2.17

$$M\frac{d^2x}{dt^2} + \frac{B\,dx}{dt} + Kx = U \qquad \sum f = 0$$

$$\dot{x}_1 = x_3 \qquad \dot{x}_2 = x_4$$

$$m_1 \overset{\ddot{x}_1}{\frac{d^2x_1}{dt^2}} + B\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) + K_1(x_1 - x_2) + K_2 x_1 = U$$

$$m_2 \overset{\ddot{x}_2}{\frac{d^2x_2}{dt^2}} + B\left(\overset{x_4}{\frac{dx_2}{dt}} - \overset{\dot{x}_3}{\frac{dx_1}{dt}}\right) + K_1(x_2 - x_1) + K_3 x_2 = 0$$

$$\begin{aligned} x_3 &= \dot{x}_1 \\ x_4 &= \dot{x}_2 \end{aligned} \quad \overset{d/dt}{\Longrightarrow} \quad \begin{aligned} \dot{x}_3 &= \ddot{x}_1 \\ \dot{x}_4 &= \ddot{x}_2 \end{aligned}$$

$$m_1 \dot{x}_3 = U - B(x_3 - x_4) - K_1(x_1 - x_2) - K_2 x_1$$

$$\dot{x}_3 = \frac{U - B(x_3 - x_4) - x_1(K_1 + K_2) + K_1 x_2}{m_1}$$

$$= \frac{-(K_1+K_2)}{m_1}x_1 + \frac{K_1}{m_1}x_2 - \frac{B}{m_1}x_3 - \frac{B}{m_1}x_4 + \frac{U}{m_1}$$

$$m_2 \dot{x}_4 = -B(x_4 - x_3) - K_1(x_2 - x_1) + K_3 x_2$$

$$\dot{x}_4 = \frac{K_1}{m_2}x_1 + \frac{(K_3 - K_1)}{m_2}x_2 + \frac{B}{m_2}x_3 - \frac{B}{m_2}x_4$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-(K_1+K_2)}{m_1} & \frac{K_1}{m_1} & \frac{-B}{m_1} & \frac{-B}{m_1} \\ \frac{K_1}{m_2} & \frac{K_3-K_1}{m_2} & \frac{B}{m_2} & \frac{-B}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix} \{U\}$$

MODELO ESTADO
(STATE MODEL)

$y_1 = x_1$
$y_2 = x_2$

$$\underset{2\times 1}{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}} = \underset{2\times 4}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}} \underset{4\times 1}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [u]$$

5)

identity of matrix

$$G_p = C(SI - A)^{-1} B$$

$$[SI-A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -\frac{2}{RC} & \frac{1}{RC} \\ \frac{1}{RC} & -\frac{1}{RC} \end{bmatrix} = \begin{bmatrix} s+\frac{2}{RC} & -\frac{1}{RC} \\ -\frac{1}{RC} & s+\frac{1}{RC} \end{bmatrix}$$

determinant

$$\det(SI-A) = \left(s+\frac{2}{RC}\right)\left(s+\frac{1}{RC}\right) - \frac{1}{(RC)^2}$$

adjunta

$$\operatorname{adj}(SI-A) = \begin{bmatrix} s+\frac{1}{RC} & \frac{1}{RC} \\ \frac{1}{RC} & s+\frac{2}{RC} \end{bmatrix}$$

$$(SI-A)^{-1} = \frac{\operatorname{adj}(SI-A)}{\det(SI-A)}$$

$$G_p = \frac{1}{\left(s+\frac{2}{RC}\right)\left(s+\frac{1}{RC}\right) - \frac{1}{(RC)^2}} \times \overset{C}{\underset{1\times 2}{\begin{bmatrix} 0 & 1 \end{bmatrix}}} \overset{\text{adjunta}}{\underset{2\times 2}{\begin{bmatrix} s+\frac{1}{RC} & \frac{1}{RC} \\ \frac{1}{RC} & s+\frac{2}{RC} \end{bmatrix}}} \overset{[B]}{\begin{bmatrix} \frac{1}{RC} \\ 0 \end{bmatrix}}$$

$$G_p = \frac{1}{\left(s+\frac{2}{RC}\right)\left(s+\frac{1}{RC}\right) - \frac{1}{RC^2}} \times \frac{1}{(RC)^2} \qquad \begin{bmatrix} \frac{1}{RC} & \frac{s+2}{RC} \end{bmatrix} \begin{bmatrix} 1/RC \\ 0 \end{bmatrix} = \frac{1}{(RC)^2}$$

$$= \frac{1}{s^2 + s\,\frac{1+2}{RC} + \frac{2}{(RC)^2} - \frac{1}{RC^2}} \times \frac{1}{(RC)^2}$$

$$= \frac{1}{s^2 + s\frac{3}{RC} + \frac{1}{(RC)^2}} \times \frac{1}{(RC)^2} = \frac{1}{s^2(RC)^2 + s \times 3(RC) + 1}$$

$M\Omega = 10^6 \qquad \mu\Omega = 10^{-6} \qquad RC = 2$

$$= \frac{1}{4s^2 + 6s + 1} = \frac{0{,}25}{s^2 + 1{,}5s + 0{,}25} \quad //$$