acossairt / **traffic_air_quality_modeling** Public

Dynamical systems modeling of transportation systems and their impacts on urban air pollution. Case of interest: New Delhi, India.

0 stars 0 forks Branches Tags Activity

Star Notifications

Code Issues Pull requests Actions Projects Security Insights

1 Branch 0 Tags Go to file Go to file Code

acossairt separate travel 4 into demo and full 46eb3f1 · last week

.ipynb_checkpoints	julia implementation of trav...	2 months ago
images	move pictures to images fol...	2 weeks ago
julia_model	separate travel 4 into demo...	last week
xpp.app/Contents	copied all the contents of x...	7 months ago
xppaut_model	reorganized folders	last month
.DS_Store	separate travel 4 into demo...	last week
.gitignore	removed (most) unnecessar...	7 months ago
README.md	Update README.md	2 weeks ago
changes.txt	copied all the contents of x...	7 months ago
xppaut	copied all the contents of x...	7 months ago
xppaut.1	copied all the contents of x...	7 months ago
xppaut_version7	copied all the contents of x...	7 months ago

README

traffic_air_quality_modeling

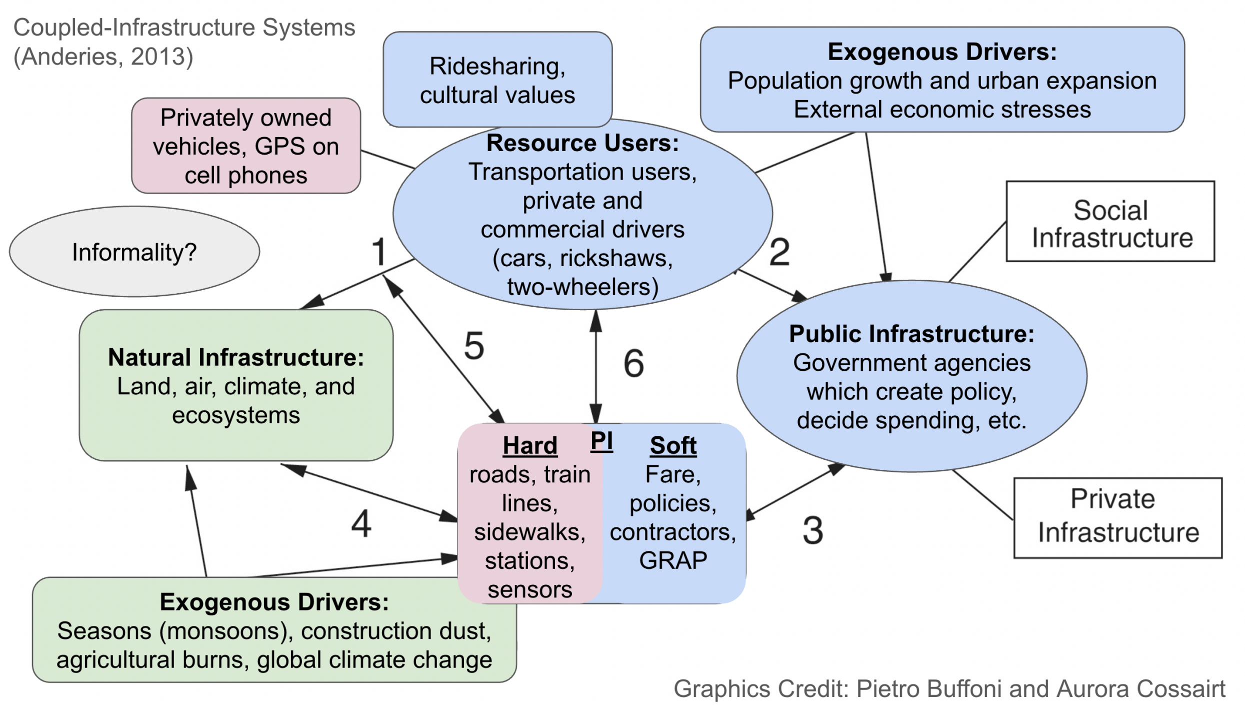
Dynamical systems modeling of transportation systems and their impacts on urban air pollution. Case of interest: New Delhi, India.

