Section 4.2

Problems: 5, 6

Problem 5

a). $\lim_{x\to 2}(3x+4) = 10$

Proof. For any $\epsilon > 0$, we let $\delta = \frac{\epsilon}{3}$, we see that

$$\begin{aligned} |x-2| &< \frac{\epsilon}{3} \\ \implies |3x-6| &< \epsilon \\ \implies |(3x+4)-10| &< \epsilon \end{aligned}$$

Consequently, we are done. □

b). $\lim_{x\to 0} x^3 = 0$

Proof. For any $\epsilon > 0$, we let $\delta = \sqrt[3]{\epsilon}$, we see that

$$\begin{aligned} |x-0| &< \sqrt[3]{\epsilon} \\ \implies |x|^3 &< \epsilon \\ \implies |x^3-0| &< \epsilon \end{aligned}$$

Consequently, we are done. □

Problem 6

a). *Proof.* True. Let the function be $\lim_{x\to c} f(x) = L$. We see that for δ_s such that $0 < \delta_s < \delta$, we have $|x-c| < \delta_s \implies |x-c| < \delta$. And we already know that the orginal δ is a suitable response to the particular ϵ challenge, thus δ_s also works (since $|x-c| < \delta \implies |f(x)-L| < \epsilon$). □

b). False.
Consider the piece-wise function as counter-example:

$$f(x) = \begin{cases} x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

We see that $\lim_{x\to 0} f(x) = 1$, but $1 \neq f(a) = 0$.

Section 4.3

Problems: 1, 4, 6

Problem 1

a).

b).

Problem 4

a).

Problem 6

a).

b).

Section 4.4

Problems: 1

Problem 1

a).

b).

c).