Physics 52C Laboratory Manual
Data Analysis with Applications to Atomic Physics

2012 Edition

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HAYDEN
HM
McNEIL

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 978-0-7380-4904-5

Hayden-McNeil Publishing
14903 Pilot Drive
Plymouth, MI 48170
www.hmpublishing.com

Rosendahl 4904-5 S12

Preface

The Physics 52C Lab is the last course in a three-quarter introduction to experimental physics. The overarching goal of the course is to introduce the basic concepts and techniques of data analysis.

Experiments from atomic and nuclear physics provide data for the application of these techniques. The curriculum for the course is summarized in the following table.

Week	Reading	Data Analysis	Experiment
1	1, 2.1-2.4, 4.1	Systematic/random error	Electron gyroradius
2	4	Standard deviation	Franck-Hertz (quantization)
3	5 (omit 5.6-5.7)	Normal distribution	Radioactive counting
4	2, 3, 11.4	Error propagation	Gamma decay in lead
5	8.1-8.5	Least-squares fitting	Photon energy
6	2.6, 8.6, 9	Advanced least-squares fitting	Radioactive half-life
7	7	Weighted average	Balmer series (H spectrum)
8	–	Correlation & regression	Electron gyroradius
9	10-12	Distributions and chi-squared test	Background radiation
10	–	Review	Final

The required texts for this course are this lab text and *An Introduction to Error Analysis: the Study of Uncertainties in Physical Measurements* (2nd edition) by John R. Taylor (both available at the bookstore). **A calculator or laptop computer with statistical functions (mean, standard deviation, least-squares fit, regression analysis) is also required.**

Administrative policies

The administrative policies for this course are the same as for the 52B lab which precedes this course. We review these policies below.

Prior to your section, use Webwork to complete the Webwork questions.

Use your UCInet ID and password to log in:

(http://homework.ps.uci.edu/webwork/)

- Assignments are normally due at the beginning of your scheduled laboratory section. The first week is an exception: the Week 1 assignment is due at the beginning of your Week 2 section.

- The material in these assignments is explained in the weekly lectures.

- Note that answers to the odd problems in *An Introduction to Error Analysis* appear in the back of the textbook. Use these to check your procedure.

- You can also ask your T.A. for assistance. (If you cannot attend your T.A.'s office hours due to a scheduling conflict, you may attend the office hours of another T.A.)

- Only one attempt is allowed on multiple choice questions, so answer carefully!

- On questions with a numerical answer, you are allowed three attempts. (To accommodate rounding issues, an answer is considered correct if it is within 1% of the value computed by the Webwork program.)

- Each assignment is worth 20 points.

During the laboratory section, you perform the experiment with a partner and document your results in a blue book. You may use a calculator or computer for your analysis and graphs whenever convenient. Guidelines for the lab writeup are described below.

1. *Objective.* Briefly summarize what you are trying to do. What science princple is being tested (e.g., conservation of charge)? What test is performed?

2. *Procedure.* Indicate how you made measurements, if not clearly spelled out in the lab manual. Use schematic pictures to explain measurements of length. Describe the particularities of your apparatus (e.g., the position of a source or the temperature of an oven).

3. *Data.* Record data with the proper number of significant figures. The number of significant digits should correspond to your uncertainty in the measurement. Always write down raw data (e.g., number of counts and time interval) before converting to desired quantity (e.g., count rate). Neat tables help.

4. *Analysis.* Show which formula you are using. Write out numbers once in a sample calculation [e.g., $v = l/t = (11.3\,\text{cm}/0.94\,\text{sec}) = 12.0\,\text{cm/sec}$] – this helps you spot errors – but leave scratch work out of your report.

5. *Conclusion.* A good conclusion is both quantitative and qualitative. The quantitative part of the conclusion is usually expressed as a comparison between the expected quantity and the measured quantity (including uncertainty). For example, you might say "the measured value of $h/e = (4.34 \pm 0.15) \times 10^{-14}$, while the theoretical value is 4.14×10^{-14} J-s/C."

6. *Reflection Questions.* Include the answers to these questions with your report.

Grades are based on Webwork scores (20%), lab report scores (60%), and a final examination (20%). To excel in this course, you must study the material in *An Introduction to Error Analysis*. You should also read through the Laboratory Exercise before you arrive at class and work out any formulas you will need. Your T.A. grades your work using standards established by the instructor and should return it to you at the subsequent section. For each T.A., the lab scores will be adjusted to have the same mean, so don't worry if your T.A. grades more harshly than the others. Historically, approximately 20% of the students receive A's, approximately 40% receive B's, and approximately 30% receive 's.[1] Final grades are assigned by the instructor. If you have trouble with your T.A., contact the instructor. Suggestions for improvements to the course are encouraged.

Make-up Policy

No make-up labs will be given under any circumstances.

[1] Your instructor may depart from this distribution.

- Whether you miss a lab or not, the lowest lab report score will be dropped from your record. Material from every experiment (and Webwork assignment) appears on the final, however, so you are **strongly encouraged to perform every experiment** if at all possible. If you must miss an experiment, study the material carefully in the lab manual on your own.

- Whether you miss a Webwork assignment or not, the lowest Webwork score will be dropped from your record. If problems with your computer or the Webwork program cause you to miss an assignment, that is the score that will be dropped.

- If you must miss two labs during the quarter due to a serious illness or injury or death in your immediate family, present written documentation on offical stationary to the instructor. You will receive the average score you receive for lab reports and quizzes during the quarter for the second week you missed.

- Students who miss three or more labs due to circumstances beyond their control will receive an incomplete.

Holidays

- *Veteran's Day (Fall), Martin Luther King Jr. (Winter), President's Day (Winter), and Memorial Day (Spring)*

  All lab meetings missed due to these holidays are re-scheduled for Saturday of the same week, at the regularly scheduled time.

- *July 4 (Summer), and Labor Day (Summer)*

  Verify with the course instructor the date your missed lab meeting will be re-scheduled on.

Radiation Safety in the Physics Teaching Laboratories

In a series of experiments on quantum physics, the student necessarily comes in contact with radioactive sources, either while studying the properties of the nucleus itself or when using the sources to obtain energetic beams of alpha or beta particles. As is well known, radiation can be harmful to humans and therefore precautions must be taken against undue exposure to it and in the handling of radioactive materials.

Radiation is harmful to living organisms because it destroys individual cells by ionization, and also because it may induce genetic changes. It seems established that low levels of radiation do not produce permanent injury, but the effect is assumed to be cumulative. A genetic change, on the other hand, can be produced by low-level radiation as well as by high-level radiation. It should not be forgotten that human beings have always been exposed to cosmic rays and natural radioisotopes.

The rem (roentgen equivalent man), defined as the amount of any radiation which when absorbed by man will produce the same biological effects as the absorption of 1 roentgen (the loss in air of 78 ergs/gm), is the standard for comparison. The radioactive sources in the student laboratory are rated at less than 5 mrem/hour at a one centimeter distance. The required procedure to handle these sources involves a brief few-second contact with the plastic disc encapsulating the source followed by remaining at least 40 cm distant to operate the counting equipment. The total exposure during a 3 hour laboratory period is:

Handling: 3 sec $\times$ 5 mrem/hr = 0.004 mrem

@ 40 cm: 3 hrs $\times$ 5 mrem/hr = 0.009 mrem

Total Exposure = 0.013 mrem

This value is well below the established permissible dose for an unrestricted area which is stated as 2 mrem in one hour or 100 mrem in seven consecutive days. For further comparison, a 1 microcurie Co^{60} source at a distance of 1 meter produces radiation of 0.0013 mrem/hr.

The most serious danger to a person working in the laboratory is that of taking internally even a very minute amount of radioactive substance. Student laboratories have sealed sources which are encapsulated in plastic to eliminate this hazard.

To the Instructor and Teaching Assistants

- Students may have difficulty completing the laboratory in the allotted time. A brief (2-5 minute) introduction to unfamiliar equipment often helps. While the students are working, circulate throughout the room the entire period to speed their progress.

- Some steps in the lab manual require your signature. In some cases, a check is needed at a crucial juncture to prevent the student from wasting time with a defective apparatus. In others, the goal is to check that the students actually observe the phenomenon (minimize cheating).

- You cannot help the students or grade effectively unless you have performed the experiment yourself. Perform the entire experiment in a blue book during the preparation period. The instructor should check that *all* of the T.A.'s have completed the experiment and that *all* of the equipment is operational.

- All of the written finals should be graded together rather than by individual section. Not all sections are created equal.[2] A common exam provides an objective basis for unequal grading between sections.

Acknowledgments

Much of the material in this lab manual is derived from the 5E Laboratory Manual by W. Molzon *et al.*

[2] One year in 52C, a particularly outstanding section averaged $\sim 80\%$ while the other sections averaged $\sim 50\%$.

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Week 1

Systematic and Random Errors: e/m

In this experiment, a measurement of the deflection of electrons in a magnetic field provides an introduction to the distinction between systematic and random error. You will infer the ratio of the electric charge e to the mass m of free electrons from the orbit of electrons in a magnetic field. An electron-beam tube supplies a (nearly) monoenergetic beam of electrons. The electrons travel in a homogeneous magnetic field produced by two large field coils.

1.1 Theory of the Experiment

An electron moving in a homogeneous magnetic field follows a helical trajectory around the magnetic field lines. The equation of motion of the electron is given by the Lorentz relation. In the absence of an electric field, this relation is

$$\vec{F} = -e(\vec{v} \times \vec{B}), \tag{1.1}$$

where $\vec{F}$ is the force on the electron which has electric charge $-e$ and moves with velocity $\vec{v}$ in the magnetic field $\vec{B}$.

In the special case that the electron moves in an orbit that is perpendicular to the magnetic field, the helical path reduces to a circular path and the magnitude of the magnetic force becomes

$$F = evB. \tag{1.2}$$

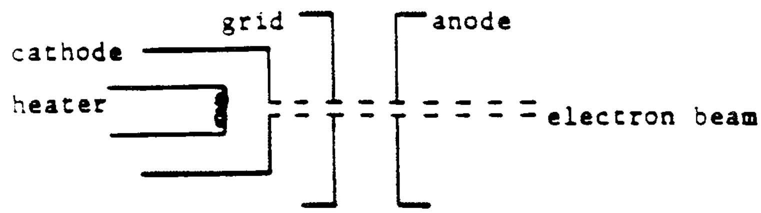

Figure 1.1: Schematic of electron-beam tube.

The centripetal force for a circular path is

$$F_{cent} = mv^2/r. \tag{1.3}$$

where m is the mass of the electron and r is the orbit radius. Therefore,

$$eB = mv/r. \tag{1.4}$$

The electrons in our experiment each have energy eV, where V is the accelerating voltage used in the electron-beam tube. Thus

$$eV = \frac{1}{2}mv^2. \tag{1.5}$$

Combining Eqs. 1.4 and 1.5 yields

$$\frac{e}{m} = \frac{2V}{B^2r^2}. \tag{1.6}$$

The electron-beam tube used in this experiment contains a cathode that is heated indirectly, a collimating grid, an anode with a small hole through it and a pair of deflection plates (Fig. 1.4.1).

Electrons "boil off" the heated cathode and are attracted by the anode which has a positive potential with respect to the cathode. Many electrons stop in the collimating grid or anode. The electrons that happen to go through the hole in the anode emerge from the back of the anode as a thin "pencil" electron beam. The kinetic energy of the electron in this beam is equal to the potential energy difference eV between the anode and the cathode.

To make the path of the electrons visible, the following elegant method is used. The large evacuated glass tube that houses the cathode and anode contains a trace of mercury, enough to have nearly saturated mercury vapor, about 1 x 10^{-3} mm. Occasionally, an electron from the beam which has a kinetic energy of about 300 eV collides with a mercury atom, causing it to become an excited atom since this requires only 10.4 eV. The excited mercury atom decays back to its ground state emitting some photons in the process including the one that has the characteristic blue color of mercury light. The decay of the excited mercury atom is fast, precluding substantial diffusion. Thus, the blue halo in the glass tube marks the path of the electrons.

Note that the electron-beam tube together with its socket can be rotated nearly 90°; this facilitates alignment of the apparatus.

The uniform magnetic field is produced by a coil arrangement known as a Helmholtz coil. A Helmholtz coil is a pair of parallel coaxial circular coils in series that are separated from each other by a distance equal to the radius of the coils. This pair of coils produces an approximately uniform magnetic field in the space between the coils. Along the axis of a thin flat coil with n turns of radius R at a distance x from the plane of the coil the magnetic field B due to a current I in the coil is

$$B = \frac{\mu_0 R^2 n I}{2(R^2 + x^2)^{3/2}}, \tag{1.7}$$

where B is directed perpendicular to the plane of the coil and is expressed in tesla, I is in amperes, R and x are in meters, and the magnetic permeability of free space is $\mu_0 = 4\pi \times 10^{-7}\text{Tesla} \cdot \text{m/amp}$.

For a pair of parallel identical coils, separated by a distance l, the total magnetic field at a distance x along the axis is (Fig. 1.2)

$$B = \frac{\mu_0 R^2 n I}{2} \left[\frac{1}{(R^2 + x^2)^{3/2}} + \frac{1}{[R^2 + (l - x)^2]^{3/2}} \right]. \tag{1.8}$$

For $x = l/2$ and $R = l$ (configuration of a true Helmholtz coil), this gives

$$B = \frac{\mu_0 R^2 n I}{(R^2 + l^2/4)^{3/2}} = \frac{\mu_0 I n}{(5/4)^{3/2} R}. \tag{1.9}$$

As shown in Fig. 1.3, the magnetic field is practically uniform near the center of a Helmholtz coil.

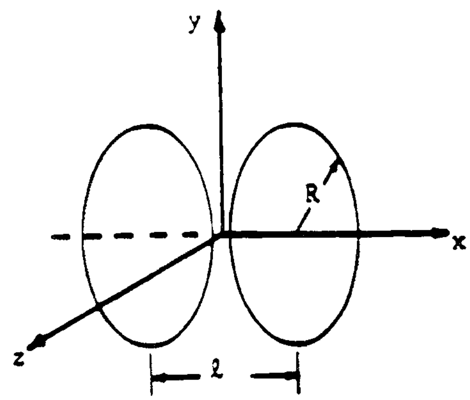

Figure 1.2: Helmholtz coil geometry.

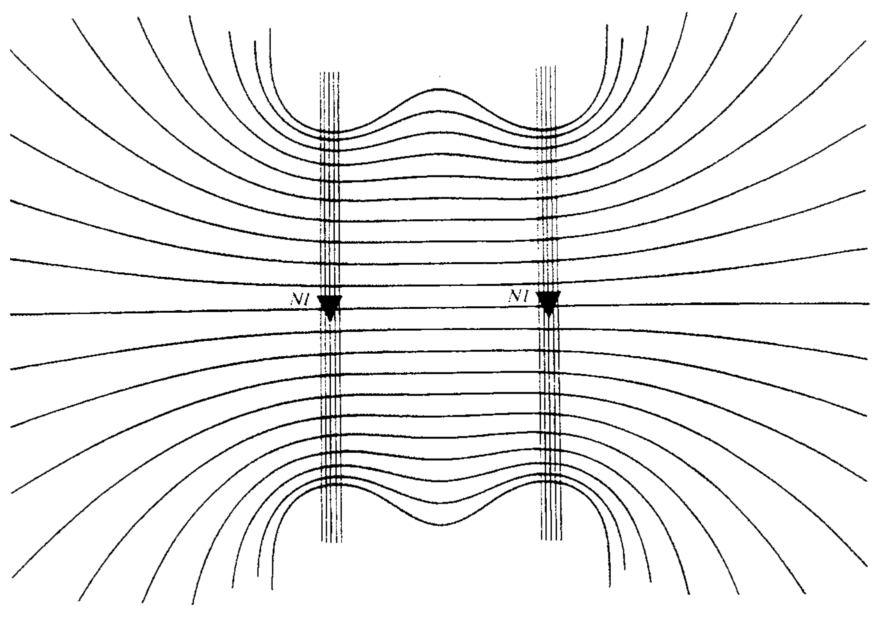

Figure 1.3: Magnetic field lines of a Helmholtz coil in the (r,x) plane (from Barger and Olsson).

1.2 Data Analysis Overview

1.2.1 Terminology

The types of errors most commonly encountered in the laboratory are *blunders*, *definitional* errors, *systematic* errors, and *random* errors. The *precision* of a measurement is determined by the size of random errors, while all four types of errors contribute to the *accuracy*.

blunder A blunder is simply a mistake, an illegitimate error that can be remedied by repeating a measurement (or its recording) or by correcting an erroneous calculation. Recording the mass of a 89.6 g object as 98.6 g or calculating $8/(1/2) = 4$ are examples of blunders.

definitional error A definitional error occurs when there is vagueness in the definition of some quantity. For example, suppose you want to measure your height. Should the measurement be performed with shoes on? Hair pushed down? In the morning? (You are actually about 1 cm shorter at the end of a day of normal activity due to spinal compression.) Because of possible confusion associated with definitional errors, it is good practice to note explicitly the experimental conditions and the selected procedure when performing measurements.

systematic error A systematic error usually results from faulty calibration of equipment or from consistent bias on the part of the experimenter. For example, use of a meter stick with a worn end will generate systematic errors. Systematic errors consistently understate or overstate the actual (true) quantity and thus are not exposed by repeated measurements with the same instrumentation.

random error A random error results from fluctuations in the measured physical quantity, in the operation of the apparatus, or in the judgement of the experimenter. These errors are distinguished from the previous three types by their inevitability. Random errors originate in the uncertainties inherent in any measurement process.

For example, when using a ruler with 1 cm tick marks, random errors of roughly 1 mm are unavoidable.[1]

[1] One might argue that with a fine enough scale random errors could be eliminated.

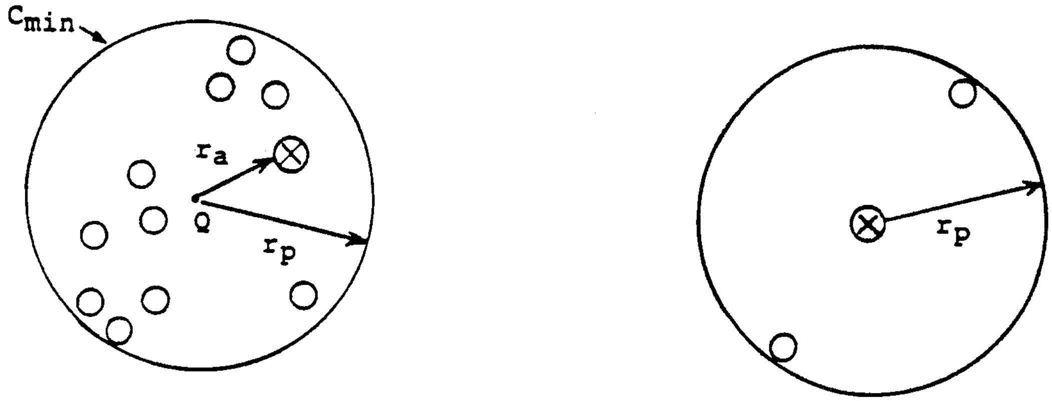

Figure 1.4: (a) A possible distribution of 10 darts. The bull's-eye is marked by the X and the average position of the darts by Q. The distance r_a between X and Q is a measure of the accuracy of the dart thrower. The radius of the circle that encloses all of the tosses r_p is a measure of the precision. (b) Lucky distribution after two tosses.

One way to reduce the magnitude of random errors is to use an instrument with better resolution. Another way to reduce the uncertainty associated with random errors is to repeat the measurement several times.

accuracy and precision The accuracy of a measurement is a measure of how close the measured value is to the accepted or "true" value. Obviously, all types of errors can contribute to the accuracy of a measurement. Precision, on the other hand, refers only to the reproducibility of the measurement. Reproducibility is governed primarily by random errors. To illustrate the distinction between accuracy and precision, consider a person playing darts. Suppose that after $n = 10$ throws, the darts are distributed as illustrated in Fig. 1.4a. The accuracy of the dart-thrower is determined by the average distance of the darts from

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But, in practice, there is always *some* limit to the resolution of the measurement.

Perfect experimental technique can eliminate systematic errors but it cannot eliminate random errors.

the bull's-eye (r_a). The precision of the dart-thrower is given by the scatter of the darts with respect to this average position (r_p). The precision r_p reflects the throwers ability to hit consistently the same spot, even if it is *not* the bull's-eye. The accuracy r_a reflects the tendency of the dart-thrower to aim below and to the left of the target. Typically, if systematic errors are not too large, the accuracy of a measurement is comparable to the precision. A maxim of error analysis is: "the accuracy is justified by the precision." This maxim is not *always* true, however. Consider the dart throw in Fig. 1.4b for $n = 2$. In this case, r_p is considerable but $r_a = 0$; the perfect accuracy is *not* justified by the precision and thus is not a reliable indication of the thrower's tendencies. (The probability that the next dart will strike the bull's-eye is very low even though the perfect accuracy suggests the opposite.) In practical terms, measurements must be repeated several times to rule out luck as an important factor in determination of the accuracy.

1.3 Webwork Questions

Reading Assignment: Chapter 1, 2.1, 2.4, 4.1 of An Introduction to Error Analysis.

Please review the general remarks on the Webwork program in the Preface.

1. Understand the derivation of Eq. 1.6.

2. Make a reasonable estimate of the uncertainty in reading the scale in Figure 1.3 of *An Introduction to Error Analysis.*

3. In Fig. 2.2 of *An Introduction to Error Analysis*, which measurements are in satisfactory agreement with theory and which are unsatisfactory?

4. Problem 2.6 of *An Introduction to Error Analysis.*

5. A student measures the radius of a beam five times and obtains measurements of 5.0 cm, 4.8 cm, 2.4 cm, 5.2 cm and 5.2 cm. Which type of error is the 2.4 cm datum?

6. A student measures the radius of the electron beam five times and gets the results (all in cm) 5.0, 5.1, 5.1, 4.9, 5.0. What type of error is responsible for the variation in these data?

