MULTIPLE RESPONSE REGRESSION MODELS

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INTRODUCTION

- Linear regression with one response (LR)
- Linear regression with two or more responses (MRLR)

INTRODUCTION

- Linear regression with one response (LR)

```
m1 <- lm(y1 ~ x1+x2+x3, data = d1)
```

- Linear regression with two or more responses (MRLR)

```
m12 <- lm(c(y1,y2) ~ x1+x2+x3, data = d1)
```

INTRODUCTION

- Differences LR vs MRLR
- When should be used?
- How to build a MRLR?
- Selecting predictors
- Making predictions

INTRODUCTION

Examples

- 4 responses, 2 predictors
- 2 responses, 5 predictors

INTRODUCTION

- MRLR is a linear regression model with two
  or more responses
- Responses are correlated
  share same predictors

EXAMPLE 1 - mtcars

Dataframe of 11 car attributes

Miles/(US) gallon	`mpg`
Number of cylinders	`cyl`
Displacement (cu.in.)	`disp`
Gross horsepower	`hp`
Rear axle ratio	`drat`
Weight (1000 lbs)	`wt`
1/4 mile time	`qsec`
Engine (0 = V-shaped, 1 = straight)	`vs`
Transmission (0 = automatic, 1 = manual)	`am`
Number of forward gears	`gear`
Number of carburetors	`carb`

EXAMPLE 1 - mtcars

mpg	disp	hp	wt	cyl	am	carb	qsec	vs	gear	drat
21	160	110	2.62	6	1	4	16.46	0	4	3.9
21	160	110	2.875	6	1	4	17.02	0	4	3.9
22.8	108	93	2.32	4	1	1	18.61	1	4	3.85
21.4	258	110	3.215	6	0	1	19.44	1	3	3.08
18.7	360	175	3.44	8	0	2	17.02	0	3	3.15
18.1	225	105	3.46	6	0	1	20.22	1	3	2.76
14.3	360	245	3.57	8	0	4	15.84	0	3	3.21
24.4	146.7	62	3.19	4	0	2	20	1	4	3.69
22.8	140.8	95	3.15	4	0	2	22.9	1	4	3.92
19.2	167.6	123	3.44	6	0	4	18.3	1	4	3.92
17.8	167.6	123	3.44	6	0	4	18.9	1	4	3.92

EXAMPLE 1 - mtcars

mpg	disp	hp	wt	cyl	am	carb
21	160	110	2.62	6	1	4
21	160	110	2.875	6	1	4
22.8	108	93	2.32	4	1	1
21.4	258	110	3.215	6	0	1
18.7	360	175	3.44	8	0	2
18.1	225	105	3.46	6	0	1
14.3	360	245	3.57	8	0	4
24.4	146.7	62	3.19	4	0	2
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24.4	146.7	62	3.19	4	0	2
22.8	140.8	95	3.15	4	0	2
19.2	167.6	123	3.44	6	0	4
17.8	167.6	123	3.44	6	0	4

cyl as a factor
(categorical variable)

lm(mpg ~ cyl + am + carb, mtcars)

mpg	disp	hp	wt	cyl	am	carb
21	160	110	2.62	6	1	4
21	160	110	2.875	6	1	4
22.8	108	93	2.32	4	1	1
21.4	258	110	3.215	6	0	1
18.7	360	175	3.44	8	0	2
18.1	225	105	3.46	6	0	1
14.3	360	245	3.57	8	0	4
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lm(disp ~ cyl + am + carb, mtcars)

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mpg	disp	hp	wt	cyl	am	carb
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19.2	167.6	123	3.44	6	0	4
17.8	167.6	123	3.44	6	0	4

EXAMPLE 1 – mtcars model with 4 responses and 3 predictors

mpg	disp	hp	wt	cyl	am	carb
21	160	110	2.62	6	1	4
21	160	110	2.875	6	1	4
22.8	108	93	2.32	4	1	1
21.4	258	110	3.215	6	0	1
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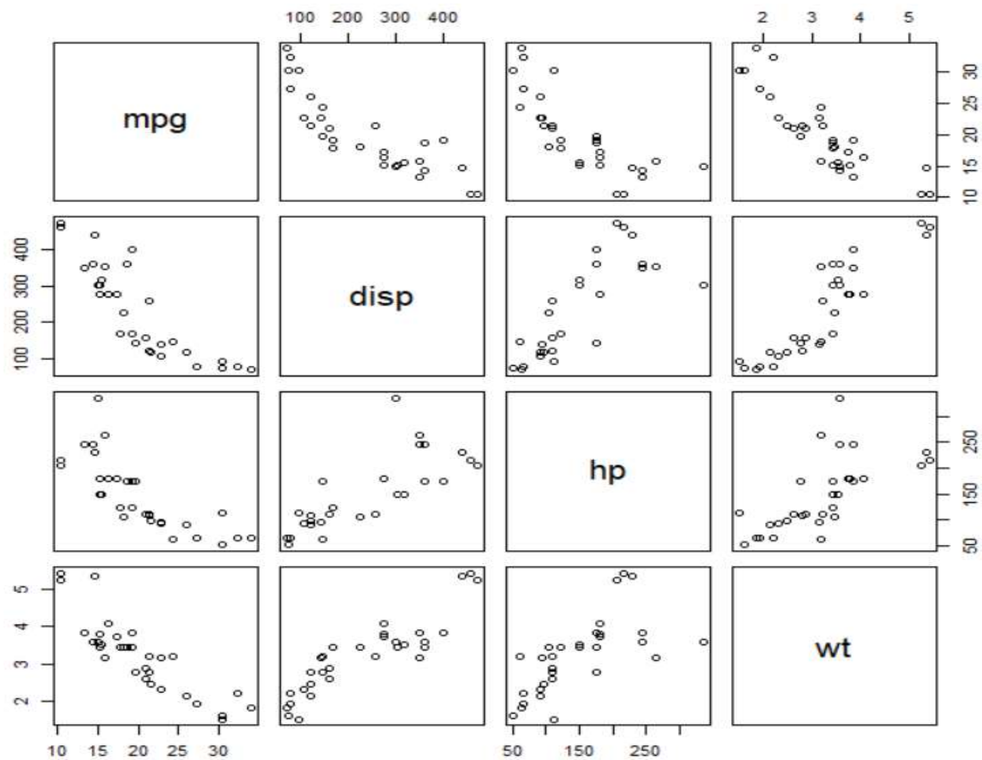
19.2	167.6	123	3.44	6	0	4
17.8	167.6	123	3.44	6	0	4

