Symbolic Bug Finding for Asynchronous Hyperproperties

Workshop on Asynchronous Hyperproperties

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Part I: Motivation

Motivating Example

Let's look at a simple voting protocol for two candidates A and B...

$$
\begin{array}{|l|}
\hline
\quad \mathit{countA} \leftarrow 0 \\
\quad \mathit{countB} \leftarrow 0 \\
\quad \textbf{loop} \\
\qquad \textbf{input } \mathit{vote} \in \{A, B\} \\
\qquad \textbf{if } \mathit{vote} = A \textbf{ then} \\
\qquad\quad \mathit{countA} \leftarrow \mathit{countA} + 1 \\
\qquad \textbf{if } \mathit{vote} = B \textbf{ then} \\
\qquad\quad \mathit{countB} \leftarrow \mathit{countB} + 1 \\
\hline
\end{array}
$$

Motivating Example

... there could be a *tiny bug* in its implementation that plays in the favor of candidate B (whoops)

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\end{array}
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1. This bug is surprisingly difficult to catch with traditional bug-finding tools without giving a full formal specification of the voting protocol.
2. Even without a precise specification, it is clear that this code cannot be correct because it does not treat candidates A and B equally.

"candidates A and B should be treated equally" is an example of a hyperproperty
$\Longrightarrow$ it requires comparing several executions of the voting protocol

Specifying Hyperproperties in HyperLTL

To formally specify hyperproperties, we can use the logic HyperLTL, an extension of LTL with trace quantification.

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Temporal relations

$$\varphi ::= P \mid \varphi \land \varphi \mid \Box\varphi \mid \varphi \;\mathsf{U}\; \varphi \mid \ldots$$

where P is a predicate over program variables labeled by a trace variable τ

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Importantly, the alternation of universal and existential quantification enables a concise specification that does not refer to the inputs of the protocol

Synchronization Problems (i)

Consider the following slightly modified version of the voting protocol:

$countA \leftarrow 0$
$countB \leftarrow 0$
loop
  input $vote \in \{A, B\}$
  if $vote = A$ **then**
    $countA \leftarrow countA - 1$
    $countA \leftarrow countA + 2$
  if $vote = B$ **then**
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  if $vote = B$ **then**
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- → This version is functionally equivalent to the initial protocol
- → However, it trivially violates the equal opportunities property

Synchronization Problems (ii)

→ In the HyperLTL specification of equal opportunities, the invariant

$$countA_{\tau_1} = countB_{\tau_2} \wedge countA_{\tau_2} = countB_{\tau_1}$$

is required to hold at every computation step

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- Since updating the score for A takes one more step than updating the score for B, the invariant will always be violated temporarily
- To avoid this problem, we need an asynchronous logic enabling to specify how the different executions are aligned before being compared

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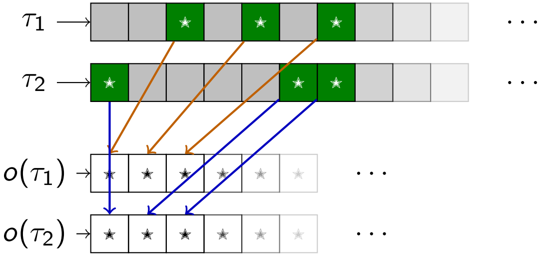

Example

ℓ_0: $countA \leftarrow 0$
ℓ_1: $countB \leftarrow 0$
ℓ_2: **loop**
ℓ_3: **input** $vote \in \{A, B\}$
ℓ_4: **if** $vote = A$ **then**
ℓ_5: $countA \leftarrow countA - 1$
ℓ_6: $countA \leftarrow countA + 2$
ℓ_7: **if** $vote = B$ **then**
ℓ_8: $countB \leftarrow countB + 1$

$$\boxed{\forall \tau_1 : @\ell_3, \exists \tau_2 : @\ell_3, \Box(countA_{\tau_1} = countB_{\tau_2} \land countA_{\tau_2} = countB_{\tau_1})}$$

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Our goal

$\rightarrow$ a fully automated bug-hunting technique for $\forall^*\exists^*$ asynchronous hyperproperties expressed in OHyperLTL

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Refutation is not (much) simpler

→ requires finding a trace and a proof that, for this trace, no second trace exists that satisfies the specified relation

Part II: Symbolic Execution for Asynchronous Hyperproperties

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$\rightarrow$ no existing approach can fully automatically find counterexamples to $\forall\exists$ hyperproperties in asynchronous, infinite-state systems

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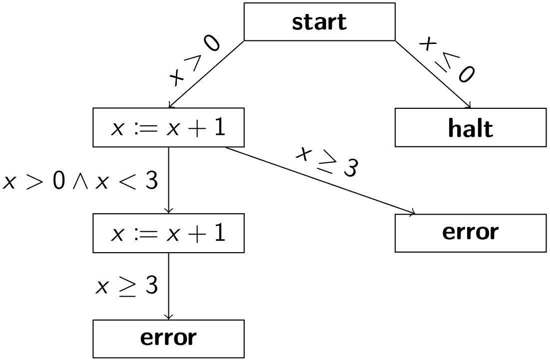
while $x > 0$
  $x := x + 1$
  assert $x < 3$



