Price-based Optimization of Serial Robot Manipulators Under Payload and Workspace Constraints

Akansel Cosgun, Harvey Lipkin and Henrik I. Christensen

Abstract— **We investigate tradeoffs between price and performance for geometric design of serial manipulators. We present a design procedure that aims to minimize the cost of a serial robot manipulator that satisfies target workspace and payload constraints. The design is conducted in two steps. In the first step, Denavit-Hartenberg parameters of the robot are determined so that the end effector can reach every point in a target rectangular prism. This optimization routine minimizes the sum of link lengths and offsets via a variant of Simulated Annealing method. Second step involves choosing the actuators from a parts database to find the minimal cost robot that can lift a target load within every point in the target prism. Successful cost guided design optimization examples of a planar 2R and a spherical wrist 6R robot are presented. Instead of designing manipulators so that it performs well everywhere in the reachable workspace, we consider only a limited workspace volume so that less expensive parts can be used to satisfy the payload constraint.**

I. Introduction

Optimal design of robots are tied closely to industrial robots and have numerous real-world applications. Very interesting optimal design problems have been addressed in the literature, as we briefly survey in Section II. However, to our knowledge, none of the prior works focussed on minimizing the cost of the manipulators while satisfying certain constraints. Most industrial robotic manipulators are designed to be precise, fast and powerful. Although such designs allow a manipulator to carry out a wide array of different tasks, the requirements for speed, precision and power lead to expensive hardware components. Recently, a relatively cheaper robot called Baxter is released. This robot has a two 7 DOF arm robot, is safe to work in human environments, is programmed by demonstration and has a $25,000 price tag. We believe that a significant reduction in the robotic manipulator prices would boost manufacturing industry and pave the way for manipulators break into the consumer market. It is often the case that a company, especially a Small or Medium size Business, underutilizes an expensive manipulator by using it for tasks that does not require high power or accuracy. Such tasks include assembly, material handling, painting and pick and place operations. Robotic manipulators are often used for repetitive tasks where the working workspace is fixed. It is therefore desirable to design robots according to task and workspace requirements so that the cost can be reduced.

We present an optimization process that aims to design the minimum-cost serial robot manipulator that satisfies 2 goal constraints:

1) Reachable Workspace: A rectangular bounding box in the reachable workspace must satisfy a minimum area/volume.
2) Payload: Robot must be able to carry a minimum mass at every point in that rectangular bounding box.

The optimization is performed in two steps on an initial robot design:

1) Geometric Optimization: Minimize robot link lengths while satisfying Workspace constraint.
2) Parts Optimization: With fixed robot geometry, motors and gearboxes are chosen so that it satisfies Payload constraint.

A. Cosgun is a Ph.D. student at Georgia Tech, Atlanta, GA.

H. Lipkin and H. Christensen are faculty with Institute for Robotics and Intelligent Machines (IRIM) at Georgia Tech, Atlanta, GA.

II. Related Works

Optimization of robot parameters can be done using different criteria, including but not limited to kinematic [1] and dynamic [2] design of parallel mechanisms, Inverse Kinematics learning [3], practical stiffness identification of links [4], system stiffness and dexterity [5] and robot skin placement [6]. Often times, the design process requires determination of the workspace. Analytical methods [7] for workspace analysis as well as sampling based methods [8] are existing in the literature. Guo [9] and Gao [10] have focussed on analytical optimization and classification of workspace of parallel manipulators.

Task-Oriented Design paradigm is used for mechanism design under geometric constraints in efforts such as [11], [12]. Chang [12] aims to guarantee completion of at least a set of tasks with high priority. In this work, in order to develop a robot manipulator for a wheelchair, first the set of desired tasks are enumerated. The required execution times and payload for each task is specified. To determine the DH parameters, a kinematic design algorithm called Grid Method is used. Given the kinematic design, maximum torque values are determined for an intended application. In [13] and [14], Kazi and Merk describe a software to simulate and optimize robot designs. Given initial robot parameters and design criteria, first the kinematic design is optimized. Afterwards, the performance of the system and dynamic properties are evaluated using dynamic simulation.

There has been some work to model robotic manipulators to either analyze the properties of a system or to guide design. In [15], hysteresis of a compliant arm and backlash in the joints are modeled. In [16], a mathematical model of a SCARA robot, including servomotor dynamics and gearboxes was developed and presented with dynamic simulation.