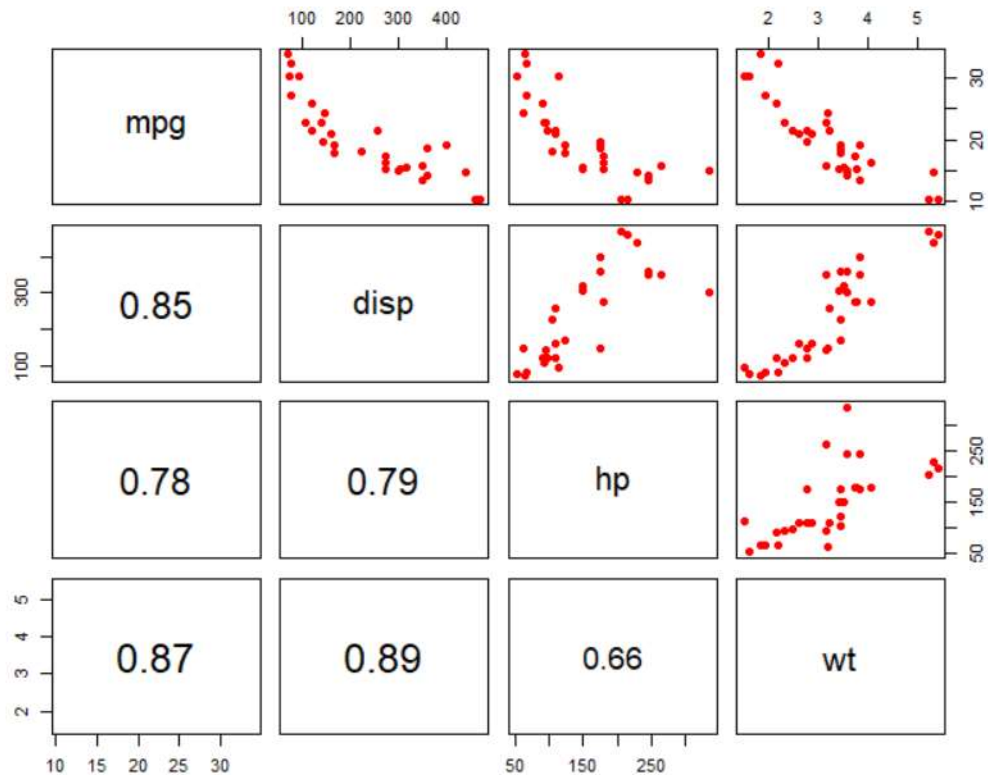
EXAMPLE 1 – mtcars responses are correlated

```
d1 = mtcars[,c("mpg","disp","hp","wt")]
pairs(d1)
```



EXAMPLE 1 – mtcars responses are correlated

```
pairs(d1,lower.panel = panel.cor,
        pch=19,col="red")
```



MRLR vs MLR

If the responses are correlated,

MRLR model will result in more accurate predictions

EXAMPLE 1 – mtcars building the model

```
y  <-  mtcars[,c("mpg","disp","hp","wt")]
y  <-  as.matrix(y)
mv1 <- lm(y ~ cyl + am + carb, mtcars)
```

mpg	disp	hp	wt
21	160	110	2.62
21	160	110	2.875
22.8	108	93	2.32
21.4	258	110	3.215
18.7	360	175	3.44
18.1	225	105	3.46
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EXAMPLE 1 – mtcars building the model

```
y  <-  mtcars[,c("mpg","disp","hp","wt")]
y  <-  as.matrix(y)
mv1 <- lm(y ~ cyl + am + carb, mtcars)

summary(mv1)
```

mpg	disp	hp	wt
21	160	110	2.62
21	160	110	2.875
22.8	108	93	2.32
21.4	258	110	3.215
18.7	360	175	3.44
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EXAMPLE 1 - mtcars

```
Response mpg

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  25.3203     1.2238  20.690  < 2e-16 ***
cyl6         -3.5494     1.7296  -2.052 0.049959 *
cyl8         -6.9046     1.8078  -3.819 0.000712 ***
am            4.2268     1.3499   3.131 0.004156 **
carb         -1.1199     0.4354  -2.572 0.015923 *

Residual standard error: 2.805 on 27 degrees of freedom
Multiple R-squared:  0.8113,    Adjusted R-squared:  0.7834
F-statistic: 29.03 on 4 and 27 DF,  p-value: 1.991e-09
```

EXAMPLE 1 - mtcars

```
Response disp

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  134.325     21.836   6.152 1.42e-06 ***
cyl6          61.843     30.860   2.004   0.0552 .  
cyl8         218.991     32.256   6.789 2.72e-07 ***
am           -43.803     24.086  -1.819   0.0801 .  
carb           1.726      7.768   0.222   0.8258    

Residual standard error: 50.05 on 27 degrees of freedom
Multiple R-squared:  0.858,	Adjusted R-squared:  0.8369 
F-statistic: 40.78 on 4 and 27 DF,  p-value: 4.537e-11
```

EXAMPLE 1 - mtcars

```
Response hp

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  46.5201    10.4825   4.438 0.000138 ***
cyl6          0.9116    14.8146   0.062 0.951386
cyl8         87.5911    15.4851   5.656 5.25e-06 ***
am            4.4473    11.5629   0.385 0.703536
carb         21.2765     3.7291   5.706 4.61e-06 ***

Residual standard error: 24.03 on 27 degrees of freedom
Multiple R-squared:  0.893,	Adjusted R-squared:  0.8772
F-statistic: 56.36 on 4 and 27 DF,  p-value: 1.023e-12
```

