Conservation Laws

Charge Conservation

- The electric charge must balance between the initial and final states of a reaction. As assumed also in chemical and nuclear reactions.
- For example, a neutron cannot decay as *n* → *p*, but it can decay as:
  - $n \rightarrow p + e^- + \nu$, since the proton and electron have opposite electric charge.
  - Why there is a third particle present is a different story, which we will come to soon
- Other examples illustrating charge conservation:
  - $e^+ + e^- \rightarrow e^+ + e^-$ but not $e^+ + e^- \rightarrow e^- + e^-$
  - $p + p \rightarrow p + p + p + \bar{p}$ but not $p + p \rightarrow p + p + \bar{p}$
  - $\pi^0 \rightarrow \gamma + \gamma$ but not $\pi^0 \rightarrow \gamma$ (but for a different region!)

Energy and Momentum Conservation

- In relativistic physics, both **energy** and **momentum** are always conserved
- For example, nuclei cannot decay if the mass of the final state products is larger than the initial ones
    - You would have to add energy somehow in a reaction to create the extra mass
- For example, a decay that cannot take place in free space is: $p \to n + e^+$
    - The neutron is more massive than the proton, so in the rest frame of the proton it would violate energy conservation for the proton to decay this way.
    - Reactions could proceed, provided there is enough initial kinetic energy in collision products
        - For example: $e^+ + e^- \to \mu^+ + \mu^-$, even though $m_\mu = 207 m_e$
- The reason the decay $\pi^0 \to \gamma$ is not allowed, but $\pi^0 \to \gamma + \gamma$ is, has to do with the fact that momentum cannot be conserved in the first case.
    - In the rest frame of the pion, you cannot have a photon at rest. It takes two of them to conserve momentum

Baryon Number Conservation

- The proton and neutron, but also all other **baryons** (hadrons that are fermions), have a quantum number associated with them that is conserved known as baryon number (*B*).

- This is the explanation for why the free proton is stable (lifetime $> \approx 10^{30}$ years!) and does not decay for example as:

  - $p \rightarrow e^{+} + \pi^{0}$

- We take baryons to have *B* = +1, and anti-baryons to have *B* = −1, and require the total number to balance on each side of the reaction equation.

- For example, the decay:

  - $\Delta^{++} \rightarrow p + \pi^{+}$

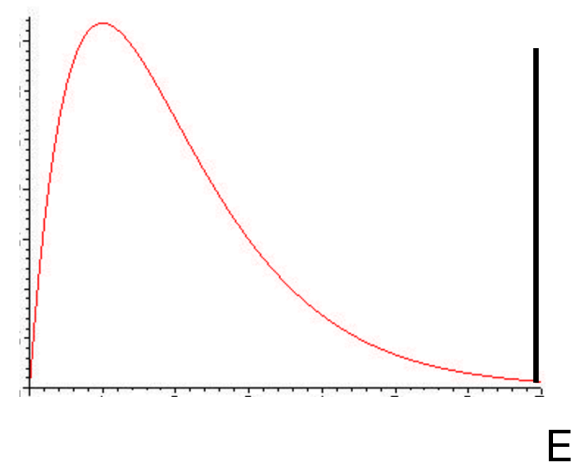
  - The Delta is a spin-3/2 baryon with *B* = +1 like the proton in the final state. The pion, which is a meson, has *B* = 0.

Beta Decay

- The weak nuclear force is responsible for many of the radioactive decays of elements, via beta decay (electron emission)
  - For example, ${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + e^-$, which conserves electric charge and energy
  - Inside the nucleus this is the decay of the neutron: $n \rightarrow p + e^-$
- But, if there are only two decay products, the decay electrons should be **monoenergetic** (single, fixed energy)
  - In rest frame of nucleus, the two outgoing particles must have equal and opposite momentum (momentum conservation)
  - Energy-momentum conservation dictates that electron energy should be equal to the mass difference between the neutron and proton: 939.57 – 938.27 = 1.3 MeV (with the proton also picking up some small amount of kinetic energy)
- However, this is not what is observed!

Beta Decay

- Instead the kinetic energy of the electron shows a range of values, not a single value, that qualitatively looks like:
  - Not a monoenergetic spike at 1.3 MeV
  - *Is momentum not conserved*?
- To explain this observation and thus avoid a crisis with a fundamental conservation law in physics, Wolfgang Pauli in 1930 proposed a third particle, the **neutrino** (as coined by Enrico Fermi, which means “little neutral one”), in the decay:
  - $n \rightarrow p + e^- + \nu$
  - Added a new electrically neutral particle, *v*, that had not yet been detected
- However, it took until 1956 before the neutrino was directly observed in an experiment at a nuclear reactor by Cowen and Reines.



The Neutrino

- The neutrino has no electric charge, is spin-½ like other leptons such as the electron, and has (nearly) zero mass
    - In fact by angular momentum (spin rules), need a third spin-1/2 particle
- It is also stable (doesn't decay)
- Enrico Fermi developed a theory of beta decay around it
- It does not interact via the electromagnetic force or the strong nuclear force, only the weak nuclear force
- Because of this, it is extremely penetrating
- Need to produce large numbers of neutrinos to have a chance to detect one
    - Reactors, atmosphere, sun

Muon Decay

- Now consider the decay of the mu lepton (muon), surely it should yield monoenergetic electrons, right?
  - $\mu^- \rightarrow e^- + \nu$
- Actually no! It's still not monoenergetic.
- What's going on?
- Propose that neutrinos come in different types
  - **Flavors**, actually, the proper scientific term for "type" 😉
- So the actual reaction is:
  - $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$
- Therefore, there is a neutrino flavor corresponding to each charged lepton flavor: e, μ, τ