7. A multiple choice question based on Fig. 4.1 of *An Introduction to Error Analysis.*

8. A multiple choice question on parallax, which is discussed on p. 97 of *An Introduction to Error Analysis.*

1.4 Laboratory Procedure

1.4.1 Equipment

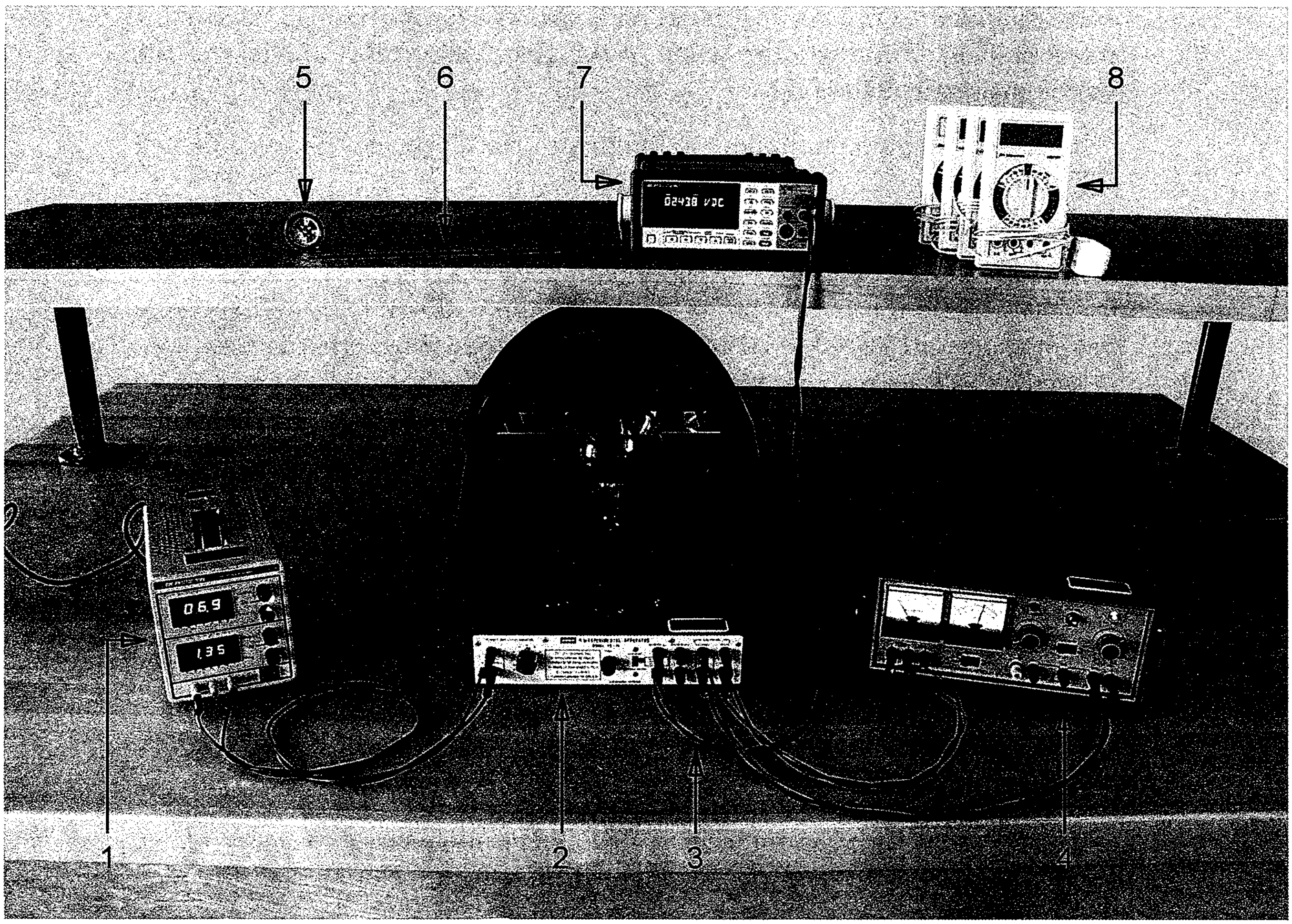


Figure 1.5: Equipment setup for Week 1.

The following equipment is to be set up at all 11 stations, except Item 8.

1- Low voltage power supply (BK 1715 or equivalent)
2- e / m apparatus
3- (9) Banana-banana cables
4- High voltage power supply
5- Compass
6- 30 cm transparent ruler
7- Digital multimeter (BK 2831E or equivalent)
8- (4) Extra digital multimeters (Wavetek HD 110, stored at TA station)

1.4.2 Preliminaries

Orient the Helmholtz coil to eliminate the influence of the earth's magnetic field on the experiment. Use a magnetic compass to determine the direction of the earth's magnetic field. Rotate the coils such that the plane of the coils is parallel to the vertical plane that one can draw through the earth's magnetic field. For this orientation of the Helmholtz coil, the effect of the earth's magnetic field is zero.

Connect the power supplies and meters to the electron tube as indicated in Fig. 1.5.

Check that the polarities agree with the ones marked on the front panel of the tube holder. The coils are powered by a 6-9 V DC, 2 A power supply. The electron tube uses a separate filament supply of 6.3 V AC. The supply for the anode is 150-300 V DC at 10 mA. Note that the current is measured on the low voltage power supply. The accelerating voltage can be monitored conveniently using the two jacks on the front panel. Nothing should be connected to the terminal jacks labeled "DEFLECT PLATES". They are not used in this experiment.

Place the hood over the Helmholtz coil to make your own darkroom for easy observation of the electron beam. Turn on the heater supply and allow two minutes for the filament to heat up. Apply 150 V to the anode. This should produce a visible electron beam. If you cannot see the beam, increase the voltage slightly but **do not exceed 200 V.**

Turn on the current through the Helmholtz coil. **Do not exceed 1.5 A through the coil.**

Observe that the electron beam is being bent into a circle. *Very carefully*, rotate the glass tube and observe a beautiful helix. Notice the direction of the helix-axis. Next, rotate the glass tube such that the plane of the electron beam is exactly parallel to the plane of the Helmholtz coil. Using the focus control, obtain a well-defined electron beam. If necessary, make a fine adjustment of the electron tube orientation so the electron beam, after traveling a full circle, will pass between the two metal struts which feed the power to the filament. Notice the effect of an increased accelerating voltage on the radius r of the electron beam. Also notice the effect of increasing the current in the Helmholtz coil.

1.4.3 Determine e/m

Goal: Measure the radius of the electron beam to deduce the value of e/m.

Set the current I through the Helmholtz coil at some fixed value, say 1 A. Measure the radius R of the Helmholtz coil and calculate the magnetic field halfway between the two coils from Eq. 1.9. The number of windings n of each coil is 130.

Measure the radius of the electron orbit r. (Use the anti-parallax mirror to position your head perpendicular to the beam on both sides of the orbit.) Calculate e/m from Eq. 1.6. Compare with the accepted value.

1.4.4 Discuss possible errors

Goal: Carefully assess sources of error in the experiment.

Make a list of all possible sources of experimental error. Classify each error in your list as random or systematic. If an error is random, try to minimize its impact by repeating the corresponding measurement several times. If appropriate, you may use one of the extra multimeters to assess the impact of any systematic errors. As a final step, rank the likely impact of each of the errors in your list, from most important to least.

1.5 Reflection Questions

1. Why does the radius of the orbit get smaller when the current increases? (Explain in words.)
2. Why does the radius of the orbit get larger when the accelerating voltage increases? (Explain in words in terms of momentum and force.)
3. Can the errors in your list account for the discrepancy between your measured value of e/m and the accepted value?

Week 2

Standard Deviation: Franck-Hertz

In this experiment you will measure the quantization of the electron energy states of atomic mercury. The methods used are based on the Nobel Prize winning efforts of J. Franck and G. Hertz in 1914. By making many measurements and calculating the standard deviation of the data, an accurate estimate of the random error is obtained.

2.1 Theory of the Experiment

The total energy (potential plus kinetic) of an electron bound to its atomic nucleus is negative because the attractive Coulombic potential (negative) has greater magnitude than the kinetic energy (positive). Quantum theory predicts that bound electrons can have only discrete values of total energy, rather than an infinitely fine continuum of values. In contrast, free (unbound) electrons, which have a non-negative total energy, can have any value of total energy.

A source of bombarding electrons can be used to excite the outermost, bound electrons of mercury vapor to higher (but still bound) energy states. Fig. 2.1 is an energy-level diagram for atomic mercury, indicating the many excited states lying above the ground state. Although each of these six states is accessible to the atom the transitions from the energy state at 4.9 eV and 6.7 eV back to the ground state are by far the most probable.[1] Such

[1]Calculating such probabilities is typically treated in an upper division course in quan-

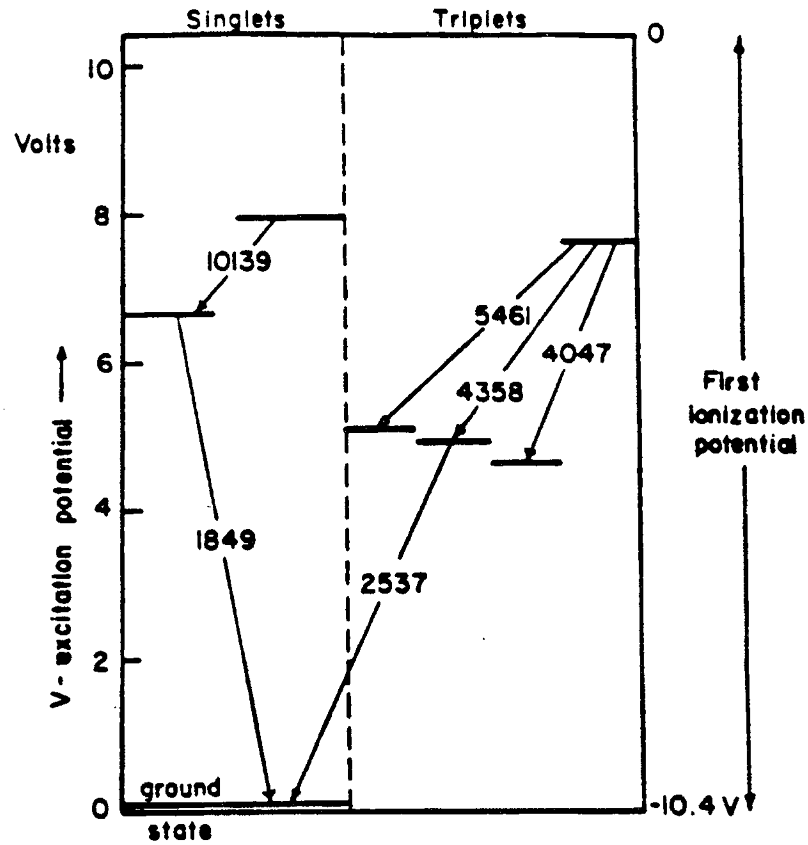


Figure 2.1: "Energy-level diagram for the mercury atom, showing the relation between excitation potentials and the ionization potential. The two main *resonance* lines connect excited states with the ground state. Wavelengths of several optical transitions are shown in angstroms." (from Livesey)

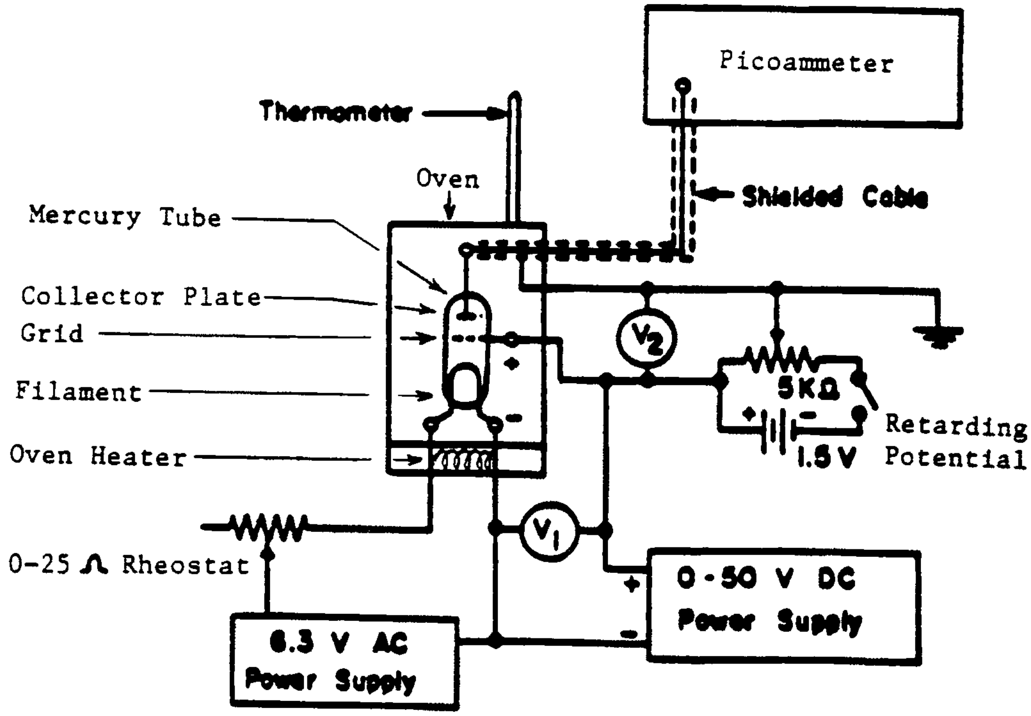


Figure 2.2: Overview schematic of Franck-Hertz apparatus.

transitions are evident by emission of photons with wavelengths 2537 Å and 1849 Å and can, in principle, be measured by a spectrometer.

The fact that only the 4.9 eV and not the 6.7 eV transition is observed is explained later in this discussion. This 4.9 eV transition will be observed by measuring the kinetic energy and current of the bombarding electrons after they have inelastically scattered from the mercury gas.

Fig. 2.2 is an overview schematic of the apparatus. Mercury inside a sealed tube is vaporized to an appropriate density by thermostatically controlling the oven temperature at about 150 C. Inside of the sealed tube, bombarding electrons are "boiled off" the cathode (negative filament) and accelerated towards the perforated anode (positive grid) by an electric field

tum mechanics.

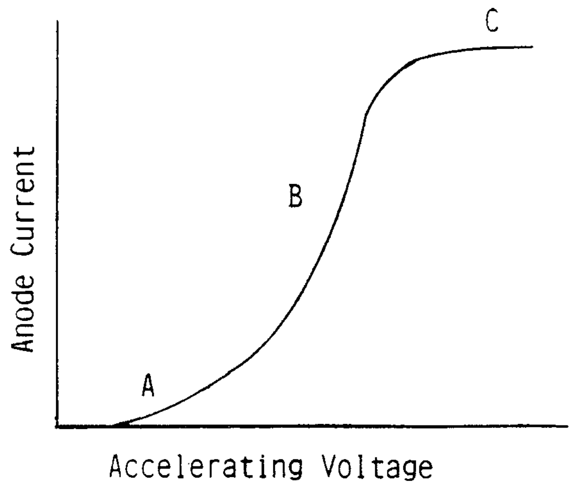


Figure 2.3: Anode current versus accelerating voltage for an evacuated tube such as a diode.

(0-25 volts). A retarding potential (1.0 volt) is applied between the grid and collecting anode (plate) to ensure only bombarding electrons with a minimum threshold of kinetic energy contribute to the current measured by the picoammeter, (10^{-10} amps). As the bombarding electrons are accelerated toward the grid, they scatter inelastically with the mercury atoms with an abundance proportional to the atomic density of mercury gas and its atomic cross sectional area. The *mean free path*, (average distance between collisions) which is inversely proportional to the density and cross section, is on the order of 10^{-2} cm for mercury vapor at 150 C. Thus, the bombarding electrons, in traveling the 1 cm distance between filament and grid, have a high probability of collision with a mercury atom.

To better understand the physics of the bombarding electron's journey from filament to collecting plate, consider first an evacuated tube (such as a simple diode). Fig. 2.3 plots the anode (collecting plate) current versus accelerating voltage. An obvious feature of Fig. 2.3 is the horizontal offset, from zero, of the accelerating potential. This is due to the contact potential difference between cathode and anode. The cathode and anodes are made of different metals and the energy required to remove electrons from

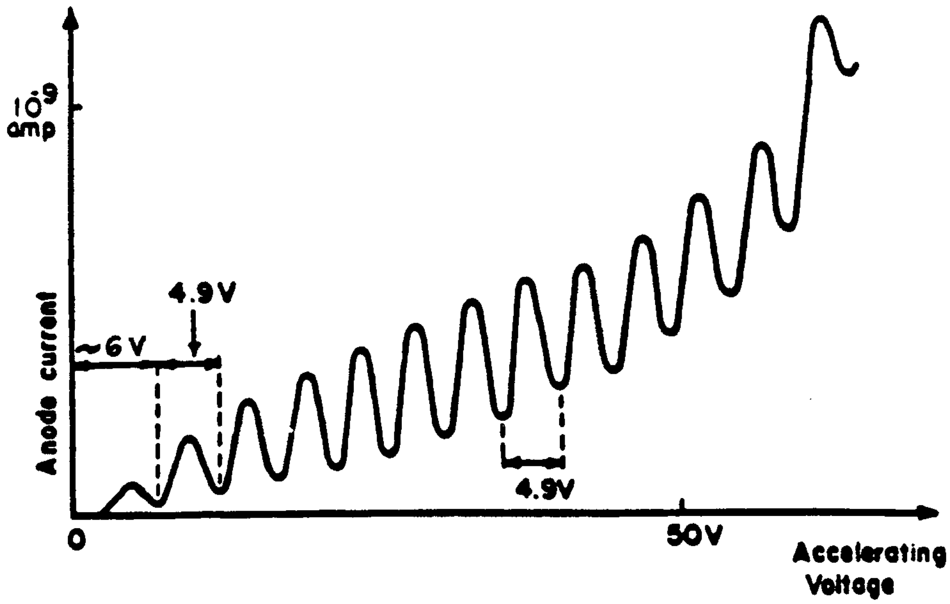


Figure 2.4: Typical characteristics of the Franck-Hertz tube.

the anode is greater than that for the cathode. This effect is present for the mercury filled tube used in the experiment and offsets the accelerating potential by $\approx$ 1 volt. In region A of Fig. 2.3, the slope of the curve is less than in region B because some of the electrons emitted from the cathode are repelled backwards by their slowly moving predecessors (an effect known as space charge limitation) and the positively charged grid actually captures some of the slower electrons. In region B, a monotonic increase in collector current results from an increased accelerating potential. In region C, there is a plateau in collector current because *all* the emitted electrons reach the collector and any subsequent increase in potential will not produce any more current.

Fig. 2.4 shows a plot of anode current versus accelerating voltage for a tube filled with mercury vapor as in this experiment. As the accelerating voltage increases, more and more electrons are able to surpass the retarding potential and reach the plate, resulting in an increased current. When the electrons have less than $\approx$ 4.9 eV of kinetic energy, they collide elastically with the mercury atom and transfer only negligible energy. When the accelerating voltage reaches ≈ 4.9 V, electrons now collide *inelastically* with the atom and transfer their kinetic energy in quanta of 4.9 eV and consequently lack sufficient energy to get past the retarding potential. A marked drop in the collector current results. The drop in current is somewhat gradual because the bombarding electrons boiled off the filament (in a process known as thermionic emission) have a distribution of velocities and thus a distribution of kinetic energies. Note that the first minimum occurs not at 4.9 V but at 4.9 V plus the contact potential difference between cathode and anode. The width of an individual minimum is determined by the size of the retarding potential. As the accelerating potential increases past the first minimum at ≈ 6 V, the collector current increases until the accelerating voltage is large enough that the bombarding electrons can undergo two successive collisions, and the current again drops. Such repeated inelastic scattering continues with increased accelerating potential and explains the 4.9 V spacing between successive minima. Because the lifetime of the excited state is very short (10^{-8}s), it is relatively improbable that atoms in the excited state will be excited to still higher energy levels. Furthermore, because the bombarding electrons start with very little kinetic energy as they are accelerated through the electric field, they give up quanta at the first opportunity for inelastic scattering at 4.9 eV. This is why the 6.7 eV state discussed earlier isn't observable in the experiment.

2.2 Webwork Questions

Reading Assignment: Chapter 4 of An Introduction to Error Analysis.

1. Problem 4.1 of *An Introduction to Error Analysis.*

2. Problem 4.13.

3. In the experiment, the bombarding electrons do not transfer very much energy to the mercury atoms in elastic collisions because mercury atoms are much heavier than electrons. From momentum and energy conservation, it can be shown that the energy lost in a head-on elastic collision is $\Delta K/K_0 = 4mM/(m+M)^2$ where ΔK and K_0 are the change in electron kinetic energy and the initial kinetic energy and m and M are the masses of the electron and mercury atom. What is the fractional

loss of kinetic energy by the electron in a head-on elastic collision with a mercury atom?

4. Assuming that the first excited state of the mercury atom is about 5.0 eV above the ground state, what is the maximum amount of energy an electron with 4.0 eV of kinetic energy can transfer inelastically to a mercury atom with which it collides? A 6.0 eV electron?

5. The cathode at temperature T "boils off" electrons with an average kinetic energy $\bar{K} = 2kT$ where k is Boltzmann's constant. The cathode in this experiment is at $T \simeq 2500$ K. What is the average kinetic energy (in eV) of such electrons? (1 eV =11600 K.)

