	Grupo Disciplinar de Controlo (ADEEEA) **TF – TESTE FINAL** Controlo de Sistemas	**Ref.ª: LRTF01** **Data: 19-dezembro-2019**

ENUNCIADO

I - PARTE TEÓRICA

Considere dois sistemas de 1ª ordem (FT1 e FT2), em que as suas respostas temporais $y_1(t)$ e $y_2(t)$, foram obtidas com um escalão de posição na entrada de cada sistema (Figura 1):

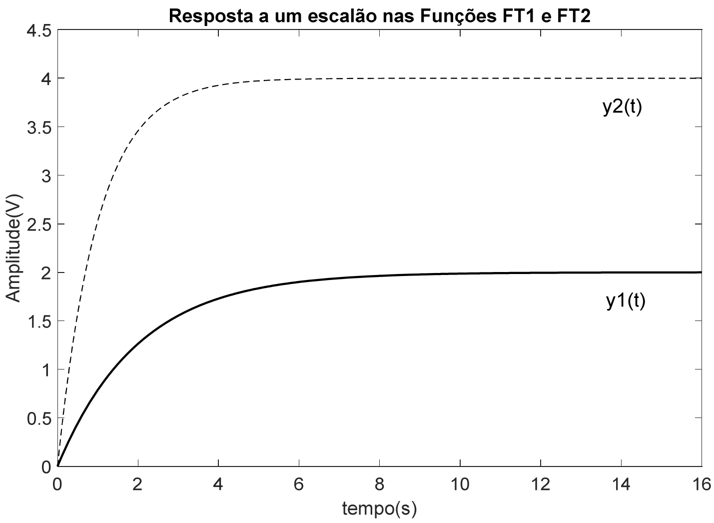

FIGURA 1

Modelo de estado de um circuito elétrico (duplo compensador de atraso de fase simples com efeito de carga), com R_1=R_2=R e C_1=C_2=C

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{2}{RC} & \dfrac{1}{RC} \\ \dfrac{1}{RC} & -\dfrac{1}{RC} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{RC} \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

FIGURA 2

(2,0) 1 – Com base nas respostas temporais apresentadas na Figura 1, determine as funções de Transferência (FT1 e FT2). Desenhe o Mapa polo-zero dos 2 sistemas (FT1 e FT2) no mesmo diagrama.

(4,0) 2 – Com base no Modelo de Estado apresentado na Fig.2, obtenha a função de transferência na forma literal e na forma numérica, considerando: R=1kΩ e C=5mF.

II - PARTE PRÁTICA

Considere o seguinte sistema hidráulico com 3 tanques interativos, com entrada em q_1 e saída em q_4:

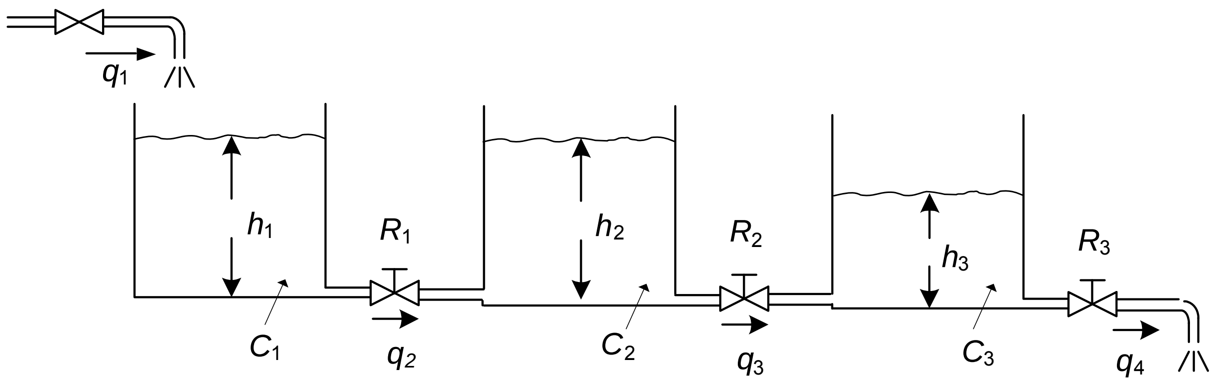

Figura 3

(4,0) 3 – Determine o Modelo de Estado do sistema de nível (Figura 3), considerando como variáveis de estado $x_1 = h_1$, $x_2 = h_2$, $x_3 = h_3$.

