Resolução ficha 1 CS -

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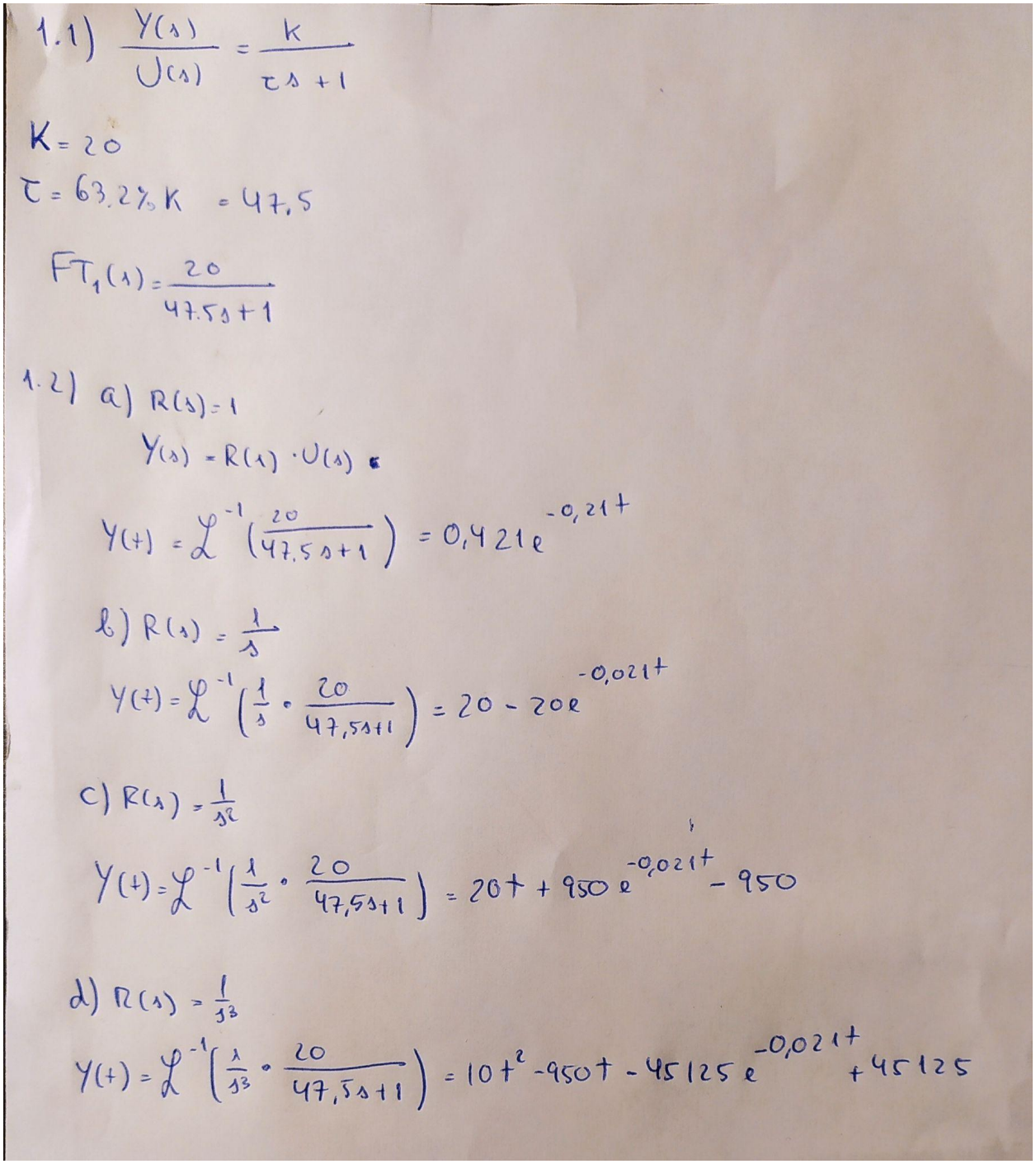
Parte 1