6. Sketch what Fig 2.4 would look like if the electrons left the cathode without any kinetic energy.

2.3 Laboratory Procedure

2.3.1 Equipment

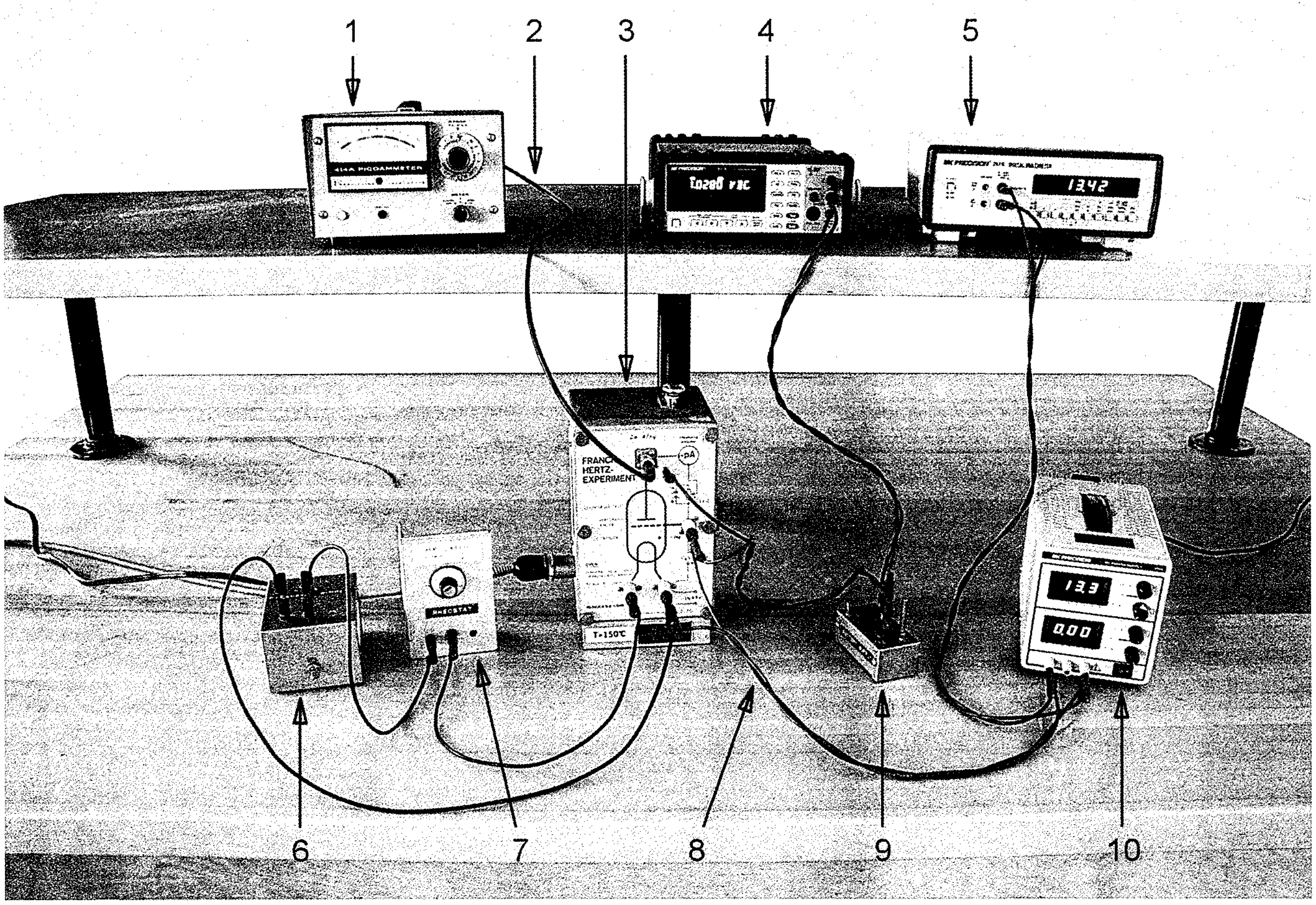


Figure 2.5: Equipment setup for Week 2.

The following equipment is to be set up at all 11 stations.

1- Picoammeter
2- BNC-BNC cable
3- Franck-Hertz apparatus w/ power cord
4- Digital multimeter (BK 2831E or equivalent)
5- Digital multimeter (BK 2831C or equivalent)
6- 6.3 V power supply
7- 25 Ohm, 25 Watt rheostat (variable resistor)
8- (11) Banana-banana cables
9- 1.5 V battery supply
10- Low voltage power supply (BK 1715 or equivalent)

2.3.2 Excitation potential of mercury

Goal: Measure the discrete energy gap between the ground and first excited state of mercury.

Caution! The hot apparatus can burn you!

FAMILIARIZATION: Before anything else, take a few minutes to look through the oven window and observe its components. Refering to Fig. 2.2, be sure you can identify the oven's coiled wire heater and thermostat control knob. Inside of the mercury filled Franck-Hertz tube, identify the filament, grid, and plate. Note that the front panel of the oven also has a useful schematic of the Franck-Hertz tube.

CIRCUIT: Make sure all of the equipment is switched off. Carefully wire the circuit shown in Fig. 2.2, then check it a second time.

OVEN: Make sure the bottom of the thermometer is level with the middle of the Franck-Hertz tube. Plug in the oven, and adjust the thermostat so that the oven stays near 150 C (or the specified temperature on your apparatus).[2] Do not trust any marking on the thermostat; use the thermometer to establish temperature. It will take about 10 minutes for the temperature to stabilize. Since the oven leaks some of its heat to the surroundings, the thermostat will continually turn the oven on and off during the experiment in an effort to maintain the established temperature. The picoammeter reading is unreliable when the oven heater is on, so avoid making measurements whenever the heater is on.

VOLTAGES AND CURRENTS:

a. Verify that the 6.3 V filament power supply is off. Turn the rheostat (25 ohm variable resistor) fully counter-clockwise for minimum current.

b. Adjust the accelerating (grid) potential to 5 volts.

c. Adjust the retarding potential to 1 volt.

d. Turn on the picoammeter, set the meter to measure "–" current (remember you're measuring electron current), and set the range to 0.1 nanoamps (0.1 x 10^{-9} amps).

e. Turn on the filament power supply. Now, VERY SLOWLY turn the rheostat clockwise to increase filament current until the collector current

[2]Newer tubes require higher operating temperature.

reads between 0.01 and 0.02 nanoamps on the picoammeter. A correctly adjusted filament emits a faint orange glow.

WARNING: The rheostat must be adjusted slowly to allow for the time lag between the increase in filament current and the corresponding increase in thermionic emission from the filament. If the filament current is set too high, a plasma discharge will occur within the Franck-Hertz tube, as evidenced by a BLUE GLOW. The resulting high currents can damage the picoammeter as well as the tube itself. If a BLUE GLOW should occur, immediately shut off the filament power supply, turn the rheostat fully counter-clockwise, and return to step (e).

TEST APPARATUS: Vary the accelerating voltage between 10-12 volts to find a maximum. If you cannot see a maximum, ask your T.A. to check your apparatus. Note: the fine gain control is nonlinear. To obtain variations of ~ 0.1 V, use the second half of the range.

TAKING DATA: The apparatus is now adjusted to study the excitation potential of mercury. Start with the accelerating voltage set to zero, and record accelerating voltage and anode current as you incrementally increase grid voltage through at least 5 anode current minima and 5 current maxima. You will need to regularly adjust the scales on the 50 volt power supply and picoammeter. To find a maxima or minima, watch the current meter while making fine adjustments to the accelerating voltage dial.

REPRODUCIBILITY CHECK: As a rough estimate of uncertainty in the measurement, repeat one of the measurements a few times. [Be sure to perform a "blind" check without paying attention to the previous value(s).]

After gathering data, turn everything off, including the oven.

2.3.3 Data Analysis

Goal: Calculate the excitation potential of the first excited state..

- Make a plot of anode current versus accelerating voltage.
- Calculate the difference in voltage ΔV between successive minima and between successive maxima.
- Use the ΔV measurements to calculate the mean and standard deviation. This is your measured value of the excitation potential of mercury, including uncertainty.

- Compare the measured value to the expected value.

2.4 Reflection Questions

1. Is your calculated value of σ close to your rough estimate of the uncertainty in the measurement? Should it be?

2. Why does the procedure we used of averaging several measurements of *both* maxima and minima better than a single measurement of the excitation potential? Why is it better than just measuring the minima?

3. In this experiment, you measured the current as a function of voltage. Does the Franck-Hertz tube obey Ohm's law? Explain.

Week 3

Normal Distribution: Radioactive Counting

In this experiment, radioactive counting is used to explore the properties of the Normal[1] distribution.

3.1 Theory of the Experiment

3.1.1 The Geiger-Mueller tube

Probably the most common detector of ionizing radiation is the Geiger-Mueller, or Geiger, counter. This is usually a cylindrical tube with a small wire along the axis of the tube, such as that in Fig. 3.1. It is filled with a gas, usually neon or argon, at a pressure of a few cm of mercury. The wire (anode) is usually at a voltage several hundred volts positive with respect to the grounded outer surface (cathode). One end of the cylinder is usually a thin window made of a material such as mica, so that weakly penetrating radiation such as β particles (electrons) can enter the active region of the Geiger tube.

Under normal operation, a single particle passing through the tube ionizes either gas atoms or atoms at the surface of the window or cathode, forming a free electron and a positive ion. The free electrons are accelerated in the electric field toward the anode, losing energy by excitation of the gas atoms. Near the anode, the electric field is sufficiently high that the electron gains

[1] Also called the Gauss distribution.

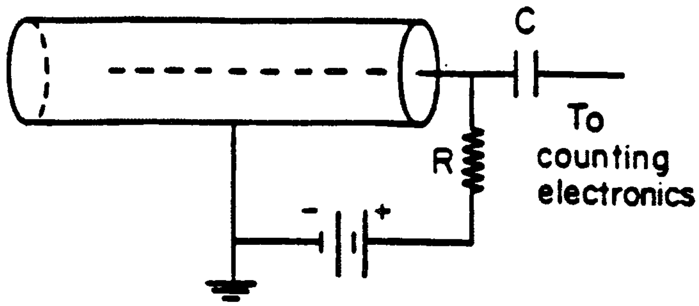


Figure 3.1: Typical Geiger tube.

enough energy before colliding with a gas atom that it can ionize additional atoms. An avalanche (or discharge) is produced in the radial direction. The discharge also propagates axially along the wire, as photons emitted by excited atoms ionize distant atoms and these newly liberated electrons generate their own avalanches. The signal induced on the anode wire causes a current to flow through the resistor attached to the anode. The current is sufficiently large to cause a voltage drop $\Delta V = IR$ which lowers the potential of the anode. This lower potential, combined with the space charge of positive ions in the vicinity of the wire, lowers the electric field to the point where the discharge ceases. The electric field then sweeps the ions and the remaining electrons out of the tube and the anode returns to the original potential. The tube is now ready to respond to the next particle that enters the active region. All this takes place in a very short time and the resulting voltage pulse is transmitted to the counting electronics through the capacitor C.

If the counting rate of a Geiger tube is plotted against the voltage across the tube, a curve like Fig. 3.2 results. The tube does not operate below some voltage V_1 and the counting rate rises rapidly above V_1. Above the *threshold voltage* V_2 the counting rate is essentially constant over a *plateau* region which may extend over a range of a few hundred volts. Above a voltage V_3 the counting rate rises abruptly, a continuous discharge soon sets in and the tube is destroyed. This occurs near 1 kV for most tubes.

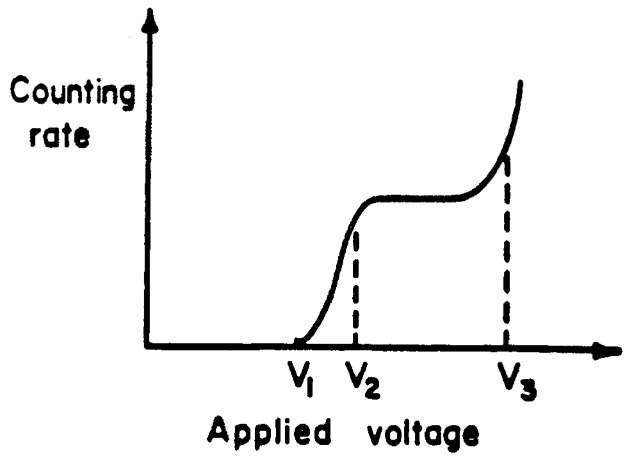


Figure 3.2: Count rate versus applied voltage for a Geiger tube.

3.1.2 Stochastic nature of radioactivity

An atom is composed of electrons, protons, and neutrons. The number of protons Z defines the element and its position in the periodic table. In a neutral atom, the number of negatively charged electrons equals the number of positively charged protons. The number of electrons and protons determine the chemical properties of the atom. Chemical properties are manifestations of electrical and magnetic forces. The number of neutrons N has virtually no effect on the chemical properties of the atom but does affect the atomic weight A of the atom. Since neutrons and protons have nearly identical masses and weigh much more than electrons, $A = N + Z$. Neutrons and protons constitute the nucleus and are held together by nuclear forces. A given element is described by the notation ${}^{A}E$, where E is the name of the element (determined by the charge Z) and A is the atomic weight. For example, ${}^{16}\text{O}$ signifies an oxygen atom ($Z = 8$) with $16 - 8 = 8$ neutrons.[2] Nuclei with the same number of protons but different numbers of neutrons are termed *isotopes.*

Just as some atoms are stable (e.g., gold) while others are not (e.g., fluorine), some nuclei persist indefinitely while others tend to rearrange them-

[2]Some authors place the atomic weight after the element, O^{16}.

selves. Atomic stability depends on the arrangement of electrons in shells. Nuclear stability depends on the arrangement of protons and neutrons in shells. Unstable nuclei that spontaneously decay into other nuclei are termed *radionuclides* and the phenomenon is called *radioactivity*.

The radiation emitted by a radionuclide is generally either an alpha (a ^4He nucleus), a beta (an energetic electron), or a gamma (a high-energy photon). The ^{137}Cs and ^{60}Co sources used in this experiment emit betas and gammas.

All evidence to date strongly supports the assumption that radioactive atoms decay completely independently of one another and that standard probability theory can be applied. An important result in probability theory is that, if the number of expected random events $\bar{n}$ in a given time interval is sufficiently large, the distribution of events approaches the normal distribution,

$$P(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(n-\bar{n})^2/2\sigma^2}. \tag{3.1}$$

Also, for radioactive decay, the standard deviation is $\sigma = \sqrt{\bar{n}}$. The main objective of this experiment is to compare the observed distribution of the number of radioactive decays in a given time interval to that predicted by Eq. 3.1.

3.1.3 Safety

Three factors determine the *dose*, or amount of radiation, received by someone who handles radioactive materials: the strength of the source, the duration of exposure, and the proximity of the source. The strength of the source is determined by the number of radioactive nuclei; it is standard practice to choose the weakest source for the task at hand (in our case, acquiring a statistically significant number of counts). Since nuclei decay randomly, dose is minimized by minimizing the duration of exposure. Since radioactive decay is isotropic, the dose falls off rapidly as one moves away from the source ($\Phi \propto r^{-2}$). A fourth factor is the type of particle emitted by the radionuclide.

The following rules should be followed when handling radioactive sources in the laboratory.

- Handle the sources as little as possible (minimize close exposure). When counting samples, do not stand beside the source (maximize distance). Set unused samples 1-2 meters away.

- Avoid direct contact with the skin. Grasp the sample by the non-radioactive exterior of the case. Wash your hands following the laboratory. Many radionuclides are harmless unless ingested or inhaled.
- Mouth pipetting, eating, drinking, and smoking are prohibited. Many radionuclides are harmless unless ingested.
- Return all sources to the closed, labeled container handled by your T.A.
- Should an accident or spill occur, report it immediately to your T.A.

The rem is the unit used by health physicists to measure exposure to radiation. Natural background for an "average" person (actually, background varies greatly with altitude and geology) in the USA is 300 mrem per year. The radioactive sources in the student laboratory are rated at less than 5 mrem/hour at a one centimeter distance. A typical exposure during a 3-hour laboratory period is 0.02 mrem. The following table lists the exposure one receives in a year doing several common activities. Obviously, if properly handled, the dose received in this laboratory is well below natural background.

The rem unit is slowly being replaced by the internationally accepted unit *Sievert* (Sv), where 1 Sv = 100 rem.

Activity	*Approximate Exposure (mrem)*
Smoking cigarettes	2000
Residing in Colorado Springs for a year	200
Diagnostic medical X-rays	140

3.2 Webwork Questions

Reading Assignment: Chapter 5 (omit 5.6-5.7) of An Introduction to Error Analysis.

1. Consider Fig. 3.2. In normal operation, the Geiger tube is operated in the plateau region above V_2. What is wrong with operation at other voltages?
2. Multiple choice question on radiation safety.

3. Problem 5.4 (first question).
4. Problem 5.4 (second question).
5. Problems 5.6 and 5.7.
6. Multiple choice question on the effect of X and σ on the normal distribution G.
7. Problem 5.21.

3.3 Laboratory Procedure

3.3.1 Equipment

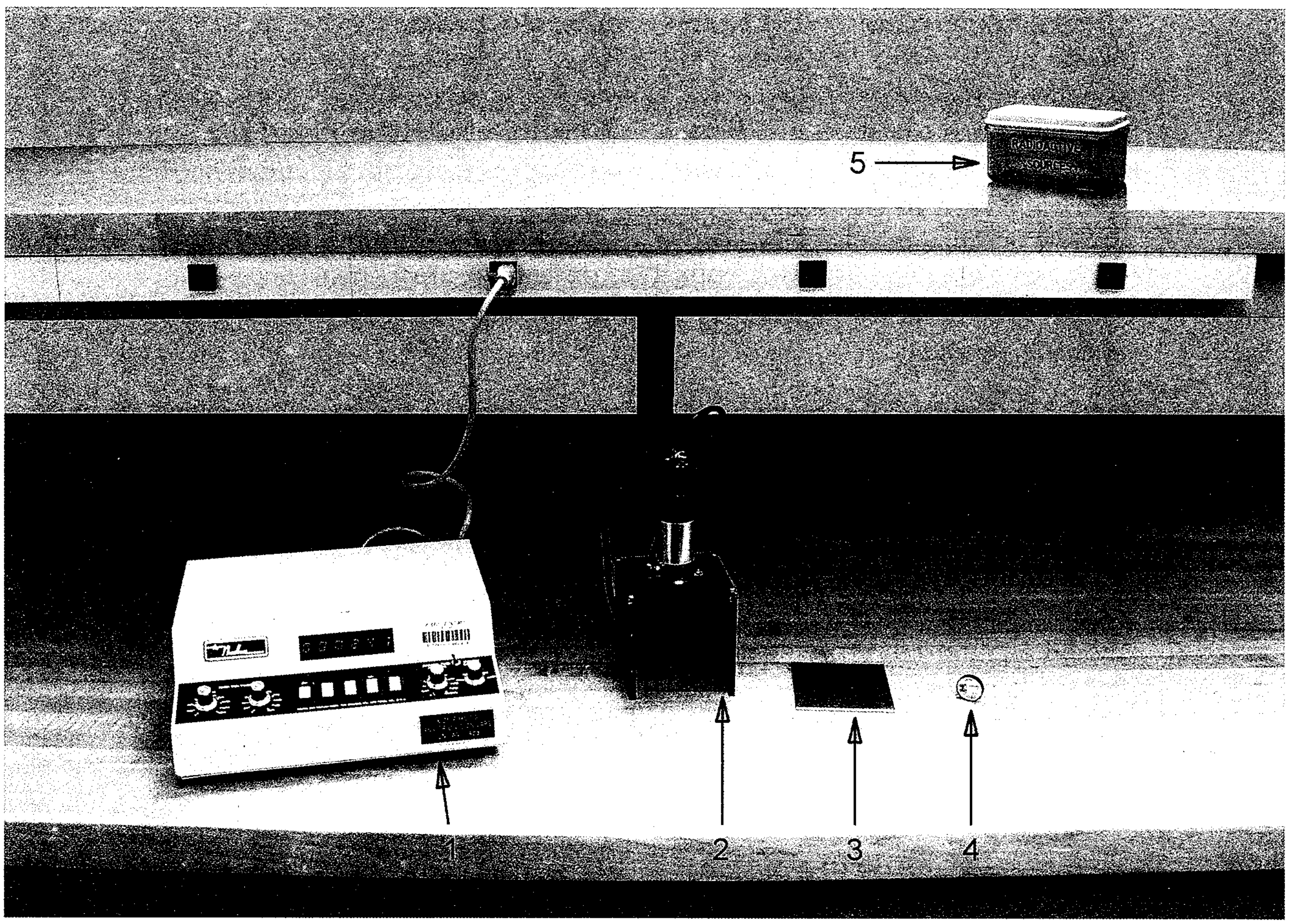


Figure 3.3: Equipment setup for Week 3.

The following equipment is to be set up at all 11 stations, except for Item 5.

1- Nuclear scaler
2- Geiger tube w/ mount
3- Solid aluminum plate
4- Cesium 137 <u>or</u> Cobalt 60 sources
5- Storage box for sources. TA will distribute and collect all sources, and keep box at TA station.

3.3.2 Geiger tube operating voltage

Goal: Determine the voltage characteristic of your Geiger tube.

With the scaler turned off, connect the Geiger tube to the rear panel BNC connector with a coaxial cable. **Make sure the high voltage control is turned to zero, i.e., all the way counterclockwise.** Place a radioactive source on the table below the Geiger tube. Remember that the mica end window of the Geiger tube is very thin and easy to puncture. Turn on the scaler, depress the start-stop button and increase the voltage until the tube begins to respond. Now start with a slightly smaller voltage and take the necessary data to determine the characteristic of your Geiger tube.[3] Take data at reasonable voltage intervals but **do not exceed 900 volts.** Plot the data and choose an operating voltage in the middle of the plateau (typically $\sim$ 800 V). Set the voltage at this value and leave it there for the remainder of the experiment.

3.3.3 Gaussian distribution

Goal: Measure the random distribution produced by radioactive decay.

Position the source so that the number of counts in a 6 second interval is approximately 100. The time interval must be accurately known so that all the fluctuation in the count rate is due to the random nature of radioactive decay. When $\bar{n}$ is as large as 100, the expected distribution is the normal distribution given by Eq. 3.1. Take data for at least 100 intervals and record it in your blue book. Next, analyze the data as follows.

- Determine the average number of counts $\bar{n}$ and the standard deviation σ of the distribution.

- Organize the data into bins. Choose $b = \sigma/2$ as your bin width; i.e., make a table of the number of counts between $\bar{n}$ and $\bar{n} + \sigma/2$, $\bar{n} + \sigma/2$ and $\bar{n} + \sigma$ and between $\bar{n}$ and $n - \sigma/2$, $\bar{n} - \sigma/2$ and $\bar{n} - \sigma$, etc. (a total of approximately 14 bins).

- Plot the binned data as a histogram.

[3] For many tubes, the higher voltage V_3 in Fig. 3.2 exceeds 900 V.

- Use the measured mean and standard deviation to calculate the expected gaussian distribution, and plot it on top of your histogram. For bins of width b, the expected probability is approximately $bG(x)$.

- The following table lists the expected values for a normal distribution.[4] Add your data to the table and compare your results quantitatively with the expected values.