EXAMPLE 1 - mtcars

```
coef(mv1)

                  mpg      disp         hp         wt
(Intercept) 25.320303 134.32487 46.5201421  2.7612069
cyl6        -3.549419  61.84324  0.9116288  0.1957229
cyl8        -6.904637 218.99063 87.5910956  0.7723077
am           4.226774 -43.80256  4.4472569 -1.0254749
carb        -1.119855   1.72629 21.2764930  0.1749132


m1 <- lm(mpg ~ cyl + am + carb, data = mtcars)

coef(m1)
(Intercept)        cyl6        cyl8          am        carb
  25.320303   -3.549419   -6.904637    4.226774   -1.119855
```

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m1 <- lm(mpg ~ cyl + am + carb, data = mtcars)

coef(m1)
(Intercept)        cyl6        cyl8          am        carb
  25.320303   -3.549419   -6.904637    4.226774   -1.119855
```

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am           4.226774 -43.80256  4.4472569 -1.0254749
carb        -1.119855   1.72629 21.2764930  0.1749132


m2 <- lm(disp ~ cyl + am + carb, data = mtcars)

coef(m2)
(Intercept)        cyl6        cyl8          am        carb
  134.32487    61.84324   218.99063   -43.80256     1.72629
```

EXAMPLE 1 – prediction

Predict attributes (mpg, disp, hp, wt) of a car with

- 6 cylinders
- 4 carburetors
- automatic transmission

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- 4 carburetors
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```
newdata <- data.frame(cyl=factor(6,levels=c(4,6,8)),am=1,carb=4)
```

EXAMPLE 1 – confidence and prediction intervals

```
# confidence interval
predict(mv1, newdata, interval="confidence")
#         mpg     disp      hp       wt
# 1 21.51824 159.2707 136.985 2.631108
```

EXAMPLE 1 – confidence and prediction intervals

```
# confidence interval
predict(mv1, newdata, interval="confidence")
#        mpg     disp      hp       wt
# 1 21.51824 159.2707 136.985 2.631108

# prediction interval
predict(mv1, newdata, interval="prediction")
#        mpg     disp      hp       wt
# 1 21.51824 159.2707 136.985 2.631108
```

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```
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# prediction interval
predict(mv1, newdata, interval="prediction")
#         mpg     disp      hp       wt
# 1 21.51824 159.2707 136.985 2.631108
```

No intervals

EXAMPLE 1 – confidence and prediction intervals

```
predictmlm <- function(object,newdata,level=0.95,
                    interval = c("confidence","prediction"))
{
 form <- as.formula(paste("~",as.character(formula(object))[3]))
 xnew <- model.matrix(form, newdata)
 fit <- predict(object, newdata)
 Y <- model.frame(object)[,1]
 X <- model.matrix(object)
 n <- nrow(Y)
 m <- ncol(Y)
 p <- ncol(X) – 1
```

EXAMPLE 1 – confidence and prediction intervals

```
# alpha correction
alpha = 1 - level
level = 1 - m*alpha
sigmas <- colSums((Y - object$fitted.values)^2) / (n - p - 1)
fit.var <- diag(xnew %*% tcrossprod(solve(crossprod(X)), xnew))
if(interval[1]=="prediction") fit.var <- fit.var + 1
const <- qf(level, df1=m, df2=n-p-m) * m * (n - p - 1) / (n - p - m)
vmat <- (n/(n-p-1)) * outer(fit.var, sigmas)
lwr <- fit - sqrt(const) * sqrt(vmat)
upr <- fit + sqrt(const) * sqrt(vmat)
```

EXAMPLE 1 – confidence and prediction intervals

```
if(nrow(xnew)==1) {
  ci <- rbind(fit, lwr, upr)
  rownames(ci) <- c("fit", "lwr", "upr") }
 else {
  ci <- array(0, dim=c(nrow(xnew), m, 3))
  dimnames(ci) <- list(1:nrow(xnew), colnames(Y), c("fit", "lwr", "upr") )
  ci[,,1] <- fit
  ci[,,2] <- lwr
  ci[,,3] <- upr}
ci
}
```

EXAMPLE 1 – confidence and prediction intervals

```
# confidence interval

predictmlm(mv1,newdata, interval="confidence")
#          mpg      disp        hp       wt
#fit 21.51824 159.27070 136.9850 2.631108
#lwr 17.79607  92.85711 105.1024 1.957935
#upr 25.24041 225.68430 168.8676 3.304281
```

EXAMPLE 1 – confidence and prediction intervals

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predictmlm(mv1,newdata, interval="confidence")
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#lwr 17.79607  92.85711 105.1024 1.957935
#upr 25.24041 225.68430 168.8676 3.304281
```

EXAMPLE 1 – confidence and prediction intervals

```
# prediction interval

predictmlm(mv1,newdata, interval="prediction")
#         mpg       disp        hp       wt
#fit 21.51824 159.270705 136.98500 2.631108
#lwr 12.45592  -2.425511  59.36086 0.992143
#upr 30.58056 320.966921 214.60914 4.270072
```

EXAMPLE 1 – prediction

Predict attributes (mpg, disp, hp, wt) of the following cars

mpg	disp	hp	wt	cyl	am	carb
				4	no	2
				6	yes	4
				8	yes	6

EXAMPLE 1 – prediction

Predict attributes (mpg, disp, hp, wt) of the following cars

mpg	disp	hp	wt	cyl	am	carb
				4	no	2
				6	yes	4
				8	yes	6

```
newdata <- data.frame(cyl=factor(c(4,6,8), levels=c(4,6,8)),
                                am=c(0,1,1), carb=c(2,4,6))
```

EXAMPLE 1 – confidence and prediction intervals