Lepton Number

- The leptons carry a conserved quantum number referred to as **lepton number**
- An electron or electron neutrino on one side of a reaction equation must be balanced by the same on the other side
- Same also for muon and tau lepton numbers (i.e. 3 lepton numbers in total)
- Each lepton flavor has its own separate conserved lepton number: L_e, L_μ, and L_τ.
- Antiparticles have negative lepton number
- So for $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$
  - $L_e : 0 = 1 \quad -1 \quad +0$
  - $L_\mu : 1 = 0 \quad +0 \quad +1$
- And for $\tau^- \rightarrow \mu^- + \bar{\nu}_\mu + \nu_\tau$
  - $L_\mu : 0 = 1 \quad -1 \quad +0$
  - $L_\tau : 1 = 0 \quad +0 \quad +1$

Strangeness

- Since the early days of particle accelerators, a certain class of particles was discovered that had a strange property:
  - They were produced copiously in hadron collisions via the strong nuclear force
  - But rather than decay quickly with lifetimes of $\approx 10^{-22}$ s, characteristic of the strong force, they lived for an anomalously long time: $\approx 10^{-10}$ s, that is more in line with the weak force.
  - These particles were observed to be produced only in pairs:
  - But the slow decays include:

$$\pi^+ p \to K^+ \Sigma^+$$
$$\pi^- p \to K^+ \Sigma^-$$
$$\pi^- p \to K^0 \Sigma^0$$
$$\pi^- p \to K^0 \Lambda^0$$

$$\Sigma^+ \to n\pi^+$$
$$\Sigma^+ \to p\pi^0$$
$$\Lambda^0 \to p\pi^-$$
$$\Lambda^0 \to n\pi^0$$

$$K^+ \to \mu^+ \nu_\mu$$
$$K^+ \to \pi^+ \pi^0$$
$$K^+ \to \pi^+ \pi^+ \pi^-$$

Note that Σ and Λ are baryons, and that baryon number is conserved

Strangeness

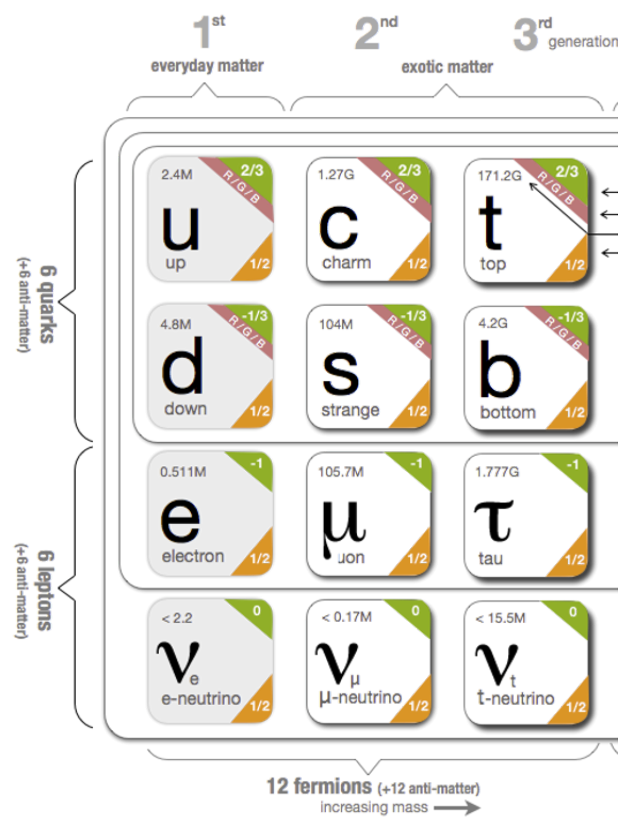
- What was concluded was that these strange particles had a conserved quantum number called **strangeness** when produced in strong (or EM) collisions, but strangeness is not conserved in the weak decays of these particles.
- K^+ and K^0 have strangeness $S = +1$
- Σ^+ and Λ^0 have strangeness $S = -1$
- The antiparticles of the above particles have opposite strangeness, namely that $S = -1$ for K^- and $\bar{K}^0$, and $S = +1$ for Σ^-.
- Some particles even have larger values of strangeness:
  - Ξ^- : $S = -2$
  - Ω^- : $S = -3$
- Only the weak force violates strangeness conservation (or “flavor” conservation).

Charm, Beauty, Truth

- As accelerators reached even higher energies, additional particles were observed that also could be produced in strong or EM interactions, but had only slow, weak decays
  - These include the D mesons (D^0, $D^\pm$) and B mesons (B^0, $B^\pm$) to name but a few
- Thus, additional quantum numbers appears to be conserved in EM and strong interactions like strangeness, which historically have been called **charm** and **beauty** (now "bottom")
- It was eventually experimentally determined that strangeness, charm, and beauty all can be identified as properties of **unique quarks** that are the constituent building blocks of hadrons
- These are the strange (s), charm (c), and bottom (b) quarks, joining the up (u) and down (d) quarks
- In 1995 the top quark (t), also known as truth, joined this family as well upon its discovery

Standard Model Particles

- Therefore the quarks can be arranged into 3 flavor generations very much like the leptons:
  - All spin-1/2
  - Generally increasing mass with each generation
  - Fractional electric charge for the quarks
  - Also antiparticles for each



$q = +2/3\ e$

$q = -1/3\ e$

$q = -e$

$q = 0$

Which Conservation Laws Violated, If Any?

1. $p + \bar{p} \rightarrow \pi^+ + \pi^- + \pi^+$
2. $\Delta^{++} \rightarrow p + \pi^+$
3. $\pi^- \rightarrow \tau^- + \gamma$
4. $K^0 \rightarrow \pi^+ + \pi^-$
5. $p + \nu_e \rightarrow e^+ + \Lambda + K^0$
6. $p + \pi^- \rightarrow \Lambda + \bar{\Sigma}^0$