Interval	Probability
$\bar{n}$ to $\bar{n} + \sigma/2$	.1915
$\bar{n} + \frac{\sigma}{2}$ to $\bar{n} + \sigma$	.1498
$\bar{n} + \sigma$ to $\bar{n} + \frac{3\sigma}{2}$	.0919
$\bar{n} + \frac{3\sigma}{2}$ to $\bar{n} + 2\sigma$	.0440
$\bar{n} + 2\sigma$ to $\bar{n} + 5/2\sigma$	.0166
$\bar{n} + \frac{5\sigma}{2}$ to $\bar{n} + 3\sigma$	.0048
greater than $\bar{n} + 3\sigma$	.0014

3.3.4 Standard Deviation

Goal: Verify that $\sigma \simeq \sqrt{\bar{n}}$.

In a counting experiment, σ should be equal to the square root of the average number of counts.

Increase the count rate to about 1000 counts for a 6-second interval and record the counts in 10 such intervals. Calculate the mean and standard deviation for this case.

At this same counting rate, record the number of counts in 10 intervals each 1 minute long (average number of counts $\sim$ 10,000). Calculate the mean and standard deviation for this case.

Finally, position the source far from the counter so that $\bar{n} \simeq 10$ in a six-second interval. Record the number of counts ten times; calculate $\bar{n}$ and σ.

Prepare a table of $\bar{n}$, σ, $\sqrt{\bar{n}}$, $\sigma/\bar{n}$ and $\sigma/\sqrt{\bar{n}}$ for the four cases: $\bar{n} \simeq 10$, 100, 1000, and 10,000. Is $\sigma \simeq \sqrt{\bar{n}}$?

[4] The probability is symmetric about $\bar{n}$.

3.4 Reflection Questions

1. Notice how erratically the number of counts increases while counting. Why doesn't it increase steadily?

2. Why don't your data in Sec. 3.3.2 change as smoothly as in Fig. 3.2?

3. Did your data in Sec. 3.3.3 agree very well with the predicted normal distribution? What could you do to improve the agreement?

4. A reasonable measure of the accuracy of the measurement is the fractional variation in the number of counts $\sigma/\bar{n}$. Ideally, to achieve excellent temporal resolution, one would use very short time bins (Δt) but, in practice, the bin duration Δt is determined by the required accuracy. For example, in an x-ray measurement, one may have to sacrifice accuracy to obtain better temporal resolution because the flux of photons is limited. How large does $\bar{n}$ need to be to achieve $\sim 10\%$ uncertainty in a counting experiment? To achieve $\sim 1\%$ error?

Week 4

Error Propagation: Gamma Absorption

This week, the propagation of uncertainties in physical measurements is illustrated by a study of the absorption of gamma rays in lead.

4.1 Theory of the Experiment

When radioactivity was first discovered, early workers had not yet identified the radiation so they called the three readily detected particles α, β, and γ. We now know that an alpha is a helium nucleus, a beta is an electron, and a gamma is a high energy photon (hard X-ray) but the names have stuck. These particles are emitted by unstable nuclei when they undergo radioactive decay. In this laboratory, you will distinguish alphas, betas, and gammas by their most easily observed feature: their *range* (or penetration depth). Another important distinction between the particles is that while alphas and gammas are emitted at characteristic, single energy levels, beta particles are emitted over a spectrum (continuous distribution) of energy. Americium-241 emits a $\sim$ 5 MeV alpha, thorium-232 emits betas over a $\sim$ MeV range of energy, and cobalt-60 emits a $\sim$ 0.3 MeV beta and two $\sim$ 1.2 MeV gammas. As you will learn in this laboratory, alphas are easily stopped, gammas are fairly penetrating, and betas are intermediate. Another common product of radioactive decay, the neutron, is even more penetrating than gammas.[1]

[1]Neutrinos are also emitted but they hardly interact with matter at all.

Alphas are readily absorbed. Since an alpha is a helium nucleus, it is relatively massive and travels relatively slowly. It is positively charged, so when it enter matter it loses energy to the atomic electrons in the material. Since alphas are heavier than electrons, alphas travel in nearly a straight line while gradually slowing down (like a bowling ball in a sea of marbles). Most alphas from a given source type travel the same distance in a solid (called the *range*). Alphas slow down so quickly that they can only penetrate a few centimeters of air.

Since betas are electrons, they are faster than alphas. They also slow down primarily through collisions with electrons but, since they are relatively light, their path in solids is tortuous rather than straight. Betas can travel a few meters in air.

The distance a particle travels in a solid tends to be inversely proportional to the density of the solid ρ. Consequently, the product of the density and a typical penetration depth varies much less from material to material than the depth itself. The product of density and distance, ρx, is called the *absorber thickness*. For alphas, if the absorber thickness exceeds a certain value, virtually no particles will penetrate the solid, while for smaller values of ρx, nearly all of the particle penetrate. Because betas from a given source are emitted over a spectrum of energies as noted earlier, they will not exhibit the sharp cut-off in transmission just described for alphas. In fact, even a thin absorber begins to block the betas and the transmission falls off continously (although *not* exponentially) with increasing absorber thickness.

Unlike alphas and betas, gammas are uncharged and do not lose energy electrostatically. As a consequence, they are more penetrating than either betas or alphas. They are stopped differently too: photons are either converted to another particle or they pass through the atom unchanged. Thus, unlike alphas, the number of gammas steadily decreases as a beam of gammas passes through a solid. When a beam of gamma rays pass through a thickness of a material dx, the decrease in intensity dI is proportional to the incident intensity

$$dI = -\mu I dx, \tag{4.1}$$

where μ is the absorption coefficient in units of cm^{-1}. Integrating Eq. 4.1, we find that the intensity of a beam of monoenergetic gamma rays decreases exponentially with distance,

$$I = I_o e^{-\mu x}. \tag{4.2}$$

There are several processes responsible for the attenuation of the gamma-ray beam: the photoelectric effect, Compton scattering, and pair formation. In the photoelectric effect, all the energy $h\nu$ of the photon (remember that a gamma ray is a high frequency photon) is transferred to an electron bound in an atom. This electron is then ejected from the atom with a kinetic energy $E_k = h\nu - E_o$ where E_o is the binding energy of the electron. This process removes the photon from the incident gamma ray beam. Below 500 keV, the photoelectric effect gives the largest contribution to the absorption coefficient.

The main contribution to the absorption coefficient in lead for energies in the range from 0.5 and 5 MeV is Compton scattering. As explained in any textbook on modern physics, Compton scattering is a collision between an electron and a photon. This process differs from the photoelectric effect in that the photon is not fully absorbed. Some energy is transferred to the electron, thus decreasing the photon energy and scattering it from its initial direction. The change in direction removes the photon from the incident gamma ray beam and effectively contributes to the decrease in the intensity.

At still higher energies, pair formation becomes the principal mechanism for gamma ray absorption. In this process, a gamma ray with sufficient energy can disappear and an electron and positron are created by the process

$$\gamma \rightarrow e^- + e^+. \tag{4.3}$$

Pair production cannot occur for $h\nu < 2m_oc^2$ where m_o is the rest mass of the electron. Since $m_oc^2 = 0.51$ MeV, pair production can begin at energies above 1.02 MeV. This process cannot occur in free space, because energy and momentum cannot both be conserved. It occurs only in the strong electric field near a nucleus. In lead, the attenuation due to pair production becomes larger than that due to Compton scattering above approximately 5 MeV.

It is possible to calculate the attenuation due to all these processes. It is found that the total absorption coefficient μ is a strong function of energy. Fig. 4.1 is a plot of μ versus E taken from *Nuclear Physics* by I. Kaplan. From a measurement of μ and the curve in Fig. 4.1, it is possible to approximately determine the energy of the gamma rays.

There are many ways of expressing the absorption coefficient. One of the more convenient ways is in terms of the *half-thickness*, i.e., the thickness of lead needed to reduce the intensity to half its initial value. Letting $I/I_o = \frac{1}{2}$ in Eq. 4.2 and taking the logarithm

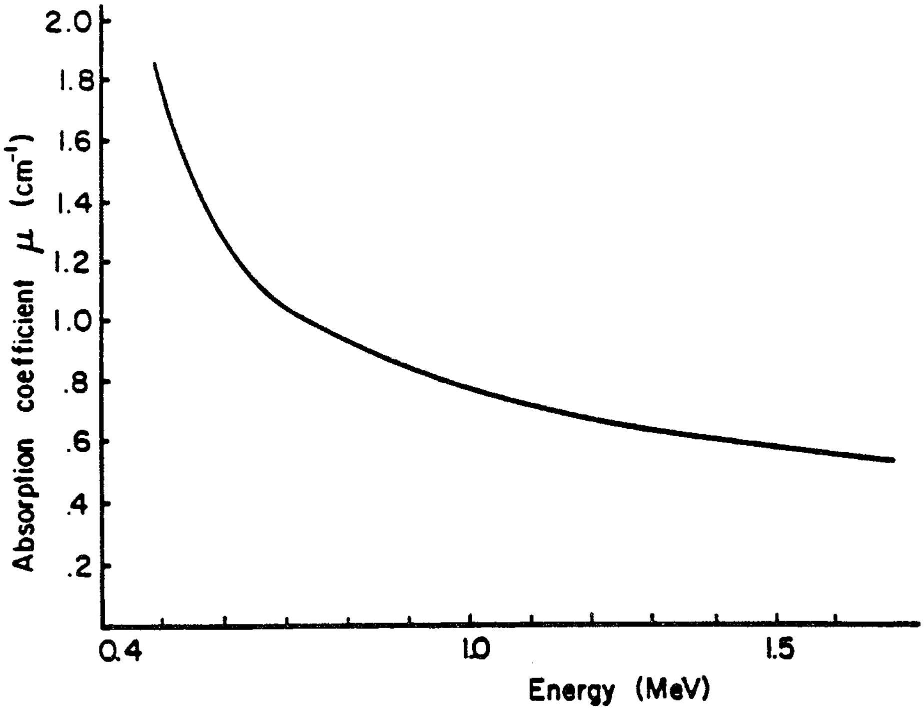


Figure 4.1: Total absorption versus energy for gamma ray attenuation by lead.

$$\ln \frac{1}{2} = \mu x_{\frac{1}{2}}$$

or

$$\mu = \frac{0.693}{x_{\frac{1}{2}}}. \tag{4.4}$$

Another way of expressing the attenuation coefficient is in terms of the *cross section*, σ (not to be confused with the standard deviation). If an electron had an effective radius r_e, then the cross sectional area of the electron would be $\sigma = \pi r_e^2$. For most nuclear processes, the convenient unit of area is 10^{-24} cm^2, known as a *barn*. Now let us relate the cross section of the electron to the attenuation coefficient μ. The precise definition of the cross section σ for scattering by a single object is the number of scattered particles divided by the number of incident particles per unit area; σ has the units of an area and can be thought of as the geometric cross section of the particle. If the density of scatterers is n particles per unit volume, there will be ndx scatterers per unit area of a material with thickness dx. Then the probability of scattering in that thickness is $P = \sigma ndx$. To obtain the probability of scattering in a length x of some absorber, let I_o be the intensity falling on the material and dI be the change in intensity. Then

$$-dI(x) = PI(x) = I(x)\sigma ndx \tag{4.5}$$

or

$$I(x) = I_o e^{-\sigma nx}. \tag{4.6}$$

Comparing this result to Eq. 4.2, we can equate $\mu = n\sigma$. Since it is the electrons in the lead which are responsible for the scattering, the density n is that of electrons

$$n = \frac{\rho NZ}{A}, \tag{4.7}$$

where ρ is the density of Pb in g/cm^3, N is Avogadro's number 6.02×10^{23} particles/mole, Z is the atomic number, and A is the weight of one mole in grams. We can then make the identification

$$\mu = \frac{\sigma\rho NZ}{A}. \tag{4.8}$$

From a measurement of μ and values of ρ, N, Z, and A which can be found in Table 4.1, you can calculate the cross section for gamma ray scattering. At the energy of gamma rays emitted by Co^{60} (the isotope used in this experiment), the principle scattering process is Compton scattering and the measured cross section will be approximately the Compton scattering cross section.

The decay scheme of Co^{60} is shown in Fig. 4.2. Co^{60} decays to an excited state of Ni^{60} by beta emission. The excited state decays to the ground state by 2 successive gamma ray emissions. The gamma rays have nearly the same energy and experimentally will appear as monoenergetic gamma rays of energy approximately 1.2 MeV.

Material	**At. No.**	**At. Wt.**	**Density**	**Comments**
Concrete	low Z	–	–	cheap for neutrons
Aluminum	13	27.0	2.70	
Iron	26	55.8	7.87	in steels
Mercury	80	200.6	13.5	low radioactivity
Lead	82	207.2	11.35	popular for gammas

Table 4.1: Properties of common radioactive shielding materials.

4.2 Webwork Questions

Reading Assignment: 2.5, 2.7-2.9, Chapter 3, and 11.4 of An Introduction to Error Analysis.

1. Problem 3.15.
2. Problem 3.24.
3. Problem 3.25.
4. Problem 3.26.
5. If $q = \cos(4\theta)$, where θ is given in radians, what is δq if $\theta = 0.20 \pm 0.02$? (Hint: Use the general formula, Eq. 3.47.)
6. Problem 11.20.
7. Use Fig. 4.1 and Eq. 4.8 to find the cross section of the electrons in lead for 1.0 MeV gamma rays.

4.3 Laboratory Procedure

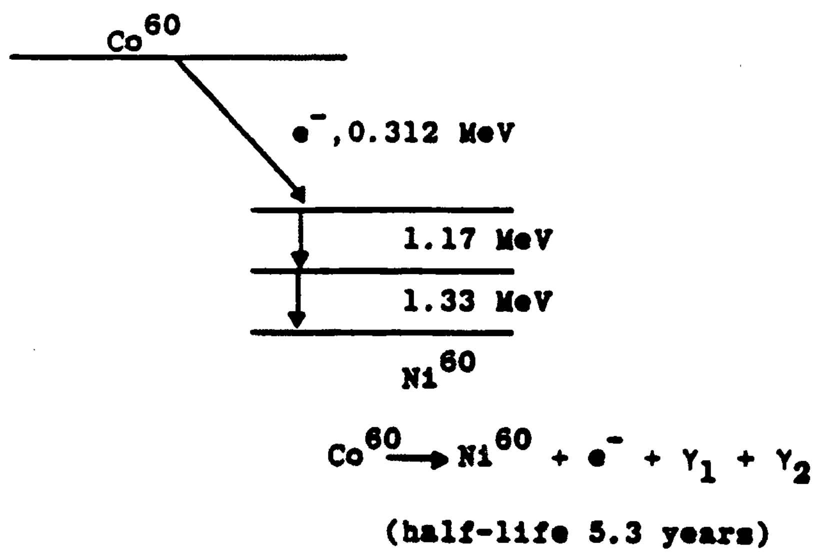


Figure 4.2: Co^{60} decay scheme.

4.3.1 Equipment

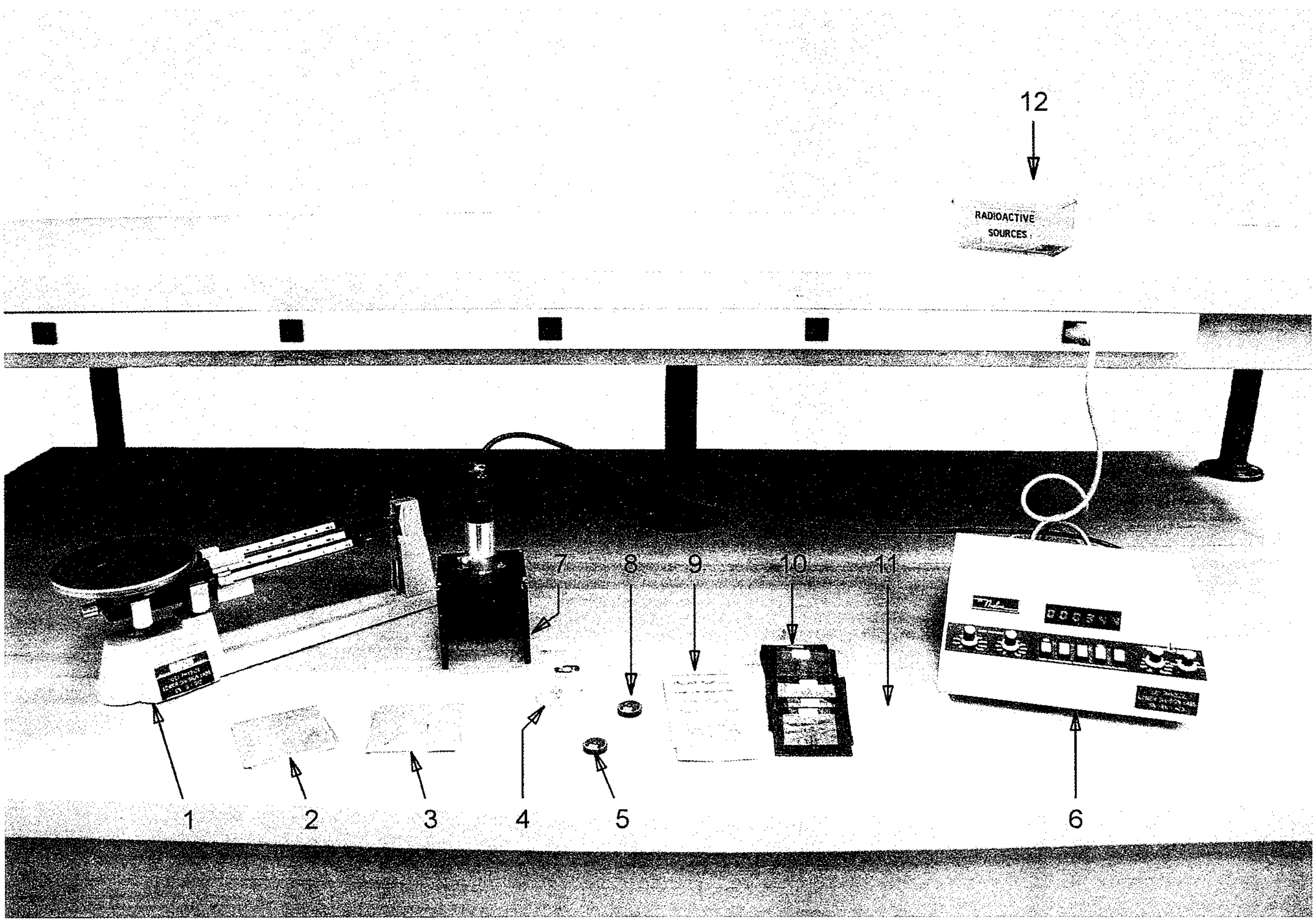


Figure 4.3: Equipment setup for Week 4.

The following equipment is to be set up at 11 stations, except for Item 12.

1- Balance
2- (2) Solid aluminum plates
3- (2) Holed aluminum plates
4- Smoke detector (Americium 241)
5- Cobalt 60 source
6- Nuclear scaler
7- Geiger tube w/ mount
8- Strontium 90 source
9- Aluminum sheets (1,2,4,8,32, and 64 sheets, each sheet .025 mm thick)
10- Lead sheets (enough to make measurements every 1/16 inch up to 7/8 inch thickness)
11- Weighing paper (5 sheets)
12- Storage boxes for sources. TA will distribute and collect all sources, and keep box at TA station

4.3.2 Background

Goal: Measure the background.

Put your sources far from your counter. Turn on the power supply to the operating point of the Geiger tube.[2] Put a source by your detector and make sure it works. Remove the source and measure the background using a 5 minute count. Record the background count rate and its uncertainty.

4.3.3 Alpha range

Goal: Observe the short range of an alpha particle.

Turn on the scaler and set the voltage at the operating point of the Geiger tube. Place the americium as close to the detector as possible. Place a sheet of weighing paper between the source and the detector. Describe the result. Add more paper. The americium source emits some X-rays in addition to alphas.

4.3.4 Beta range

Goal: Study the absorption of betas qualitatively.

Mount the strontium source below the detector. You may want to use a bit of scotch tape to hold the thorium source in place as you add absorbers. Find the thickness of aluminum that approximately halves the number of detected counts. Try a few other thicknesses. Does the signal decrease exponentially with absorber thickness?

4.3.5 Gamma range

Goal: Measure the absorption coefficient for gamma radiation from Co^{60}.

Place a Co^{60} source on an aluminum plate and insert it in the slot of the Geiger tube stand that is fourth from the bottom. Place another solid aluminum plate in a slot just above. The second plate is thick enough to absorb all the beta particles emitted by Co^{60} but will not appreciably attenuate

[2] If you are using a different apparatus than you used last week, select 800 V.

the gamma radiation. Place the lead absorbers on the uppermost aluminum plate. For each thickness of lead, count for 5 minutes or 500 counts, whichever comes first. Take data at every 1/16 in. of lead up to a thickness of 7/8 inch. For each measurement, record the absorber thickness, the number of counts and time, the measured count rate and uncertainty, and the corrected (for background) count rate and uncertainty.[3]

Plot the count rate (corrected for background) versus absorber thickness on semilog graph paper. Be sure to include error bars on your graph. Draw a straight line that goes through most of the data and use this to find the half-thickness $x_{1/2}$. Make a reasonable estimate of the uncertainty in $x_{1/2}$. Compute the absorption coefficient μ, including uncertainty. From this, find the gamma-ray energy, including uncertainty. Compare with the expected value.

From your measured absorption coefficient μ, calculate the effective cross section of the electrons in the lead.

4.4 Reflection Questions

1. In living tissue, radiation damages the cell because the energy transferred to the material creates ions and free radicals. The damage is greatest when the energy is absorbed over a short distance. Use this information and your experiments to explain this paradox: gamma sources are the most dangerous radiation type outside the body but alpha sources are the most dangerous type inside the body (ingested).

2. Why is the uncertainty in the background rate $\sqrt{B}/t$ rather than $\sqrt{B/t}$? (Here B is the number of background counts and t is the counting interval.)

3. Why are the uncertainties in the measured rate and the background rate added in quadrature when finding the uncertainty in the corrected rate?

4. Why is it convenient to plot your data on semilog graph paper? Sketch what the data look like on ordinary (linear) graph paper.

[3]To speed your work, calculate the various quantities in the table while you are acquiring data.

5. Why doesn't the straight line go through all of the error bars on the semilog plot? What percentage of the error bars should the line pass through?

Week 5

Least-Squares Fitting: Photon Energy

In this experiment, the ratio of Planck's constant h to the charge on the electron e is measured using light-emitting diodes (LEDs). Application of the technique known as least-squares fitting to the data yields h/e.