```
predictmlm(mv1, newdata, interval="prediction")
#, , fit
#        mpg     disp         hp       wt
#1 23.08059 137.7774  89.07313 3.111033
#2 21.51824 159.2707 136.98500 2.631108
#3 15.92331 319.8707 266.21745 3.557519
```

mpg	disp	hp	wt	cyl	am	carb
23.08	137.77	89.07	3.11	4	no	2
21.51	159.27	136.98	2.63	6	yes	4
15.92	319.87	266.21	3.55	8	yes	6

EXAMPLE 1 – prediction

predicted

mpg	disp	hp	wt	cyl	am	carb
23.08	137.77	89.07	3.11	4	no	2
21.51	159.27	136.98	2.63	6	yes	4
15.92	319.87	266.21	3.55	8	yes	6

EXAMPLE 1 – confidence and prediction intervals

```
predictmlm(mv1, newdata, interval="prediction")
, , lwr
        mpg       disp        hp       wt
1 13.871941 -26.529667  10.19560 1.445604
2 12.455915  -2.425511  59.36086 0.992143
3  6.728061 155.802679 187.45471 1.894514
, , upr
       mpg     disp       hp       wt
1 32.28925 302.0846 167.9507 4.776462
2 30.58056 320.9669 214.6091 4.270072
3 25.11856 483.9387 344.9802 5.220524
```

EXAMPLE 1 – confidence and prediction intervals

```
predictmlm(mv1, newdata, interval="prediction")
, , lwr
        mpg       disp        hp       wt
1 13.871941 -26.529667  10.19560 1.445604
2 12.455915  -2.425511  59.36086 0.992143
3  6.728061 155.802679 187.45471 1.894514
, , upr
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1 32.28925 302.0846 167.9507 4.776462
2 30.58056 320.9669 214.6091 4.270072
3 25.11856 483.9387 344.9802 5.220524
```

lower boundaries

upper boundaries

EXAMPLE 1 – predictions

lower boundary

mpg	disp	hp	wt	cyl	am	carb
19.01	137.77	89.07	3.11	4	no	2
17.79	159.27	136.98	2.63	6	yes	4
11.88	319.87	266.21	3.55	8	yes	6

predicted

mpg	disp	hp	wt	cyl	am	carb
23.08	137.77	89.07	3.11	4	no	2
21.51	159.27	136.98	2.63	6	yes	4
15.92	319.87	266.21	3.55	8	yes	6

mpg	disp	hp	wt	cyl	am	carb
27.14	137.77	89.07	3.11	4	no	2
25.24	159.27	136.98	2.63	6	yes	4
19.95	319.87	266.21	3.55	8	yes	6

upper boundary

EXAMPLE 1 – new MRLR predict function

Function predictmlm()

can be found in our Github sites

EXAMPLE 2 – Health care data (Johnson, Wichern)

Drug, *amitriptyline*, is prescribed as an antidepressant.
Possible side effects

- irregular heartbeat
- abnormal blood pressure
- irregular waves on the electrocardiogram
- others

Data from patients who were admitted to the hospital ER after an overdose follow

EXAMPLE 2 – Health care data (Johnson, Wichern)

TOT	AMI	GEN	AMT	PR	DIAP	QRS
3389	3149	1	7500	220	0	140
1101	653	1	1975	200	0	100
1131	810	0	3600	205	60	111
596	448	1	675	160	60	120
896	844	1	750	185	70	83
1767	1450	1	2500	180	60	80
807	493	1	350	154	80	98
1111	941	0	1500	200	70	93
645	547	1	375	137	60	105
628	392	1	1050	167	60	74
1360	1283	1	3000	180	60	80
652	458	1	450	160	64	60
860	722	1	1750	135	90	79
500	384	0	2000	160	60	80
781	501	0	4500	180	0	100
1070	405	0	1500	170	90	120
1754	1520	1	3000	180	0	129

```
TOT  Total TCAD plasma level
AMI  Amount of amitriptyline
     present in TCAD plasma level
GEN  1 female, 0 male
AMT  Amount of antidepressant
     at time of overdose
PR   PR wave measurement
DIAP Diastolic blood pressure
QRS  wave measurement
```

EXAMPLE 2 – Health care data (Johnson, Wichern)

TOT	AMI	GEN	AMT	PR	DIAP	QRS
3389	3149	1	7500	220	0	140
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1111	941	0	1500	200	70	93
645	547	1	375	137	60	105
628	392	1	1050	167	60	74
1360	1283	1	3000	180	60	80
652	458	1	450	160	64	60
860	722	1	1750	135	90	79
500	384	0	2000	160	60	80
781	501	0	4500	180	0	100
1070	405	0	1500	170	90	120
1754	1520	1	3000	180	0	129

TOT Total TCAD plasma level
AMI Amount of amitriptyline present in TCAD plasma level
GEN 1 female, 0 male
AMT Amount of antidepressant at time of overdose
PR PR wave measurement
DIAP Diastolic blood pressure
QRS wave measurement

EXAMPLE 2 – Health care data (Johnson, Wichern)

d1

TOT	AMI	GEN	AMT	PR	DIAP	QRS
3389	3149	1	7500	220	0	140
1101	653	1	1975	200	0	100
1131	810	0	3600	205	60	111
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781	501	0	4500	180	0	100
1070	405	0	1500	170	90	120
1754	1520	1	3000	180	0	129

```
TOT  Total TCAD plasma level
AMI  Amount of amitriptyline
     present in TCAD plasma level
GEN  1 female, 0 male
AMT  Amount of antidepressant
     at time of overdose
PR   PR wave measurement
DIAP Diastolic blood pressure
QRS  wave measurement
```

EXAMPLE 2 – Health care data (Johnson, Wichern)

Model with two responses

```
mlm1 <- lm(cbind(TOT, AMI) ~ GEN + AMT + PR + DIAP + QRS,d1)
```

Models with one response

```
m11 <- lm(TOT ~ GEN + AMT + PR + DIAP + QRS,d1)
m22 <- lm(AMI ~ GEN + AMT + PR + DIAP + QRS,d1)
```

EXAMPLE 2 – Health care data (Johnson, Wichern)