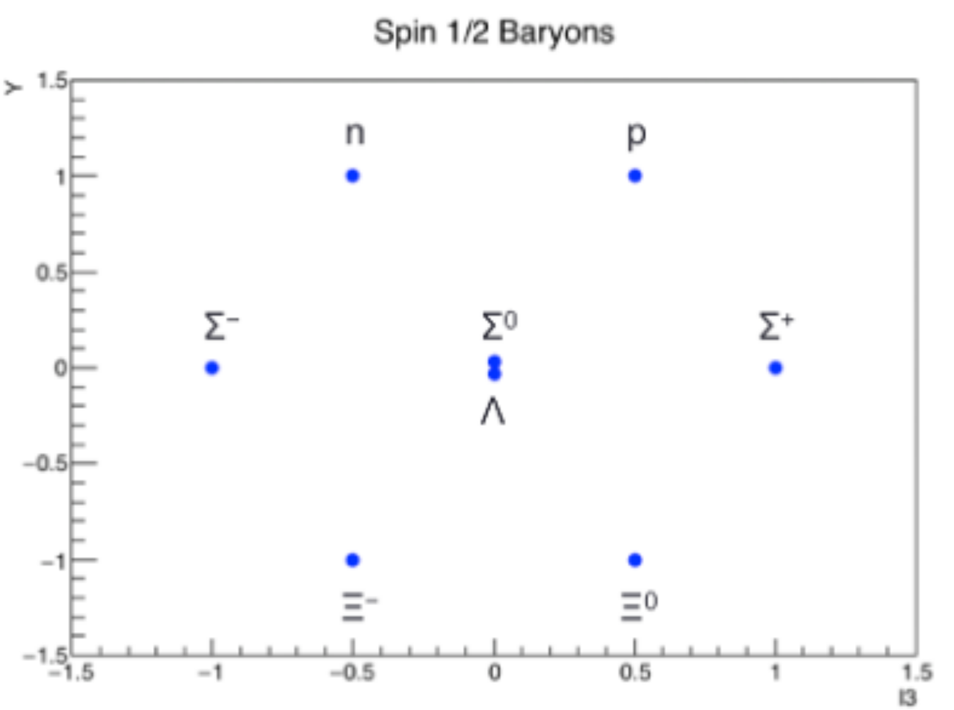
Quark Model & Hadron Structure

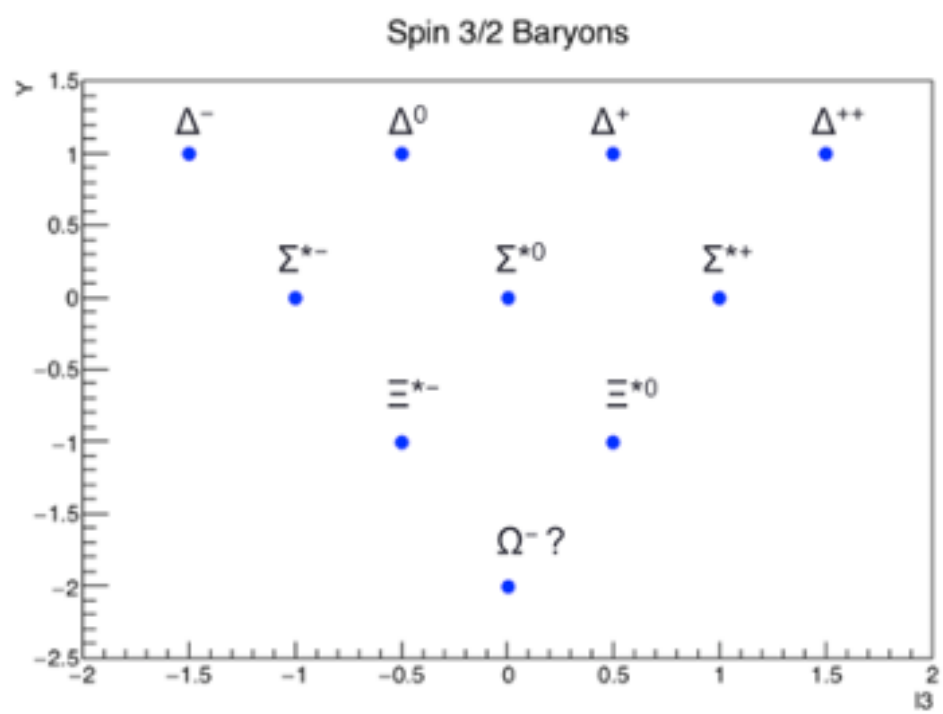
- In 1964 Murray Gell-Mann and George Zweig propose a theory for the apparent structure in the particle multiplets and to explain the particle zoo
- In this **Quark Model**:
- **Baryons** are bound states of 3 quarks (qqq), and are fermions
- **Mesons** are bound states of a quark and an anti-quark ($q\,\bar{q}$), and are bosons
    - *However, note that the top quark decays too promptly to form a bound state*

