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Special Relativity

PHY 3101 Modern Physics

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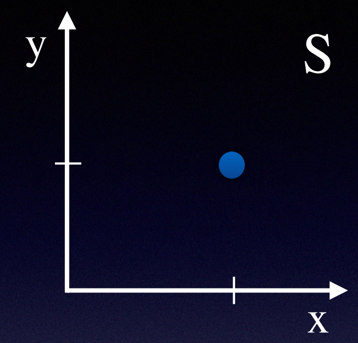
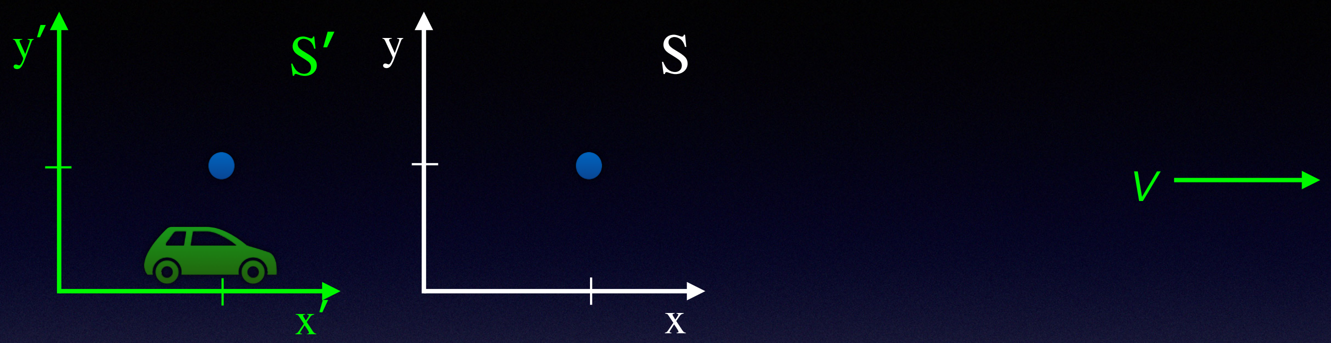
Newtonian Relativity



- Galileo and Newton described the motion of objects with respect to a particular **reference frame**, which is basically a **coordinate system** attached to a particular observer
- A reference frame in which Newton's Laws hold is called an **inertial frame**. It is a frame that is *not accelerating*
- **Newtonian Principle of Relativity** (Galilean Invariance):
  - If Newton's Laws hold in one inertial frame, they also hold in a reference frame moving at a constant velocity relative to the first frame. So the other frame is also an inertial frame.

Galilean Transformation



- Transform from frame S′ to S
  - $x = x' + vt'$
  - $y = y'$
  - $z = z'$
  - $t = t'$

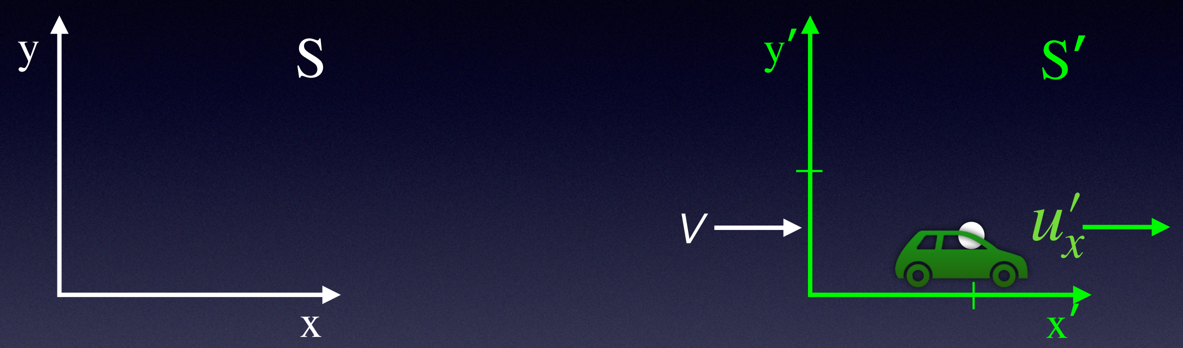
universal time

- Transform from frame S to S′
  - $x' = x - vt$
  - $y' = y$
  - $z' = z$
  - $t' = t$

Symmetric expressions, only sign of v changes

Addition of Velocities

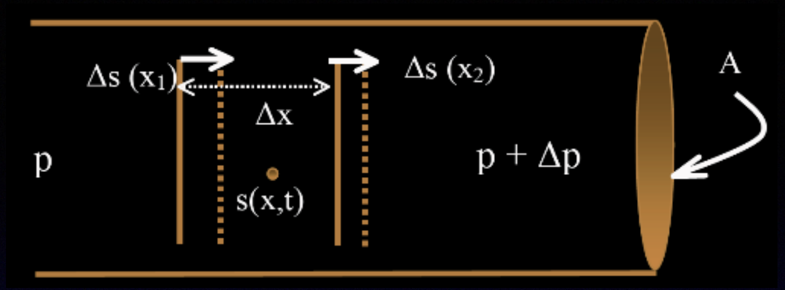
- Consider tossing a ball forward from a moving car. What is the velocity of the ball in the frame of reference of the road?



