Data Analysis **AE/ME 599 Fall 2020** **Instructor: P. Kreth**
Homework #3 **Due: 10/01/2020**

Note: *Show all necessary steps in determining your solution.* ***Grade:*** / 80

[5 pts] 1. Given the following system, $\ddot{y} + 12\dot{y} + 35y - 3 = 0; \quad y(0) = 0; \quad \dot{y}(0) = 2$, determine the:

(a) free response

(b) forced response

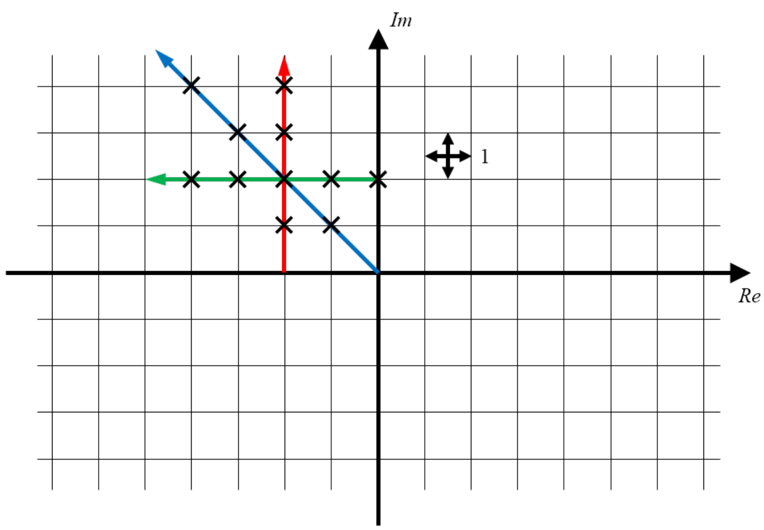
(c) total response

(d) steady-state response

(e) transient response

[3 pts] 2. Use analytical methods (i.e., work with pen and paper) to determine the frequency response function of a physical system with a weighting function $h(\tau) = Ae^{-a\tau}$, where $a > 0$.

[12 pts] 3. Plot the impulse response function $h(t)$ against the quantity $\omega_n t$ for each of the pole locations shown below in the complex plane. Make one graph for each colored line (total of 3 graphs), and identify that graph as having a constant damping ratio, time constant, or oscillation frequency, etc. Note that these need their complex conjugate pairs (omitted from this figure for simplicity).



[25 pts] 4. A first-order system with an impulse response function of $h(t) = \frac{1}{a}e^{-t/a}$ is excited by a single square wave pulse $x(t)$, defined below.

$$x(t) = \begin{cases} 1 & 0 < t \leq 2a \\ -1 & 2a < t \leq 4a \\ 0 & \text{otherwise} \end{cases}$$

If the time constant is $a = 2$ [s],

(a) Determine the system output $y(t)$ by applying the convolution integral **analytically** (i.e., using pen and paper). Only use online tools or MATLAB/MathCAD/Wolfram/etc. to check your work. *HINT: be careful to use the proper integration limits for when your delay is* $0 < t \leq 2a$, $2a < t \leq 4a$, *and* $t > 4a$.

(b) Discretize the system output $y(t)$ through numerical convolution methods (e.g., in MATLAB).

(c) Produce a graph (e.g., in MATLAB) that shows the analytical and numerical solutions as well as the forcing function. Ensure that you have a sufficient duration plotted such that the full response can be seen (e.g., 20 seconds).

(d) Interpret your results. Does everything make physical sense? Explain.

[30 pts] 5. A second-order system with an impulse response function of $h(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_d t)$ is excited by the truncated step function input $x(t)$, defined below.

$$x(t) = \begin{cases} 2 & 0 \le t \le 2T \\ 0 & \text{otherwise} \end{cases}$$

Given the following parameters, $\omega_n = 1$ [rad/s], $\zeta = 0.2$, $\omega_d = \omega_n\sqrt{1-\zeta^2}$, and $T = \frac{2\pi}{\omega_d}$ [s].

(a) Determine the system output $y(t)$ by applying the convolution integral **analytically** (i.e., using pen and paper). Only use online tools or MATLAB/MathCAD/Wolfram/etc. to check your work. *HINT: you can use the principles of homogeneity and superposition to your advantage, solving only a unit step response analytically. If you do this, you can multiply the response by 2 to account for the magnitude of $x(t)$ and express the truncation as a shifted negative step response. This can save some work, but it can be a difficult exercise resulting in more headache if you can't sort out the terms.*

(b) Discretize the system output $y(t)$ through numerical convolution methods (e.g., in MATLAB).

(c) Produce a graph (e.g., in MATLAB) that shows the analytical and numerical solutions as well as the forcing function. Ensure that you have a sufficient duration plotted such that the full response can be seen (e.g., 40 seconds).

(d) Interpret your results. Does everything make physical sense? Explain.

[5 pts] 6. Given the convolution integral $y(t) = \int_{-\infty}^{\infty} u(\tau)h(t-\tau)\,d\tau$ and the definition of the Fourier transform $Y(\omega) = \int_{-\infty}^{\infty} y(t)e^{-j\omega t}\,dt$, prove that $Y(\omega) = U(\omega)H(\omega)$.