The latest version of the model (as of November 13, 2024) is `travel_4_1021.ode` in XPPAUT and `travel_4.ipynb` in Julia. It involves two patches connected by two (bidirectional) corridors.

Introduction

The transportation system in the National Capital Territory (NCT) of Delhi, India is a

The transportation system in the National Capital Territory (NCT) of Delhi, India is a complex network of vehicles, transit lines, and users of multiple types. Rising air pollution associated with this transportation system is a major public health challenge, and recent studies have estimated that poor air quality in Delhi reduces residents' life expectancy by about 6 years \citep{ghude2016premature}. This system is bounded by the municipal boundaries of NCT, but the boundaries are permeable to the movement of air, vehicles, and people across political boundaries. We conceptualize the transportation system as consisting of *vehicles* (e.g. cars, trains, motorcycles, auto-rickshaws, pedestrians) which travel between *patches* (e.g. residential areas, business districts) along *corridors* (e.g. highways, train lines) and emit pollutants at a certain rate depending on their travel speed and congestion levels. To conceptualize the wider social-ecological context, we can use the Coupled-Infrastructure Systems (CIS) framework shown in \autoref{fig:CIS_framework} \citep[][]{anderies2015understanding}.



Research Questions

Our objective is to build a general model of urban mobility systems, with NCT New Delhi as a prototyping case, to gain insight into the relationship between mobility infrastructure configurations, governance policies, and air quality. Specific research questions include:

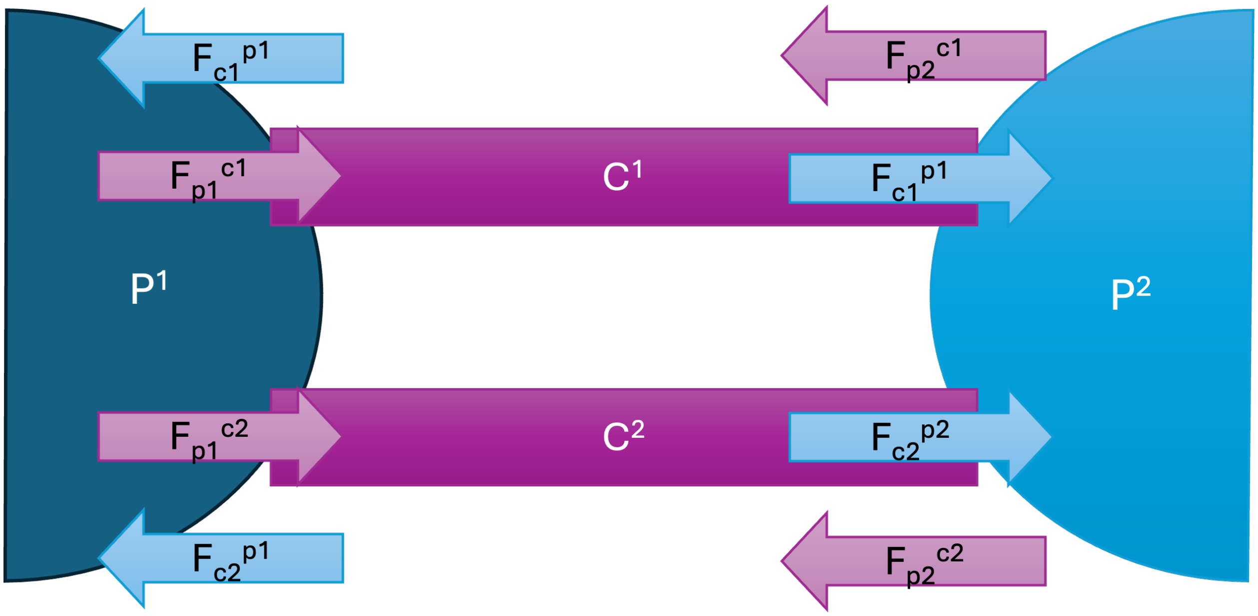
- How does the overall pollution level compare for a small mobility infrastructure system (few patches, few corridors) vs a large mobility infrastructure system (many patches, many corridors)?
- What is the impact on air pollution of increasing capacity of existing roads (decrease the parameter β^k, defined below)?
- What is the impact on air pollution of increasing road connectivity by adding new

corridors while keeping the same number of patches (increase parameter K, defined below)?