5.1 Theory of the Experiment

In 1905 Einstein explained a phenomenon called the photoelectric effect by assuming that light is composed of photons of energy $h\nu$. A typical apparatus is shown in Fig. 5.1. The photon energy is absorbed by a single electron in the irradiated metal. Electrons with sufficient energy to escape the metal and cross the opposing electric field produce a current through the tube. If the retarding voltage V is too large, no current flows through the circuit. At a certain voltage, called the *extinction voltage* V_x, current just begins to flow in the circuit. At this voltage the electron energy equals the photon energy so

$$h\nu = eV_x + e\phi \tag{5.1}$$

The quantity $e\phi$ is the energy required to escape the metal; it is a property of the surface material and is called the work function. By measuring the voltage V_x for several frequencies of light, an experimentalist can measure the constants h/e and ϕ.

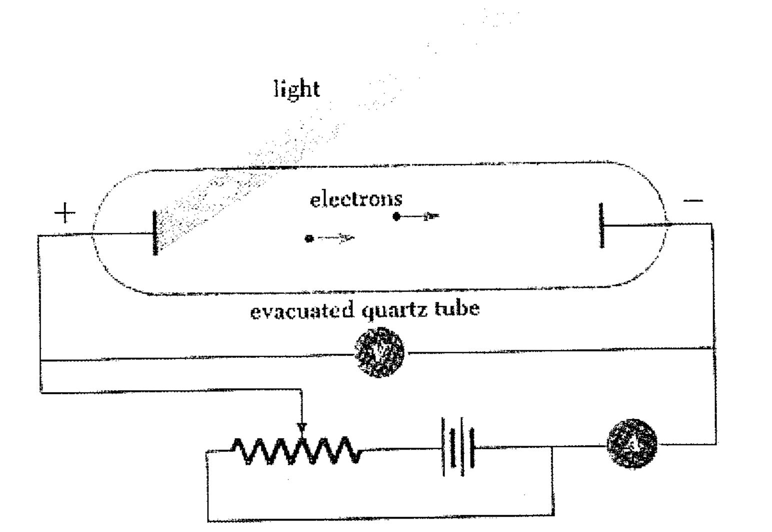


Figure 5.1: Typical apparatus for observing the photoelectric effect. A beam of monoenergetic light irradiates the metal "photocathode" of an evacuated tube. Electrons with sufficient energy to overcome the bias V are collected at the anode and an ammeter measures the current flowing through the circuit.

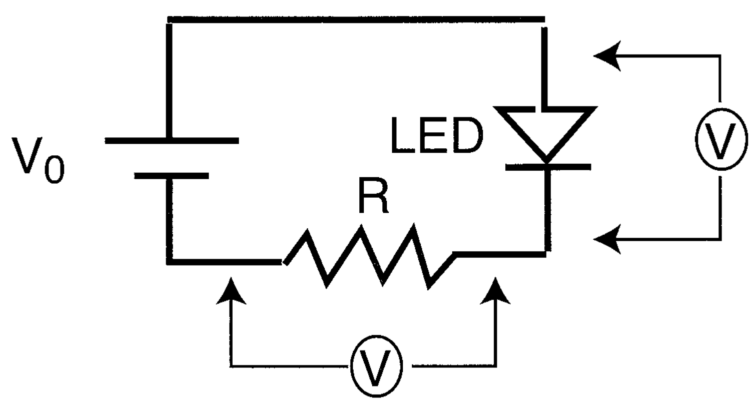


Figure 5.2: Schematic for LED apparatus.

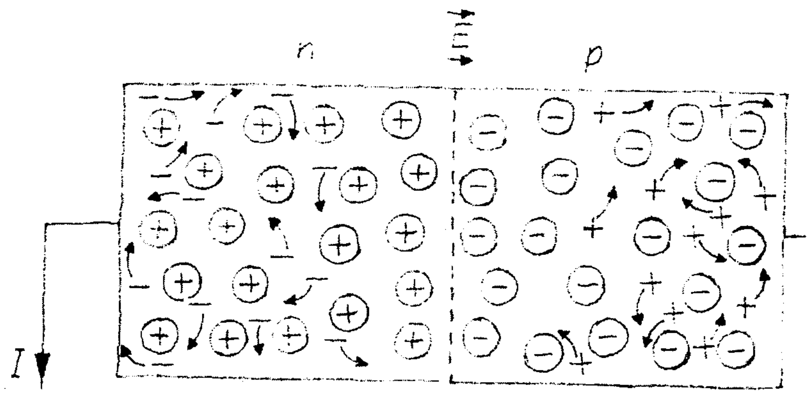


Figure 5.3: Schematic diagram of a p-n junction. On the left, the semiconductor is doped with impurities that contain more electrons than the basic crystal structure, so additional electrons are available in this n-type (for *negative* charge carriers) region. On the right, the semiconductor is doped with impurities that contain less electrons than the basic crystal structure, so a "hole" occurs where there is a missing electron. The hole acts like a positively charged electron, so this is the p-type (for *positive* charge carriers) region. In an LED, light is emitted when holes and electrons recombine at the junction.

This week's experiment could be called "the inverse photoelectric effect." In a light-emitting diode (LED), the electrons gain energy from the electric field, causing a photon to be emitted (rather than absorbed). The circuit diagram for the experiment is shown in Fig. 5.2. When the bias voltage $V_0 = 0$, virtually no current flows through the diode. As the bias voltage is increased, more energy becomes available to the charge carriers in the LED. Current begins to flow when the electrostatic energy just equals the photon energy so

$$eV_x = h\nu - e\phi. \tag{5.2}$$

Here V_x is the voltage when current just begins to flow through the diode and $e\phi$ is associated with additional energy losses (or gains) in the diode. As the bias voltage is further increased above V_x, more current flows through the diode and the brightness of the LED increases.

Physically, the LED is a semiconductor with a p-n junction (Fig. 5.3).

Holes and electrons recombine at the junction, releasing energy. In silicon and most other semiconductors, this energy is transfered to the crystal as heat. An LED is made of a special semiconducting material (such as gallium-phosphate) that radiates the released energy as a photon.

The actual physical process is complicated. Not all energy is radiated as light. The radiated light is not monoenergetic, although most of the light is close to a dominant wavelength. Also, to obtain a variety of colors, different semiconducting materials must be employed, so the actual physical mechanism differs in different LEDs. Nevertheless, the basic behavior of an LED is governed by the simple energy conservation equation, Eq. 5.2.

In the experiment, you will measure V_x for six different LEDs. From these data, you will infer the fundamental constant h/e and compare it to the accepted value.

5.2 Webwork Questions

Reading Assignment: 8.1-8.5 of An Introduction to Error Analysis.

1. What is the accepted value of h/e?

2. For what value of voltage V_x do you expect a red LED with wavelength of 635 nm to begin to conduct? (Assume $\phi = 0$.)

3. Problem 8.1.

4. Problem 8.5.

5. Problem 8.15.

5.3 Laboratory Procedure

5.3.1 Equipment

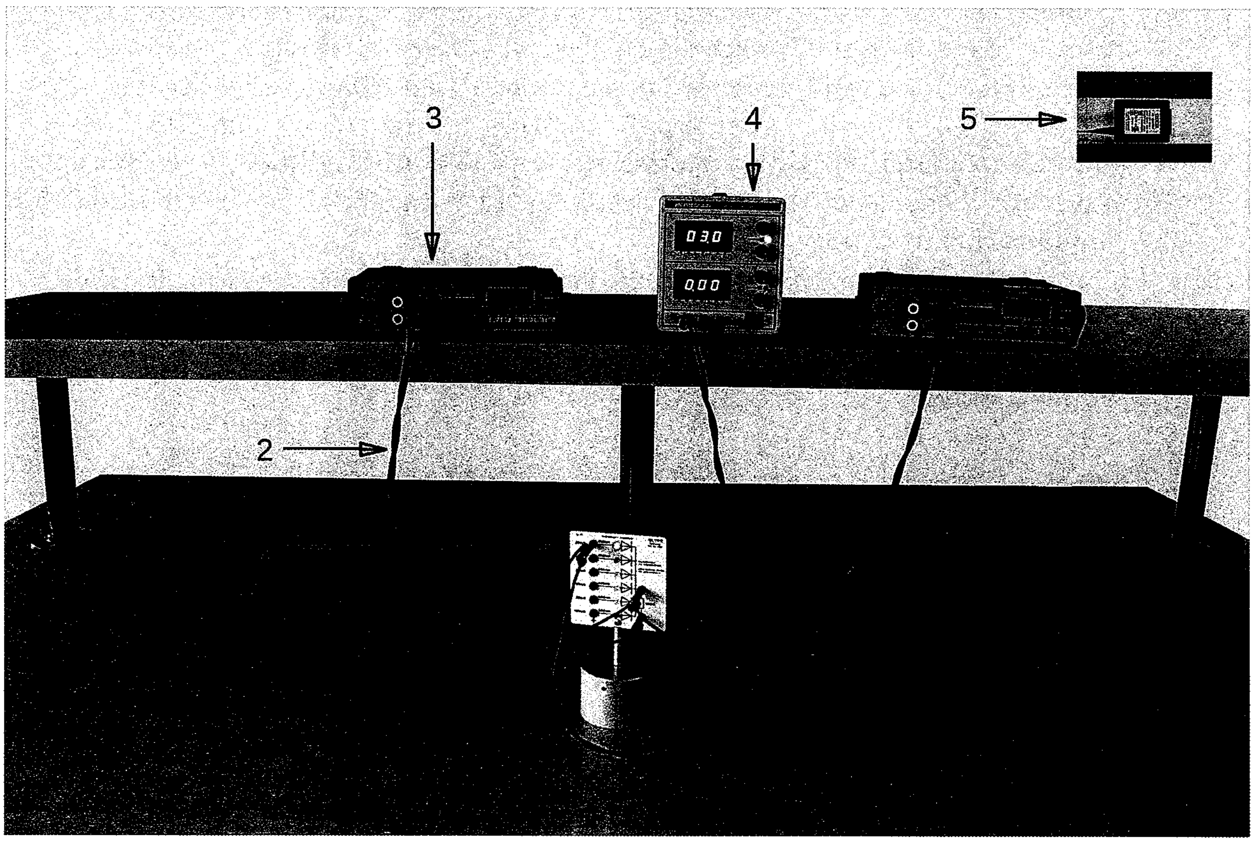


Figure 5.4: Equipment setup for Week 5.

The following equipment is to be set up at all 11 stations.

1- Light emitting diode board w/ support base
2- (6) Banana-banana cables
3- (2) Digital multimeters (BK 2833 or equivalent)
4- Low voltage power supply (BK 1715 or equivalent)
5- (2) Power adapters for BK 2833

5.3.2 Diode I-V Characteristics

Goal: Measure the $I - V$ curve of the 635 nm LED.

To avoid burning out the LEDs, **the DC supply voltage should never exceed 3 V** in this experiment.

Connect the DC power supply to the resistor-LED series circuit. (Fig. 5.2). Use the red LED initally. Begin with a low value of voltage. Monitor the voltage across the LED with one multimeter; monitor the current in the circuit by measuring the voltage across the 100 Ω resistor. (Before you connect the circuit use a multimeter to measure the actual resistance of the 100 Ω resistor.) Turn on the power supply and turn up the voltage until the LED glows moderately brightly. Notice the variation in brightness with diode current. Turn down the voltage until the LED barely glows: what is the minimum current you can see?

Measure the current in the circuit for 15-20 values of bias voltage. Include several measurements in the region where the LED just becomes conducting. Plot your current vs. diode voltage data twice: on a linear graph and on a semilog graph.

5.3.3 Dependence of voltage on wavelength

Goal: Measure the turn-on voltage V_x for all six LEDs.

Choose a small but non-zero value of current where the diode is definitely conducting. Call the diode voltage that produces this value of current the "turn-on" voltage V_x. Measure V_x for all six LEDs and tabulate your results. Also tabulate the value of current that produces visible glowing of the LED.

5.3.4 Least-squares fitting

Goal: Determine h/e.

Plot the turn-on voltage V_x versus the frequency ν. Calculate a least-squares fit to the data, including errors in the fit coefficients A and B. Add the best-fit line to your graph. Also draw dotted lines that represent $B \pm \delta B$. Compare the measured value of h/e with the accepted value.

5.4 Reflection Questions

1. Explain why an LED's brightness is proportional to the electric current flowing through it.
2. Why must an LED that produces green photons have a larger voltage drop than an LED that produces red photons? (Explain in words.)
3. Which LEDs were easiest to see? Hardest? Why?
4. Explain the sign of the intercept of the least-squares fit in terms of energy balance. (Hint: in the photoelectric effect, the intercept is the work function of the metal.)
5. Consider the dotted lines on the graph that you produced in Sec. 5.3.4. Does the calculated uncertainty in B seem reasonable?

Week 6

Advanced Least-Squares Fitting: Half-Life

Linear least-squares fitting can be applied to nonlinear functions; also data with unequal uncertainties can be treated. The *correlation coefficient* is a measure of the quality of the fit. In this experiment, two radioactive counting experiments provide data for these extensions of least-squares fitting.

6.1 Theory of the Experiment

A basic property of radioactive decay is its statistical, random nature. In a sample of unstable nuclei, it is not possible to say which of the nuclei will decay first or what direction any emitted particles will head. The average properties of the sample can be described, however. One of the most important properties is the decay rate. In a sample with N radioactive nuclei, the number of nuclei decays as

$$\frac{dN}{dt} = -\lambda N, \tag{6.1}$$

where t is the time and λ is a constant that is characteristic of the radionuclide. This equation has the solution

$$N = N_0 e^{-\lambda t}, \tag{6.2}$$

where N_0 is the number of nuclei at $t = 0$. Equation 6.2 states that (if no new radionuclides are being created) the number of radionuclides in a sample will steadily decrease in an exponential fashion. The decay constant

λ is useful, but it is customary to describe the rate of decay of a radionuclide in terms of its *half-life* $t_{1/2}$, which is the amount of time required for half of the radionuclides in a sample to decay. With the substitutions $N = N_0/2$ and $t = t_{1/2}$ in Eq. 6.2, we find that the relationship between the half-life and λ is

$$\frac{N}{N_0} = \frac{1}{2} = e^{-\lambda t_{1/2}}; \tag{6.3}$$

$$t_{1/2} = \frac{\ln 2}{\lambda}. \tag{6.4}$$

In an actual experiment, we usually measure the decay rate $R = -dN/dt$ rather than the number of radionuclides directly. (We measure particles ejected when the nucleus decays.) Because the decay rate is proportional to the number of radionuclides, however (Eq. 6.1), the decay rate also decreases with the same half-life as the number of nuclei N.

Another property of radioactive decay is that it is *isotropic*, that is, particles ejected by the nucleus are emitted uniformly in all directions. Recall that, for isotropic emission, the flux of particles Φ falls off inversely with the square of the distance from the source d, $\Phi \propto d^{-2}$. In Part 6.3.2 of the experiment, the dependence of the flux on distance is measured.

6.2 Webwork Questions

Reading Assignment: 2.6, 8.6, and Chapter 9 of An Introduction to Error Analysis.

1. Suppose you measure the kinetic energy K of some (non-relativistic) ions as a function of their speed v. You want to determine the mass m of the ions. How can you use a least-squares fit to determine m?

2. Problem 8.10.

3. Problem 8.10 (graph).

4. Problem 8.25.

5. Problem 9.11.

6. Use your calculator or computer to calculate the correlation coefficient r for the five pairs of measurements in Problem 9.8.

6.3 Laboratory Exercise

6.3.1 Equipment

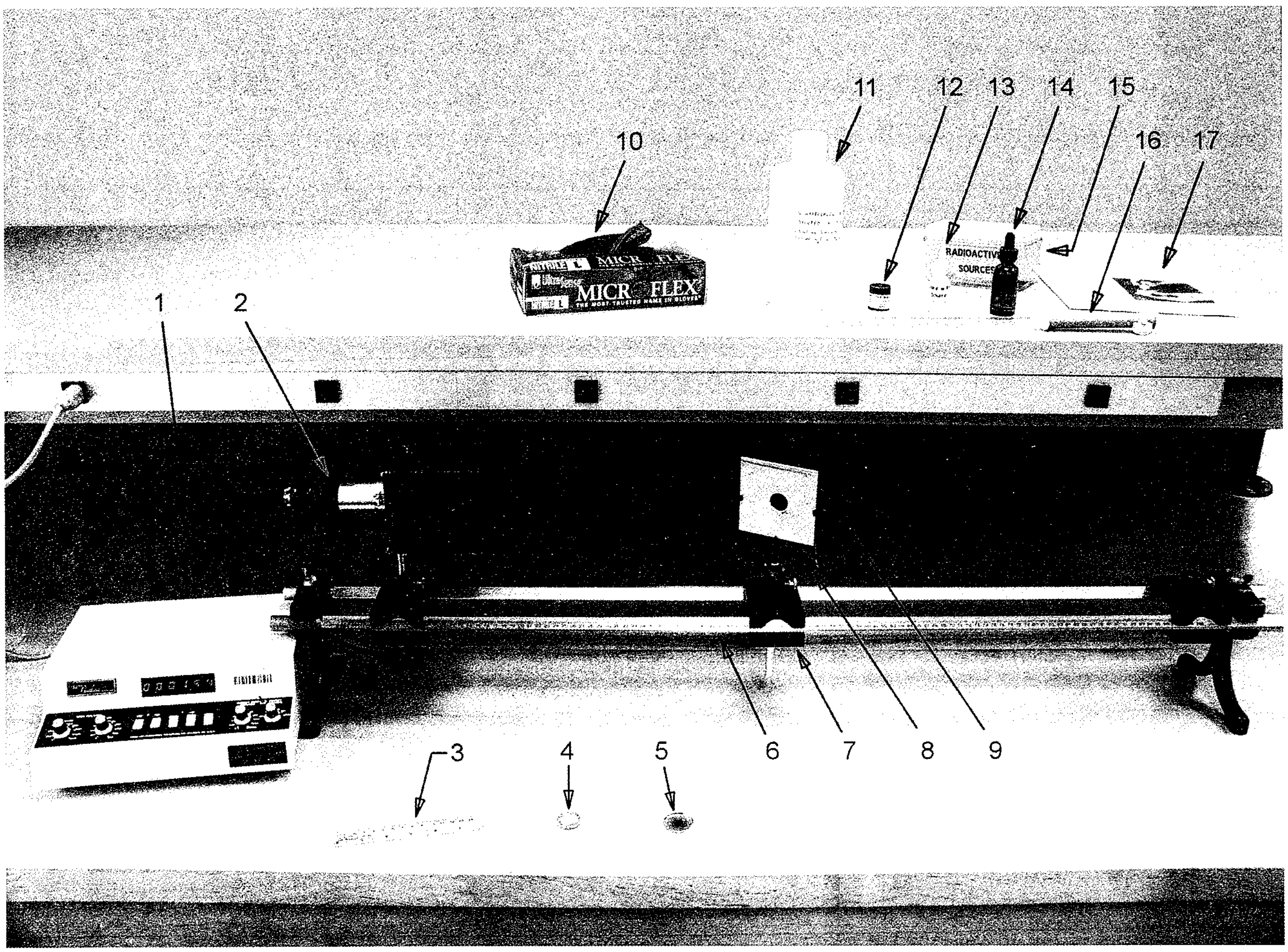


Figure 6.1: Equipment setup for Week 6.

The following equipment is to be set up at 11 stations, except for Items 10-17, which are to be kept at the TA Station.

1- Nuclear scaler
2- Geiger tube w/ mount, set-up as shown
3- Transparent ruler
4- Cobalt 60 <u>or</u> Cesium 137 source
5- Planchet (small metal dish w/ rim)
6- Optical bench
7- (3) Carriages
8- Lens holder
9- Holed aluminum plate
10- Latex gloves
11- HCl/NaCl Elute
12- Cs/Ba Minigenerator
13- Elute squeeze bottle
14- Eyedropper bottle
15- Radioactive sample box
16- Pipette
17- Minigenerator instructions

6.3.2 Distance dependence

Goal: Measure the number of radioactive decay counts versus distance.

TO AVOID DAMAGING THE GEIGER COUNTER, SET YOUR VOLTAGE AT 800 VOLTS. DO NOT DISCONNECT THE CABLE CONNECTING THE SUPPLY TO THE DETECTOR. Put the cobalt or cesium source[1] next to the detector to verify that the Geiger counter is operational. Remove the source and measure the background. Select your most intense source and mount it in the collapsible lens holder placed through a sliding carriage atop the optical bench. Use the aluminum rod to mount the Geiger stand horizontally through the sliding carriage. This arrangement allows you to vary the distance, d, between source and detector while maintaining their "on axis" alignment. At 5 cm intervals between $d = 10$ and 30 cm, measure the number of counts during 5 minute periods.

- Correct the data for background. Compute the uncertainty. (Remember to propagate the errors properly–see Week 4.)
- Make a table with $1/d^2$, corrected count rate, uncertainty, and weight. Graph the data, including error bars.
- Calculate the weighted least-squares fit to your data using Eqs. (8.37)-(8.39). Draw the best-fit line on your graph.
- Calculate the correlation coefficient r.
- Are your data consistent with isotropic decay? (Use Appendix C.)

6.3.3 Half-life

Goal: Measure the half-life of ^{137m}*Ba.*

Most of the radioisotopes used in our laboratory have very long half-lifes so that the decay in activity is negligible during a lab period. To measure the half-life, your TA will give you a short-lived isotope, ^{137m}Ba. Set up your scaler before getting your sample. When you are ready, get your isotope and slide it under the detector. To measure the activity versus time, count for 6 seconds once each minute for ten minutes (11 measurements total).

[1] These sources have long half-lives so the number of decay events does not change appreciably during a lab period.

- Correct the data for background. Calculate the uncertainties.
- Plot the data on a semilog plot, including error bars.
- Calculate the least-squares fit to the data.[2] Draw the fit on the graph. Also calculate the uncertainty in B.
- From the fit coefficient B, calculate the half-life of ^{137m}Ba, including uncertainty. The accepted half-life is 2.6 minutes. Compare your measurement to the accepted value.

6.4 Reflection Questions

1. The three main safety principles in handling radioactive sources are a) minimize the duration of exposure, b) maximize the distance from the source, and c) maximize the amount of shielding (material) between you and the source. For each of these principles, name an experiment that you have performed in this course that demonstrates the wisdom of the principle.
2. In Sec. 6.3.2, you plotted the data versus d^{-2}. Why would the analysis fail if you used d itself?
3. What assumption about the weights are made in an ordinary least-squares fit? In words, why does it make better sense to use the measured weights?

