Derivatives / Integrals

Exponentials

e form	$f(x)$	$f'(x)$	$\int f(x)dx$
	e^x	e^x	$e^x + C$
$e^{\ln(b)\cdot x}$	b^x	$\ln(b)\cdot b^x$	$\frac{1}{\ln(b)}\cdot b^x + C$
	$\ln\lvert x\rvert$*	$\frac{1}{x}$	???
$\frac{\ln(x)}{\ln(b)}$	$\log_b(x)$	$\frac{1}{\ln(b)\cdot x}$	???

*: $\ln(x)$ has the same derivative; the absolute value is used to give a larger domain for the integral of $\frac{1}{x}$.

Trig

$f(x)$	$f'(x)$	$\int f(x)dx$
$\tan(x)$	$\sec^2(x)$	$-\ln\lvert\cos(x)\rvert + C$
$\cot(x)$	$-\csc^2(x)$	$\ln\lvert\sin(x)\rvert + C$
$\sec(x)$	$\sec(x)\cdot\tan(x)$	???
$\csc(x)$	$-\cot(x)\cdot\csc(x)$	???

Some Rules

- $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$

Trig Identities

Pythagorean

- $\sin^2(x) + \cos^2(x) = 1$
- $1 + \cot^2(x) = \csc^2(x)$
- $\tan^2(x) + 1 = \sec^2(x)$

Euler's Formula (just derive them lol)

$e^{i\theta} = \cos(\theta) + i\sin(\theta)$

Double-Angle

- $\cos(2x) = \cos^2(x) - \sin^2(x)$

$$= 2\cdot\cos^2(x) + 1$$
$$= 1 - 2\cdot\sin^2(x)$$

- $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Power Reduction

- $\sin^2(x) = \frac{1-\cos(2x)}{2}$
- $\cos^2(x) = \frac{1+\cos(2x)}{2}$
- $\tan^2(x) = \frac{1-\cos(2x)}{1+\cos(2x)}$

Angle-Sum

- $\cos(x\pm y) = ???$ $\sin(x\pm y) = ???$

Hint: $e^{x+y} = e^x e^y$

Differential Equations

- General and specific solutions (initial conditions?)

Separable Equations

Form:

$$y' = f(x)\cdot g(y) \text{ ; alternatively, } \frac{dy}{dx} = f(x)\cdot g(y)$$

Solve:

$$\int \frac{dy}{g(y)} = \int f(x)dx$$

First-order Linear

Form:

$$y' + a(x)\cdot y = f(x)$$

Solve: Multiply the whole equation by the integrating factor,

$$u(x) = e^{\int a(x)dx}$$

$$u\cdot y' + u'\cdot y = f(x)\cdot u(x)$$

$$(u\cdot y)' = f(x)\cdot u(x)$$

$$u\cdot y = \int f(x)\cdot u(x)dx$$

$$y = \frac{\int f(x)\cdot u(x)dx}{u}$$

Series

Common Types

Type	Form	Formula (if applicable)	Converges when
Geometric	$\sum_{n=0}^{\infty} ar^n$	$\frac{a}{1-r}$	$\lvert r\rvert < 1$

Properties of Convergeence

Let $A \cong B$ indicate A and B have the same convergence behavior.

- $\sum_{n=a}^{\infty} s_n \cong \sum_{n=b}^{\infty} s_n$
- $a \cdot \sum_{n=c}^{\infty} s_n \cong b \cdot \sum_{n=c}^{\infty} s_n$

Tests

Let a_n, b_n be sequences.

Test	Conditions	Result
Divergence	$\lim_{n\to\infty} a_n \neq 0$	$\sum_{n=c}^{\infty} a_n$ diverges
Monotone Convergence Theorem	1. a_n has upper (lower) bound b on $[m, \infty)$ 2. a_n is monotonically increasing (decreasing) on $[m, \infty)$	$\lim_{n\to\infty} a_n$ converges
Integral	1. $a_n \geq 0 \ \forall\ n \in [c, \infty)$ 2. a_n is decreasing on $[c, \infty)$ 3. a_n is integrable on $[c, \infty)$	$\int_c^{\infty} a_n dn \cong \sum_{n=c}^{\infty} a_n$
Comparison	1. $a_n \geq b_n \geq 0 \ \forall\ n \in [c, \infty)$ 2. $\sum_{n=c}^{\infty} a_n$ converges	$\sum_{n=c}^{\infty} b_n$ converges
Limit Comparison Test	$\lim_{n\to\infty} \frac{a_n}{b_n} = L$	$L \neq 0 \in \mathbb{R} \Rightarrow \sum_{n=c}^{\infty} a_n \cong \sum_{n=c}^{\infty} b_n$ $L = 0 \Rightarrow \sum_{n=c}^{\infty} a_n < \sum_{n=c}^{\infty} b_n$ $L = \infty \Rightarrow \sum_{n=c}^{\infty} a_n > \sum_{n=c}^{\infty} b_n$
Ratio test	$\lim_{n\to\infty} \lvert\frac{a_{n+1}}{a_n}\rvert = L$	$L > 1 \Rightarrow \sum_{n=c}^{\infty} a_n$ diverges $L < 1 \Rightarrow \sum_{n=c}^{\infty} a_n$ converges