Baryons

$q = +2/3\ e$

$q = -1/3\ e$

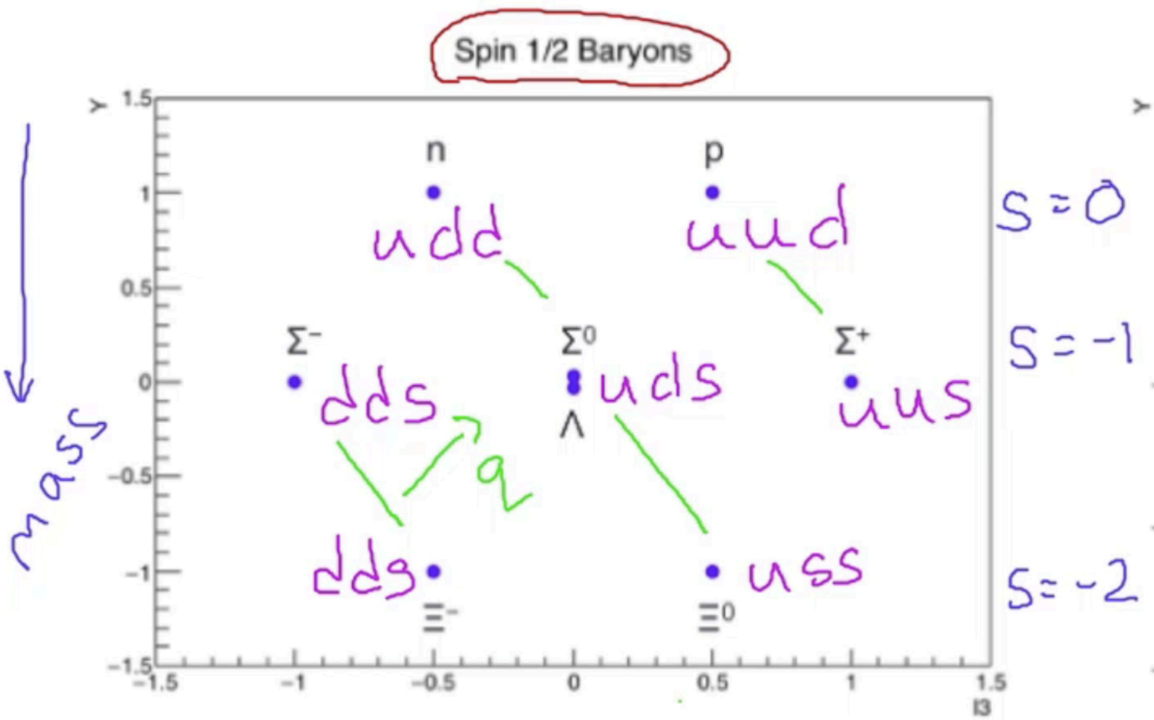


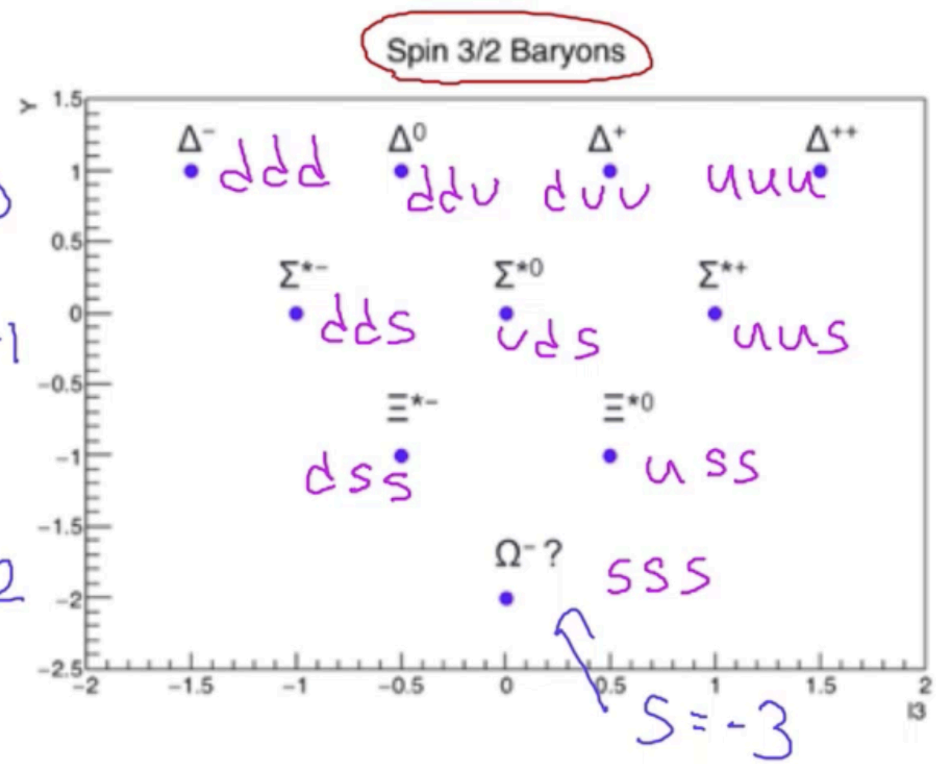


Baryons

u (up): q = +2/3 e

d (down), s (strange): q = −1/3 e

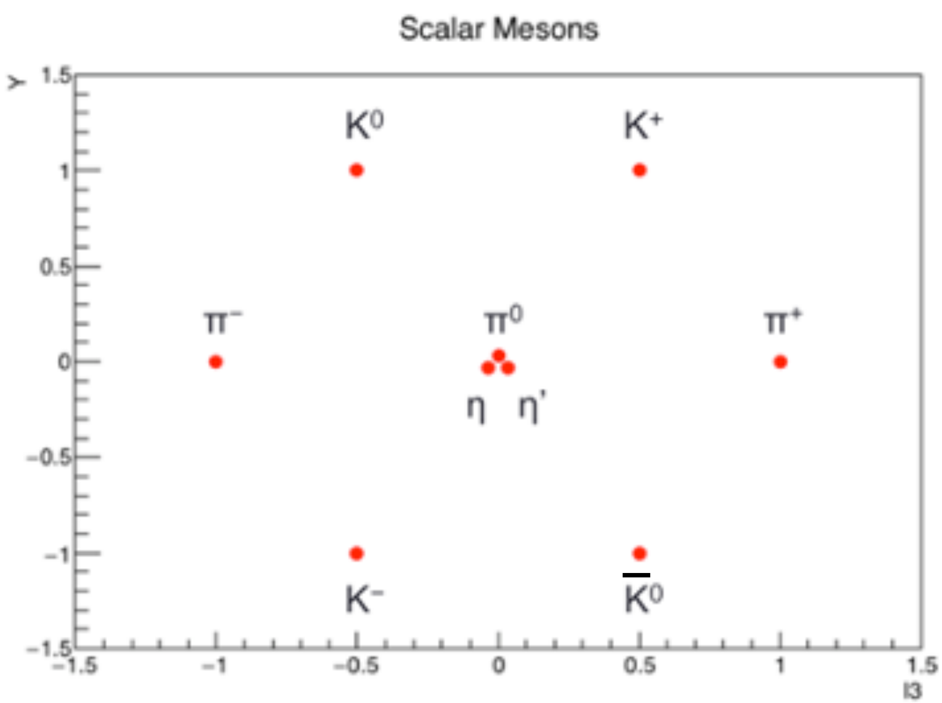


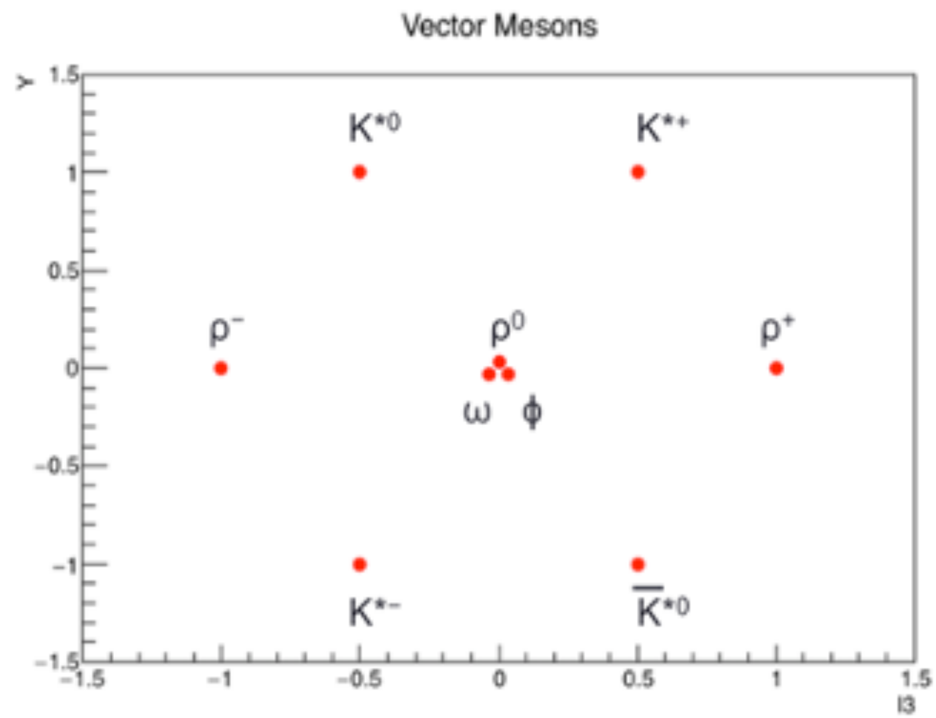


Mesons

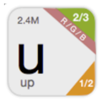
$q = +2/3\ e, \quad \bar{q} = -2/3\ e$