III. Problem Definition

We are interested in finding low-cost robot design solutions to satisfy certain workspace and payload constraints. The problem is to find the cheapest serial robot that can carry a specific amount of load in a space at least as large as a defined rectangular prism. In that regard, the workspace constraint is a rectangular prism such that every point inside it must be reachable by the robot. The rectangular prism P_{req} is defined by aspect ratios a, b, c and a minimum required volume V_{req}, however its exact position in robot's workspace W is not specified. Therefore in the final design, P_{req} must be encapsulated somwhere in W. The second constraint is that at every point in P_{req}, the robot should be able to lift a specified mass. In order to find the size of the workspace and rectangle prisms inside it, we need a way to analyze workspace.

IV. Design Approach

We divide the design process into two steps: geometric design that finds the Denavit-Hartenberg (DH) parameters and actuator selection that finds the minimum required torques of joint motors. The abstraction to two steps greatly reduces the complexity of computation because robot geometry is fixed after first step. In this section we present the two steps of design procedure in detail.

A. Estimation of Largest Rectangular Prism in Workspace

At every iteration of the geometric design, the workspace of the robot should be analyzed to check if it passes the workspace rectangular prism constraint. Therefore a tool for estimating the workspace shape and volume is required for a given robot design π.

The problem of finding the workspace boundaries of a general robot design analytically is a hard problem and it can be computationally costly. Since there will be a number of iterations, the computational cost of the workspace estimation method is important. A rough estimation of the workspace volume is sufficient for our optimization purposes, therefore we utilize a sampling-based approach. As stated in the introduction section, one of our design constraints is to fit a minimum-size bounding box to the reachable workspace. The method explained below is used to find a rectangular prism with maximum volume in the workspace.

The task space around the robot is discretized by a 3D grid G for estimation of W. For the sake of generality, the Cartesian space can be discretized uniformly but with different proportions a_x, a_y, a_z along Cartesian axes. There are N_0 number of grid cells for every dimension, yielding step sizes of a_x/N_0, a_y/N_0 and a_z/N_0 along x, y and z axes. For a given workspace grid G, we are looking to find the largest rectangular prism with given aspect ratios a, b, c. Since the grid is sampled using different proportions a_x, a_y, a_z, the problem is reduced to finding the largest cube filled with 1's in a 3D binary matrix G. We adopt the solution by Pashkevich [17], which describes a computationally efficient dynamic programming method to find the largest cubic submatrix of 1's in a 3D binary matrix.

B. Geometric Optimization

The goal of the geometric design is to get alter DH parameters of a given robot so that the cost of the robot links are minimized while satisfying the workspace prism constraint. We assume that we are given a "seed" robot, meaning that a robot to start optimization on. Therefore the degrees of freedom as well as initial DH parameters of the robot are given. For a given robot design, we can estimate the largest rectangular prism that can fit into the workspace as described in Section IV-A. We use the largest rectangular prism finder as a black box function $f_{largest}(\pi) = P_{max} \subseteq W(\pi)$.

We assume that the cost of a robot link i is proportional to its length:

$LinkCost(l_i) \propto (|a_i| + |d_i|)$

Therefore we minimize the cost of all robot link across all robot designs π:

$\pi = \arg\min \, (LinkCosts(\pi))$

where $LinkCosts(\pi) = \sum_{i=1}^{DOF} (|a_i| + |d_i|)$

Satisfying the rectangular prism constraint:

$P_{req} \subseteq f_{largest}(\pi)$

In order to have the solution to optimization problem stated above, we used a variant of Simulated Annealing local search method [18] with random-restarts.

At every iteration, the algorithm chooses a random move, which is defined by a small change in one of the design variables. For this new incremental design π_{new}, the maximum rectangular prism with given aspect ratios is calculated. If this prism is smaller than P_{req}, the algorithm halts. If it is larger than P_{req}, then we look at the change in the volume of the prism ΔV. If $\Delta V \leq 0$, we accept the move. If $\Delta V > 0$ we accept the move with probability $e^{\Delta Et}$. We restart the simulated annealing optimization process a number of times because of the highly random nature of the algorithm in hopes to reach the global minimum.

The optimization variables were link lengths a_i and offsets d_i. We did not add twist angles α_i to design variables so that the designer is able to provide a template of the robot design to the optimizer.

