Automated Hyperbug Finding

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Part I: Motivation

Motivating Example

Let's look at a simple voting protocol for two candidates A and B...

$$
\begin{array}{ll}
\ell_0: & countA \leftarrow 0 \\
\ell_1: & countB \leftarrow 0 \\
\ell_2: & \textbf{loop} \\
\ell_3: & \quad \textbf{input } vote \in \{A, B\} \\
\ell_4: & \quad \textbf{if } vote = A \textbf{ then} \\
\ell_5: & \quad\quad countA \leftarrow countA + 1 \\
\ell_6: & \quad \textbf{if } vote = B \textbf{ then} \\
\ell_7: & \quad\quad countB \leftarrow countB + 1
\end{array}
$$

Motivating Example

… there could be a *tiny bug* in its implementation that plays in the favor of candidate B (whoops)

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2. Even without a precise specification, it is clear that this code cannot be correct because it does not treat candidates A and B equally.

"candidates A and B should be treated equally" is an example of a hyperproperty
$\Longrightarrow$ it requires comparing several executions of the voting protocol

Specifying Hyperproperties in HyperLTL

To formally specify hyperproperties of software systems, we use the logic HyperLTL, an extension of LTL with trace quantification.

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Temporal relations

$$\varphi ::= P \mid \varphi \land \varphi \mid \Box \varphi \mid \varphi \ \mathsf{U} \ \varphi \mid \ldots$$

where P is a predicate over program variables labeled by a trace variable τ

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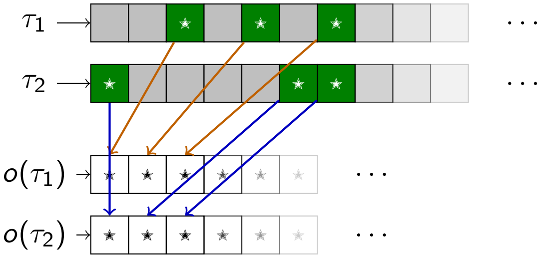

OHyperLTL in Action

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Importantly, the alternation of universal and existential quantification enables a concise specification that does not refer to the inputs of the protocol

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Our goal

$\rightarrow$ a fully automated bug-hunting technique for $\forall^*\exists^*$ asynchronous hyperproperties expressed in OHyperLTL

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Challenges

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Refutation is not (much) simpler

→ requires finding a trace and a proof that, for this trace, no second trace exists that satisfies the specified relation

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$\rightarrow$ no existing approach can fully automatically find counterexamples to $\forall\exists$ hyperproperties in asynchronous, infinite-state systems

Part II: Symbolic Execution for Asynchronous Hyperproperties

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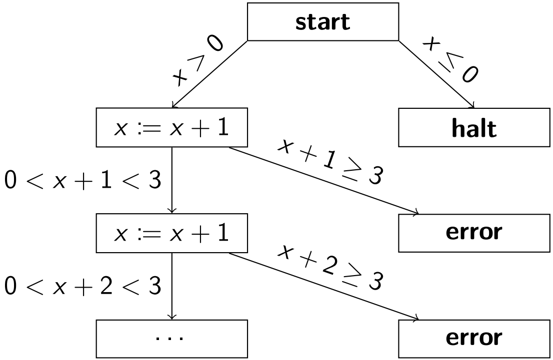

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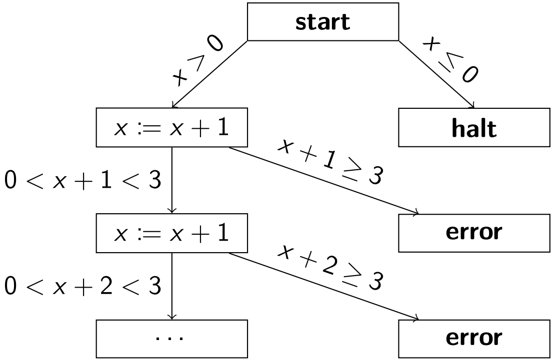
- → logical representation of program paths
- → reduces bug finding to SMT solving



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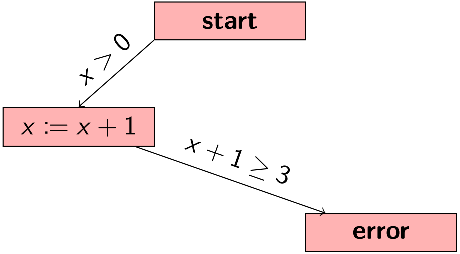
$0 < x \land x + 1 \geq 3$ is sat
$\implies$ bug found!