1.3)

a)

```
>> ilaplace(20/(47.5*s+1))

ans =

(8*exp(-(2*t)/95))/19
```

b)

```
>> ilaplace(20/(47.5*s^2+s))

ans =

20 - 20*exp(-(2*t)/95)
```

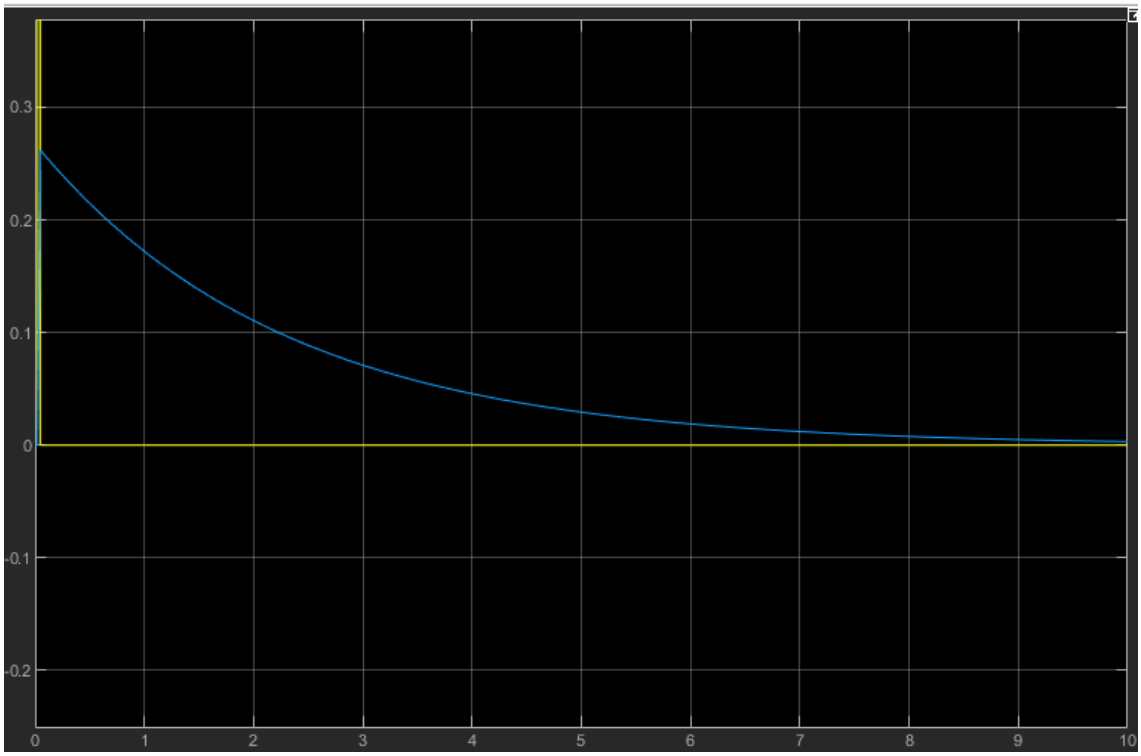
c)

```
>> ilaplace(20/(47.5*s^3+s^2))

ans =

20*t + 950*exp(-(2*t)/95) - 950
```

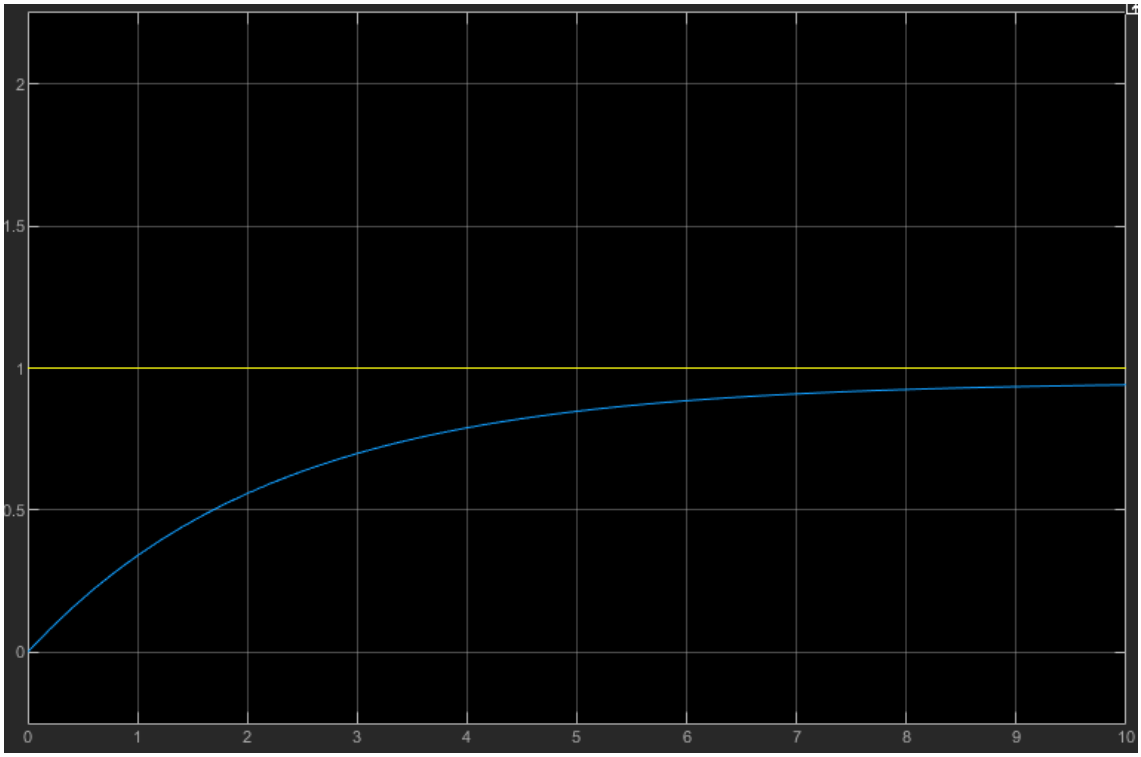
d)

```
>> ilaplace(20/(47.5*s^4+s^3))

ans =

10*t^2 - 45125*exp(-(2*t)/95) - 950*t + 45125
```