C. Actuator Selection

The first part of the design procedure fixes the DH parameters, which yield a specified largest workspace prism P where $P_{req} \subseteq P$. We now choose the actuators so that the payload requirement is satisfied in every point within P. Joint torques due to a load in the end effector can be found using the equation:

$$Q = J^T W.$$

The payload requirement is a stationary mass m_{req}. Then the selected actuators must satisfy the following inequality:

$$Q_p > {}_0J^T(q)_0W, \, \forall p \text{ in } P$$

where ${}_0W = [0 \; 0 \; -m_{req}g \; 0 \; 0 \; 0]^T$ is the wrench due to the load.

There are different actuation schemes for robot manipulators to provide the necessary torques. One of the most common schemes is using a DC motor and a gearbox pair. Our approach is based on having DC motors and gearboxes placed in the joints.

We created a database of 200 DC motors and 200 gearboxes from the product list of Maxon [19]. An entry for a DC motor consist of product ID, Nominal torque (maximum continuous torque) and price. An entry for a gearbox consist of product ID, gear ratio, Nominal torque (maximum continuous torque) and price. We do not consider the weight of the parts in our analysis and assume that any motor can be paired with any gear. For a pair of motor M_i and gear G_j, the maximum deliverable torque is calculated by:

$$\tau_{ij} = min(GR_j.\tau_{M_i}, \tau_{G_j}).$$

The cost of the pair is the sum of the cost of parts. In

The constraint for the actuator at joint k is that the selected pair τ_{ij} has to exceed the required torque due to payload constraint: $\tau_{ij} > Q_P[k]$. After minimum torque requirements are found, we search through every list of catalogue to find the pair with minimum cost that satisfy the constraint above.

The design of the robot is therefore finalized by the selection of actuators after the geometric design.

V. Results

We implemented our price-based design optimization method using the Robotics Toolbox [20] in MATLAB. We present resulting optimal designs for a 2R planar and 6R spherical wrist robot.

A. 2R Planar Robot

- Problem: Find a 2R planar robot with minimal cost that can carry a mass m in a rectangle shape of at least $0.5m^2$ with an aspect ratio of 2:1

The initial and optimized DH parameters of the 2R planar robot is given in Figure 1. Analytical IK has 2 solutions in the general case and we always pick the elbow up configuration. The algorithm is run with discretizion precision $N_0 = 60$, number of optimizer restarts is 40.

The optimized lengths came out to be very close. Theoretically, the maximum interior rectangle of any aspect ratio is found when $a_1 = a_2$ so that the workspace has a filled circle shape. However, since the optimization algorithm relies on random actions (i.e. sampling of incremental changes of link lengths), some error margin is expected.

i	a_i	d_i	α_i	θ_i
1	0.5^+	0	0	θ_1^*
2	0.5^+	0	0	θ_2^*

i	a_i	d_i	α_i	θ_i
1	0.3574^+	0	0	θ_1^*
2	0.3629^+	0	0	θ_2^*

Fig. 1. Initial (left) and optimal (right) DH parameters of the 2R planar robot.

After geometric optimization, the price of the motors and gearboxes is found for a payload constraint m. In Figure 3, we plot the minimum payload vs minimal robot cost that satisfies the constraint.

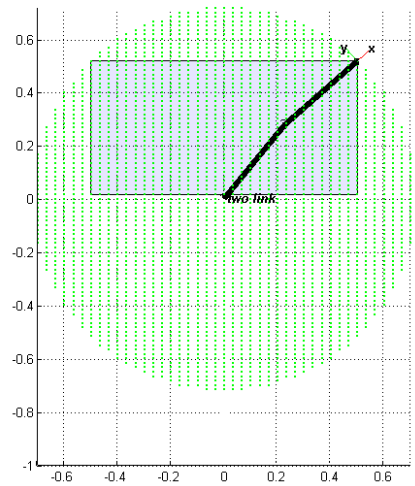

Fig. 2. Illustration of the optimized 2R planar robot. The green dots indicate reachable workspace. The workspace constraint $0.5m^2$ rectangle with an aspect ratio of 2:1 is illustrated.

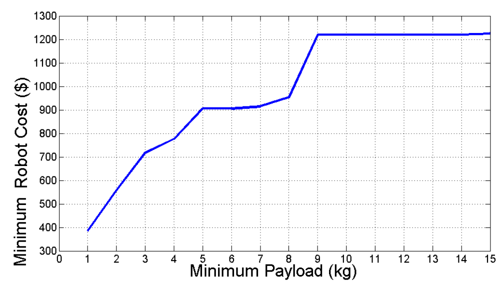

Fig. 3. Minimum Payload vs. Minimum robot cost plotted for 2R robot. Note that the price increases with payload up to 10kg, then stays constant.