- $x = x' + vt'$
- $\dfrac{dx}{dt} = \dfrac{dx'}{dt} + v$
- $\boxed{u_x = u'_x + v}$

It's just the sum of the velocity of the ball in the car's frame and the velocity of the car with respect to the road's frame

Sound Waves



- Sounds waves are longitudinal pressure waves that propagate through matter

$$\Delta p(x,t) = \Delta p_m \sin(kx \pm \omega t + \phi) = \Delta p_m \sin\left[k(x \pm vt) + \phi\right]$$

- $v = \sqrt{\dfrac{B}{\rho}}$ B = bulk modulus of material, ρ = mass density
- For air at STP (20°C and 1 atm) v= 343 m/s = 765 mph
- Sound propagates at this speed relative to the air (matter), not the source
- Without matter, as in space, *no one can hear you scream!* 😱

EM Waves

- Traveling wave solution to Maxwell's equations

$$\mathbf{E}(\mathbf{x},t)=\mathbf{E}_m\sin(\mathbf{k}\cdot\mathbf{x}\pm\omega t-\phi) \qquad \mathbf{B}(\mathbf{x},t)=\mathbf{B}_m\sin(\mathbf{k}\cdot\mathbf{x}\pm\omega t-\phi)$$

$\mathbf{E}_m,\mathbf{B}_m =$ amplitude of electric and magnetic fields

$\omega =$ angular frequency (rad/s) $= 2\pi f$

$f =$ cyclic frequency (s^{-1}, Hz)

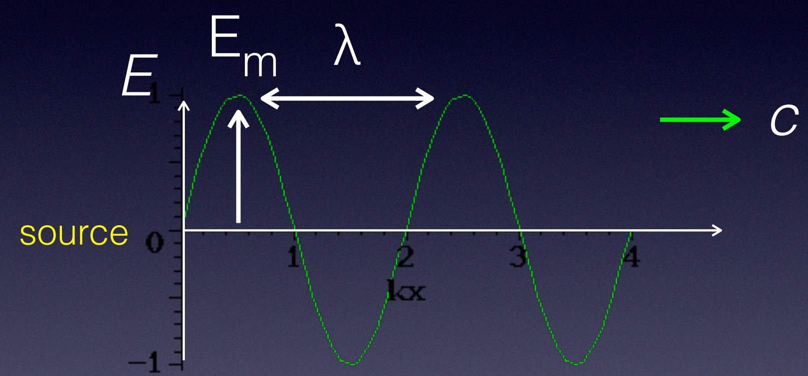
$T =$ period (s) $= 1/f$

$k =$ angular wavenumber (m^{-1}) $= 2\pi/\lambda$

$\lambda =$ wavelength (m)

$$v=\frac{\omega}{k}=\frac{1}{\sqrt{\varepsilon_0\mu_0}}=c \quad \text{phase velocity of wave (m/s)}$$

$\phi =$ possible phase constant



$$\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/Nm^2}$$
$$\mu_0 = 4\pi\times10^{-7}\ \mathrm{Ns^2/m^2}$$
$$\Rightarrow c = 2.998\times10^{8}\ \mathrm{m/s}$$

Firm prediction of Maxwell's Eqns.

Michelson-Morley Experiment

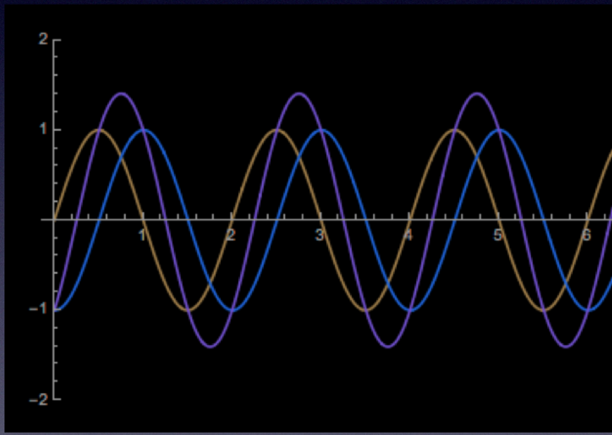
- Video demo of speed of light, and propagation in a vacuum
  - Light does travel at 3×10^8 m/s, and does not need air
- But do EM waves need some sort of "ether" to propagate, like sound needs air?
- If so, then the measured speed of light, for example from the time to propagate between two points, will depend on the speed of the observer through the ether
  - In fact, maybe you could catch up to the speed of light, equivalent to breaking the sound barrier?
- And if the speed of light depends on the observer's reference frame, to which frame do Maxwell's equations apply?
  - Do we need a different equation for every reference frame?

Michelson-Morley Experiment

- The question of whether ether exists was settled by the Michelson-Morley experiment(s) in 1887.