- (Future work) How does information flow along links in the CIS to inform infrastructure investment decisions by PIP?

Model Background

The concept for this model is shown in \autoref{fig:conceptual_diagram} for a case with two patches and two connective corridors.



In general, suppose there are N patches, up to K corridors connecting each pair, and 1 total vehicles (this is a normalized value; we can think of it as a density). The state variables and parameters are listed below:

State Variables

- P^i: population in patch i (units: vehicles)
- $C^k[i, j]$: population in corridor k, heading from patch i to patch j (units: vehicles)

These state variables are countable populations which could in theory be measured at a point in time.

Parameters

- $\beta^k[i, j]$: Road congestion factor (inverse road capacity) for corridor k. Analogous to ``resistance'' in an electrical wire. Think of congestion as the ratio of vehicles on the road to the jam capacity of that road: $C^k[i, j] \ / \ C^k_{jam}[i, j]$. Therefore, $\beta^k[i, j] = 1 \ / \ C^k_{jam}[i, j]$, and $\beta^k[i, j] = 0$ would imply infinite road capacity. (units: 1/ vehicles)

- α^i: aversion to congestion for travelers from patch i. Given $\beta_k C^k_{i,j}$, $\uparrow \alpha \Rightarrow \downarrow$ new travelers. (unitless)
- $O^P[i]$: Percentage of population in patch i who want to travel. (unitless) \end{itemize}

These parameters describe infrastructure capacity (such as the width of a highway) and social behavior of travelers. These quantities are likely to change on a time scale of months to years, whereas our model is focused on traffic flows on a time scale of minutes and hours, so we can treat the above quantities as parameters.

Conservation law

Here the total number of vehicles is conserved.

Relationships between variables

Because of the conservation law, the change in number of vehicles in any node (a patch, corridor, urban sponge, etc.) is determined by *flow-in* minus *flow-out*. The speed of fluxes into and out of corridors is density dependent.

- $F^{\to k}[i,j]$: flux from patch i into corridor k, with an ultimate destination of patch j.
- $F^{k \to}[i,j]$: flux from corridor k into patch j, where the origin was i.

Formal Model

For a general system of N patches, and up to K connecting corridors for each pair of patches, we will use the following notation:

The number of vehicles in each patch P^i is given by $P[i]$.

$$P \quad = \begin{pmatrix} 1 \\ 2 \\ \dots \\ n \end{pmatrix}$$

Number of vehicles in a corridor C^k heading from patch i to patch j is given by $C^k[i,j]$. For example, suppose maximum of one connecting corridor for each pair of patches, and assume no self-connected patches, the matrix C^k is:

$$C^k \quad = \begin{pmatrix} 0 & 1 \to 2 & 1 \to 3 & \dots & 1 \to n \\ 2 \to 1 & 0 & 2 \to 3 & \dots & 2 \to n \\ \dots & \dots & \dots & \dots & \dots \\ n \to 1 & n \to 2 & n \to 3 & \dots & 0 \end{pmatrix}$$

If $K > 1$, we may have multiple matrices $C^k[i, j]$ (one for each k), or the C matrix may be three dimensional with indices $[i, j, k]$.