Symbolic Execution for OHyperLTL$_{\text{safe}}$

Given: OHyperLTL$_{\text{safe}}$ hyperproperty $\forall\tau_1.\exists\tau_2.\Box\varphi$

Idea: use two symbolic execution engines to find a model for $\exists\tau_1.\forall\tau_2.\neg\Box\varphi$

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Can be generalized to $\forall^* \exists^*$ hyperproperties through product constructions

The good news

Theorem (Soundnesss)

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Refuting asynchronous $\forall\exists$ hyperproperties of infinite-state systems is undecidable.
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Corollary (Incompleteness)

Symbolic Execution for $OHyperLTL_{\mathsf{safe}}$ is necessarily incomplete.

What is the problem?

Intuition: using symbolic execution to find a model for $\exists \tau_1 . \forall \tau_2 . \neg \Box \varphi$ requires to enumerate all possible symbolic paths of a given observation length

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ℓ_0: **loop**
ℓ_1: **while**$(\dots)$ $\{\dots\}$
ℓ_2: **observe**

Symbolic paths of observational lengths k are of the form $(\ell_0(\ell_1)^\star \ell_2)^k$ (i.e., there are infinitely many of them)

Relative completeness for observable programs

Definition (Observable Programs)

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Theorem

Relative Completeness Symbolic Execution for $\mathrm{OHyperLTL}_{\mathrm{safe}}$ *is a complete hyperbug finding method for observable programs (assuming symbolic paths can be expressed in a decidable first-order theory).*

Part III: Evaluation

Experimental results - ORHLE benchmarks

Class	Type	Program	FO	Bug found	# Combinations	Runtime
$\forall\exists$	Other	`draw-once`	✓	✓	1	0.001 s
$\forall\exists$	Refinement	`simple-nonrefinement`	✓	✓	1	0.001 s
$\forall\exists$	Other	`do-nothing`	✓	✓	1	0.001 s
$\forall\forall\exists$	Generalized non-interference	`nondet-leak2`	✓	✓	2	0.001 s
$\forall\forall\exists$	Generalized non-interference	`simple-leak`	✓	✓	1	0.001 s
$\forall\forall\exists$	Generalized non-interference	`smith1`	✓	✓	2	0.003 s
$\forall\forall\exists$	Generalized non-interference	`nondet-leak`	✓	✓	2	0.003 s
$\forall\forall\exists$	Delimited release	`parity-no-dr`	✓	✓	2	0.003 s
$\forall\forall\exists$	Delimited release	`wallet-no-dr`	✓	✓	2	0.003 s
$\forall\exists$	Refinement	`conditional-nonrefinement`	✓	✓	4	0.006 s
$\forall\exists$	Refinement	`add3-shuffled`	✓	✓	6	0.009 s
$\forall\forall\forall\exists$	Delimited release	`conditional-no-dr`	✓	✓	8	0.013 s
$\forall\forall\exists$	Delimited release	`median-no-dr`	✓	✓	4	0.016 s
$\forall\forall\forall\exists$	Generalized non-interference	`conditional-leak`	✓	✓	48	0.074 s
$\forall\exists$	Refinement	`loop-nonrefinement`	✗	✗	N/A	∞

Experimental results - our own benchmarks

Class	Program	Bug found	# Observations	# Combinations	Runtime
$\forall\exists$	`even_odd`	✓	1	1	0.001 s
$\forall\exists$	`factor2`	✓	2	2	0.001 s
$\forall\exists$	`for_loop_simple`	✓	1	2	0.010 s
$\forall\exists$	`linear_equation`	✓	22	22	0.023 s
$\forall\exists$	`monotonic_increase`	✓	7	7	0.029 s
$\forall\exists$	`escalating`	✓	7	747	0.103 s
$\forall\forall\exists$	`secret_pin_leak`	✓	8	11	0.103 s
$\forall\exists$	`escalating_2`	✓	7	1707	0.190 s
$\forall\forall$	`obs_determinism`	✓	4	86	0.408 s
$\forall\exists$	`no_primes_above_31397`	✓	1	201	1.203 s
$\forall\forall\exists$	`secret_pin_leak_2`	✓	3	248	1.972 s
$\forall\exists$	`exponential_branching_1`	✓	1	1024	2.216 s
$\forall\exists$	`exponential_branching_2`	✓	1	2048	4.040 s