Series

A_n is the alternating term, $(-1)^n$

$f(x)$	MacLaurin series	Radius of Convergene
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$	∞
$\sin(x)$	$\sum_{n=0}^{\infty} A_n \cdot \frac{x^{2n+1}}{(2n+1)!}$	∞
$\cos(x)$	$\sum_{n=0}^{\infty} A_n \cdot \frac{x^{2n}}{(2n)!}$	∞
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n$	1

Exam Brain

- Taylor Series of f, centered at a:

$$\sum_{n=0}^{\infty} \frac{f^n(a) \cdot (x-a)^n}{n!}$$

- $\int uv' = uv - \int u'v$

Series

Common Types

Type	Form	Formula (if applicable)	Converges when
Geometric	$\sum_{n=0}^{\infty} ar^n$	$\frac{a}{1-r}$	$\lvert r \rvert < 1$

Properties of Convergeence

Let $A \cong B$ indicate A and B have the same convergence behavior.

- $\sum_{n=a}^{\infty} s_n \cong \sum_{n=b}^{\infty} s_n$
- $a \cdot \sum_{n=c}^{\infty} s_n \cong b \cdot \sum_{n=c}^{\infty} s_n$

Tests

Let a_n, b_n be sequences.

Test	Conditions	Result
Divergence	$\lim_{n \to \infty} a_n \neq 0$	$\sum_{n=c}^{\infty} a_n$ diverges
Monotone Convergence Theorem	1. a_n has upper (lower) bound b on $[m, \infty)$ 2. a_n is monotonically increasing (decreasing) on $[m, \infty)$	$\lim_{n \to \infty} a_n$ converges
Integral	1. $a_n \geq 0 \ \forall \ n \in [c, \infty)$ 2. a_n is decreasing on $[c, \infty)$ 3. a_n is integrable on $[c, \infty)$	$\int_c^{\infty} a_n dn \cong \sum_{n=c}^{\infty} a_n$
Comparison	1. $a_n \geq b_n \geq 0 \ \forall \ n \in [c, \infty)$ 2. $\sum_{n=c}^{\infty} a_n$ converges	$\sum_{n=c}^{\infty} b_n$ converges
Limit Comparison Test	$\lim_{n \to \infty} \frac{a_n}{b_n} = L$	$L \neq 0 \in \mathbb{R} \Rightarrow \sum_{n=c}^{\infty} a_n \cong \sum_{n=c}^{\infty} b_n$ $L = 0 \Rightarrow \sum_{n=c}^{\infty} a_n < \sum_{n=c}^{\infty} b_n$ $L = \infty \Rightarrow \sum_{n=c}^{\infty} a_n > \sum_{n=c}^{\infty} b_n$
Ratio test	$\lim_{n \to \infty} \lvert \frac{a_{n+1}}{a_n} \rvert = L$	$L > 1 \Rightarrow \sum_{n=c}^{\infty} a_n$ diverges $L < 1 \Rightarrow \sum_{n=c}^{\infty} a_n$ converges