For fluxes from patches into corridors, we also need a matrix with three dimensions i, j, and k. $F^{\to k}[i, j]$ means flux from patch i into corridor k, with an ultimate destination of patch j.

$$F^{\to k}[i,j] \quad = \begin{pmatrix} 0 & p1 \to ck(\to p2) & p1 \to ck(\to p3) & \dots & p1 \to ck(\to pn) \\ p2 \to ck(\to p1) & 0 & p2 \to ck(\to p3) & \dots & p2 \to ck(\to pn) \\ \dots & \dots & \dots & \dots & \dots \\ pn \to ck(\to p1) & pn \to ck(\to p2) & pn \to ck(\to p3) & \dots & 0 \end{pmatrix}$$

Meanwhile, $F^{k\to}[i, j]$ means fluxes from corridor k into patch j, where the origin was i.

$$F^{k\to}[i,j] \quad = \begin{pmatrix} 0 & (p1 \to)ck \to p2 & (p1 \to)ck \to p3 & \dots & (p1 \to)ck \to pn \\ (p2 \to)ck \to p1 & 0 & (p2 \to)ck \to p3 & \dots & (p2 \to)ck \to pn \\ \dots & \dots & \dots & \dots & \dots \\ (pn \to)ck \to p1 & (pn \to)ck \to p2 & (pn \to)ck \to p3 & \dots & 0 \end{pmatrix}$$

Governing differential equations

The system of differential equations governing this system can be written:

$$\begin{aligned} \frac{dP^i}{dt} &= \sum_{k=1}^{K} \sum_{j=1}^{N} F^{k\to}[j,i] - \sum_{k=1}^{K} \sum_{j=1}^{N} F^{\to k}[i,j] \\ \frac{dC^k_{ij}}{dt} &= \sum_{k=1}^{K} F^{\to k}[i,j] - \sum_{k=1}^{K} F^{k\to}[i,j] \end{aligned}$$

with the conservation law:

$$1 \quad = \sum_{i=1}^{N} P[i] + \sum_{k=1}^{K} \sum_{i=1}^{N} \sum_{j=1}^{N} C^k[i,j]$$

The equations for fluxes into and out of corridors are as follows.

Fluxes from patches to corridors

$$F^{\to k}[i,j] \quad = \exp(-\alpha[i] * \beta^k[i,j] * C^k[i,j]) * O^P[i] * P[i]$$

This equation states that the number of vehicles which flow from patch i into corridor k with a final destination of patch j depends positively on the current number of vehicles in patch i who want to leave ($O^P[i] * P[i]$), and inversely on the current congestion level (

$\beta^k[i,j]C^k[i,j]$), with the strength of aversion to congestion controlled by the factor $\alpha[i]$.

Fluxes from corridors to patches

$$F^{k\rightarrow}[i,j] \quad = \exp(-\beta^k[i,j] * C^k[i,j]) * C^k[i,j]$$

This equation says that number of vehicles flowing out of corridor k into patch j, with an origin of patch i, is proportional to the number of cars on the road $C^k[i,j]$ and inversely related to the congestion on the road $\beta^k[i,j] * C^k[i,j]$. There is no factor controlling sensitivity to congestion, because we assume anyone already traveling in a corridor will exit (reaching their final destination) at the earliest opportunity.

Predicting emissions from traffic density

Given a traffic density $C^k[i,j]$ on a given corridor which has jam density $C^k_{jam}[i,j] = 1 \ / \ \beta^k[i,j]$, we can compute average speeds from an alternative version of **Greenshield's model** and then predict emissions / vehicle using the \textbf{U-shaped curve}.

Traffic density to average speed

First, to compute average vehicle speeds, we will use an alternative to Greenshield's law. The typical Greenshield's model states:

$$v^k[i,j] = v^k_{free}[i,j] - \left(\frac{v^k_{free}[i,j]}{C^k_{jam}[i,j]}\right) C^k[i,j]$$

where $v^k_{free}[i,j]$ is the free-flow speed and $C^k_{jam}[i,j]$ is the jam density in corridor k connecting patches i and j. This formula codifies a linear relationship between velocity and density, where average speeds go to zero exactly at $C^k[i,j] = C^k_{jam}[i,j]$, and the average speed is the free flow velocity $v^k[i,j]$ only in the edge case where $C^k[i,j] = 0$.