```
Model with two responses
    mlm1 <- lm(cbind(TOT, AMI) ~ GEN + AMT + PR + DIAP + QRS,d1)

    Assumption:  TOT, AMI are correlated

Models with one response
    m11 <- lm(TOT ~ GEN + AMT + PR + DIAP + QRS,d1)
    m22 <- lm(AMI ~ GEN + AMT + PR + DIAP + QRS,d1)

    Assumption:  TOT, AMI are not related
```

EXAMPLE 2 – Health care data (Johnson, Wichern)

Model with two responses

```
mlm1 <- lm(cbind(TOT, AMI) ~ GEN + AMT + PR + DIAP + QRS,d1)
```

Assumption: TOT, AMI are correlated

```
cor(d1[,1:2])
                TOT       AMI
        TOT 1.0000000 0.9760717
        AMI 0.9760717 1.0000000
```

EXAMPLE 2 – Health care data (Johnson, Wichern)

```
Jointly test predictors
Anova(mlm1)
#Type II MANOVA Tests: Pillai test statistic
#     Df test stat approx F num Df den Df   Pr(>F)
#GEN   1   0.65521   9.5015      2     10 0.004873 **
#AMT   1   0.69097  11.1795      2     10 0.002819 **
#PR    1   0.34649   2.6509      2     10 0.119200
#DIAP  1   0.32381   2.3944      2     10 0.141361
#QRS   1   0.29184   2.0606      2     10 0.178092
```

EXAMPLE 2 – Health care data (Johnson, Wichern)

```
Jointly test predictors
Anova(mlm1)                                          library car
#Type II MANOVA Tests: Pillai test statistic
#     Df test stat approx F num Df den Df   Pr(>F)
#GEN   1   0.65521   9.5015      2     10 0.004873 **
#AMT   1   0.69097  11.1795      2     10 0.002819 **
#PR    1   0.34649   2.6509      2     10 0.119200
#DIAP  1   0.32381   2.3944      2     10 0.141361
#QRS   1   0.29184   2.0606      2     10 0.178092
```

EXAMPLE 2 – Health care data (Johnson, Wichern)

```
Jointly test predictors
Anova(mlm1)                                            library car
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#      Df test stat approx F num Df den Df   Pr(>F)
#GEN    1   0.65521   9.5015      2     10 0.004873 **
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#PR     1   0.34649   2.6509      2     10 0.119200
#DIAP   1   0.32381   2.3944      2     10 0.141361
#QRS    1   0.29184   2.0606      2     10 0.178092
```

EXAMPLE 2 – Health care data (Johnson, Wichern)

```
m1 <- lm(TOT ~ GEN + AMT + PR + DIAP + QRS, data = d1)
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.879e+03  8.933e+02  -3.224 0.008108 **
GEN          6.757e+02  1.621e+02   4.169 0.001565 **
AMT          2.848e-01  6.091e-02   4.677 0.000675 ***
PR           1.027e+01  4.255e+00   2.414 0.034358 *
DIAP         7.251e+00  3.225e+00   2.248 0.046026 *
QRS          7.598e+00  3.849e+00   1.974 0.074006 .
```

EXAMPLE 2 – Health care data (Johnson, Wichern)

```
Jointly test predictors

Anova(mlm1)                                           library car

#Type II MANOVA Tests: Pillai test statistic

#     Df test stat approx F num Df den Df   Pr(>F)

#GEN   1   0.65521   9.5015      2     10 0.004873 **

#AMT   1   0.69097  11.1795      2     10 0.002819 **

#PR    1   0.34649   2.6509      2     10 0.119200

#DIAP  1   0.32381   2.3944      2     10 0.141361

#QRS   1   0.29184   2.0606      2     10 0.178092
```

EXAMPLE 2 – Health care data (Johnson, Wichern)

Simplify the model

```
mlm2 <- update(mlm1, . ~ . - PR - DIAP - QRS)
```

or

```
mlm2 <- lm(cbind(TOT, AMI) ~ GEN + AMT,d1)
```

EXAMPLE 2 – Health care data (Johnson, Wichern)

```
Predict TOT and AMI for a female with 1200 mg. of overdose
(consider two linear regression models)

    newval <- data.frame(GEN = 1, AMT = 1200)
    m11 <- lm(TOT ~ GEN + AMT,d1)
    m22 <- lm(AMI ~ GEN + AMT,d1)
```

EXAMPLE 2 – Health care data (Johnson, Wichern)

Predict TOT and AMI for a female with 1200 mg. of overdose
(consider two linear regression models)

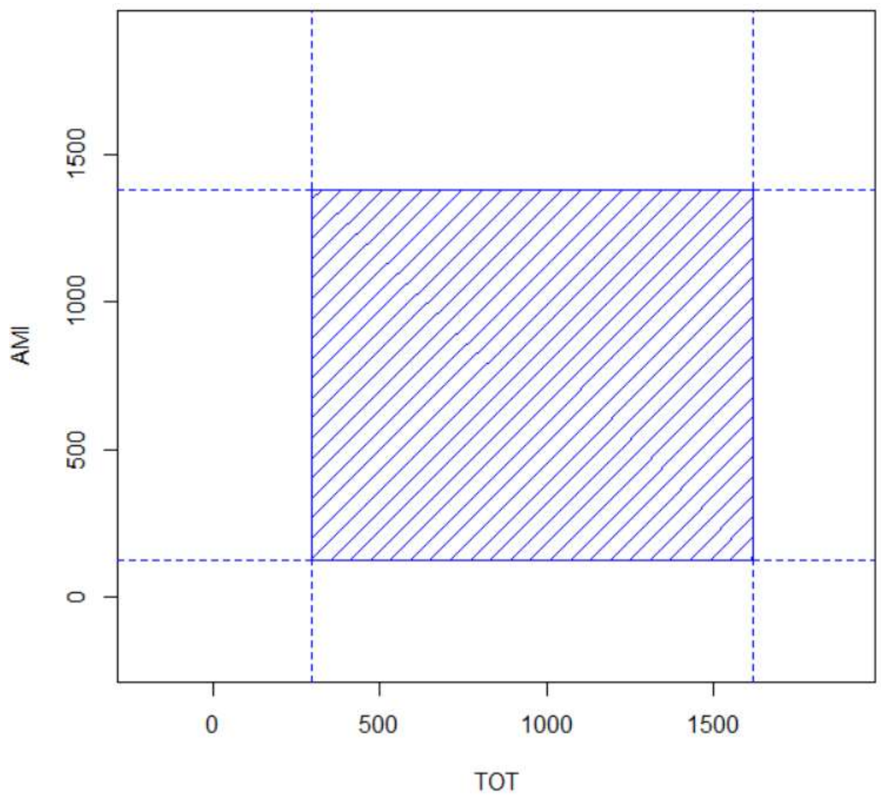
```
predict(m11,newval,level=0.90,interval="predict")
#        fit      lwr      upr
#1 958.5473 297.8818 1619.213