(2,0) 4 – Desenhar o diagrama de blocos inicial do sistema da Figura 3 (utilizar as equações iniciais).

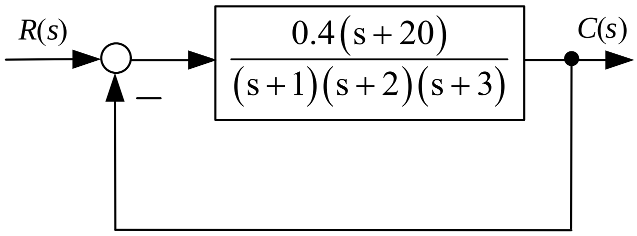

Figura 4

(2,0) 5 – Determine o erro forçado do sistema da Figura 4, para uma entrada do tipo escalão de posição.

(4,0) 6 – Analise a estabilidade do sistema da Figura 4, a partir do critério de estabilidade do Diagrama do Lugar Geométrico das Raízes (*Root-Locus*).

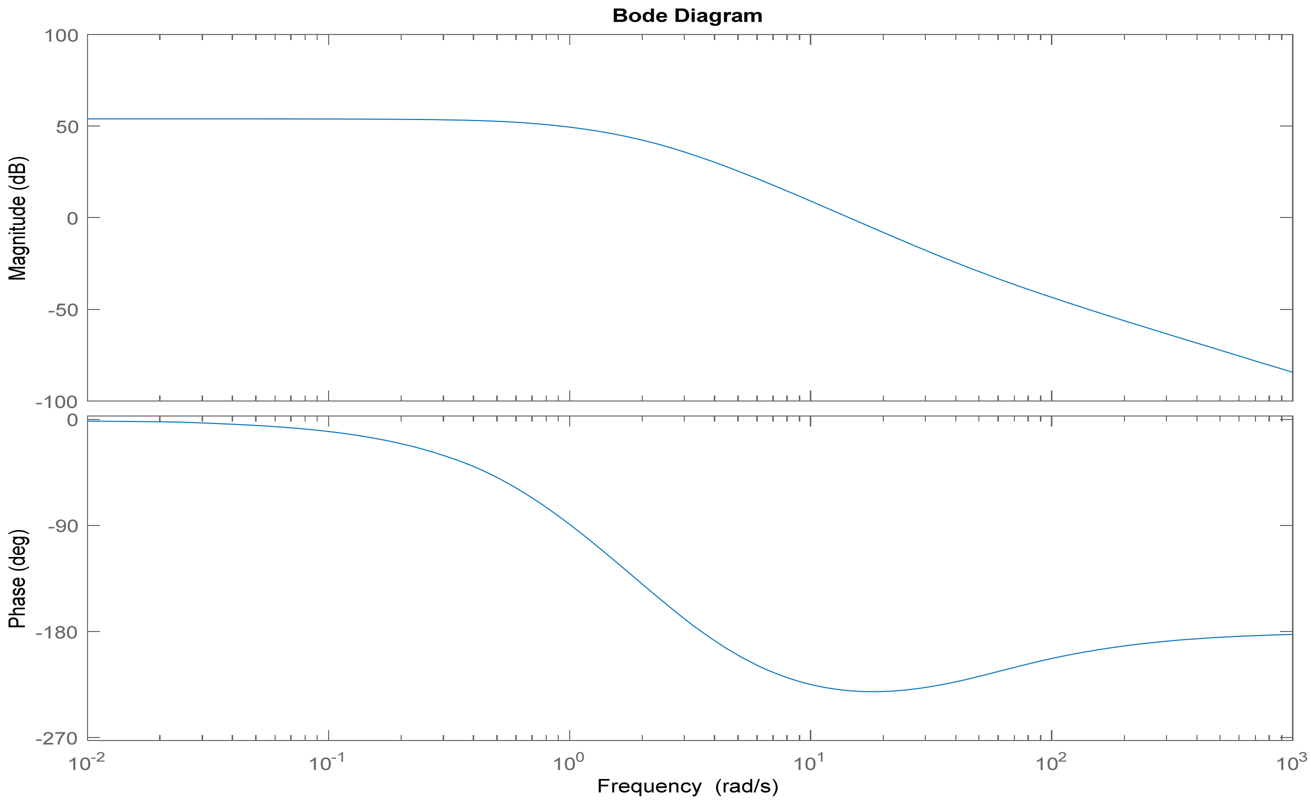

Figura 5

(2,0) 7 – Com base no Diagrama de amplitude e de fase, referentes a uma FTCA (Figura 5), determine graficamente a margem de ganho e a margem de fase. Conclua sobre a estabilidade. ***Nota:*** *(Marcar G_m e P_m diretamente no enunciado)*

NOTAS FINAIS - Para a resolução da prova atenda às seguintes notas:
1 - Deverá apresentar todas as justificações a cálculos realizados.
2 - O enunciado é entregue juntamente com ou sem a folha de prova.

