Homework 2

ME586 Robot Manipulators

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First, the normal vector n_e^b is determined. This is simple: It will always remain normal to the base XY coordinate frame, and will thus always be along the vector $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

Next, the sliding vector is determined. Since both joints are parallel, this orientation vector can be found by rotating the position of the sliding vector when $\vartheta_1 = \vartheta_2 = 0$ (which is $\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$) by the sum of the two angles $\vartheta_1 + \vartheta_2$. From this point forward, the shorthand $c_n = \cos\vartheta_n, s_n = \sin\vartheta_n$ will be used.

$$s_e^b = \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} s_{12} \\ -c_{12} \\ 0 \end{bmatrix}$$

The approach vector is determined in a similar way. When $\vartheta_1 = \vartheta_2 = 0$, the approach vector is $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, and this is rotated by the sum of the two axis angles.

$$s_e^b = \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} c_{12} \\ s_{12} \\ 0 \end{bmatrix}$$

Finally, the position vector is determined. This can be calculated as the sum of two vectors. The first is the translation from the base to the first joint, and the second is the translation from the first joint to the second joint. Both of these can be calculated as rotations of those translation vectors when $\vartheta_1 = \vartheta_2 = 0$. The first joint is rotated by ϑ_1, while the second joint is rotated by the sum of ϑ_1, ϑ_2 since the rotation is relative to when both joints are at 0.

$$T_0^1 = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a_1 c_1 \\ -a_1 s_1 \\ 0 \end{bmatrix}, T_1^2 = \begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a_2 c_{12} \\ a_2 s_{12} \\ 0 \end{bmatrix}$$

$$\Rightarrow p_e^b = T_0^1 + T_1^2 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix}$$

Therefore, the whole equation can be written:

$$T_e^b(q) = \begin{bmatrix} n_e^b & s_e^b & a_e^b & p_e^b \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & s_{12} & c_{12} & a_1 c_1 + a_2 c_{12} \\ 0 & -c_{12} & s_{12} & a_1 s_1 + a_2 s_{12} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$