predict(m22,newval,level=0.90,interval="predict")
#        fit      lwr      upr
#1 754.0677 127.2403 1380.895
```

EXAMPLE 2 – Health care data (Johnson, Wichern)

```
predict(m11,newval,interval="predict")
#        fit      lwr      upr
#1 958.5473 297.8818 1619.213

predict(m22,newval,interval="predict")
#        fit      lwr      upr
#1 754.0677 127.2403 1380.895
```



EXAMPLE 2 – Health care data (Johnson, Wichern)

Predict TOT and AMI for a female with 1200 mg. of overdose

(use the multiple response regression model)

```
newval <- data.frame(GEN = 1, AMT = 1200)
predict(mlm2,newval,interval="predict")
#        TOT      AMI
#1 958.5473 754.0677
```

no prediction interval

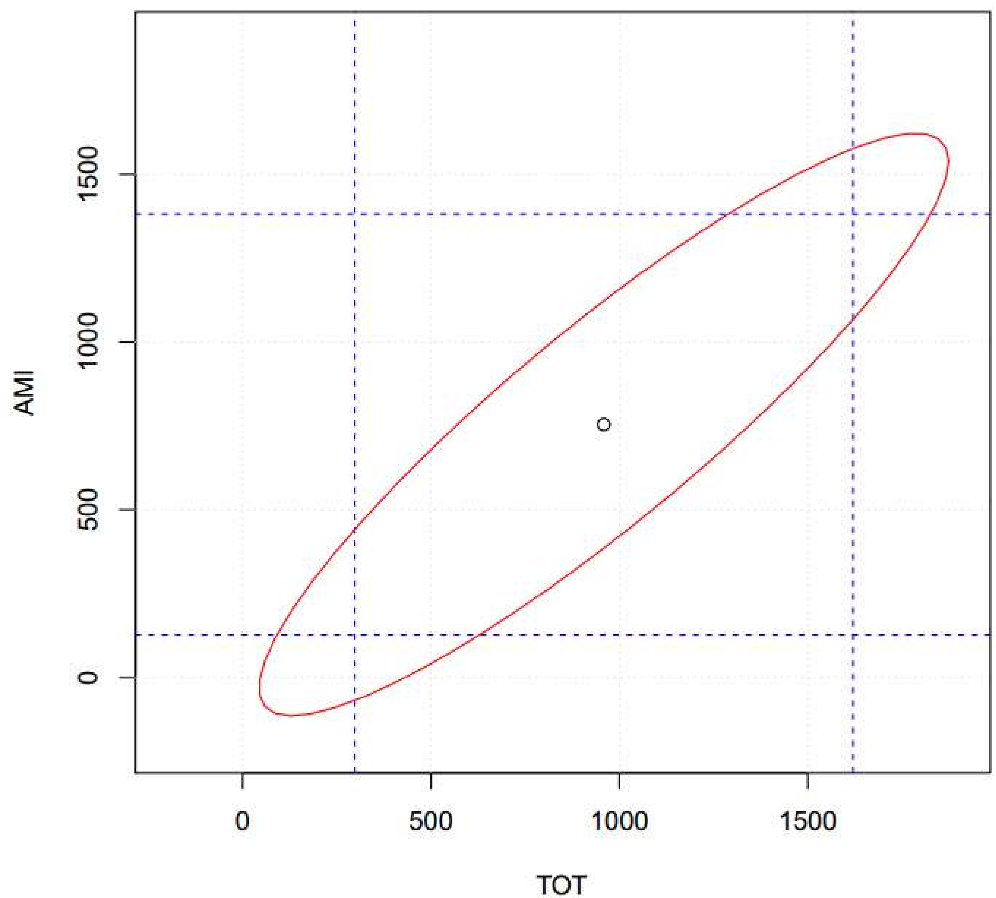
EXAMPLE 2 – Health care data (Johnson, Wichern)

```
For two-response model

plotellipse <- function(mod,newdata,level = 0.95,
                  interval = c("confidence","prediction"))
{
    .. .
}