B. 6R Arm With Spherical Wrist

- Problem: The problem is to find the 6R spherical wrist robot with a spherical wrist with minimal cost that can carry a mass m in a rectangular prism of at least $V_{req} = 0.03m^3$ with an aspect ratio of 1:1:1, which is a cube with edge length of about 31cm.

The initial design is taken to be the Puma 560 robot. Since there is no publicly available software package that computes the IK of a general 6R robot, we are demonstrating a spher 6R spherical wrist robot because it has analytical IK solution. The algorithm is run with discretizion precision $N_0 = 15$, number of optimizer restart option is disabled due to the long execution time. The solution is found in 23 iterations. Initial and optimal DH parameters of the 6R robot are given in Figure 4. Figure 5 illustrates resulting optimal robot and the rectangular prism workspace constraint.

For the payload analysis, if we computed J on the 6th joint, there would be no torques generated for the last 3 joints since the last three axes intersect. We therefore introduce a tool offset of 0.1m in $+y_0$ direction in world coordinates, so that torques are generated when there is a tool offset at the end effector. Moreover, joints torques for joints 0 and 6 were always zero because joint 0 provides motion around z_0

i	a_i	d_i	α_i	θ_i
1	0^+	0^+	$\pi/2$	θ_1^*
2	0.4318^+	0^+	0	θ_2^*
3	0.0203^+	0.15^+	$-\pi/2$	θ_3^*
4	0	0.4318^+	$\pi/2$	θ_4^*
5	0	0	$-\pi/2$	θ_5^*
6	0	0	0	θ_6^*

i	a_i	d_i	α_i	θ_i
1	-0.0411^+	0.0256^+	$\pi/2$	θ_1^*
2	0.27^+	0.0151^+	0	θ_2^*
3	0.005^+	0.0415^+	$-\pi/2$	θ_3^*
4	0	0.2481^+	$\pi/2$	θ_4^*
5	0	0	$-\pi/2$	θ_5^*
6	0	0	0	θ_6^*

Fig. 4. Initial (left) and optimal (right) DH parameters of the 6R spherical wrist robot.

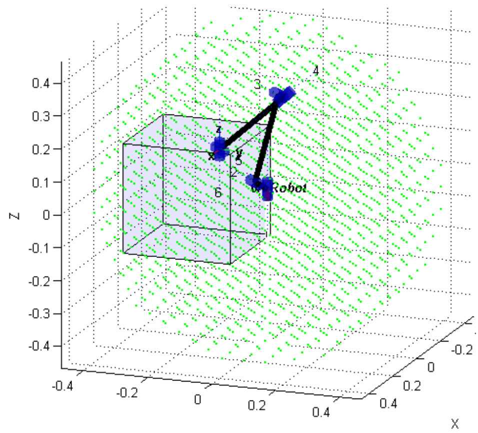

Fig. 5. Illustration of the optimized 6R spherical wrist robot. The green dots indicate reachable workspace. The workspace constraint $0.03m^3$ rectangular prism with an aspect ratio of 1:1:1 is illustrated.

direction and in a static case, there are no torques required. The same is true for joint 6 as the load in end effector does not create any torque around z_6. Under these conditions, the minimum price for motor-gearbox pairs are found. Figure 5 plots payload vs price. According to our results, the motors and gearboxes of a 6R robot with a minimum payload of 5kg costs about 2500 dollars. This is a very optimistic estimation on cost because of the fact that we neglect the robot weight as well as the actuator weight.

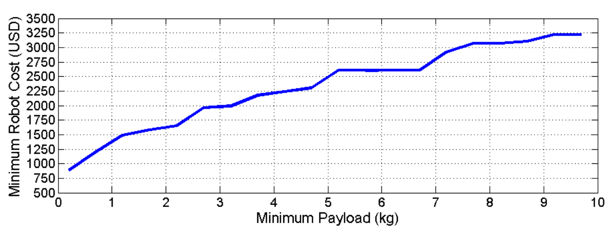

Fig. 6. Minimum Payload vs. Minimum parts cost plotted for a 6R robot with spherical wrist. The cost increases linearly with payload.