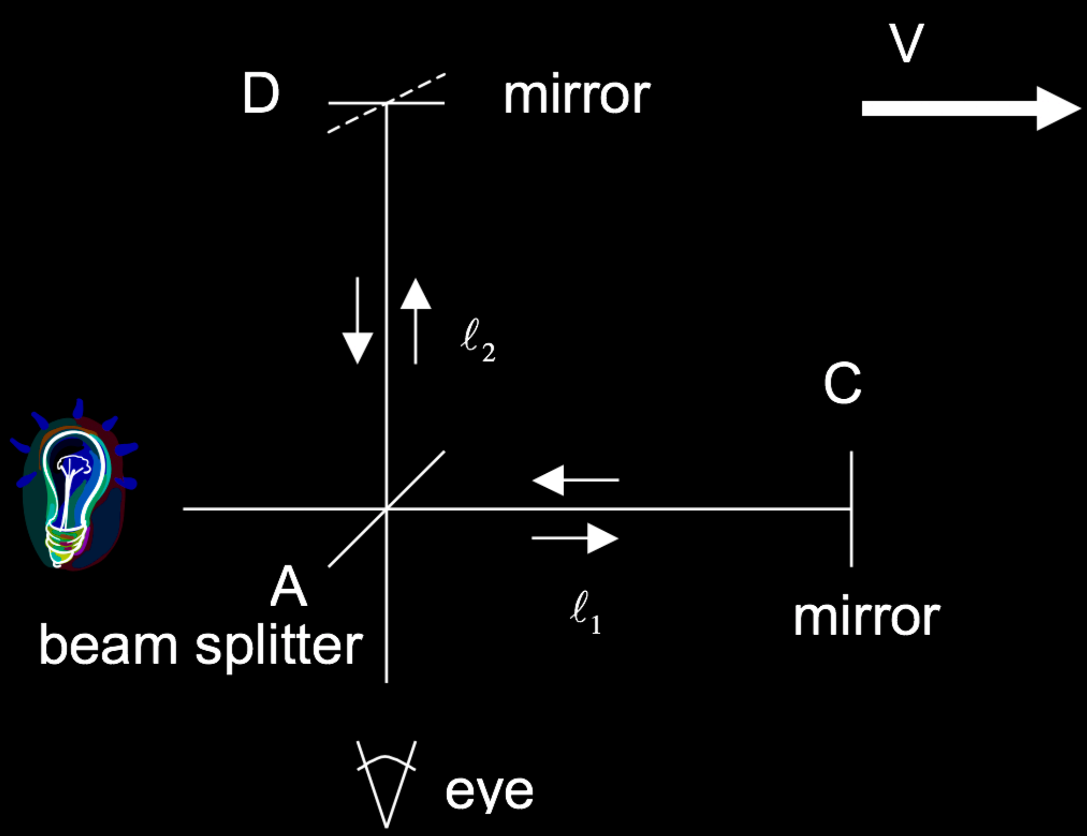
- An **interferometer** was used to separate a light beam into two paths of possibly different length and then recombined.

  - If two light waves are completely in phase, then the amplitude of each wave adds **constructively**. If they are completely out of phase, the amplitudes subtract **destructively**.



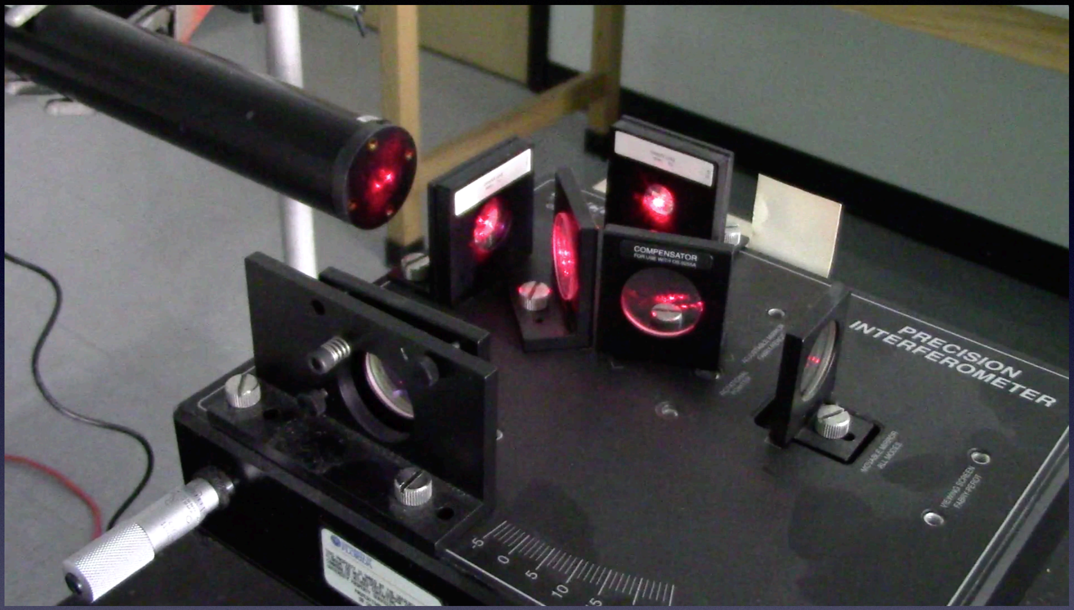
- If light travels at the speed *c* with respect to the ether, it will take a different amount of time to propagate the length of one of the arms of the interferometer depending on the direction the interferomater travels through the ether (i.e. how the Earth travels through the ether)