$q = -1/3\ e, \quad \bar{q} = +1/3\ e$



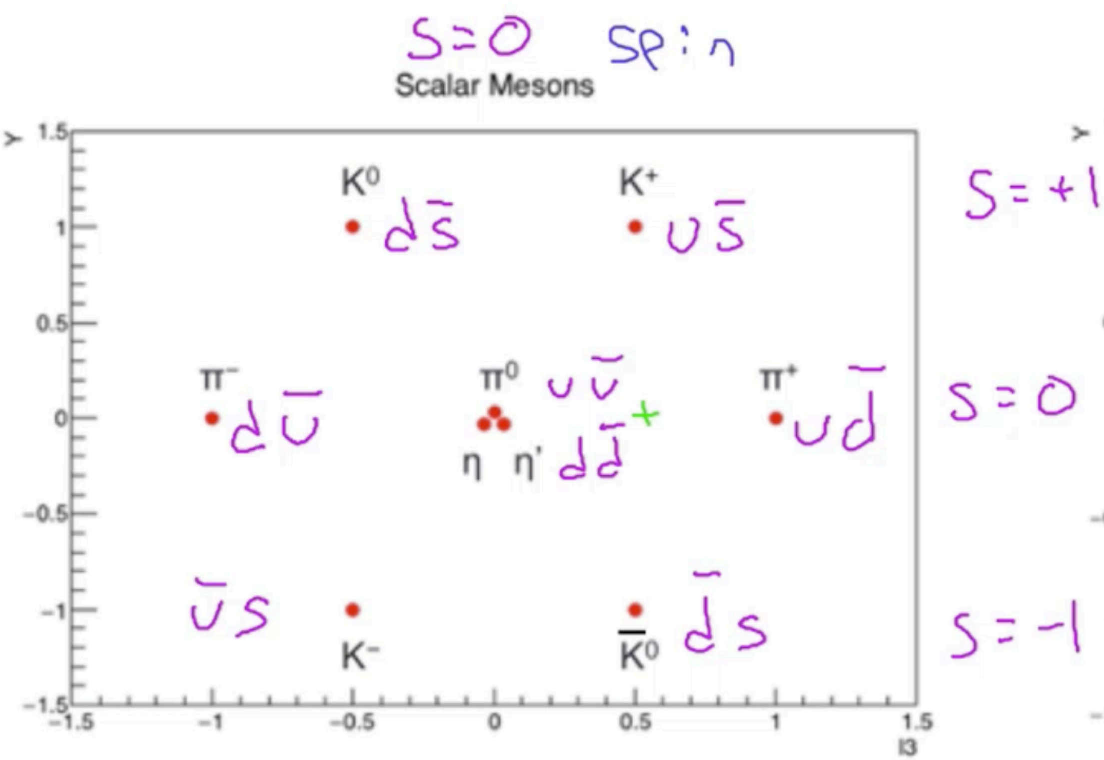


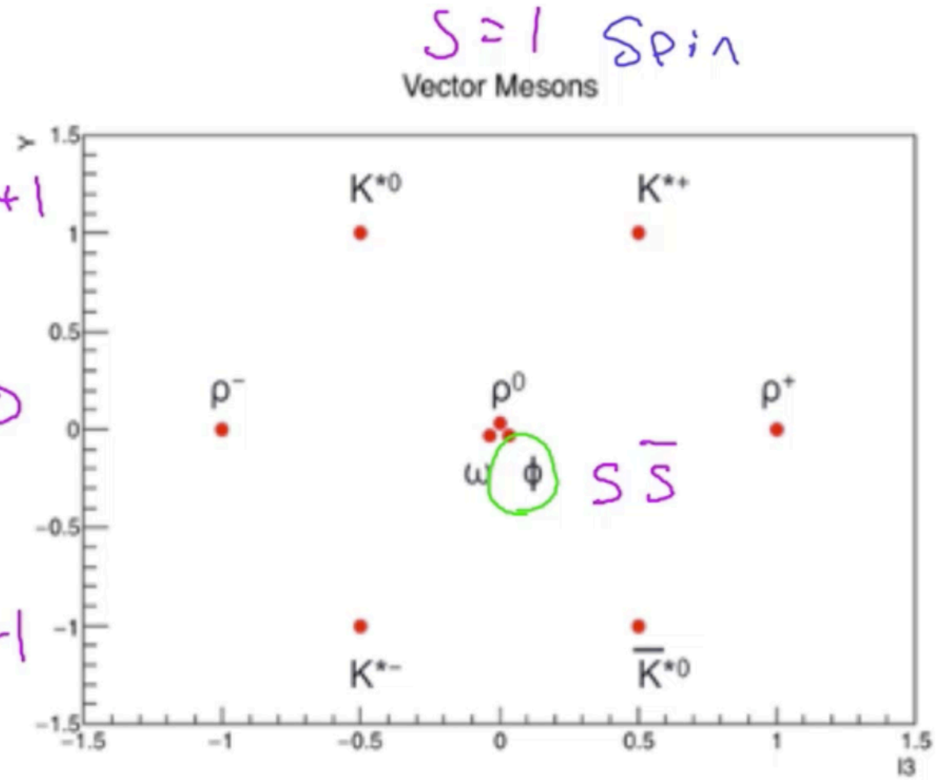
Mesons



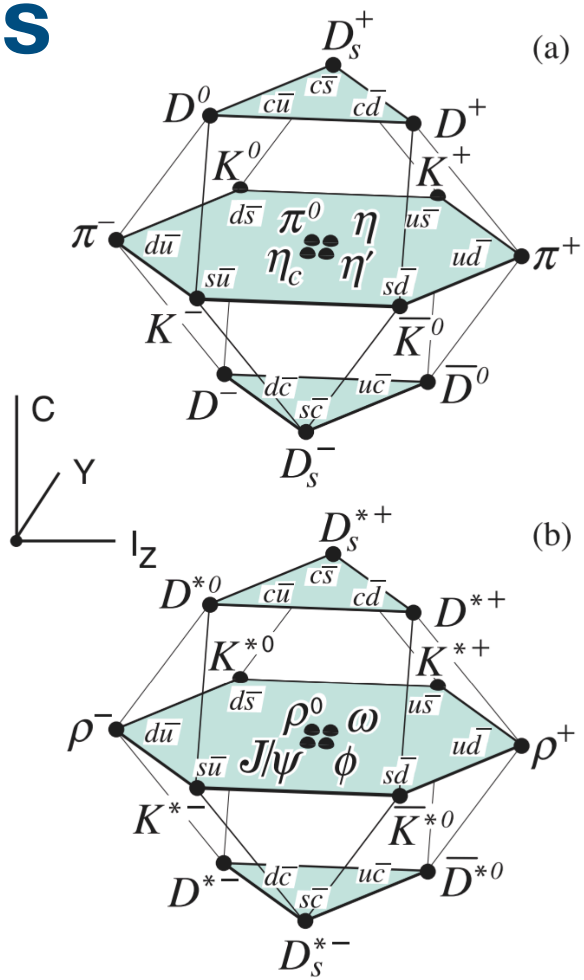
$q = +2/3$ e, $\bar{q} = -2/3$ e

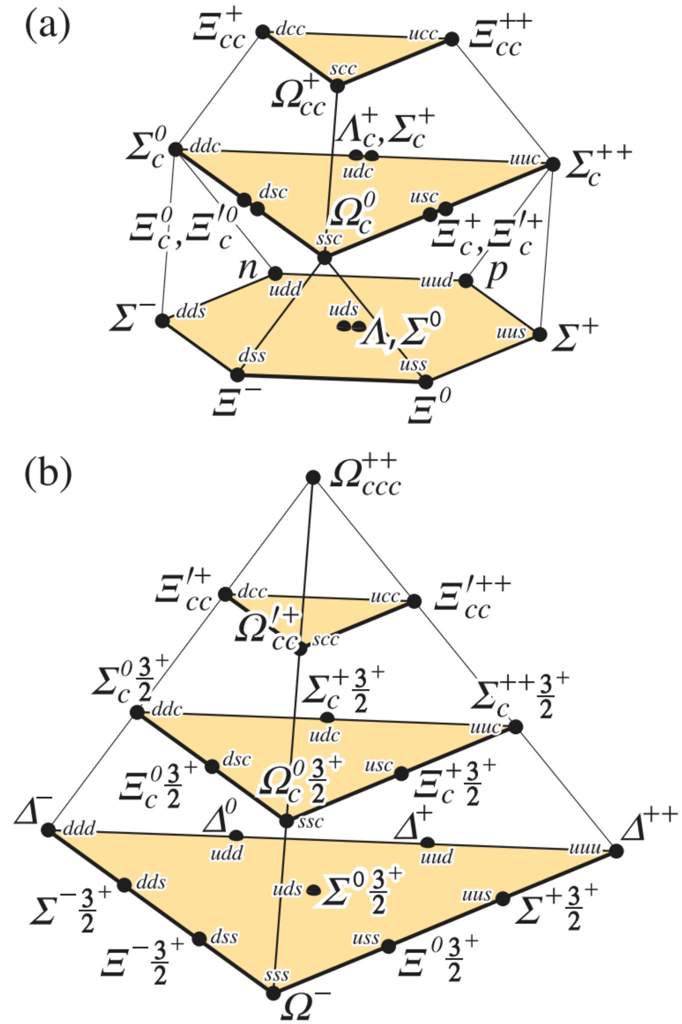
$q = -1/3$ e, $\bar{q} = +1/3$ e





Charmed Mesons and Baryons





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Experimental Evidence for Quarks

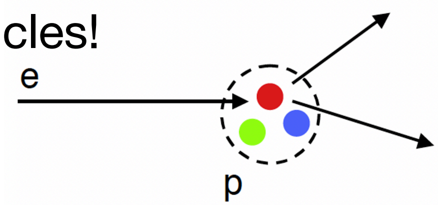
- Rutherford scattering formula for elastic scattering of point-like electrically charged particles like the proton or neutron is violated at high energy
  - An extended charge distribution leads to "**form factors**" in the scattering cross section
- At even higher energies, the proton and neutron breaks up!

