Section 1.2

Problems: 3ab, 8, 10ac

Problem 3

a). False. We define $A_n = \{n, n+1, n+2, \dots\}$ for all $n \in \mathbb{N}$. We see that $A_1 \supseteq A_2 \supseteq A_3 \dots$, however $\bigcap_{n=1}^{\infty} A_n = \emptyset$, thus is not infinite. the proof below would establish that $\bigcap_{n=1}^{\infty} A_n$ is indeed empty.

Proof. We will use a proof by contradiction. Assume $a \in \bigcap_{n=1}^{\infty} A_n$, we know that $a \in A_n$ for all $n \in \mathbb{N}$. But wee see that $A_{a+1} = \{a+1, a+2, a+3, \dots\}$, and that $a \notin A_{a+1}$. Thus we have arrived at our contradiction. $\square$

b). True.

Problem 8

a). Example: $f : \mathbb{N} \to \mathbb{N}$, $f(x) = x + 1$. It is not surjective since no element of the domain maps to 1 in the co-domain.

b). Example: $f : \mathbb{N} \to \mathbb{N}$, $f(x) = \lceil \log x \rceil$

c). Example: $f : \mathbb{N} \to \mathbb{Z}$, $f(x) = (-1)^x \lceil \frac{x}{2} \rceil$

Problem 10

a). False. If $a = b$, we see that $a = b < b + \epsilon$ for some $\epsilon > 0$. However, we see that $a \not< b$, and thus the statement is false.

c). True.

Proof. We will first establish the forward direction of the proof. We know that $b < b+\epsilon$ for all $\epsilon > 0$, and so $a \leq b < b + \epsilon$. Consequently, $a < b + \epsilon$. Now we will show the converse of this statement using contradiction. That is, we assume $a < b + \epsilon$ for all $\epsilon > 0$ and $a > b$. Because $a > b$, we know that there exists some $r \in \mathbb{R}$ such that $r = a - b$. We see that $a > b + \frac{r}{2}$ as $\frac{r}{2} < r$. We now arrived at our contraction because we know that $a < b + \epsilon$ for all $\epsilon > 0$. $\square$

Section 1.3

Problems: 2, 6, 8

Problem 2

a).

b).

c).

Problem 6

a).

b).

c).

d).

Problem 8

a).

b).

c).

d).