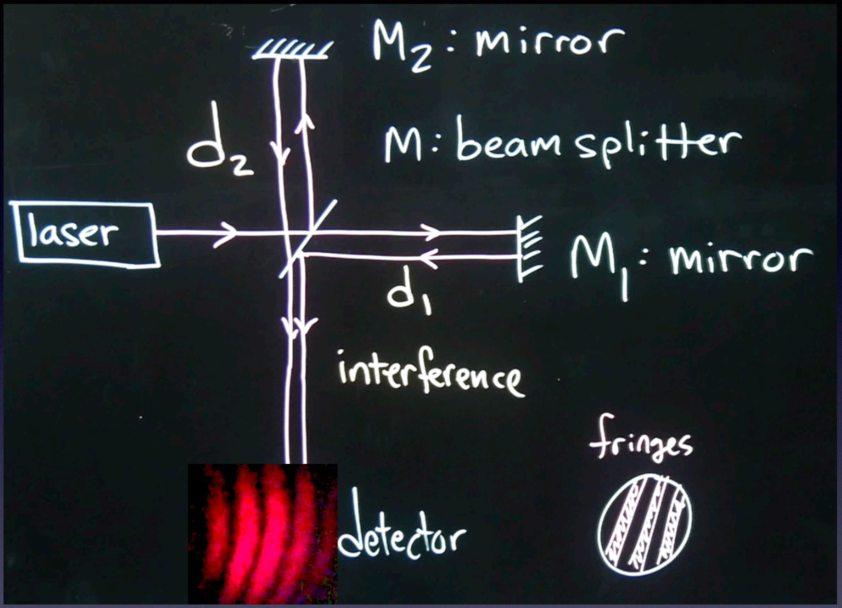
Michelson-Morley Experiment



- The speed of the device through the ether is v
- The time to travel the length of each arm will be different depending whether the arm is oriented parallel to or perpendicular to the direction of motion
- The interference will yield fringes that shift if we rotate the device by 90°

Michelson Interferometer





- Beam splitter splits laser light into two paths of differing lengths
- Beams reflect off mirrors and then recombine
- When difference in roundtrip path lengths is a multiple of the wavelength, constructive interference leads to a bright fringe: $2(d_1 - d_2) = n\lambda$

Michelson-Morley Experiment

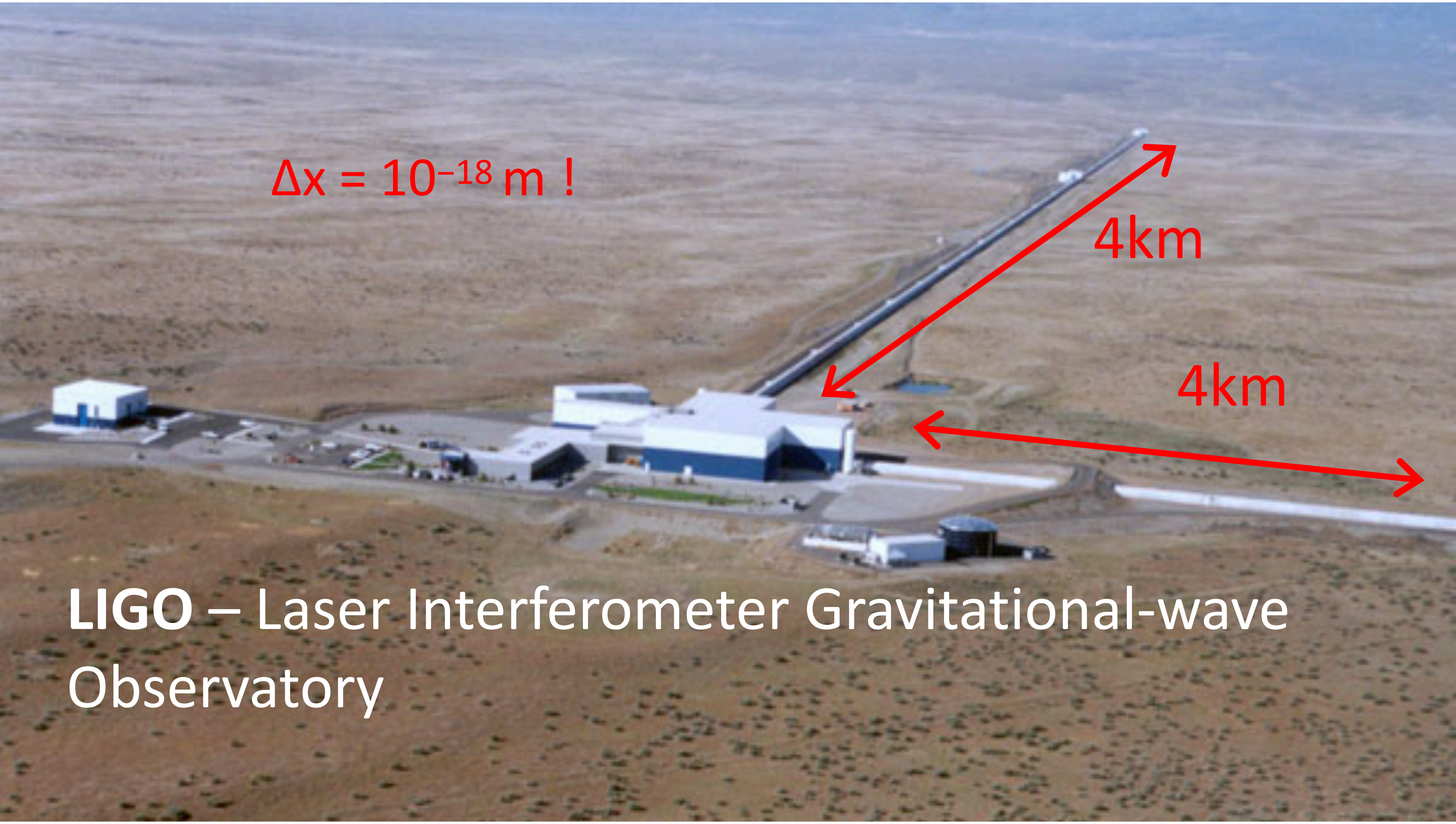
- Skipping details on calculating the difference in propagation time between the two arms, and converting this difference to a fraction of a wavelength yields:
  - $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v^2(\ell_1 + \ell_2)}{c^2\,\lambda}$