A more realistic version of this model would show nearly free flow speeds for a range of traffic density, followed by a sharp drop in average speeds at a certain congestion threshold $C^k_{thresh}[i,j]$, and then average speeds asymptotically approaching zero. We can represent such a relationship with the following function:

$$v^k[i,j] \quad = \quad -\frac{v^k_{free}[i,j]}{\pi} \arctan a(C^k[i,j] - C^k_{thresh}[i,j]) + \frac{v^k_{free}[i,j]}{2}$$

where a is a parameter that controls the speed at which congestion decreases average speeds, $C^k_{thresh}[i,j]$ is the vehicle density value for which average speeds are $\frac{1}{2} v^k free[i,j]$. If we say $C^k_{thresh}[i,j] = \frac{1}{2} C^k_{jam}[i,j]$, then an equivalent formula is:

$$v^k[i,j] \quad = -\frac{v^k_{free}[i,j]}{\pi} \arctan a(C^k[i,j] - \frac{C^k_{jam}[i,j]}{2}) + \frac{v^k_{free}[i,j]}{2}$$

Therefore, the road congestion parameter can now be written $\beta^k[i,j] = \frac{1}{2C^k_{thresh}[i,j]}$. The shape of the function \autoref{eq:alternative_greenshields} is illustrated in \autoref{fig:greenshield} in purple, compared to the original Greenshield's model \autoref{eq:greenshield} in green.

Average speeds to emission rates

With average vehicle speeds in hand, we can estimate emission rates using the U-shaped curve. This empirical function depends on particular vehicle types, engine and fuel types, and location, and it relates CO2 emission rates (grams / unit distance) with average vehicle speeds (unit distance / time). We assume that very low average speeds (e.g. 5mph) are associated with stop-and-go traffic, and therefore high emission rates. Meanwhile, very high average speeds (e.g. > 85 mph) also lead to high emissions because of the energy required to sustain high speeds, and how quickly a vehicle can travel many miles. An example is shown in \autoref{fig:u-shaped_curve} with data from California \citep{barth2009traffic}. We will eventually procure similar data for NCT Delhi -- for now, we can use the California data as a proxy.

Given an array for the U-shaped curve E_per_v, total emission rates $g \ / \ hr$ at any given time for a mass of vehicles will be emissions per vehicle (g / vehicle) times average vehicle speeds (distance / time) times density of vehicles (vehicles):

$$Emissions = E_{per_v}[v^k[i,j]] * v^k[i,j] * C^k[i,j]$$

Formal model for demo case: two patches, two corridors

For the simple system with two patches and two corridors shown in \autoref{fig:conceptual_diagram}, the system of equations is as follows:

$$\begin{aligned}
\frac{dP^1}{dt} &= F_{c1}^{p1} + F_{c2}^{p1} - F_{p1}^{c1} - F_{p1}^{c2} \\
\frac{dP^2}{dt} &= F_{c1}^{p2} + F_{c2}^{p2} - F_{p2}^{c1} - F_{p2}^{c2} \\
\frac{dC_{12}^1}{dt} &= F_{p1}^{c1} - F_{c1}^{p2} \\
\frac{dC_{21}^1}{dt} &= F_{p2}^{c1} - F_{c1}^{p1} \\
\frac{dC_{12}^2}{dt} &= F_{p1}^{c2} - F_{c2}^{p2} \\
\frac{dC_{21}^2}{dt} &= F_{p2}^{c2} - F_{c2}^{p1}
\end{aligned}$$

Conservation law:

$$1 \quad = P^1 + P^2 + C_{12}^1 + C_{21}^1 + C_{12}^2 + C_{21}^2$$

Fluxes from patches to corridors depend on 1) population of patch i interested in becoming travelers ($O^{Pi} * P^i$), 2) current levels of congestion ($\beta_1 * C_{12}^1$), and 3) how averse to congestion are the potential travelers (α_i).