plot prediction region
plotellipse(mlm2,newval,interval="prediction")
```

EXAMPLE 2 – Health care data (Johnson, Wichern)



EXAMPLE 2 – Health care data (Johnson, Wichern)

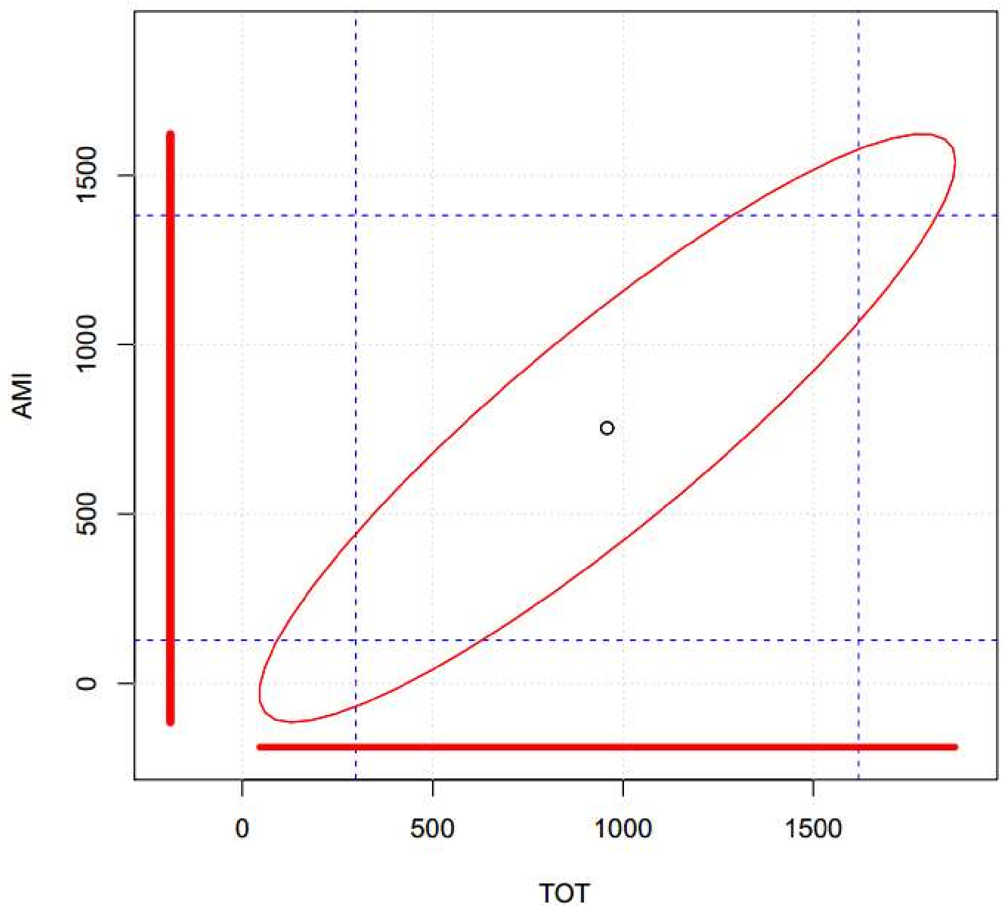
```
to plot the prediction region

    plotellipse(mlm2, newval, level, interval="prediction")