[2]For simplicity, you may neglect the variable weights if you wish.

Week 7

Weighted Average: Rydberg constant

In this experiment, you will observe the radiation produced by mercury and hydrogen lamps and use the data to infer the Rydberg constant R_H. A weighted average of the data gives a more precise value of R_H.

7.1 Theory of the Experiment

Quantum mechanics successfully predicts that atoms and molecules radiate at certain, discrete frequencies rather than a continuous spectrum of radiation. This *line radiation* depends on the mass and shell structure of the atom, on the ionization state, and on how the atom was prepared (which energy levels are occupied). Different atoms have different characteristic spectra. For example, several of the more intense visible lines in the mercury spectrum are listed in Table 7.1.

7.1.1 Balmer series

The Bohr model of hydrogen is discussed in virtually all introductory textbooks, so only a brief sketch of the theory is given here. In Bohr's model, the hydrogen atom consists of a proton with an electron circling it like a planet around the sun. In SI units, the force between the electron and proton is

$$F = \frac{1}{4\pi\epsilon_o}\frac{e^2}{r^2}. \tag{7.1}$$

Table 7.1: Prominent spectral lines for mercury.

Color	**Wavelength (Å)**
red	6908
red-orange	6234
orange	6150
yellow	5791
yellow	5770
green	5460
turquoise	4916
blue purple	4358
violet	4078
violet	4047

Since the electron moves in a circle, it experiences an acceleration,

$$a = \frac{v^2}{r} = \frac{F}{m}.$$

This leads to an expression for the velocity

$$v^2 = \frac{e^2}{4\pi\epsilon_o mr}. \tag{7.2}$$

It was Bohr's postulate that the electron's angular momentum can take on only integral values of the fundamental constant $h/2\pi$:

$$mvr = \frac{nh}{2\pi} \tag{7.3}$$

where n is an integer. Eliminating v between Eqs. 7.2 and 7.3 gives

$$r = \frac{\epsilon_o h^2 n^2}{\pi\, me^2}. \tag{7.4}$$

The total energy E of the electron is its kinetic energy T plus its electrical potential energy U,

$$E = T + U = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_o r}.$$

Using Eq. 7.2 this becomes

$$E = \frac{e^2}{8\pi\epsilon_o r} - \frac{e^2}{4\pi\epsilon_o r} = -\frac{e^2}{8\pi\epsilon_o r}. \tag{7.5}$$

Substituting Eq. 7.4 into Eq. 7.5, we find that the energy can have only the discrete values

$$E = -\frac{me^4}{8\epsilon_o^2 h^2 n^2}, \tag{7.6}$$

where $n = 1, 2, 3, \ldots$.

Bohr's second postulate was that an electron does not radiate as long as it remains in one of the states given by Eq. 7.6. Radiation only occurs when an electron undergoes a transition from one energy level to a lower one. The energy of the radiated photon, $h\nu$, is equal to the difference of the energy states,

$$h\nu = \frac{me^4}{8\epsilon_o^2 h^2}\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right),$$

where n_l corresponds to the lower state and n_u to the upper state. The radiated frequency ν is

$$\nu = \frac{me^4}{8\epsilon_o^2 h^3}\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right) \tag{7.7}$$

or, in terms of the wave length λ,

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right), \tag{7.8}$$

where $R_H = \frac{me^4}{8\epsilon_o^2 h^3 c}$ is known as the Rydberg constant.

If we would have considered the motion of the proton in the derivation, we would have found that the mass in Eq. 7.7 is not exactly the electron mass but the "reduced mass"

$$m = \frac{m_e m_p}{m_e + m_p}, \tag{7.9}$$

where m_e and m_p are the electron and proton mass, respectively.

The presently accepted value of the Rydberg constant for hydrogen is $R_H = 1.0968 \times 10^7\,\mathrm{m}^{-1}$.

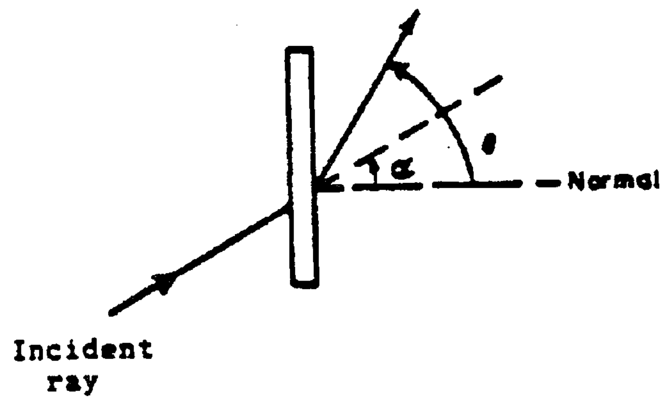


Figure 7.1: A ray of light strikes a diffraction grating at an angle α with respect to the normal of the grating. The light is diffracted into the angle θ.

The portion of the hydrogen spectra which occurs in the visible range arises from transitions which end in the $n_l = 2$ state. These transitions are known as the *Balmer series* and have wavelengths

$$1/\lambda = R_H(\frac{1}{2^2} - \frac{1}{n_u^2}), \tag{7.10}$$

where $n_u = 3, 4, 5, \ldots$ The first three lines in this series are red, turquoise, and violet in color.

7.1.2 Diffraction grating

In the experiment, you will use a diffraction grating to disperse light from the mercury and hydrogen lamps so you can see the individual colors. For a diffraction grating, the angle θ of the diffracted light is related to the wavelength of the light λ according to

$$n\lambda = d \sin\theta, \tag{7.11}$$

where d is the distance between the lines in the diffraction grating and $n = 1, 2, 3, \ldots$ is the *order* of the diffraction pattern (not to be confused with the energy level quantum number n!).

Equation 7.11 implies that measurements of the diffraction angle θ in various orders can be used to determine d if λ is known; alternatively, if d is already known an unknown wavelength can be determined from the measured diffraction angle. In the experiment, both approaches are taken. However, this simple formula is only valid if the incident ray is perpendicular to the grating surface. The more general case of a ray incident at angle α is illustrated in Fig. 7.1. For this case, the relationship between the wavelength and the diffraction angle is

$$n\lambda = d(\sin\theta - \sin\alpha). \tag{7.12}$$

To make accurate measurements of λ using Eq. 7.11, α should be made negligibly small.

7.1.3 Spectrometer Vernier scale

The Gaertner-Peck spectrometer (Fig. 7.2) includes a Vernier scale for precise measurements of angle. You will need to know how to read this scale to obtain accurate wavelength measurements. An example of an angle measurement is shown in Fig. 7.3. The main scale is on the bottom and it measures in degrees. The diagram shows a rough angular measurement of 70° and the top scale increases the precision. The top scale is called the *vernier* and it reads in minutes. You read it by looking for the point where lines on the top and bottom coincide. For example, the lines in Fig. 7.3 coincide at about ten. This says that the actual angle measurement is $70°10'$. Since there are sixty minutes in a degree, this is equivalent to $70\frac{10}{60} = 70.17°$.

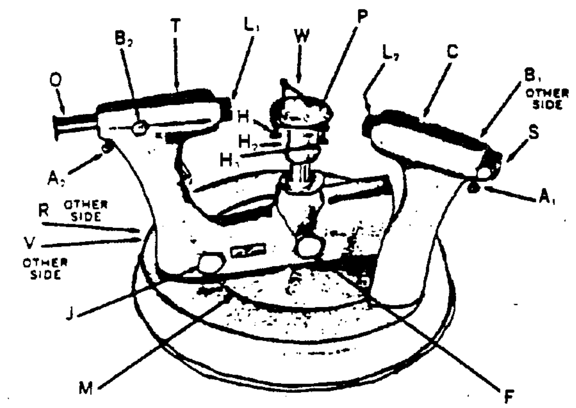
A second example of an angle measurement is in Fig. 7.4. The coarse scale says that the measurement is just under 272°. The vernier only goes up to $31'$ so, since the lower scale reads between 271.5° and 272°, we must remember to add $30'$ to the Vernier reading. Figure 7.4 reads $271°45'$.

7.2 Webwork Questions

Reading Assignment: Chapter 7 of An Introduction to Error Analysis.

1. Problem 7.1.

2. Find the weighted average and uncertainty of these three measurements of the wavelength of light emitted by a certain atom (in nm): 491 ± 8, 525 ± 20, 520 ± 10.



Overall View of the Gaertner-Peck Spectrometer

- C collimator
- S slit
- L_1, L_2 objectives
- A_1, A_2 vertical adjusting screws (shipped in separate plastic box)
- O ocular
- B_1, B_2 snap-in buttons
- M principal angle scale
- V vernier index
- R viewing lens
- F clamp screw for fine motion
- J adjusting screw for fine motion
- P prism table
- H_1, H_2, H_3 leveling screws
- T telescope

Figure 7.2: The Gaertner-Peck spectrometer. The spectrometer consists of a collimator (right side), a central stand (center), and a telescopic eyepiece (left side) mounted on a circular table. Light enters the spectrometer through an entrance slit (S). The width of the slit is adjusted with a screw (S). Once inside the collimator (C), the light passes through lenses and emerges as a beam of parallel rays. An optical element that disperses the light into its constituent wavelengths is placed on the central stand (W). In this course, a diffraction grating is employed. The height and orientation of the grating can be adjusted by the various screws underneath the stand (H, H_1, H_2). After passing through the grating, the light is collected by the telescope (T). The focus of the telescope is adjusted by sliding the ocular (O) in and out.

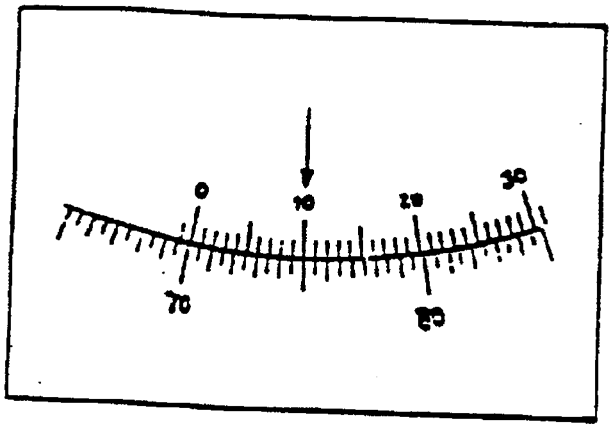


Figure 7.3: Spectrometer Vernier scale.

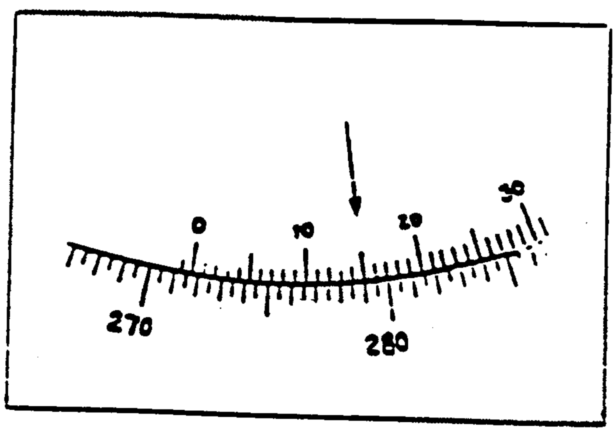


Figure 7.4: Spectrometer Vernier scale.

3. What is the measured value of angle for the two orientations of the telescope shown in Figs. 7.3 and 7.4?

4. A pair of students calibrate their diffraction grating using a mercury lamp. When the collimating slit is viewed directly by the telescope, the telescope scale reads 180°. When the telescope is rotated clockwise to view the longer-wavelength yellow line in first order, the telescope scale reads $169°59'$. When the telescope is rotated counterclockwise to view the same line in first order on the opposite side, the telescope scale reads $190°8'$. What is the spacing of lines in the diffraction grating d?

5. A pair of students view the red Balmer line from a hydrogen lamp in second order. For the clockwise orientation of the telescope, the measured angle is $156°48'$. For the counterclockwise orientation, the measured angle of the line is $203°33'$. (When the telescope views the collimating slit directly it reads 180°.) The students found that the spacing of lines in their diffraction grating was $d = 3311$ nm. What is the measured wavelength of the red Balmer line? What value of the Rydberg constant do they infer?

7.3 Laboratory Procedure

7.3.1 Equipment

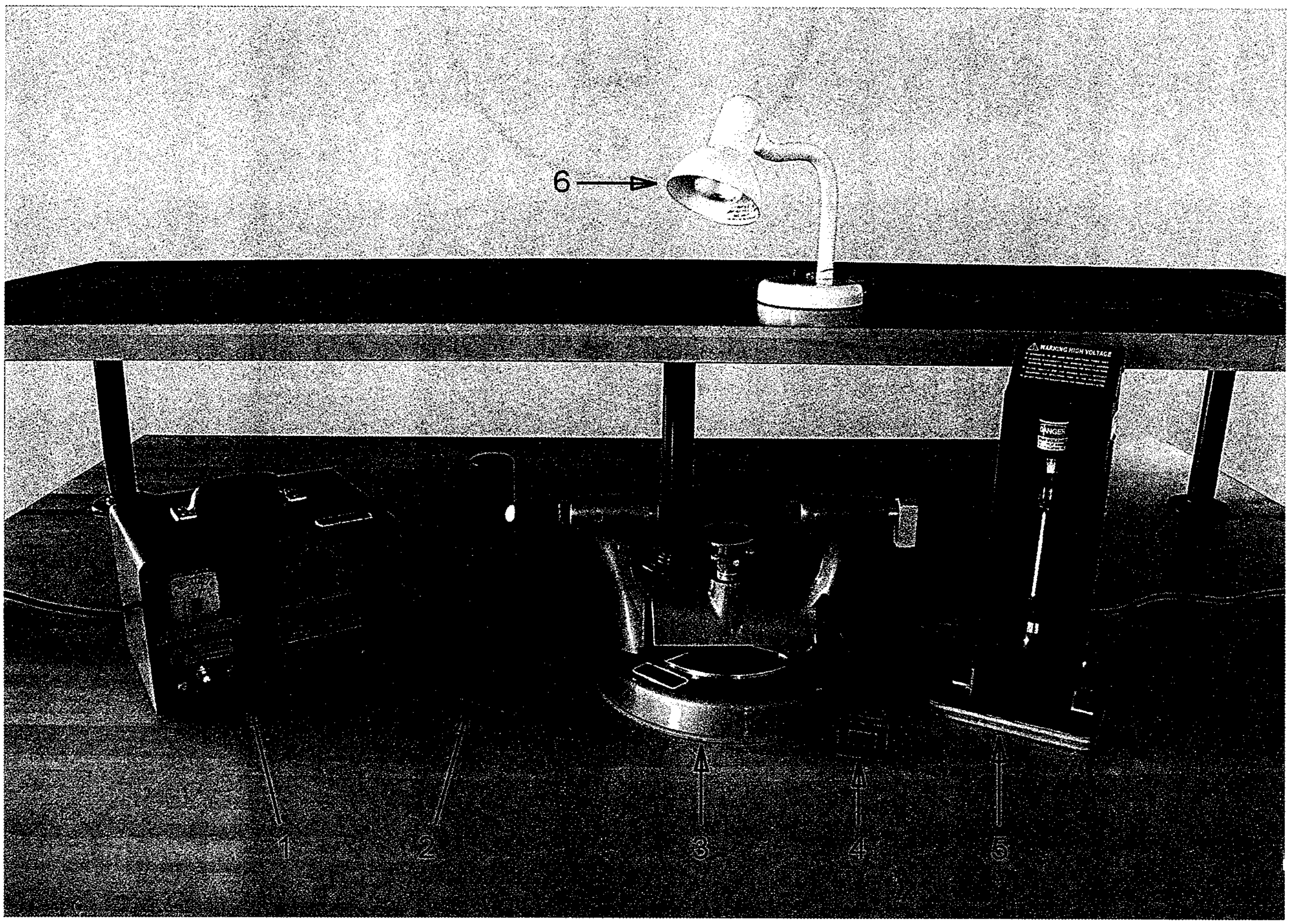


Figure 7.5: Equipment setup for Week 7.

The following equipment is to be set up at all 11 stations.

1- Power supply
2- Mercury (Hg) bulb w/ mount
3- Spectrometer
4- Diffraction grating
5- Hydrogen (H) bulb w/ power supply and cable
6- Desk lamp

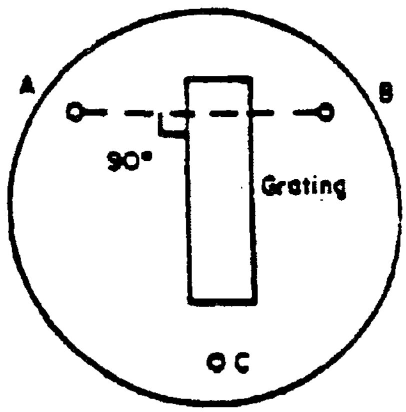


Figure 7.6: Orientation of the grating on the prism table. The positions of the leveling screws are indicated by A, B, and C. The normal to the grating should point toward the collimating slit.

7.3.2 Preliminaries

Goal: Align the diffraction grating vertically and minimize α.

The objective of this experiment is not just to measure the Rydberg constant R_H but to obtain the most accurate value consistent with the equipment available. The experiment is rather simple and you could get a value of R_H accurate to 1% in about 15 minutes. However, the spectrometer you are using is a precision instrument which, if used with care and patience should yield a value of R_H accurate to 0.1% or better. It is also expensive ($\sim$ \$3000), so **PLEASE HANDLE THE SPECTROMETER WITH CARE!**

Turn on the mercury lamp.[1] **CAUTION: THE BULBS ARE HOT!** Place the grating on the prism table. To simplify adjustments, place the grating as shown in Fig. 7.6. Illuminate the slit with the mercury lamp. Use the telescope to look straight at the slit. If the slit looks fat, use the adjacent thumbscrew to reduce the opening. Look at the edge of the slit. It should be sharply defined (not blurry). You should also see cross hairs that are well

[1] Leave the mercury lamp on until you have finished making measurements with it. A hot lamp often will not come back on until it has cooled down.

defined and parallel to the slit. If the slit or cross hairs are poorly focused, contact your T.A.

Next, check the horizontal alignment of the grating as follows. Select a prominent line in the spectrum and measure θ in first order on both sides of the slit. Also measure the angular position of the slit. (It should be close to 180° but may not be perfectly aligned.) If θ is not within 0.1° on either side, rotate the central table to minimize α.

As a final preliminary check, rotate the telescope to view the yellow doublet (a *doublet* is a pair of lines with nearly the same wavelength) in first order. If you cannot see two distinct lines, reduce the width of your slit.

7.3.3 Grating calibration

Goal: Use the known values of the mercury spectrum to measure accurately the grating spacing d.

The inverse grating spacing supplied by the manufacturer is only a nominal value. If accurate wavelength measurements are to be made, this quantity must be determined with commensurate accuracy.

Choose a prominent line in first order. Measure the angular displacement on both sides of the slit (i.e., on both sides of 180°) θ_+ and θ_-. The angle θ is $\theta = \frac{1}{2}(\theta_+ - \theta_-)$. Now measure θ for a different line. Make at least ten measurements of θ for various lines in different orders. Record your data in a table with the headings: Color, Order, θ_+, θ_-, θ, λ, d. With the use of Table 7.1, identify the wavelength of each of your lines. Use Eq. 7.11 to infer the grating spacing d for each of the measurements. Determine the average value of d and the standard deviation of your data.

7.3.4 The Balmer spectrum

Goal: Measure the wavelengths of lines in the Balmer series.

Turn on the hydrogen lamp. Note: In contrast to the mercury lamp which you should leave on until you have completed all measurements), turn off the hydrogen lamp whenever it is not in use. (This maximizes tube life.) Some of the lines are rather faint, so it is necessary to maximize the intensity of the hydrogen light. Place the spectrometer on the one-inch plywood board to align the hydrogen lamp vertically with the slit. While looking directly at the source through the telescope, shift the lamp horizontally to obtain a bright line. (You may also need to increase the slit width.) If the room is not already dark, ask your T.A. to eliminate sources of stray light by closing the blinds, doors, etc.

Ask your T.A. to check your setup and to initial your blue book. [2] Measure the wavelengths of the visible Balmer lines in the hydrogen spectrum. You should be able to identify three of the lines: bright red, blue-green, and violet. Measure the wavelength of each of the lines in as many orders as possible. Complete a table with the headings: Color, Order, θ_+, θ_-, θ, λ.

7.3.5 Rydberg constant

Goal: Infer the Rydberg constant and compare with the accepted value.

One possible way to analyze the data is to calculate R_H for each of your measurements in Sec. 7.3.4 *independently*, then calculate the mean and standard deviation for the *entire* set of measurements. As an alternative, to obtain practice using weighted averages, we treat each color line *separately*, then combine the three determinations of R_H together to obtain our final value.

- Calculate the mean and standard deviation $\bar{\lambda}$ and σ_λ of the wavelength measurements for each of the three lines. (This standard deviation is a good measure of the random error associated with uncertainty in the measurement of θ.)[3]

- Calculate the Rydberg constant R_H for each of the three lines.

- There are two sources of uncertainty in R_H: uncertainty associated with measurement of θ and uncertainty associated with the grating calibration σ_d. Add these uncertainties in quadrature to obtain the uncertainty in σ_{R_H}, $\sigma_{R_H}/R_H = \sqrt{(\sigma_d/d)^2 + (\sigma_\lambda/\lambda)^2}$, for each of the three Balmer lines.

- Use Eqs. (7.10)-(7.12) in *An Introduction to Error Analysis* to calculate the weighted average and uncertainty of these three measurements.

[2] You should see the violet line in (at least) second order.

[3] In principle, the error in λ can be found from the general error propagation formula, $\sigma_\lambda = \sqrt{(\sigma_d \partial\lambda/\partial d)^2 + (\sigma_\theta \partial\lambda/\partial\theta)^2}$ but it is difficult to estimate σ_θ accurately. A better approach is to use several measurements of λ to estimate the random error associated with the angle measurements (while temporarily ignoring σ_d), then combine the uncertainties in quadrature.

- Compare the measured value of R_H with the accepted value.