We'll anyway get practice soon on calculating such time differences…

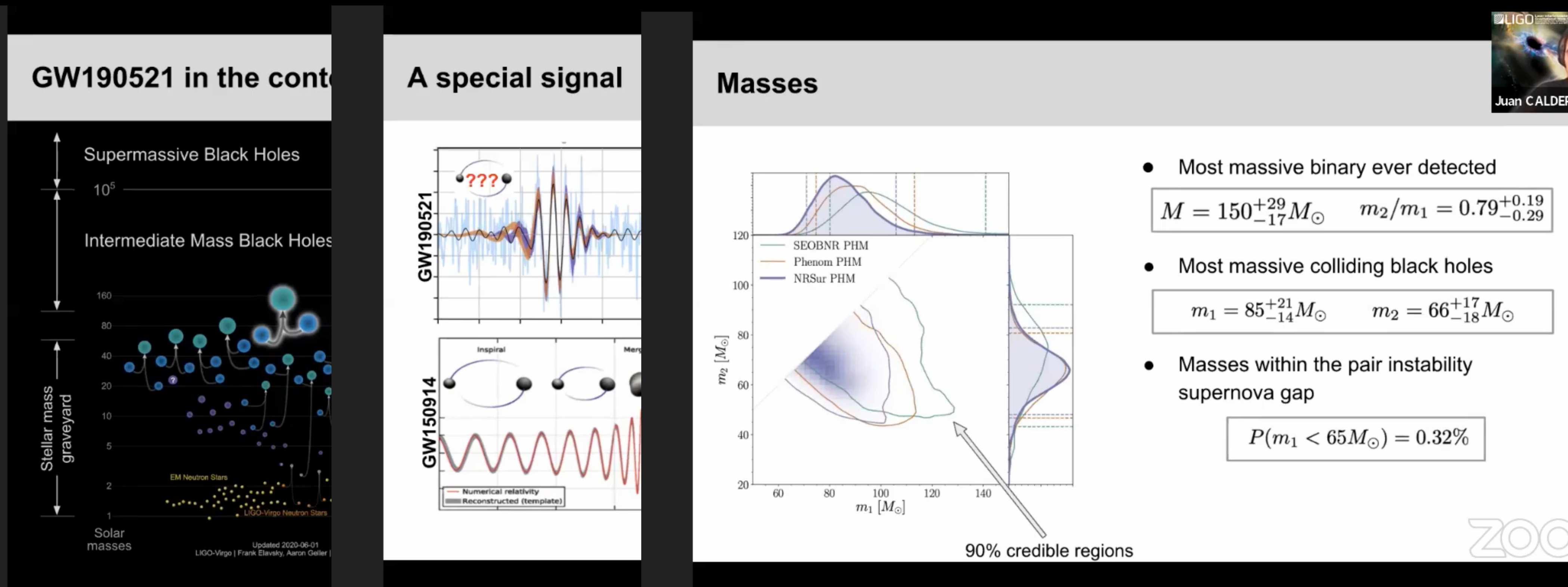
- What we expect to see in the Michelson-Morely experiment is a shift in the interference fringes as one rotates the interferometer by 90 degrees. Let's see how big a shift:
  - $\ell_1 = \ell_2 = 11\ \mathrm{m}$
  - $v = 3.0 \times 10^4\ \mathrm{m/s}$ (Earth's orbital velocity)
  - $c = 3.0 \times 10^8\ \mathrm{m/s}$
  - $\lambda = 6.0 \times 10^{-7}\ \mathrm{m}$ (red light)
  - $\Rightarrow \Delta\lambda/\lambda = 0.4$ (i.e. nearly a maximum shift!)

No evidence of an ether…

- Result: **no shift was ever seen**, even after checking at different times of the year (orbit)


Δx = 10^–18 m !
4km
4km
LIGO – Laser Interferometer Gravitational-wave Observatory

Discovery of Gravitational Waves from Colliding Black holes



Webinar 9/3/2020 LIGO/Virgo

Question

- The Earth is located a distance of 25,000 light-years from the center of our Milky Way galaxy. Its orbital period is about 240 million years. What is its orbital speed in m/s? What fraction of the speed of light is this?