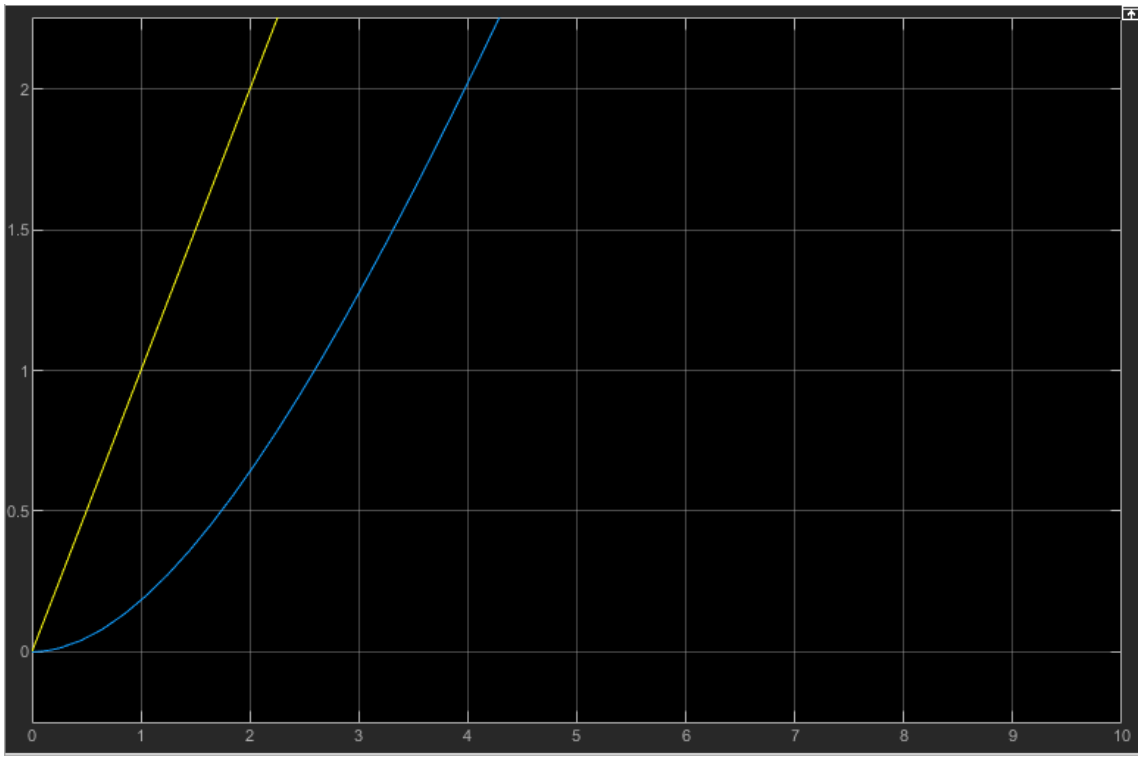
1.4)



