Data Analysis **AE/ME 599 Fall 2020** **Instructor: P. Kreth**
Homework #2 **Due: 09/17/2020**

Note: *Show all necessary steps in determining your solution.* ***Grade:*** / 20

[5 pts] 1. Later in the course, we will discuss "window functions" in spectral analysis. When we discretize or sample data from a random process, and we want to compute the Discrete Fourier Transform (DFT), we are actually portioning the data by "windowing" it. The simplest window to use is a rectangular window (a.k.a. "boxcar") as defined below for a record of length T. We will see that the window function affects the DFT accuracy. In fact, the resulting spectrum is the result of a convolution between the true spectrum and the Fourier Transform of the window function.

Determine the Fourier Transform, $X(\omega)$, of the boxcar window *analytically* (i.e., not using MATLAB, Wolfram Alpha, etc. to solve for the expression). Your answer should be expressed in simplified mathematical terms. The boxcar window can be defined as

$$x(t) = \begin{cases} A & 0 \le t \le T \\ 0 & t < 0,\ t > T \end{cases}$$

Now, use MATLAB or another plotting software to produce a graph of $X(\omega)$ in the complex plane.

Also produce graphs of the magnitude, $|X(fT)|$, and phase, $\angle X(fT)$, as a function of fT with $A = 1$ and $T = 1$ for $0 \le fT \le 10$. Note, you should solve the expression you found for both the magnitude and phase.

What happens if you change T and/or A, but keep $A \cdot T = const$?

[2 pts] 2. B&P - Problem 1.3: If a stationary random process $\{x(t)\}$ has a mean value of μ_x, what is the limiting value of the autocorrelation function $R_{xx}(\tau)$ as the time delay τ becomes long? Explain your conclusions.

[2 pts] 3. B&P - Problem 1.4: An estimate is known to have a mean square error of 0.25 and a bias error of 0.40. Determine the variance of the estimate.

[2 pts] 4. B&P - Problem 1.5: In Problem 1.4, if the quantity being estimated has a true value of $\phi = 5$, what is the normalized rms error of the estimate?

[2 pts] 5. B&P - Problem 2.1: If an input $x(t)$ produces an output $y(t) = x(t)|x(t)|$, prove that the input/output relationship is nonlinear.

[2 pts] 6. In a calibration test, 10 measurements using a digital voltmeter have been made of the voltage on a battery that is known to have a true voltage of 6.11 V. The readings are: 5.98, 6.05, 6.10, 6.06, 5.99, 5.96, 6.02, 6.09, 6.03, and 5.99. What is the bias error caused by the voltmeter? In this sample set, what was the largest *random* error?

[5 pts] 7. Write a MATLAB *function* to perform the reverse arrangement test. A "function" is not a MATLAB "script"; the *function* takes in arguments and has return variables (see the example snippet below). The function should receive a random variable as an input (i.e., a vector x) and split it into a user-defined number of sample records (another argument), N_{rec}, of equal length and return the total number of reversals of the mean-square value of each record.

Validate your code using the data in Ex. 4.4.

Then, use real data (if available) and MATLAB-generated data to test your code. First, use stationary random data. Then, use nonstationary data.

Include the code you wrote, plots of the data you used (data record snippets, histograms), etc. in your results. Explain your observations and results.

```matlab
1  % Example MATLAB Function - this adds two numbers
2
3  function output = add_example(value1, value2)
4
5  output = value1 + value2;
6
7  % The function operates via: (remove the '%' at the start of the line)
8  % x = 3;
9  % y = 2;
10 % z = add_example(x, y) % This is equivalent to z = add_example(3, 2);
11
12 % MATLAB returns: (if you didn't terminate the statement on line 10 with ';')
13 % z =
14 %     5
```