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Special Relativity

Einstein's Postulates

1. **The laws of physics, including electromagnetism, are the same in all inertial frames**

2. **Every observer measures the same value c for the speed of light (in vacuum) in all inertial frames**

The second postulate is really a consequence of the first, because if Maxwell's equations hold in all inertial frames, then the only possible value for the speed of light is c.

These postulates embody **Einstein's Special Theory of Relativity**, first published in 1905 in a paper titled "On the Electrodynamics of Moving Bodies"

- Later, gravity and acceleration would be incorporated in General Theory of Relativity

As in Newtonian Relativity, there is no way to detect absolute motion. Only the relative velocities between two inertial reference frames matters. All observers are equal!

A rocket moves away from the Earth, and the Earth moves away from the rocket

Basic Definitions

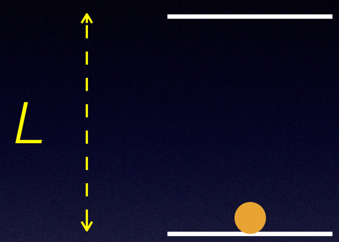
- **Events** are physical phenomena that occur *independent* of any reference frame
    - For example: a flash, explosion, return of a spaceship, or disintegration of a subatomic particle
- **Observers** record events, both the time and spatial coordinates, *in a particular reference frame*
    - For example, Mission Control in Houston marking down the time and location of the splashdown of a space capsule. The reference frame in this case is the Earth
- **Simultaneous events** occur when the light signals from two events reach an observer at the same time
- **Relativity of Simultaneity:** Two events simultaneous in one inertial frame are not simultaneous in any other frame. This is a consequence of Einstein's Postulates

Basic Definitions, Continued

- **Proper time** (t_0 or τ) is the time difference between two events occurring at the same position
- **Rest frame** is the inertial frame where two events are only separated by time. The frame in which the proper time is measured
- **Proper length** (L_0) is the distance between two positions at rest. The length measured in the rest frame.

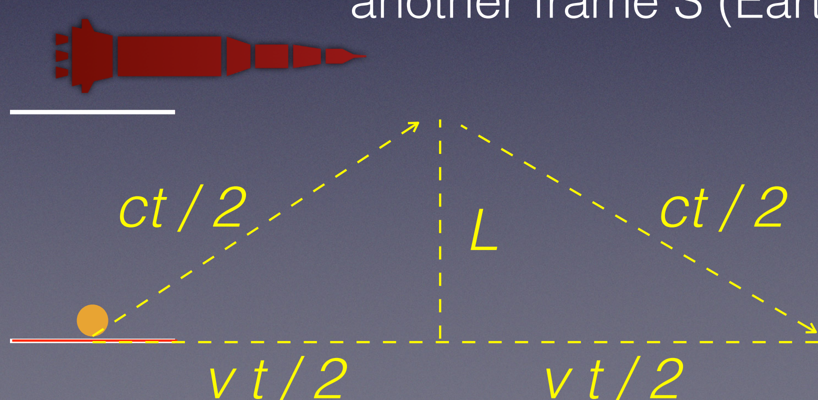
A Light Clock, and Time Dilation

frame S′



- Consider a light pulse bouncing between 2 mirrors, in the reference frame of the mirrors (frame S′). (*c* = speed of light)
- Total round trip time is $t' = \dfrac{L}{c} + \dfrac{L}{c} = \dfrac{2L}{c}$
- Now put the light clock on a spaceship traveling at speed v, but measure the roundtrip time of the light pulse from another frame S (Earth)

frame S



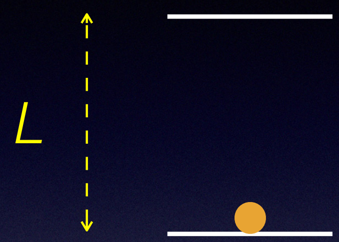
$$L^2 + v^2t^2/4 = c^2t^2/4 \quad \rightarrow L^2 = (c^2 - v^2)t^2/4$$

$$t^2 = \frac{4L^2}{c^2 - v^2} = \frac{4L^2}{c^2}\frac{1}{\left(1 - v^2/c^2\right)}$$

$$\rightarrow t = \frac{2L}{c}\frac{1}{\sqrt{1 - v^2/c^2}} = \boxed{\frac{t'}{\sqrt{1 - v^2/c^2}}}$$

Time Dilation Summary

frame S′

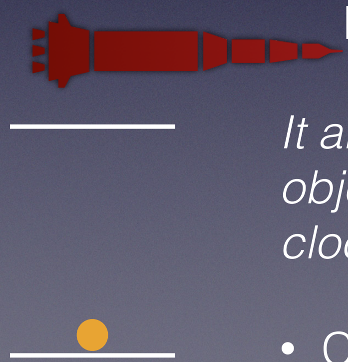


- In the rest frame of the light clock, the total round trip time is: $t' = \dfrac{2L}{c}$