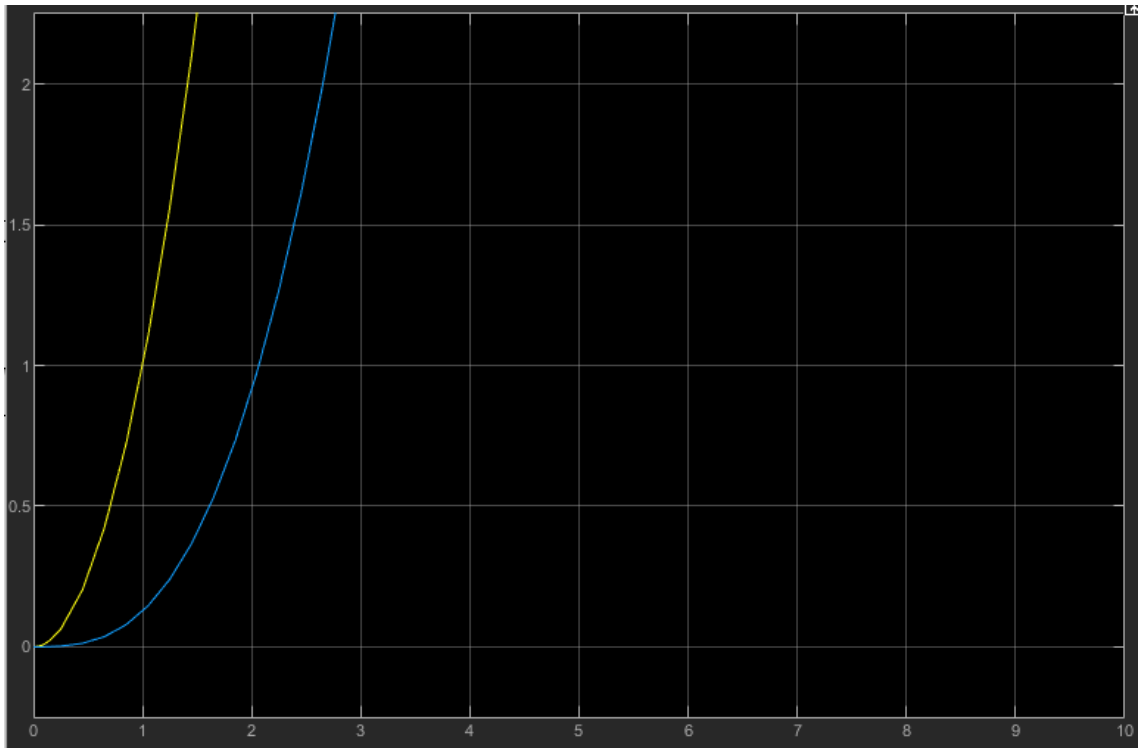
Resposta Temporal a Sinal de Entrada tipo Impulso