Nome ______________________________ Aluno nº ____________

Turma________ Semestre ________ Classificação _______(_______________) O Professor ______________

FIM

Tabela com as Regras de Construção do DLGR (RL $K > 0$ e CRL $K < 0$)

Table 8-1 Rules of Construction of Root Loci

1. $K = 0$ points	The $K = 0$ points on the complete root loci are at the poles of $G(s)H(s)$. (The poles include those at infinity.)
2. $K = \pm\infty$ points	The $K = \pm\infty$ points on the complete root loci are at the zeros of $G(s)H(s)$. (The zeros include those at infinity.)
3. Number of separate root loci	The total number of root loci is equal to the order of the equation $F(s) = 0$.
4. Symmetry of root loci	The complete root loci of systems with rational transfer functions with constant coefficients are symmetrical with respect to the real axis of the s-plane.
5. Asymptotes of root loci as $s \longrightarrow \infty$	For large values of s, the root loci ($K > 0$) are asymptotic to straight lines with angles given by $$\theta_k = \frac{(2k+1)\pi}{n-m}$$ and for the complementary root loci ($K < 0$) $$\theta_k = \frac{2k\pi}{n-m}$$ where $k = 0, 1, 2, \ldots, \lvert n-m \rvert - 1$.
6. Intersection of the asymptotes (centroids)	(a) The intersection of the asymptotes lies only on the real axis in the s-plane. (b) The point of intersection of the asymptotes on the real axis is given by (for all values of K) $$\sigma_1 = \frac{\sum \text{real parts of poles of } G(s)H(s) - \sum \text{real parts of zeros of } G(s)H(s)}{n-m}$$
7. Root loci on the real axis	On a given section on the real axis in the s-plane, root loci are found for $K \geq 0$ in the section only if the total number of real poles and real zeros of $G(s)H(s)$ to the right of the section is *odd*. If the total number of real poles and zeros to the right of a given section is *even*, complementary root loci ($K \leq 0$) are found in the section.
8. Angles of departure and arrival	The angle of departure of the root locus ($K \geq 0$) from a pole or the angle of arrival at a zero of $G(s)H(s)$ can be determined by assuming a point s_1 that is on the root locus associated with the pole, or zero, and which is very close to the pole, or zero, and applying the equation $$\angle G(s_1)H(s_1) = \sum_{i=1}^{m} \angle s_1 + z_i - \sum_{j=1}^{n} \angle s_1 + p_j = (2k+1)\pi \qquad k = 0, \pm 1, \pm 2, \ldots$$ The angle of departure or arrival of a complementary root locus is determined from $$\angle G(s_1)H(s_1) = \sum_{i=1}^{m} \angle s_1 + z_i - \sum_{j=1}^{n} \angle s_1 + p_j = 2k\pi \qquad k = 0, \pm 1, \pm 2, \ldots$$
9. Intersection of the root loci with the imaginary axis	The values of ω and K at the crossing points of the root loci on the imaginary axis of the s-plane may be obtained by use of the Routh-Hurwitz criterion. The Bode plot of $G(s)H(s)$ may also be used.
10. Breakaway points (saddle points)	The breakaway points on the complete root loci are determined by finding the roots of $dK/ds = 0$, or $dG(s)H(s)/ds = 0$. These are necessary conditions only. Alternatively, the breakaway points are determined from a tabulation using the coefficients of the characteristic equations $F(s) = 0$ and $F'(s) = 0$. The conditions are necessary and sufficient.
11. Calculation of the values of K on the root loci	The absolute value of K at any point s_1 on the complete root loci is determined from the equation $$\lvert K \rvert = \frac{1}{\lvert G(s_1)H(s_1) \rvert} = \frac{\text{product of lengths of vectors drawn from the poles of } G(s)H(s) \text{ to } s_1}{\text{product of lengths of vectors drawn from the zeros of } G(s)H(s) \text{ to } s_1}$$

(Tabela usada nas aulas)

Adaptado do livro :
Automatic Control Systems,
Benjamim C. Kuo , 3rd Edition (1975)