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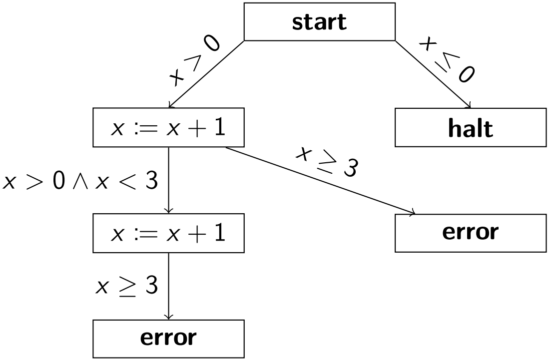
- → logical representation of program paths
- → reduces bug finding to SMT solving
- → symbolic encoding must be computable



Symbolic Execution for OHyperLTL$_{\text{safe}}$

Given: OHyperLTL$_{\text{safe}}$ hyperproperty $\forall\tau_1.\exists\tau_2.\Box\varphi$

Idea: use two symbolic execution engines to find a model for $\exists\tau_1.\forall\tau_2.\neg\Box\varphi$

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Can be generalized to $\forall^* \exists^* \ldots$ hyperproperties through product constructions

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For $\forall\exists$ hyperproperties, computable symbolic encodings do not generally exist!

Lemma

Refuting an asynchronous $\forall\exists$ hyperproperty of an infinite-state system is undecidable.

By reduction from the halting problem.

Undecidability and Incompleteness

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By reduction from the halting problem.

Corollary

Any sound bug finding algorithm for asynchronous $\forall\exists$ hyperproperties of infinite-state systems is necessarily incomplete.

Relative completeness for observable programs

Definition (Informal)

A program is observable if, in any symbolic state, there is some n such that, after n steps, the program either produces an observed system state or terminates.

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$\mathrm{OHyperLTL}_{\mathrm{safe}}$ *permits a computable symbolic encoding* $[\![\psi]\!]_{\mathcal{T}}$ *for observable input programs over some theory* $\mathcal{T}$.

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Corollary

Symbolic execution-based refutation of OHyperLTL$_{\mathsf{safe}}$ *hyperproperties is complete for observable input programs, assuming* $\mathcal{T}$ *is decidable.*

Part III: Evaluation

Experimental results - ORHLE benchmarks

Class	Type	Program	FO	Bug found	# Combinations	Runtime
$\forall\exists$	Other	`draw-once`	✓	✓	1	0.001 s
$\forall\exists$	Refinement	`simple-nonrefinement`	✓	✓	1	0.001 s
$\forall\exists$	Other	`do-nothing`	✓	✓	1	0.001 s
$\forall\forall\exists$	Generalized non-interference	`nondet-leak2`	✓	✓	2	0.001 s
$\forall\forall\exists$	Generalized non-interference	`simple-leak`	✓	✓	1	0.001 s
$\forall\forall\exists$	Generalized non-interference	`smith1`	✓	✓	2	0.003 s
$\forall\forall\exists$	Generalized non-interference	`nondet-leak`	✓	✓	2	0.003 s
$\forall\forall\exists$	Delimited release	`parity-no-dr`	✓	✓	2	0.003 s
$\forall\forall\exists$	Delimited release	`wallet-no-dr`	✓	✓	2	0.003 s
$\forall\exists$	Refinement	`conditional-nonrefinement`	✓	✓	4	0.006 s
$\forall\exists$	Refinement	`add3-shuffled`	✓	✓	6	0.009 s
$\forall\forall\forall\exists$	Delimited release	`conditional-no-dr`	✓	✓	8	0.013 s
$\forall\forall\exists$	Delimited release	`median-no-dr`	✓	✓	4	0.016 s
$\forall\forall\forall\exists$	Generalized non-interference	`conditional-leak`	✓	✓	48	0.074 s
$\forall\exists$	Refinement	`loop-nonrefinement`	✗	✗	N/A	∞

Experimental results - our own benchmarks

Class	Program	Bug found	# Observations	# Combinations	Runtime
$\forall\exists$	even_odd	✓	1	1	0.001 s
$\forall\exists$	factor2	✓	2	2	0.001 s
$\forall\exists$	for_loop_simple	✓	1	2	0.010 s
$\forall\exists$	linear_equation	✓	22	22	0.023 s
$\forall\exists$	monotonic_increase	✓	7	7	0.029 s
$\forall\exists$	escalating	✓	7	747	0.103 s
$\forall\forall\exists$	secret_pin_leak	✓	8	11	0.103 s
$\forall\exists$	escalating_2	✓	7	1707	0.190 s
$\forall\forall$	obs_determinism	✓	4	86	0.408 s
$\forall\exists$	no_primes_above_31397	✓	1	201	1.203 s
$\forall\forall\exists$	secret_pin_leak_2	✓	3	248	1.972 s
$\forall\exists$	exponential_branching_1	✓	1	1024	2.216 s
$\forall\exists$	exponential_branching_2	✓	1	2048	4.040 s