Resposta Temporal a Sinal de Entrada tipo Escalão de Posição

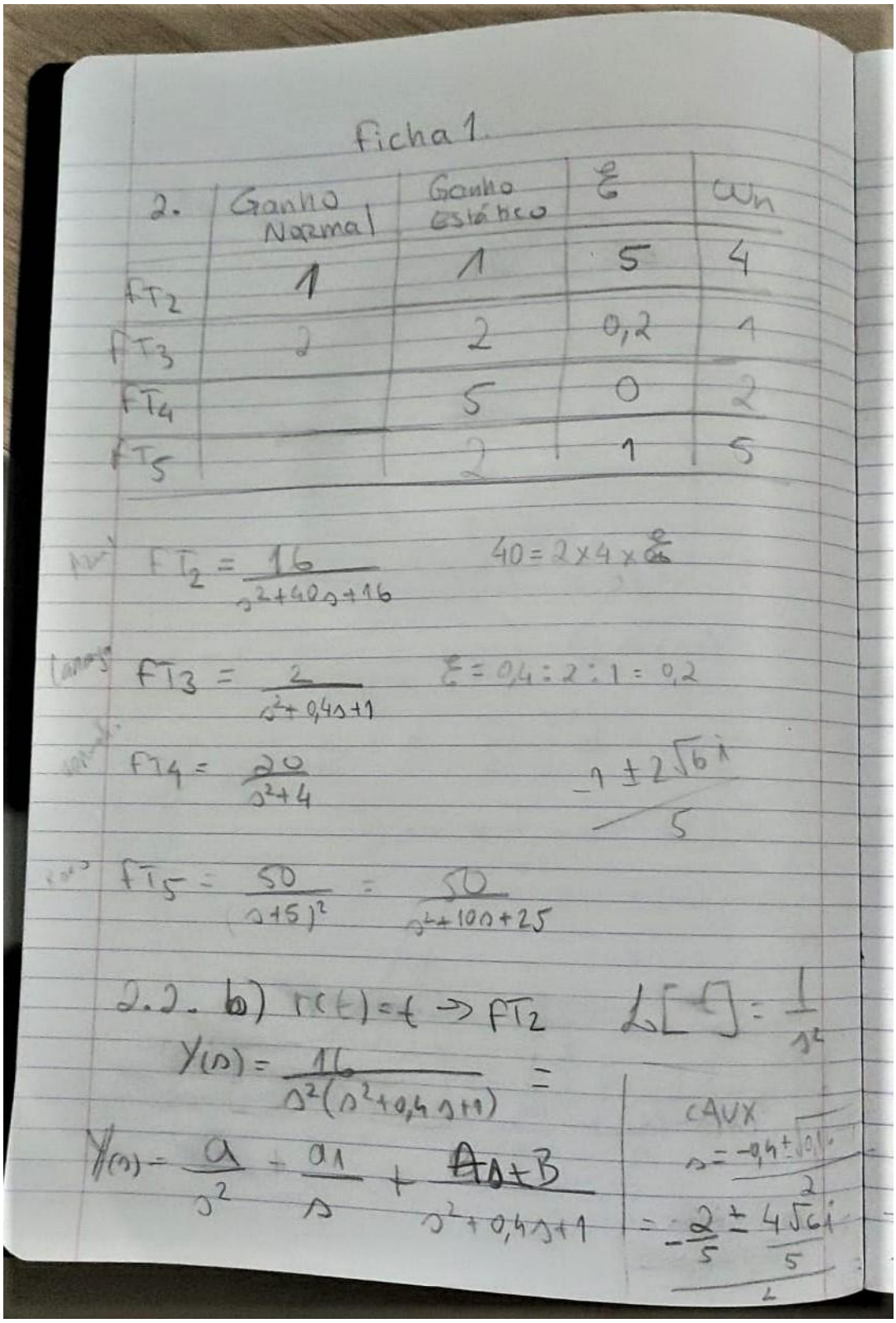


Resposta Temporal a Sinal de Entrada tipo Rampa



Resposta Temporal a Sinal de Entrada tipo Parábola

Parte 2)



Ficha 1.

2.	Ganho Normal	Ganho Estático	ξ	ω_n
FT_2	1	1	5	4
FT_3	2	2	0,2	1
FT_4		5	0	2
FT_5		2	1	5

$FT_2 = \frac{16}{s^2+40s+16}$ $\quad 40 = 2 \times 4 \times \xi$

$FT_3 = \frac{2}{s^2+0,4s+1}$ $\quad \xi = 0,4 : 2 : 1 = 0,2$

$FT_4 = \frac{20}{s^2+4}$ $\quad \frac{-1 \pm 2\sqrt{6}i}{5}$

$FT_5 = \frac{50}{(s+5)^2} = \frac{50}{s^2+10s+25}$

2.2. b) $r(t) = t \rightarrow FT_2$ $\quad \mathcal{L}[t] = \frac{1}{s^2}$

$Y(s) = \frac{16}{s^2(s^2+0,4s+1)} =$

$Y(s) = \frac{A}{s^2} + \frac{B}{s} + \frac{As+B}{s^2+0,4s+1}$

CAUX

$s = \frac{-0,4 \pm \sqrt{0,16}}{2}$

$= -\frac{2}{5} \pm \frac{4\sqrt{6}i}{5}$

$$a = \lim_{s \to 0} \frac{16}{(s^2+0{,}4s+1)} = 16$$

$$\lim_{s \to \frac{-1+2\sqrt{6}j}{5}} \frac{16}{s^2} = \lim_{s \to \frac{-1+2\sqrt{6}j}{5}} \left(\frac{a}{s^2} + \frac{c}{s}\right)(s^2+0{,}4s+1) + As + B$$

$$\Rightarrow \frac{16}{\left(\frac{-1+2\sqrt{6}j}{5}\right)^2} = 0 = A + 2A\sqrt{6}j + B \Rightarrow$$

$$\Rightarrow -\frac{365}{25} - \frac{64\sqrt{6}j}{25} = -\frac{A + 2A\sqrt{6}j}{5} + B \Rightarrow$$

$$\Rightarrow -\frac{365}{25} + \frac{64\sqrt{6}j}{25} = \frac{-5A + 10\sqrt{6}Aj + 5B}{25} =$$