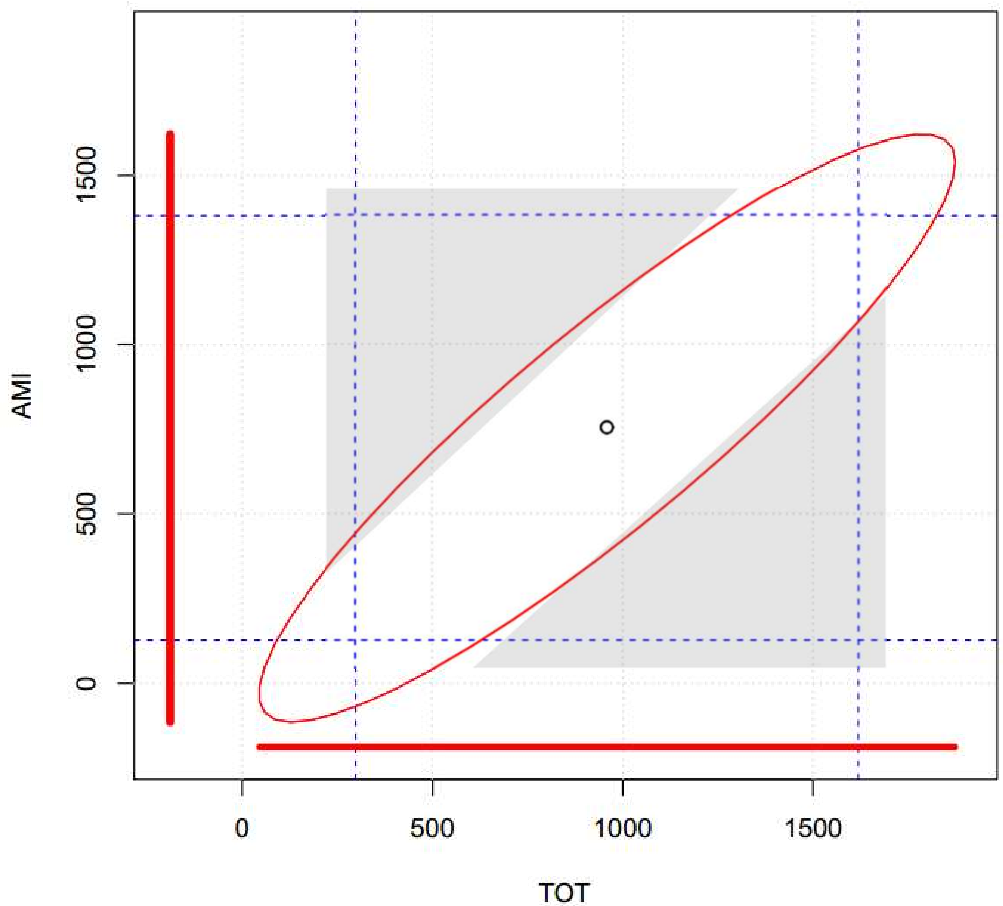
To get individual prediction intervals

    predictmlm(mlm2,newval,level=0.90,interval="prediction")
    #           TOT        AMI
    # fit  958.5473  754.06767
    # lwr  138.7838  -23.70903
    # upr 1778.3108 1531.84437
```

EXAMPLE 2 – Health care data (Johnson, Wichern)



EXAMPLE 2 – Health care data (Johnson, Wichern)



EXAMPLE 2 – Health care data (Johnson, Wichern)

For two responses, prediction region is an ellipse

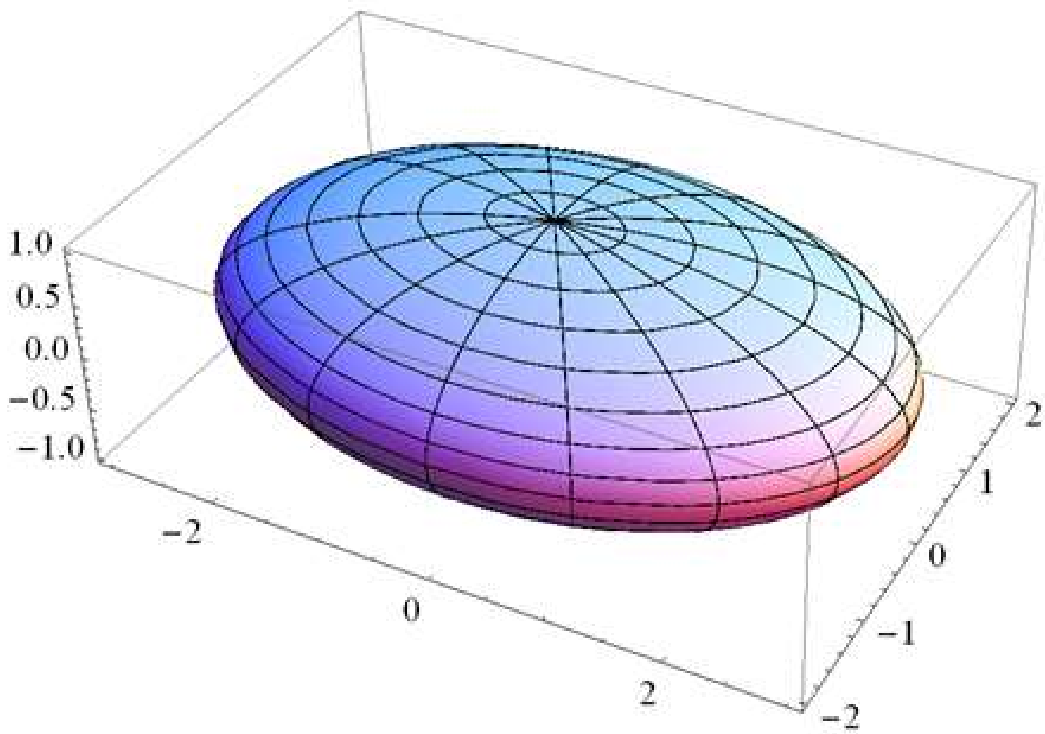
```
plotellipse(mlm2, newval, level, interval="prediction")
```

To get individual prediction intervals

```
predictmlm(mlm2,newval,level=0.90,interval="prediction")
#           TOT        AMI
# fit  958.5473  754.06767
# lwr  138.7838  -23.70903
# upr 1778.3108 1531.84437
```

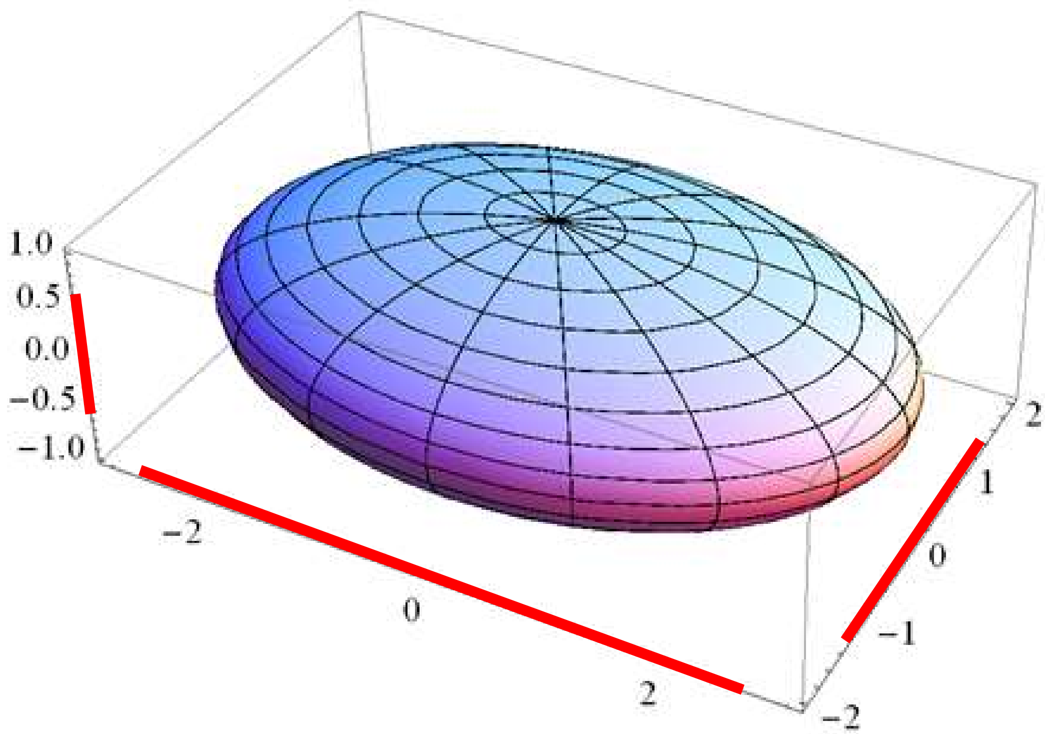
EXAMPLE 2 – Health care data (Johnson, Wichern)

For three responses - ellipsoid



EXAMPLE 2 – Health care data (Johnson, Wichern)

For three responses – prediction intervals



For more than two responses

- do not plot the prediction region
- but find prediction intervals

Conclusions

CONCLUSIONS

- MRLR is a linear regression model with two
  or more responses
- Should be used when responses are correlated
  and share same predictors
- MRLRs do not provide prediction intervals
  we provide predictmlm()

PERSONAL DATA

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PERSONAL DATA

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MULTIPLE RESPONSE REGRESSION MODELS

Thank you!

MULTIPLE RESPONSE REGRESSION MODELS

Questions?