But at least need one baryon produced!

  - **Deep inelastic scattering** of leptons off hadrons: $e^- + p \rightarrow e^- + X$
- Leads to "structure functions" (F_1, F_2) in the scattering cross section:
- $$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \frac{1}{\nu} \left[\cos^2(\theta/2) F_2(x, Q^2) + \sin^2(\theta/2) \frac{Q^2}{x m_N^2} F_1(x, Q^2) \right]$$

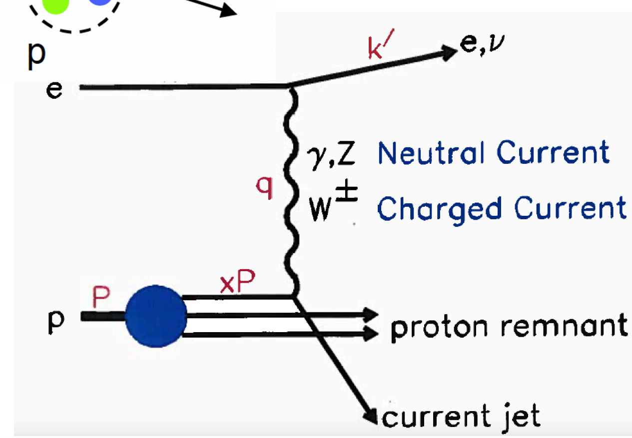
Experimental Evidence for Quarks

- But, point-like scattering is restored if we assume that hadrons themselves are composed of point-like particles!



  - Dubbed "**partons**" by Richard Feynman

- These constituents carry a fraction *x* of the momentum of the proton or neutron



- Incoming electron can undergo Rutherford scattering off of the electrically charged constituents

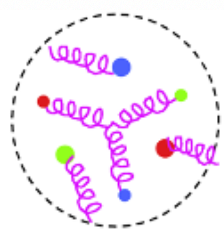
- Approx. independent of exchanged squared momentum Q^2
  - Thus independent of the de Broglie wavelength, i.e. point-like

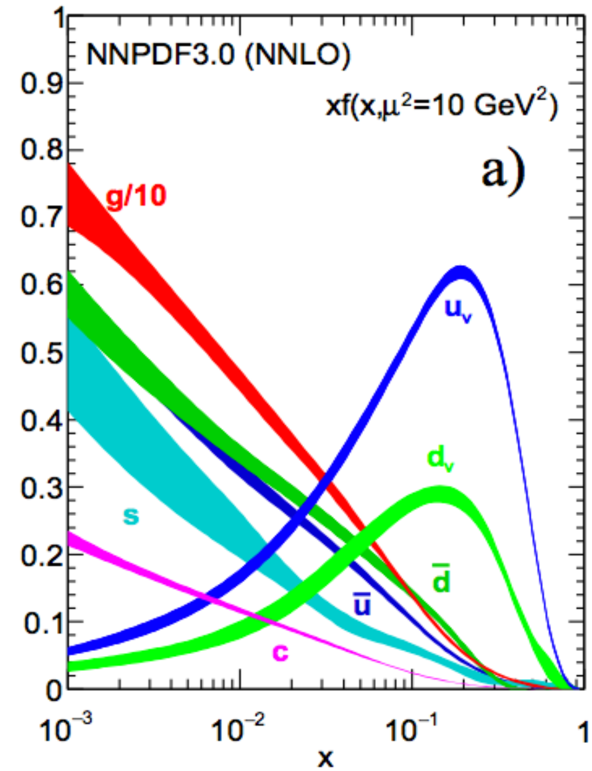
- Can rewrite structure functions in terms of the probability density of finding a parton with momentum fraction x, its squared electric charge, and x:

- $\boxed{F_2(x,Q^2) = \sum_{i=1}^{N_f} x\, e_i^2\, f_i(x)}$, which for just u and d in proton: $F_2(x,Q^2) = x\left[\left(\frac{2}{3}\right)^2 u(x) + \left(-\frac{1}{3}\right)^2 d(x)\right]$

The Parton Distribution in the Proton

- In reality, the proton is comprised not just of u and d quarks, and they do not have each 1/3 of the proton momentum
- There is a distribution of momentum fractions, as compiled from many experiments:
  - The valance quarks are u and d
  - But there is a "sea" of other quarks, and antiquarks
  - The structure function becomes:
  - $F_2(x,Q^2) = x\left\{\frac{4}{9}\left[u(x)+\bar{u}(x)+c(x)+\bar{c}(x)\right]+\frac{1}{9}\left[d(x)+\bar{d}(x)+s(x)+\bar{s}(x)+b(x)+\bar{b}(x)\right]\right\}$
- And gluons carry about half of the momentum!
- The proton is complicated!

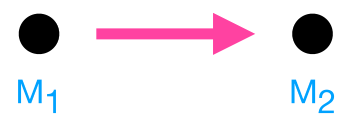




Particle Kinematics

Fixed Target Particle Collisions

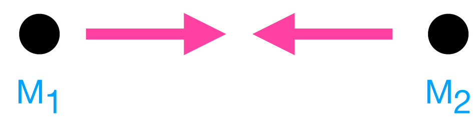
- Suppose a particle of total energy *E* collides with another particle at rest along the z-axis
    - $E_1 = E, \qquad \vec{p}_1 = p_z \hat{k},$ mass M_1 (moving particle)
    - $E_2 = M_2 c^2, \vec{p}_2 = 0,$ mass M_2 (particle at rest)