$$\Rightarrow -368 + 64\sqrt{6}j = -5A + 5B + 10\sqrt{6}Aj \Rightarrow$$

$$\Rightarrow \begin{cases} -368 = -5A + 5B \\ 64\sqrt{6} = 10\sqrt{6}A \end{cases} \Rightarrow \begin{cases} A = \frac{32}{5} = 6{,}4 \\ B = -67{,}2 \end{cases}$$

c) $\mathcal{L}\left[\frac{t^2}{2}\right] = \frac{1}{s^3}$

$$\mathcal{L}^{-1}\left[\frac{20}{s^3(s^2+4)}\right] = \frac{20}{s^3(s^2+4)} = \frac{R_{13}}{s^3} + \frac{R_{12}}{s^2} + \frac{R_{11}}{s} + \frac{As+B}{s^2+4} =$$

$$R_{13} = \lim_{s \to 0} \frac{20}{s^2+4} = 5 \qquad \frac{1}{2}\frac{d^2}{ds}\frac{0-20}{(s^2+4)^2} = -1{,}25 \times$$

$$R_{12} = \frac{1}{1}\frac{d}{ds}\left[\frac{s^3 20}{s^3(s^2+4)}\right]_{s=0} = \lim_{s \to 0}\left|\frac{-20}{(s^2+4)^2}\right| = -1{,}25$$

$$R_{11} = \frac{1}{2}\frac{d^2}{ds^2}\left[\frac{s^3 20}{s^3(s^2+4)}\right]_{s=0} = 0{,}3925$$

$-1 \pm 2\sqrt{6}i$

2.3)

```
FT2=tf([0 0 16],[1 40 16]);
 FT3=tf([0 0 2],[1 0.4 1]);
 FT4=tf([0 0 20],[1 0 4]);
 FT5=tf([0 0 50],[1 10 25]);
figure(5)
subplot(2,2,1)
 pzmap(FT2);
 title('Mapa Pólo-Zeros da FT2');xlabel ('Eixo Real/'); ylabel ('Eixo Imaginário/');
subplot(2,2,2)
 pzmap(FT3);
 title('Mapa Pólo-Zeros da FT3');xlabel ('Eixo Real/'); ylabel ('Eixo Imaginário/');
subplot(2,2,3)
 pzmap(FT4);
 title('Mapa Pólo-Zeros da FT4');xlabel ('Eixo Real/'); ylabel ('Eixo Imaginário/');
subplot(2,2,4)
 pzmap(FT5);
 title('Mapa Pólo-Zeros da FT5');xlabel ('Eixo Real/'); ylabel ('Eixo Imaginário/');


 syms s t;
 V(t)=ilaplace (2/(s^2+0.4*s+1))
figure(6);
fplot(V(t))
 title('r(t)=1')
W(t)=ilaplace (16/s^2*(s^2+40*s+16))
figure(7);
fplot(W(t))
 title('r(t)=t')
R(t)=ilaplace (20/s^3*(s^2+4))
figure(8);
fplot(R(t))
title('r(t)=t^2/2')
U(t)=ilaplace (50*s/(s^2+25)*(s^2+10*s+25))
figure(9);
fplot(U(t))
title(' r(t)=cos(5t)')
```

Parte 3)

3.1.1)