- In another frame watching the clock move by with speed v, the total round trip time is:

$$t = \gamma\, t' \quad \text{where} \quad \gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

frame S

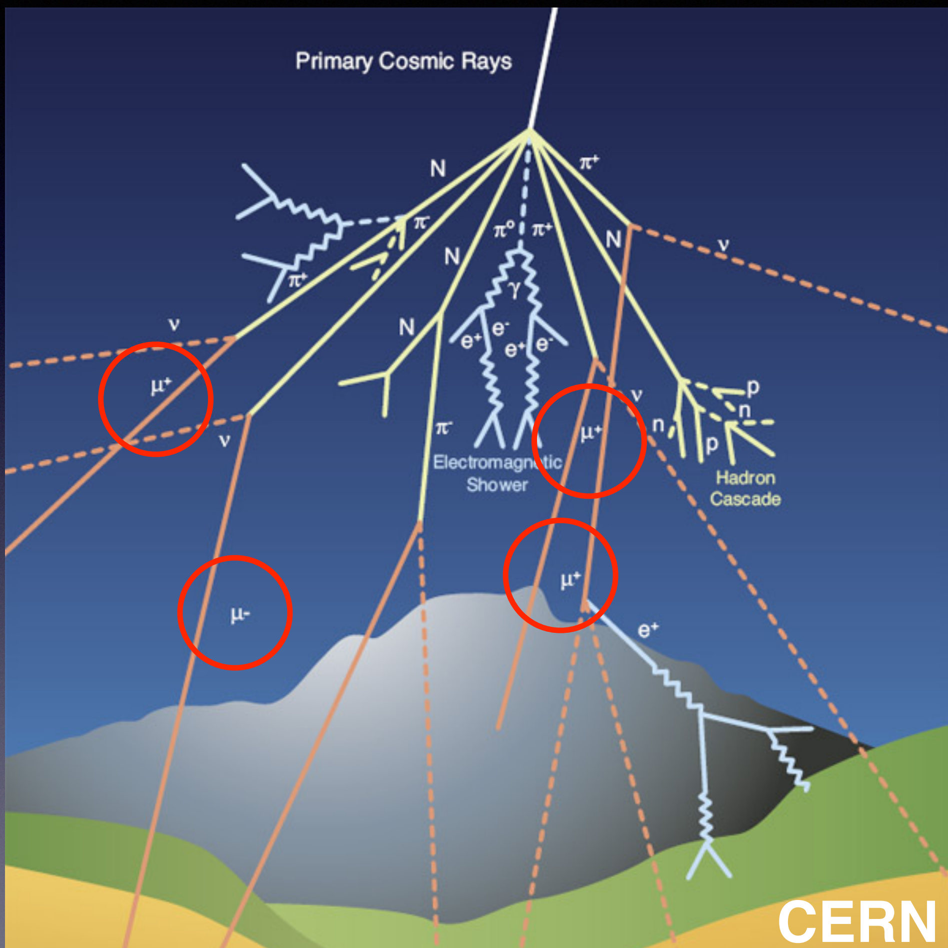


Note that $\gamma \geq 1$, so $t \geq t'$ always ("dilation")

It also doesn't matter which frame is the clock or which is the Earth! Any object that moves by with a significant velocity appears to have its clock running slow.

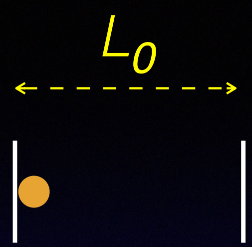
- Clocks on Earth appear slow to astronauts on rocket

Experimental Verification of Time Dilation



- Cosmic rays (protons, light nuclei) undergo nuclear collisions in upper atmosphere to create showers of subatomic particles
- Longest-lived penetrating particle is the muon (μ). Average lifetime is 2.2 microseconds
- But without time dilation, average travel distance at speed of light is $d = ct = (3 \times 10^8\,\mathrm{m/s})(2.2 \times 10^{-6}\,\mathrm{s}) = 660\,\mathrm{m}$
- With time dilation, distance is $d = \gamma\, c\, t$, and can reach kilometers and is thus detectable at the surface of the Earth
  - Flux can be studied as a function of altitude to verify time dilation dependence

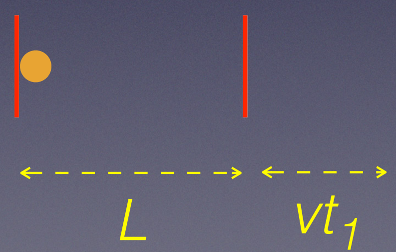
Length Contraction



- Can use a light clock also to define length, based on the time it takes light to make a round trip

- In rest frame: $L_0 = \dfrac{ct_0}{2}$

- Now put the light clock on a spaceship traveling at speed v, but measure the roundtrip time of the light pulse from another frame S (Earth)



- $t_1 =$ time out
- $t_2 =$ time back
- $t = t_1 + t_2$

- $L + vt_1 = ct_1 \rightarrow t_1 = \dfrac{L}{c - v}$
- $L - vt_2 = ct_2 \rightarrow t_2 = \dfrac{L}{c + v}$
- $t = t_1 + t_2 = \dfrac{L(c+v) + L(c-v)}{c^2 - v^2} = \dfrac{2Lc}{c^2 - v^2}$

