Section 6.5

Problems: 1a, 6

Problem 3

a).

Problem 6

Section 6.6

Problems: 2a

Problem 2

a).

Section 7.2

Problems: 4

Problem 4

We know that there exists a partition P with $L(g,P) = U(g,P)$. P can be expressed as $x_0, x_1, \ldots, x_n$, and we see that

$$\sum_{k=1}^{n} m_k(x_k - x_{k-1}) = \sum_{k=1}^{n} M_k(x_k - x_{k-1})$$
$$\implies \sum_{k=1}^{n} (M_k - m_k)(x_k - x_{k-1}) = 0$$

We know that $x_k - x_{k-1} \neq 0$, thus $M_k = m_k$. This indicates that $\sup\{g(x) : x \in [x_{k-1}, x_k]\}$ $= \inf\{g(x) : x \in [x_{k-1}, x_k]\}$. We can easily see that $f(x)$ is a constant over the interval $[x_{k-1}, x_k]$ (A rather simple proof using contradiciton), and $g(x_{k-1}) = g(x_k)$. And thus we see $g(x_0) = g(x_1) = \cdots = g(x_n)$ (via induction), and so we know that $g(x)$ is constant over the entire interval [a,b]. This means it is integrable (constant implies continuity implies integrability). And the value of $\int_a^b g = (b-a)g(c)$ for any $c \in [a,b]$.