3ª Parte

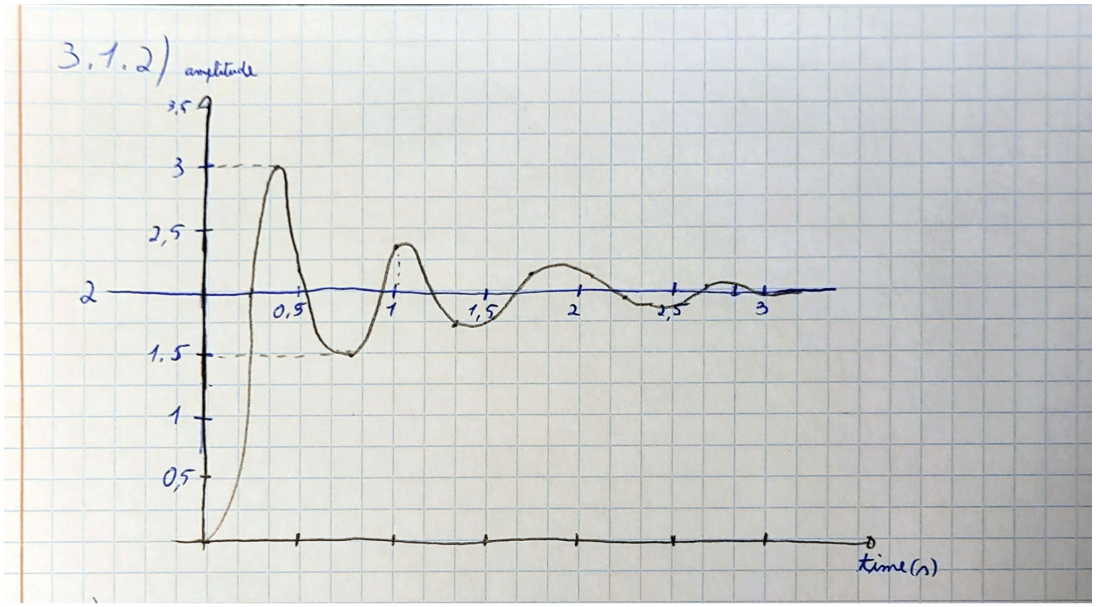
3.1) 3.1.1) $\frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n \xi s + k} = \frac{200}{s^2 + 4s + 100}$

$\omega_n = \sqrt{200}$ $\quad 2\xi\sqrt{200} = 4 \Leftrightarrow \xi = \frac{\sqrt{2}}{10}$

$\omega_a = \omega_n \sqrt{1 - \xi^2} = \sqrt{200} \times \sqrt{1 - \left(\frac{\sqrt{2}}{10}\right)^2} = 14$

- $t_p = \frac{\pi}{\omega_a} = \frac{\pi}{14} \approx 0,224$
- $t_r = \frac{\pi - \cos^{-1}\xi}{\omega_a} = \frac{\pi - \arccos\frac{\sqrt{2}}{10}}{14} = 0,122$
- $M_p = \left(e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}\right) 100\% = \left(e^{-\frac{\frac{\sqrt{2}}{10}\pi}{\sqrt{1-\frac{\sqrt{2}}{10}}}}\right) \times 100\% = 63,84\%$
- $t_d = \frac{1}{2} t_r = 0,061$
- $t_{s\_5\%} = \frac{3}{\xi\omega_n} = \frac{3}{\frac{\sqrt{2}}{10}\sqrt{200}} = \frac{3}{2}$

3.1.2)



3.2)

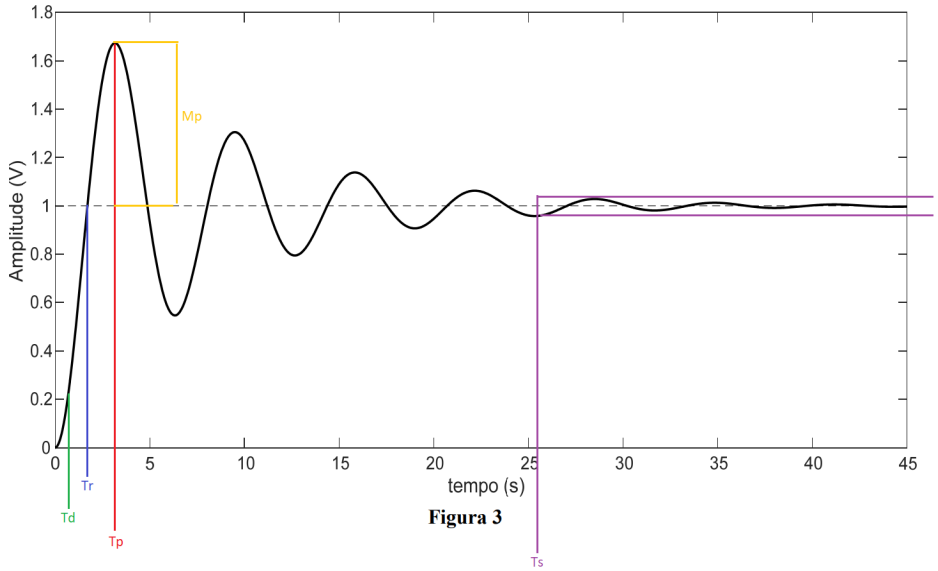


Figura 3

3.2)

3.2.1) • $t_p \simeq 3,8$ • $t_r = 1,9$

• $t_d = \frac{1}{2} t_r = 0,95$

• $t_p = \frac{\pi}{\omega_a} \Leftrightarrow \omega_a = \frac{\pi}{t_p} = \frac{\pi}{3,8} = \frac{5\pi}{19}$