$$\begin{aligned}
F_{p1}^{c1} &= \exp(-\alpha_1 * \beta_1 * C_{12}^1) * O^{P1} * P^1 \\
F_{p1}^{c2} &= \exp(-\alpha_1 * \beta_2 * C_{12}^2) * O^{P1} * P^1 \\
F_{p2}^{c1} &= \exp(-\alpha_2 * \beta_2 * C_{21}^1) * O^{P2} * P^2 \\
F_{p2}^{c2} &= \exp(-\alpha_2 * \beta_2 * C_{21}^2) * O^{P2} * P^2
\end{aligned}$$

Fluxes from corridors to patches depends on the current population in a patch

$$\begin{aligned}
F_{c1}^1 &= \exp(-\beta_1 * C_{21}^1) * C_{21}^1 \\
F_{c1}^2 &= \exp(-\beta_1 * C_{12}^1) * C_{12}^1 \\
F_{c2}^1 &= \exp(-\beta_1 * C_{21}^2) * C_{21}^2
\end{aligned}$$

$$F_{c2}^2 \quad = \exp(-\beta_1 * C_{12}^2) * C_{12}^2$$

Average speeds are then calculated as follows:

```
  using Interpolations

  # Calculate speeds from densities
  v_f = 90 # free-flow velocity, 90 km/hr
  k_jam_C1 = 1 / β₁
  k_jam_C2 = 1 / β₂
  k_half_C1 = k_jam_C1 / 2
  k_half_C2 = k_jam_C2 / 2

  # Function to calculate average speeds using alternative
Greenshield's model
  function calc_space_mean_speed_alternative(v_f, k, k_half; a=1)
    u_s = - (v_f / pi) * atan(a*(k - k_half)) + (v_f / 2)
    return u_s > 0 ? u_s : 0
  end

  # Calculate average vehicle speeds for C1 and C2
  C1_speeds = calc_space_mean_speed_alternative.(v_f, pop_C1,
k_half_C1)
  C2_speeds = calc_space_mean_speed_alternative.(v_f, pop_C2,
k_half_C2)
```