VI. Conclusion

We presented a price-based computational design procedure for serial robot manipulators under payload and workspace constraints. First, the DH parameters are determined using Simulated Annealing so that the reachable workspace includes a target rectangular prism. Then the minimum price motor-gearbox pair that satisfies the payload constraint is found using exhaustive search. We successfully demonstrate our approach on 2R and 6R serial robots. We believe that low-cost robot manipulators would be important economic enablers for the manufacturing industry in the future.

References

[1] X.-J. Liu and J. Wang, "A new methodology for optimal kinematic design of parallel mechanisms," *Mechanism and Machine Theory*, vol. 42, no. 9, pp. 1210–1224, 2007.
[2] C. Germain, S. Caro, S. Briot, P. Wenger, *et al.*, "Optimal design of the irsbot-2 based on an optimized test trajectory," in *Proceedings of the ASME 2013 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference IDETC/CIE 2013*, 2013.
[3] E. Oyama, N. Y. Chong, A. Agah, and T. Maeda, "Inverse kinematics learning by modular architecture neural networks with performance prediction networks," in *Robotics and Automation, 2001. Proceedings 2001 ICRA. IEEE International Conference on*, vol. 1. IEEE, 2001, pp. 1006–1012.
[4] A. Klimchik, S. Caro, A. Pashkevich, *et al.*, "Practical identifiability of the manipulator link stiffness parameters," in *ASME 2013 International Mechanical Engineering Congress & Exposition*, 2013, pp. 1–10.
[5] Z. Gao, D. Zhang, and Y. Ge, "Design optimization of a spatial six degree-of-freedom parallel manipulator based on artificial intelligence approaches," *Robotics and Computer-Integrated Manufacturing*, vol. 26, no. 2, pp. 180–189, 2010.
[6] D. Anghinolfi, G. Cannata, F. Mastrogiovanni, C. Nattero, and M. Paolucci, "On the problem of the automated design of large-scale robot skin," 2013.
[7] K. Goyal and D. Sethi, "An analytical method to find workspace of a robotic manipulator," *Journal of Mechanical Engineering*, vol. 41, no. 1, pp. 25–30, 2010.
[8] F. Zacharias, C. Borst, and G. Hirzinger, "Capturing robot workspace structure: representing robot capabilities," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, 2007, pp. 3229–3236.
[9] W. Guo and F. Gao, "Classification and solution space atlases for the design of planar serial manipulators," in *ASME 2008 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*. American Society of Mechanical Engineers, 2008, pp. 1231–1239.
[10] F. Gao, B. Peng, W. Li, and H. Zhao, "Design of a novel 5-dof parallel kinematic machine tool based on workspace," *Robotica*, vol. 23, no. 1, pp. 35–43, 2005.
[11] J. Wu, Q. Ge, H.-J. Su, and F. Gao, "Kinematic acquisition of geometric constraints for task centered mechanism design," in *ASME 2010 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*. American Society of Mechanical Engineers, 2010, pp. 801–810.
[12] P. Chang and H. Park, "Development of a robotic arm for handicapped people: a task-oriented design approach," *Autonomous robots*, vol. 15, no. 1, pp. 81–92, 2003.
[13] A. Kazi, G. Merk, M. Otter, and H. Fan, "Design optimisation of industrial robots using the modelica multi-physics modeling language," in *Proceedings of the 33rd ISR (International Symposium on Robotics) October*, vol. 7, 2002, p. 11.
[14] A. Kazi, G. Merk, H. Fan, M. Langelaar, and M. Otter, "Experience with realsim for robot applications," Technical Report, KUKA Roboter GmbH, Augsburg, Tech. Rep., 2002.
[15] A. Kumpf, "Explorations in low-cost compliant robotics," Master's thesis, Massachusetts Institute of Technology, 2007.
[16] M. Alshamasin, F. Ionescu, and R. Al-Kasasbeh, "Modelling and simulation of a scara robot using solid dynamics and verification by matlab/simulink," *International Journal of Modelling, Identification and Control*, vol. 15, no. 1, pp. 28–38, 2012.
[17] A. Pashkevich, D. Chablat, and P. Wenger, "Design optimization of parallel manipulators for high-speed precision machining applications," in *Information Control Problems in Manufacturing*, vol. 13, no. 1, 2009, pp. 139–144.
[18] P. van Laarhoven and E. Aarts, *Simulated annealing: theory and applications*. Springer, 1987, vol. 37.
[19] [Online]. Available: http://www.maxonmotorusa.com/maxon/view/catalog/
[20] [Online]. Available: http://petercorke.com/RoboticsToolbox.html