• $t_r = \frac{\pi - \arccos\xi}{\omega_a} \Leftrightarrow 1,9 \times \frac{5\pi}{19} = \pi - \arccos\xi \Leftrightarrow$

$\Leftrightarrow \frac{\pi}{2} = \pi - \arccos\xi \Leftrightarrow \xi = 0$

Consideramos $\xi = 0,1$

• $\omega_a = \omega_0 \sqrt{1-(0,1)^2} \Leftrightarrow \omega_0 = \frac{50\pi\sqrt{11}}{627} \approx 0,83$

• $Mp\% = \left(e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}\right) 100\% = \left(e^{-\frac{0,1\pi}{\sqrt{1-0,1^2}}}\right) 100\% = 73\%$

• $t_{s\_5\%} = \frac{3}{\xi\omega_0} = \frac{3}{0,1 \times 0,83} = \frac{3}{0,083}$

3.2.2)

$$\frac{Y(s)}{U(s)} = \frac{\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2} =$$

$$= \frac{(0,83)^2}{s^2 + 2(0,1)(0,83)s + (0,83)^2} = \frac{0,6889}{s^2 + 0,166s + 0,6889} \approx$$

$$FT = \frac{0,6889}{s^2 + 0,166s + 0,6889}$$

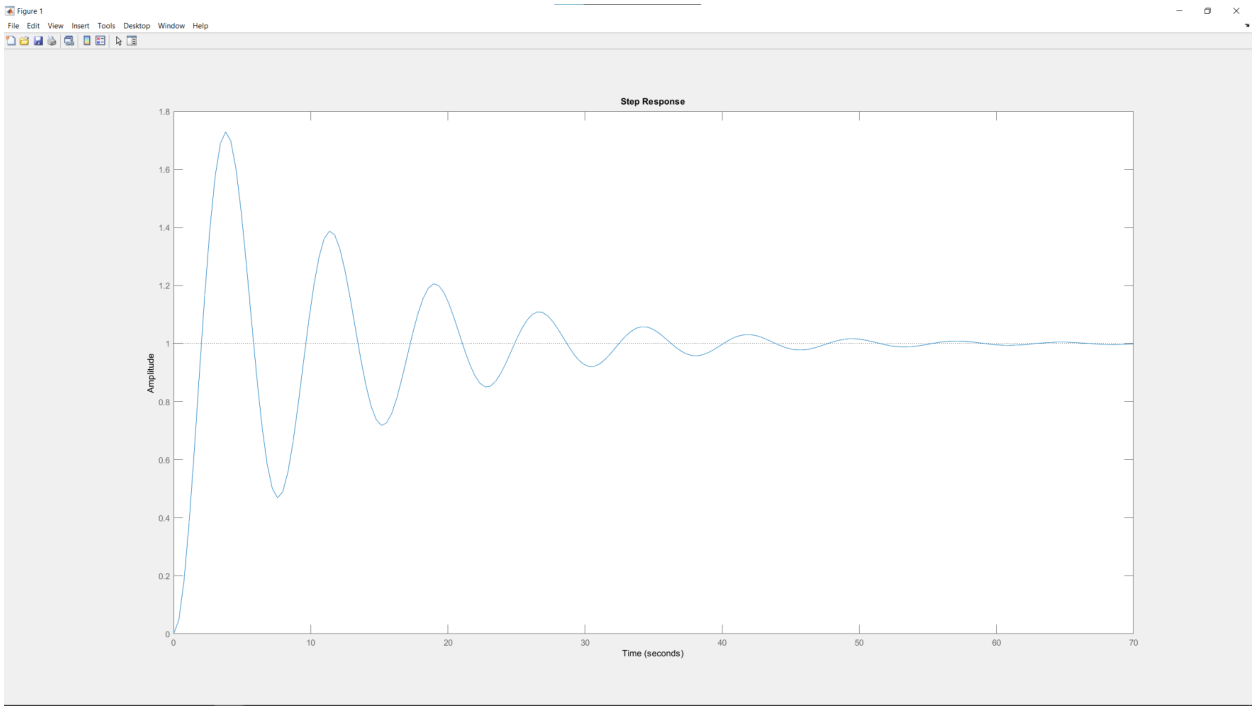


Figure 1
File Edit View Insert Tools Desktop Window Help
Step Response
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
10
20
30
40
50
60
70