7.4 Reflection Questions

1. For a spectrometer, what is the advantage of a narrow slit? Of a wide slit?

2. What properties of the mercury lamp make it a convenient calibration source for the grating?

3. In Sec. 7.3.3, identify a possible systematic error in your value of d. Does this systematic error influence your determination of R_H or does it tend to cancel out?

4. Compare the weighted average with the values of R_H you obtained from the red, turquoise, and violet lines. Did performing a weighted average improve your agreement with the accepted value of R_H?

Week 8

Multiple Regression: Electron Gyroradius

A common problem in data analysis is to find the unknown dependencies in a set of measurements. In this experiment, we revisit the e/m apparatus used in Week 1 to obtain a set of data that depends on several independent variables. These data are analyzed to determine the parametric dependencies using a technique called multiple linear regression.

8.1 Theory of the Experiment

Recall from Week 1 that the radius[1] ρ of an electron of energy eV and mass m in a uniform magnetic field B is

$$\rho = \frac{1}{B}\sqrt{\frac{2mV}{e}}. \tag{8.1}$$

The magnetic field is proportional to the current I in the coil, $B \propto I$. In the experiment, you control the electron accelerating voltage V and the coil current I and measure the resulting radius of the orbit ρ. In terms of the independent variables V and I, the dependent variable ρ is given by

$$\rho = k_1 V^{1/2} I^{-1}, \tag{8.2}$$

[1]In Week 1, the symbol r represented the orbital radius but we use the symbol ρ this week to avoid confusion with the correlation coefficient.

where k_1 is a constant. You will also measure another variable, the number N on a die, which is really a random variable with no impact on the electron orbit. If we include this parameter as a third "independent" variable, then the expected dependence of ρ on the three independent variables is

$$\rho = k_2 V^{1/2} I^{-1} N^0. \tag{8.3}$$

(k_2 is a constant.)

8.2 Regression Analysis

Two important topics in data analysis are *hypothesis testing* and *parameter fitting*. In hypothesis testing, we want to know if the data are consistent with a theoretical prediction. For example, in Week 6 you used the correlation coefficient r to check if the radiation from a radioactive source decreases as the square of the distance from the source. In parameter fitting, we find numbers that provide the best description of the experimental results; an example is the parameters A and B in a least-squares fit. Hypothesis testing is studied further next week. Parameter fitting is this week's topic.

Suppose you want to know if the stock market will rise or fall. No one has developed a complete theoretical understanding of stock prices. The mechanisms that *cause* a stock price to change are not known quantitatively. If we did know all the causes, we could base an investment strategy on our theory and get rich. In the absence of a predictive theory, we must adopt an *empirical* strategy. We compile data governing all the possible factors that influence stock prices: interest rate, employment data, temperature on Wall Street, Democratic/Republican president, etc. Then, after compiling this enormous *database*, we search for correlations between stock prices and the variables in our database. If we learn that energy stocks tend to increase when war threatens but financial stocks decrease, then we can use this information to guide our investment strategy (even if we don't understand the underlying cause of the correlation).

The standard way to search for correlations in a database is called *multiple regression*. You are already familiar with this technique: least-squares fitting is a type of regression analysis. In least-squares fitting, the dependent variable y only depends on one independent variable x. In this case, the set of points $\{x_i, y_i\}$ constitute the database. The least-squares fit is the line $y_{fit} = A + Bx$ that comes "closest" to these measured points. What do we mean by "closest?" The definition of "closest" used in linear regression is that the best line minimizes the quantity

$$\chi^2 = \sum_{i=1}^{N} \frac{(y_i - y_{fit})^2}{\sigma_{y,i}^2}. \tag{8.4}$$

(See Eq. 8.5 in *An Introduction to Error Analysis.*) The input to the calculation are the data points $\{x_i, y_i\}$; the ouputs are the fitted parameters A and B. The quality of the fit is measured by the correlation coefficient r (Chapter 9). Geometrically, $y_{fit} = A + Bx$ is the equation of the line in two-dimensional space that comes closest to the plotted error bars.

What if y depends on more than one variable? Multiple regression is the generalization of least-squares fitting to higher dimensions. Suppose that the dependent variable y depends on the independent variables $\{x, u, v, \ldots\}$. We create a database of measurements, $\{x_i, u_i, v_i, \ldots, y_i\}$. We seek the best-fit line

$$y_{fit} = A + B_x x + B_u u + B_v v + \ldots. \tag{8.5}$$

Here $A, B_x, B_u, B_v, \ldots$ are the desired parameters of the fit (corresponding to A and B). In multiple linear regression, the "closest" fit is the one that minimizes the quantity χ^2 defined in Eq. 8.4. The quality of the fit to each of the independent variables $\{x, u, v, \ldots\}$ is measured by the correlation coefficients $r_x, r_u, r_v, \ldots$. Geometrically, Eq. 8.5 gives the equation of the line that comes closest to the measured points (including error bars) in multidimensional space.

How are the best-fit parameters $A, B_x, B_u, B_v, \ldots$ calculated? Using formulas like Eqs. (8.10)-(8.12) in *An Introduction to Error Analysis.*[2] In principle, these quantities can be calculated by hand but, in practice, one uses a computer. All major data analysis software packages, including Mathematica, Excel, and MathCad, include tools for linear regression analysis. Application notes for Mathematica and Excel appear after the Reflection Questions.

Equation 8.5 assumes that y depends linearly on the independent variables but, of course, y often has a more complicated dependence. In some instances, a more complicated analysis called *nonlinear* regression must be utilized. However, with a simple change of variable, many cases can be analyzed using linear regression. For example, in Week 6 you used a linear

[2]See, for example, Chapter 9 of *Data Reduction and Error Analysis for the Physical Sciences* by Philip R. Bevington (1969).

least-squares fit to analyze both exponential decay, $\exp(-t/\tau)$, and an algebraic dependence, $(\text{distance})^{-2}$. Similar transformations are employed in multiple regression analysis. Perhaps the most important special case is a polynomial dependence on the variables,

$$y = kx^a u^b v^c \ldots \tag{8.6}$$

where k, a, b, and c are constants. This can be transformed into a linear relationship by taking the logarithm of both sides:

$$\log y = \log k + a \log x + b \log u + c \log v + \ldots. \tag{8.7}$$

Equation 8.7 is in the form of Eq. 8.5, so commercial multiple regression software can be used to find the unknown coefficients $a, b, c, \ldots$.

Equation 8.3 is a polynomial relationship. In the experiment, you will find the coefficients corresponding to a, b, c and compare them to their expected values.

Equation 8.4 contains the uncertainties $\sigma_{y,i}$ in the y measurements. To perform the regression analysis, many commercial software packages will require you to specify the uncertainty of each point (often called the *weights*).[3] For simplicity, in this experiment, you may use the default value selected by the software. Better yet, estimate the uncertainty in ρ and use this to specify the weights.

8.2.1 Multiple regression example

In Sec. 6.3.2, Jennifer and Jaywon made the following measurements for the dependence of radioactive decay counts on distance. They also measured the room temperature, which is actually an unrelated variable.

Distance d (cm)	Temperature T (C)	Counts - Background C
11	20.5	227
15	20.7	121
20	20.8	63.5
25	20.3	43.5
30	20.7	34

Jennifer and Jaywon begin by separately plotting the dependent variable C versus each of the two independent variables d and T, as shown in

[3] This feature is designed to accommodate measurements with differing uncertainties, as in Problem 8.9 of *An Introduction to Error Analysis*.

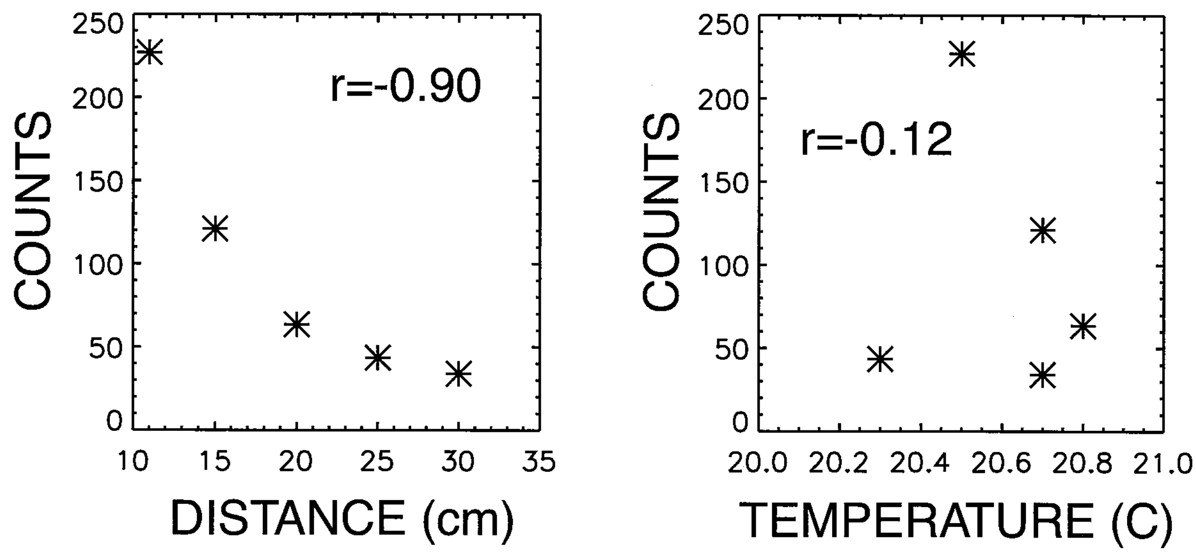


Figure 8.1: Scatter plots of the corrected number of counts versus distance and temperature. The linear correlation coefficients of the plotted data are also shown.

Fig. 8.1. They also separately calculate the correlation coefficient for $\{d, C\}$ and $\{T, C\}$. It is evident from the graphs that the dependence of the measured counts on the distance is strong, while the dependence on the temperature is weak. The calculated correlation coefficients support this visual impression. For the dependence of counts on distance, $|r|$ is nearly unity while, for the dependence on temperature, $|r|$ is nearly zero. The next step in the analysis is to analyze all of the data simultaneously using multiple regression analysis. Since they expect a polynomial dependence, Jennifer and Jaywon first take the logarithm of all of their data. They then use their favorite software to find that the exponent a for the distance dependence is $a = -1.93$; in other words, $C \propto d^{-1.93}$. This is close to the theoretically expected value of $a = -2$. The multiple regression analysis also finds $C \propto T^{-0.37}$. Their software also returns correlation coefficients for the entire fit. (These are different from the correlation coefficients when each variable is considered *separately*.) These are $r_d = -0.997$ and $r_T = -0.02$, which once again indicate that the dependence on d is strong while the dependence on T is weak. The "multiple linear correlation coefficient" for the entire fit is 0.997.

8.3 Webwork Questions

1. In the experiment, what are the expected values of a, b, c for the independent variables V, I, and N?

2. What is the expected value of k_2 in Eq. 8.3? (Hint: review Eqs. 1.6 and 1.9 of Week 1.) When you perform your regression analysis in the experiment, what is the expected value of $\log k$ in Eq. 8.7?

3. Consider the following data.

x	u	y
1.0	3.42	1.31
2.0	4.31	3.92
3.0	5.50	8.92
4.0	3.08	15.9

   (a) Calculate the linear correlation coefficient for y and x considered separately and the linear correlation coefficient for y and u considered separately. Answer for $\{x, y\}$: 0.98.

(b) Learn to use your favorite software to calculate multiple regression. (Instructions for Excel and Mathematica appear in Secs. 8.6 and 8.7.) Find the coefficients a and b. Partial answer: $a = 1.8$.

8.4 Laboratory Procedure

This week the lab report may be completed outside of class and turned in the following day.

8.4.1 Equipment

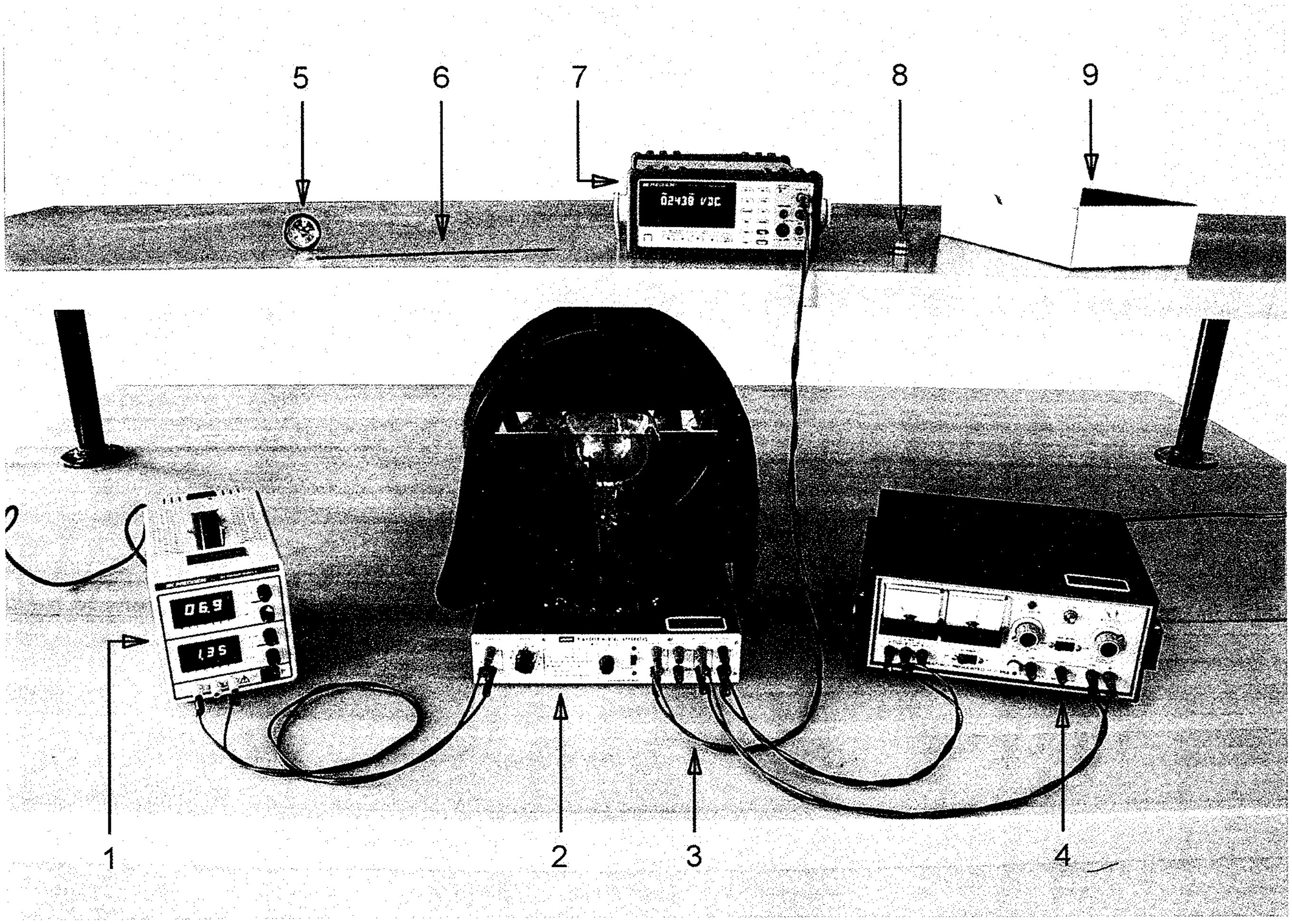


Figure 8.2: Equipment setup for Week 8.

The following equipment is to be set up at 11 stations.

1- Low voltage power supply (BK 1715 or equivalent)
2- e / m apparatus
3- (9) Banana-banana cables
4- High voltage power supply
5- Compass
6- 30 cm transparent ruler
7- Digital multimeter (BK 2831E or equivalent)
8- Die
9- Box

8.4.2 Database

Goal: Compile a V, I, N, ρ database.

Prepare to acquire data as in Sec. 1.4.2 of Week 1. Tabulate sixteen measurements of the accelerating voltage, coil current, number on the die, and electron-beam radius. Choose four values of voltage with a large variation of values (e.g., 115, 140, 165, 190 V) and four values of current with a large variation in values.[4] **Do not exceed 200 V for the voltage across the grid or 1.5 A for the current through the coil!** For each value of V and I measure ρ; also roll the die and enter the number N in your table.

Have your T.A. initial the data in your bluebook before you leave class.

8.4.3 Regression analysis

Goal: Analyze your database.

Enter the data into your favorite software package, then produce the following graphs and numbers.

1. Graph ρ versus each of the three independent variables: V, I, and N. Calculate the correlation coefficient r in each case.

2. Graph V versus I and calculate the correlation coeficient r.

3. Take the logarithm of your data and perform multiple regression analysis. The analysis should produce four numbers corresponding to $\log k$, a, b, and c in Eq. 8.7. If available, also quote the calculated uncertainties in the fits. Compare all four numbers to their expected theoretical values.

8.5 Reflection Questions

1. Graphs like those you obtained in step 1 of the previous section are often used to search for correlations in a database. Interestingly, even if the true dependence is not linear, these searches often reveal an underlying

[4] If your beam turns off at low voltage, reestablish a beam at higher voltage and current, then *gradually* lower the voltage and current in fine steps.

dependency. Consider the correlation coefficients you obtained. Which value of $|r|$ was largest? Smallest? Why does this make sense?

2. A problem in many databases is that the "independent" variables are not really independent. For example, if you had measured both the magnetic field and the coil current, the results of your regression analysis would have been more confusing.[5] This is because I and B are *correlated variables*. Consider step 2 of the analysis in the previous section. Are V and I correlated variables or are they independent?

3. Regression analysis and searches for correlations are used most often when no theory is available (or the available theories don't fit the data). Imagine that the Lorentz force law isn't known yet. What would you learn from the regression analysis you just performed?

8.6 Multiple Regression Analysis in Excel

1. The "Data Analysis" package must be installed. (It is not included in the default installation of Excel.) In Excel 2002, you do this by clicking on "Tools," then "AddIns," then "AnalysisToolPak."

2. Enter your data in a spreadsheet.

3. To produce your plots, select the "Chart Wizard" icon, then "XY (Scatter)," then the graph without connecting lines.

4. To find the separate correlation coefficients of the linear data, use the formula palette. After highlighting a box where the result will be written, click the formula box and type (for example): "= Correl(A2:A5,B2:B5)". You may use the mouse to fill the columns if you wish.

5. Have Excel compute the logarithm of each entry for you. Highlight the second element of a blank column, then enter, for example, "= log(A2)" in the formula palette (bar). After you hit Enter, the logarithm appears in the selected column. Now grab the lower right corner of the

[5] You would find that any combination of $(BI)^{-1}$ provides a good fit to the data. For example, the fit $B^0 I^{-1}$ and $B^{-1} I^0$ are both consistent with the data.

selected box and drag down the column with the mouse. Compute the logarithms of the remaining variables in adjacent columns.

6. Under the "Tools" menu, select "Data Analysis," then select "Regression." To fill the "Input Y Range:" box, highlight the appropriate column with the mouse. To fill the "Input X Range:" box, highlight *all* of the columns with independent variables.

7. The desired exponents a, b, ... appear in the SUMMARY OUTPUT sheet in the column labeled "Coefficients" and the rows labeled "X variable."

8.7 Multiple Regression Analysis in Mathematica

1. Enter your data as arrays, e.g., voltage={137.4, 137.1, ...}.

2. Create tables for plotting, e.g., data1=Table[{voltage[[i]],rho[[i]]},{i,1,16}]. Use ListPlot to create the graph.

3. Calculate the correlation coefficient, e.g., Correlation[voltage,rho].

4. Make a big table with all of the data, e.g., data=Table[{voltage[[i]],current[[i]],ndie[[i]],rho[[i]]},{i,1,16}].

5. Use the LinearModelFit command to fit the logarithm of the data, e.g., Normal[LinearModelFit[Log[data],{volt,cur,die},{volt,cur,die}]]. (Note that you must use a *different* name for the variables like "volt" in the arguments of the LinearModelFit command; otherwise Mathematica confuses it with the original array "voltage." Also, enclosing the entire command inside the Normal[...] command prevents Mathematica from hiding some of the coefficients inside of a "skeleton.")

Week 9

Distributions and The Chi-Squared Test

This week we use dice and background radiation to obtain statistical distributions. We then use the chi-squared test to see if the measured distributions are consistent with theoretical expectations.

9.1 Theory of the Experiment

Hypothesis testing is an important application of data analysis. Are the results of an experiment consistent with theoretical predictions? The chi-squared test is the most commonly used technique to answer this question. The general approach is given by Eq. (12.11) of *An Introduction to Error Analysis*,

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{O_i - E_i}{\sigma_i} \right)^2, \tag{9.1}$$

where O_i is the observed value, E_i is the expected value, and σ_i is the standard deviation of each measurement. If there were no errors in the measurements and the theory perfectly described the experimental situation, χ^2 would be zero (since $O_i = E_i$ for each measurement). If the errors are normally distributed, a typical measurement contributes about 1 to the sum, so the expected value of χ^2 is approximately the number of measurements N. If the theory doesn't describe the experimental situation, then $\chi^2 \gg N$.

Actually, the expected value of χ^2 is often somewhat smaller than N. In general, only *independent* observations O_i contribute to the sum in Eq. 9.1, so one expects $\chi^2 \simeq d$, where d is the number of independent observations. As is discussed in Sec. 12.3 of *An Introduction to Error Analysis*, the number of truly independent observations d is called the number of *degrees of freedom* and can be smaller than N. For example, theoretical predictions that arise from fits to the data don't give independent information about the validity of the theory, so they don't contribute to χ^2. As another example, consider an experiment where you flip a coin ten times, measuring "heads" six times and "tails" four times. Does $d = 2$ because there are $N = 2$ measurements? No, in this case there is only one *independent* measurement, since the number of "tails" is completely determined (constrained) by the number of "heads."

There is a special form of Eq. 9.1 that is convenient for counting experiments.[1] For counting experiments, we know that the uncertainty is given by the square root of the number of counts (Week 3). So, with the substitution $\sigma_i \simeq \sqrt{O_i} \simeq \sqrt{E_i}$, Eq. 9.1 becomes

$$\chi^2 = \sum_{i=1}^{N} \frac{(O_i - E_i)^2}{E_i}. \tag{9.2}$$

which is Eq. (12.7) of *An Introduction to Error Analysis*.