- The square of the Lorentz invariant center-of-mass energy of this collision is given by:
    - $s = E_{\text{tot}}^2 - c^2 p_{x,\text{tot}}^2 - c^2 p_{y,\text{tot}}^2 - c^2 p_{z,\text{tot}}^2$
    - $s = (M_2 c^2 + E_1)^2 - c^2 p_z^2 = M_2^2 c^4 + 2E_1 M_2 c^2 + (E_1^2 - c^2 p_z^2) = 2E_1 M_2 c^2 + M_1^2 c^4 + M_2^2 c^4$
        - If $E_1 \gg M_1 c^2, M_2 c^2, \quad s \to 2E_1 M_2 c^2$
- The total final state mass that can be created in the collision, at threshold (minimum E_1) is:
    - $M_{\text{tot}} c^2 = \sqrt{s} = \sqrt{2E_1 M_2 c^2 + M_1^2 c^4 + M_2^2 c^4}$
    - The **threshold kinetic energy** of the incident particle is $K_1 = E_1 - M_1 c^2$

Symmetric Particle Collisions

- Now suppose both particles have total energy *E* and collide head-on with each other along the z-axis. Both particles have mass *M*
  - $E_1 = E, \quad \vec{p}_1 = p_z \hat{k}$
  - $E_2 = E, \quad \vec{p}_2 = -p_z \hat{k}$



- The square of the Lorentz invariant center-of-mass energy is given by:
  - $s = E_{\text{tot}}^2 - c^2 p_{x,\text{tot}}^2 - c^2 p_{y,\text{tot}}^2 - c^2 p_{z,\text{tot}}^2$
  - $s = (E + E)^2 - c^2(p_z - p_z)^2 = 4E$
- The total mass that can be created is $M_{\text{tot}} c^2 = \sqrt{s} = 2E$
  - Grows linearly with the incident energy

Collision Example

- The main ring of the Tevatron proton accelerator at Fermilab has a maximum beam energy of 1 TeV (=1000 GeV)

- What is the center-of-mass energy when an accelerated proton collides with another proton at rest?

  - $\sqrt{s} = \sqrt{2(1000 \text{ GeV})(0.938 \text{ GeV})} = 43 \text{ GeV}$

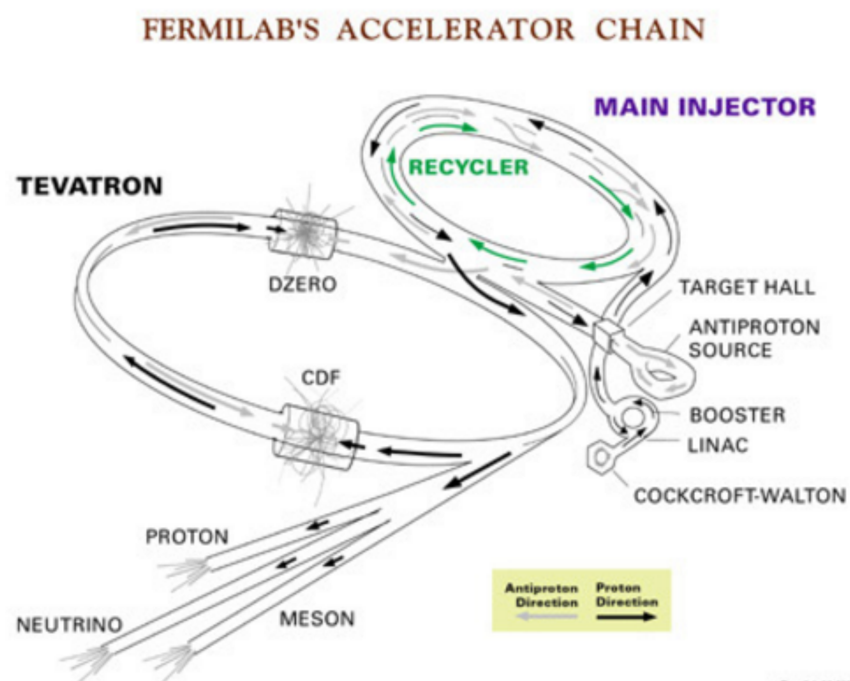
- What is the center-of-mass energy when an accelerated proton collides with an accelerated anti-proton traveling in the opposite direction?

  - $\sqrt{s} = 2E = 2000 \text{ GeV}$

    - The fixed target $\sqrt{s}$ only grows with $\sqrt{E}$, while the collider grows with *E*

  - The collider has enough center-of-mass energy to produce (and discover in 1995!) two top quarks, each with mass $m_t = 170 \text{ GeV}$

The Tevatron Proton-Antiproton collider



In operation 1985-2011.

Particle Decay Kinematics

Consider the decay of an excited kaon into a charged pion and a kaon: $K^{0*} \rightarrow K^+ + \pi^-$. What are the energies of the final particles in the parent's rest frame?

4-vector energy-momentum conservation: $p_{K*} = p_K + p_\pi \Rightarrow p_K = p_{K*} - p_\pi$

$$p_K^2 = (p_{K*} - p_\pi)^2 = p_{K*}^2 + p_\pi^2 - 2p_{K*} \cdot p_\pi$$

$$m_K^2 c^2 = m_{K*}^2 c^2 + m_\pi^2 c^2 - 2p_{K*} \cdot p_\pi \text{ (from Lorentz invariance)}$$

In K^{0*} rest frame: $p_{K*} = (m_{K*}c, 0, 0, 0)$, $p_\pi = (E_\pi / c, p_x, p_y, p_z)$

so the dot product only picks the 0th component:

$$m_K^2 c^2 = m_{K*}^2 c^2 + m_\pi^2 c^2 - 2(m_{K*}c)(E_\pi / c)$$

$$\Rightarrow \boxed{E_\pi = \frac{m_{K*}^2 c^4 + m_\pi^2 c^4 - m_K^2 c^4}{2m_{K*}c^2} \qquad E_K = \frac{m_{K*}^2 c^4 + m_K^2 c^4 - m_\pi^2 c^4}{2m_{K*}c^2}}$$

$m_{K*} = 892 \text{ MeV}/c^2$
$m_K = 494 \text{ MeV}/c^2$
$m_\pi = 140 \text{ MeV}/c^2$
$\Rightarrow E_\pi = 320$ MeV
$\Rightarrow E_K = 572$ MeV

$$\boxed{\begin{aligned} K_\pi &= E_\pi - m_\pi c^2 \\ K_K &= E_K - m_K c^2 \end{aligned}}$$