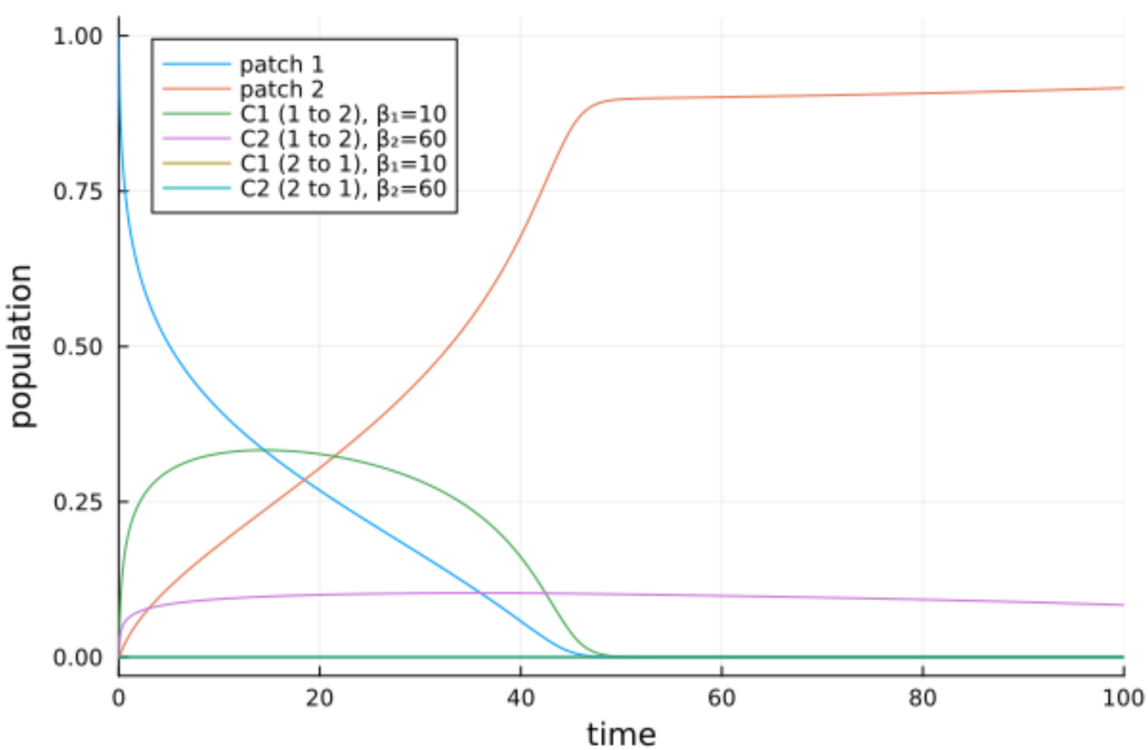
And finally, emission rates are calculated:

```
    # Make a U-shaped curve
    start = 5 # speeds in mph, from California paper
    my_step = 5
    stop = 100
    mph_to_kmh = 1.60934
    speed_arr = collect(start:my_step:stop) * mph_to_kmh # convert mph
to kmh
    emissions_arr = [1200, 950, 700, 500, 425, 350, 325, 310, 309,
308, 308, 308, 309, 320, 330, 350, 375, 400, 450, 550] * mph_to_kmh
    plot(speed_arr, emissions_arr)

    # Define an interpolation function to get emission rate (g /
distance) for particular average speeds
    interp_fn = linear_interpolation(speed_arr, emissions_arr,
extrapolation_bc=Line())

    # Function to calculate emission rate (g / hour) for given average
speeds
    function calc_emissions_from_speed(vehicle_pop_arr, my_speed_arr,
interp_fn)
```

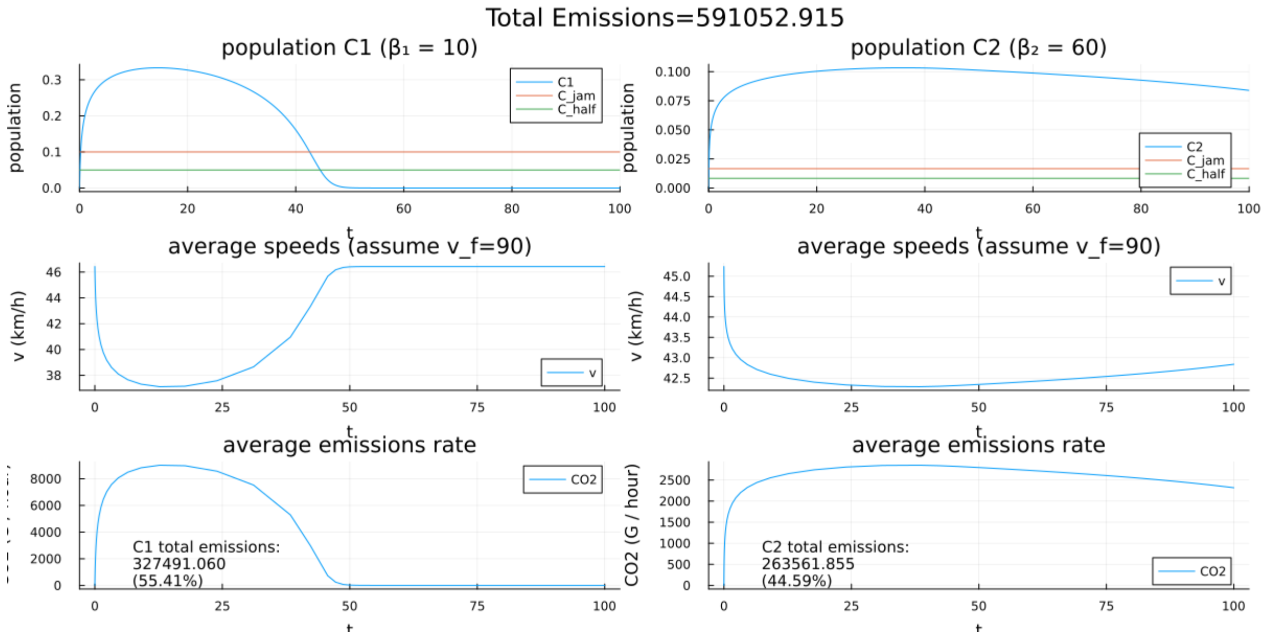
```
        interpolated_emission_per_vehicle = interp_fn(my_speed_arr)
        emissions = interpolated_emission_per_vehicle .* my_speed_arr
.* vehicle_pop_arr
        return emissions
    end

    # Calculate emissions for C1 and C2
    C1_emissions = calc_emissions_from_speed(pop_C1, C1_speeds,
interp_fn)
    C2_emissions = calc_emissions_from_speed(pop_C2, C2_speeds,
interp_fn)
```