In the first part of the experiment, we compare the results of random throws of dice with theoretically predicted results. The outcome of games of chance using fair dice are described by a type of distribution function called the *binomial* distribution, discussed in Chapter 10 of *An Introduction to Error Analysis*. The binomial distribution gives us E_i in Eq. 9.2.

In the second part of the experiment, background radiation provides the randomly generated distribution. Background radiation arises from cosmic rays and radioactive elements in the environment. Cosmic rays are high-energy particles from space. When the particles bombard the atmosphere, they often decay into a shower of other high-energy particles. A fraction of these are detected by the Geiger counter as "background" counts. Another source of background counts are betas and gammas from the decay of naturally-occurring radioisotopes.

Events that occur randomly in time with constant probability are described by a distribution function called the *Poisson* distribution, discussed in Chapter 11 of *An Introduction to Error Analysis*. If the background measured by your Geiger counter is produced by cosmic rays and environmental

[1] Unfortunately, Taylor highlights this special form as the principal definition, but it really is a special case.

radioisotopes, it should be consistent with the Poisson distribution (because these sources rarely change on the timescale of your experiment). On the other hand, the background could be caused by a malfunctioning tube or by electromagnetic noise caused by power glitches. The probability of tube arcs or power glitches usually varies in time, so your results could differ from the Poisson distribution.

You will also compare your measurements with the Gauss (or normal) distribution, which is given by Eq. (5.25) of *An Introduction to Error Analysis*. The Gauss distribution asymptotically approaches the Poisson distribution as the number of counts increases.

9.2 Webwork Questions

Reading Assignment: Chapters 10, 11(1-3), and 12 of An Introduction to Error Analysis.

1. (a) Problem 10.2. Answer for no aces: 0.694.
   (b) For the same case, compute the probabilities for the normal distribution. In this case, $X = np$ and $\sigma = \sqrt{np(1-p)}$. Partial answer: $G(0) = 0.620$.
   (c) Compute the probabilities for the Poisson distribution. Here $\mu = np$. Partial answer: $P(0) = 0.716$.
2. Problem 10.8.
3. Problem 12.3 and (12.11).
4. A problem similar to Problem 12.1.

9.3 Laboratory Exercise

9.3.1 Equipment

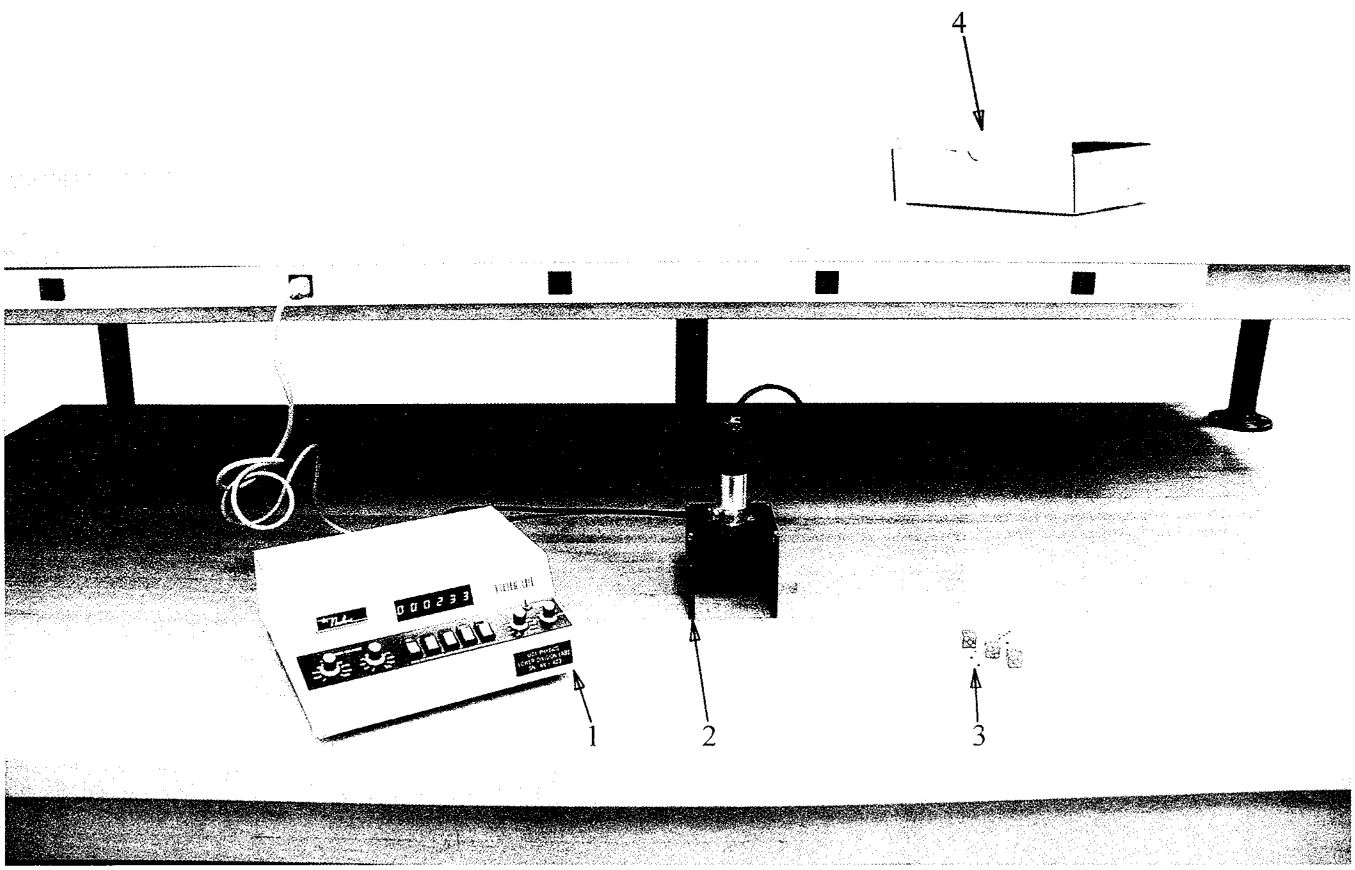


Figure 9.1: Equipment setup for Week 9.

The following equipment is to be set up at all 11 stations.

1- Nuclear scaler
2- Geiger tube w/ mount
3- (3-5) Dice (different number at each station)
4- Box

9.3.2 Dice

Goal: Compare the binomial distribution with the observed outcome.

- Choose a "lucky number" between one and six (inclusive). Roll the dice at least fifty times. After each roll, count how many of the dice came up with your lucky number. Ask your T.A. to initial your data.

- Organize your data into bins. Make a table with the number (from zero to the number of dice), the observed number of occurrences, and the expected number of occurrences (calculated from the binomial distribution).

- Graph the data (both theory and experiment on the same graph).[2]

- Calculate χ^2 using Eq. 9.2. If any of your bins have less than 4 counts, combine them with another bin before computing chi-squared.[3]

- Calculate the number of degrees of freedom and the reduced chi-squared. Are your data consistent with the expected distribution? (Answer both qualitatively and quantitatively using Table D.)

9.3.3 Geiger counter background

Goal: Determine if the background is caused by radiation.

- Set the power supply of the Geiger tube at 800 V and the scaler on "Preset Time" and 0.1 minute. Measure the background in a 6-second interval at least 50 times. Ask your T.A. to initial your data.

- Compute the mean and standard deviation for your data.

- Tabulate your data in bins. On each line, also record the predicted value for the Poisson distribution and for the normal distribution.

[2]Graph the actual measurements, not the collapsed bins used in the χ^2 calculation.

[3]According to Louis Lyons in *Statistics for nuclear and particle physicists*, "It is undesirable to have less than five events in any bin. This is because it is useful to be able to assume that the errors are Gaussian distributed, and the actual distribution approximates to a Gaussian only asymptotically (which is conventionally taken to be for a mean of five or larger)." A second, more practical, reason, is that E_i in the denominator of Eq. 9.2 can become very small for improbable events.

- Graph the data (experiment and two theories).
- Calculate χ^2 using Eq. 9.2 for both theories. If any of your bins have less than 4 counts, combine them with another bin before computing χ^2.
- Calculate the number of degrees of freedom and the reduced chi-squared for both cases. Which distributions fit the data? Which don't? (Answer qualitatively and quantitatively using Table D.) Is the measured background distribution consistent with the hypothesis that the source is cosmic rays and environmental radioisotopes?

9.4 Reflection Questions

1. Use Eq. 9.1 to explain why, when theory and experiment agree, the contribution of each bin to χ^2 is about 1.
2. The Poisson distribution provides an accurate description of a counting experiment but the binomial distribution describes games of chance. How are radioactive decay and coin tossing similar? Different?
3. Compare your graphs of the Poisson and normal distributions in Sec. 9.3.3. Qualitatively, what differences do you notice?

Week 10

Final

Note to Instructors: Information regarding the administration and content of the Final Exam can be found at:

http://undergraduate.ps.uci.edu/LabFinals/

The last week of class is devoted to a final examination on data analysis. Bring your calculator, *An Introduction to Error Analysis* and this lab manual for this "open book" test. **NOTE: The final exam will be conducted in the lecture section of 52C.** Here is a sample test from a previous year.

1. (4 pts) In the experiment to determine the value of e/m for the electron, an electron beam was accelerated to an energy eV, then the beam was deflected by the magnetic field produced by a Helmholtz coil. Collisions with the mercury vapor made the electron orbits visible. Measured quantities are the voltage, the current in the coil, and the radius of the beam. Describe one possible random error and one possible systematic error in this experiment.

2. (4 pts) In an experiment on the photoelectric effect, a pair of students measure (in SI units) $h/e = 4.34 \pm 0.10 \times 10^{-15}$. The accepted value is 4.14×10^{-15}. Assuming that the difference between their measurement and the accepted value is due to random errors, how probable is it that the discrepancy between experiment and theory will be this large?

3. (4 pts) A student is using the formula

$$\frac{e}{m} = \frac{2V}{B^2r^2} \tag{10.1}$$

to find the charge-to-mass ratio e/m of the electron. The fractional errors in the voltage, field, and radius are $\delta V/V = 1\%$, $\delta B/B = 3\%$, and $\delta r/r = 5\%$, respectively. Calculate the fractional error in e/m.

4. (5 pts) In a counting experiment, the background measured in a 5-minute interval is 171 counts and the number of counts measured with the source present in a 1-minute interval is 121 counts. With background correction, the rate due to the source alone is 86.8 counts/min. The uncertainty in the corrected rate is $\sqrt{(11)^2 + (2.6)^2} = 11.3$.

   (a) Why are the 11 and 2.6 added in quadrature? (In other words, what assumption must be satisfied to justify adding errors in quadrature?)

   (b) Where does the 11 come from? Why is this a valid estimate of this error? (Hint: consider the properties of the normal distribution.)

   (c) Where does the 2.6 come from? Why is this a valid estimate of this error?

5. (4 pts) Theoretically, the count rate of a radioactive source is expected to decrease as r^{-2}. For the measurement of the dependence of radioactive exposure on distance, Vijay and Nathan obtained the following table. Find the best-fit line for adjusted count rate versus r^{-2} for their data.

r (cm)	Adjusted Counts/min	Expected	$1/r^2$ (m^{-2})
10	530 ± 10	530	100
15	238 ± 7	236	44
20	129 ± 6	133	25
25	72 ± 5	85	16
30	42 ± 4	59	11

6. (5 pts)

   (a) Calculate chi-squared for the data in the previous problem.

   (b) Calculate the reduced chi-squared.

   (c) Interpret your results qualitatively and quantitatively. Are Vijay and Nathan's data consistent with the theoretical prediction?

7. (5 pts) A T.A. wants to persuade her students that it is important for them to do the Webwork homework so they will perform well on the laboratory experiments. She presents the following data from a previous section.

Webwork Scores	**Lab Scores**
3.3	12.8
4.8	15.7
2.4	13.0
4.0	15.7
3.0	13.8
4.8	13.0
3.4	12.6
3.0	10.9
3.5	10.9
5.1	13.0
5.0	15.0
4.2	13.5
2.4	13.7
4.0	15.0
2.0	13.5
5.7	15.0
4.8	14.0

(a) Calculate the correlation coefficient. Is the correlation significant?

(b) Even if the correlation were significant, would it necessarily prove her point? Explain.

8. (2 pts) The graph shows a figure from one of Prof. Heidbrink's publications. The error bars represent one standard deviation. The solid line is the theoretical prediction. Are the data consistent with the theory? Explain your reasoning.

Partial answers 2) It will happen 4.55% of the time. 3) 11.7%. 4a) They are independent and random. b) It is $\sqrt{121}/1$. c) $\sqrt{171}/5$. 5) (if you use weights) $A = -15.6$, $B = 5.6$. 6a) 25.3. b) 6.3. c) $< 0.1\%$, inconsistent. 7a) 0.43, not "significant" at 5% level. b) A maxim in statistics is "correlation does not prove causation." In this case, students might score better on both because they are

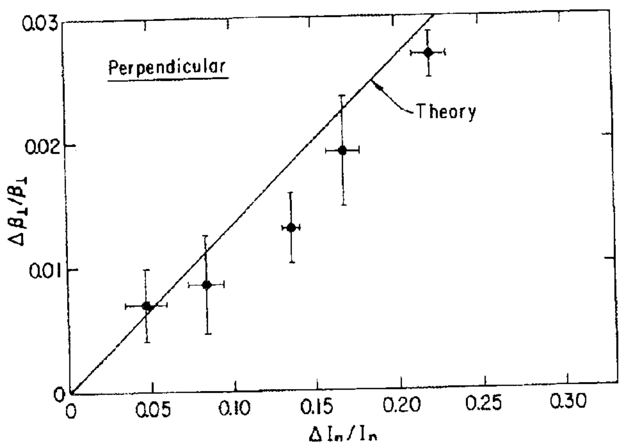


Figure 10.1:

smarter and more industrious, not because the Webwork homework helps. 8) Expect 68%, observe 60%. OK.

Appendix A

Graph Paper

- Linear, 5 Squares to the Centimeter (9 sheets)
- Semi-logarithmic, 3 Cycles $\times$ 10 to the Inch (4 sheets)
- Semi-logarithmic, 4 Cycles (1 sheet)
- Full Logarithmic, 3×3 Cycles (2 sheets)

24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

5 squares to the centimeter

5 squares to the centimeter

5 squares to the centimeter

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5 squares to the centimeter

5 squares to the centimeter

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5 squares to the centimeter

5 squares to the centimeter

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Semi - Logarithmic
3 Cycles x 10 to the inch

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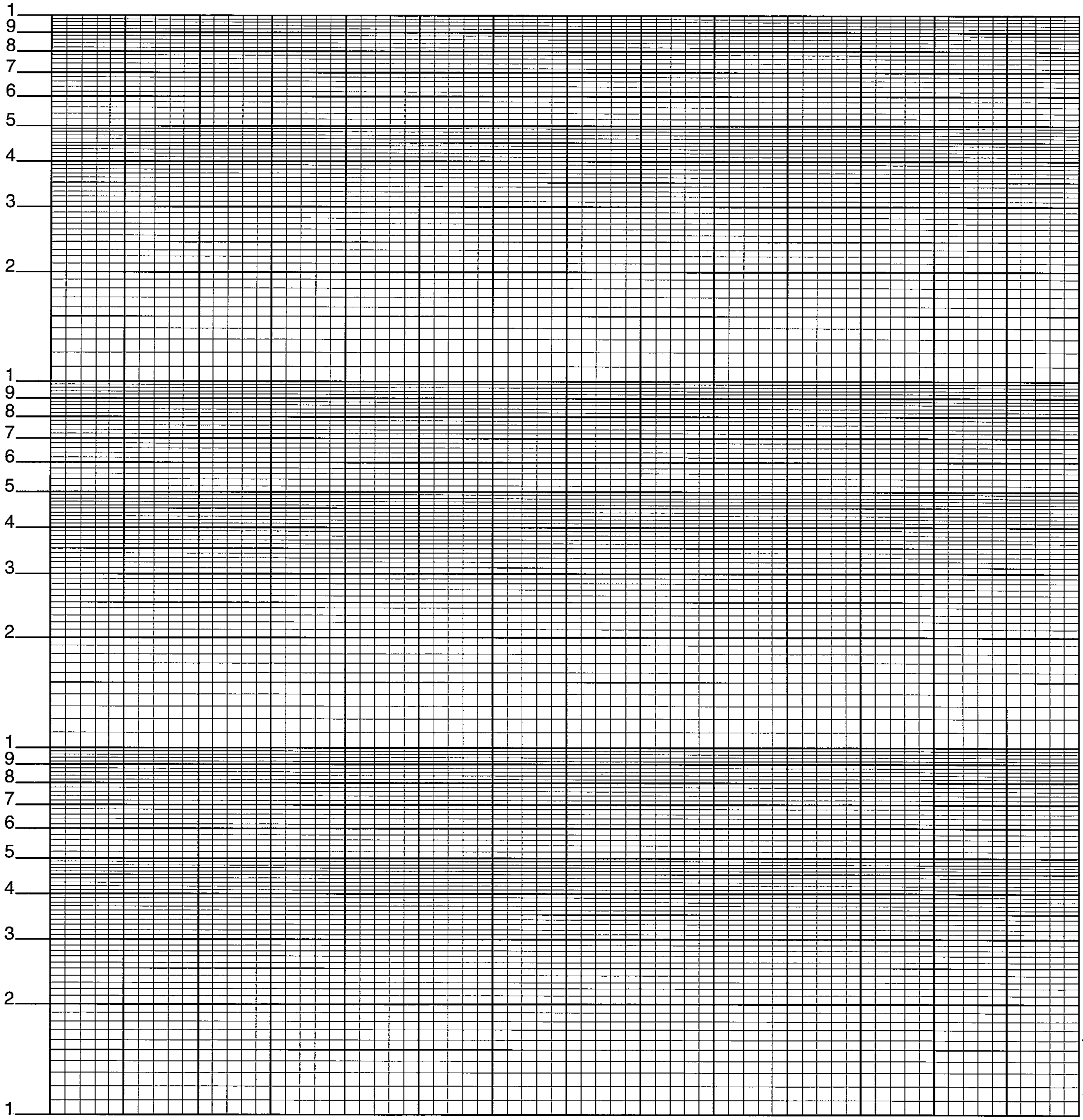
Semi - Logarithmic
3 Cycles x 10 to the inch

Semi - Logarithmic
3 Cycles x 10 to the inch



Semi - Logarithmic
3 Cycles x 10 to the inch

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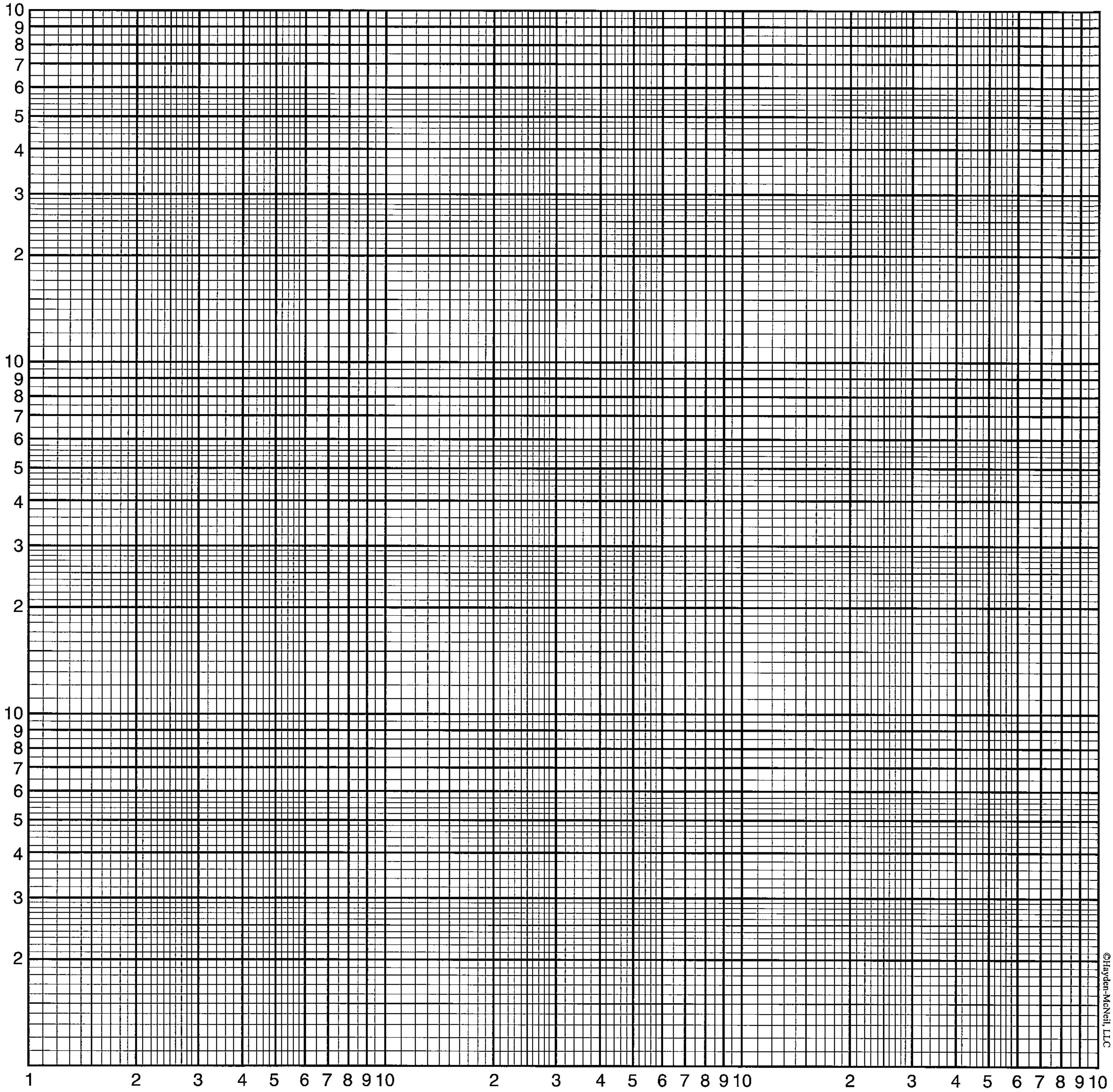
Semi - Logarithmic
4 Cycles x 10 to the inch

Full Logarithmic
3 x 3 cycles



Full Logarithmic
3 x 3 cycles