Analysis

For the demo case, we find traffic flows and resulting emissions for 100 times steps with parameters $O^p[1] = 1, O^p[2] = 0$, $\alpha_1 = \alpha_2 = 1$, and $\beta^1[1,2] = 10$, $\beta^2[1,2] = 60$. That is, everyone wants to leave patch 1, no one wants to leave patch 2, all travelers have equal tolerance for congestion, and corridor 1 has much higher capacity than corridor 2. We start with all vehicles in patch 1 and no travelers in any of the other patches or corridors. The traffic flows for this case are shown in \autoref{fig:traffic_flows_1}. We see more travelers choosing corridor 1 over corridor 2, and the rush hour (period of high traffic) in corridor 1 is about 45 time steps, compared to corridor 2's rush hour of more than 100 time steps.



The associated emissions over time for each corridor is shown in \autoref{fig:emissions}. We notice the curve for emissions rates for each corridor is almost identical to its associated population curve, with a bit of discretization. The final step (not shown) to calculate total emissions for a given part of the day is the integrate the area under the emissions curve for time a to time b.



Discussion/Conclusion

This exercise showed me the importance of dimensional analysis to developing a realistic model. At multiple phases of this project, I found myself questioning the realism of the model, or struggling to connect my first principles model with other models (such as Greenshield's model). The solution always came from a careful assessment of how to interpret each variable and parameter in terms of its units.

At the moment, I am not satisfied with this model for several reasons:

- The assumption that emissions depend on average speeds according to a simple U-shaped function rests on the logic that lower average speeds indicate stop-and-go traffic conditions. This is unsatisfying because emissions really depend on how often drivers hit the accelerator. Two trips might have the same average speed but very different emission rates depending on driving patterns.
- Additionally, the current analysis pipeline of vehicle population → average speeds → emissions implies a linear relationship between population density and emission rates at each time. This seems overly simplistic, and it doesn't suggest any interesting properties about how travel infrastructure relates to emissions.

Future versions of the model should also consider more sophisticated social-ecological interactions. For instance:

- Increasing road capacity (e.g. by widening highways) can induce additional demand (higher O^P). Our model currently does not consider non-static demand.

- Corridors that represent highways, smaller streets, and railways should likely be governed by different dynamical equations (e.g. trains depart in discrete increments, there is no continuous flow in and out).
- We should model heterogeneous traffic flows, such as highways used by cars, trucks, motorcycles, auto-rickshaws, cyclists, and pedestrians who may be using the same corridor simultaneously.

Eventually, we hope to flesh out the links in the CIS model that connect this formal model of hard urban infrastructure to policy decisions. It would be particularly useful to identify how information about road conditions and emissions flow between nodes and influence the infrastructure investment decisions of PIPs.

Releases

No releases published

Packages

No packages published

Contributors 2


acossairt


janderie J. Marty Anderies

Languages

- **Jupyter Notebook** 99.6%
- **Other** 0.4%