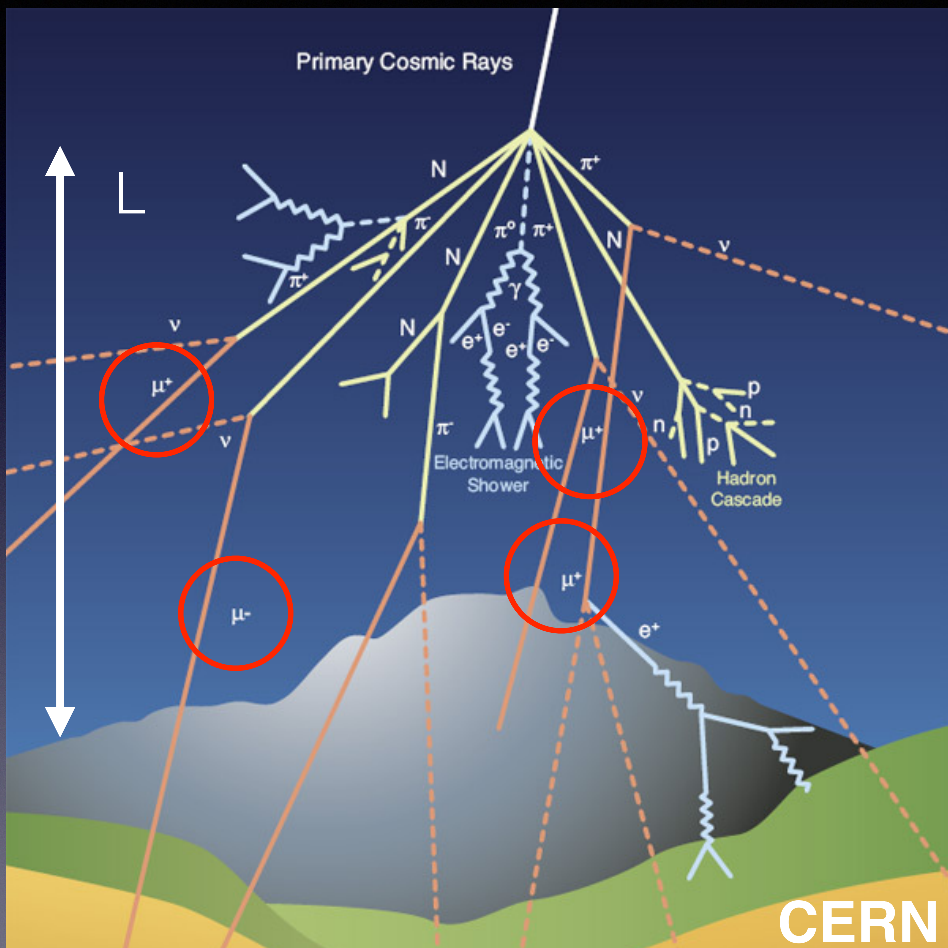
Length Contraction, Continued

- $t = t_1 + t_2 = \dfrac{2Lc}{c^2 - v^2} = \dfrac{2L}{c}\dfrac{1}{1 - v^2/c^2}$
- $L = \dfrac{ct}{2}\left(1 - v^2/c^2\right)$
- Note that from time dilation $t = \dfrac{t'}{\sqrt{1 - v^2/c^2}}$
- $\rightarrow L = \dfrac{ct'}{2}\sqrt{1 - v^2/c^2} = L_0\sqrt{1 - v^2/c^2}$
- $\boxed{L = \dfrac{L_0}{\gamma}}$ where $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

Length contraction and time dilation are really two sides of same coin

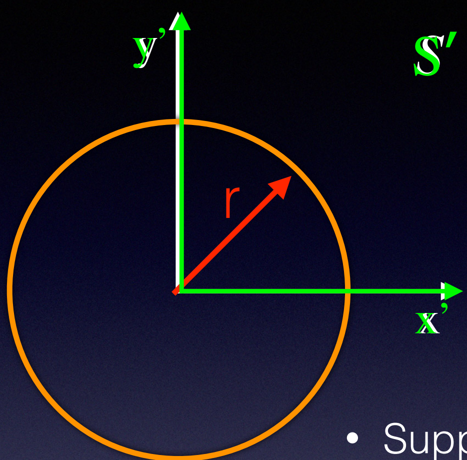
Note that $\gamma \geq 1$, so $L \leq L_0$
Length is *contracted*

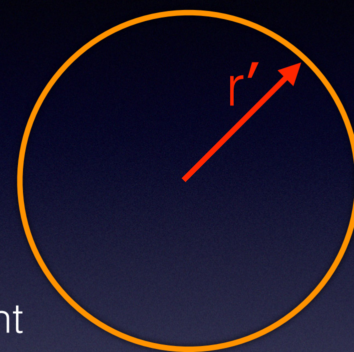
Cosmic Muons, Alternative Viewpoint



- Average muon lifetime is 2.2 microseconds
- In the muon rest frame, this implies that the average distance it can travel at near speed of light is
  $d = ct = (3 \times 10^8\,\mathrm{m/s})(2.2 \times 10^{-6}\,\mathrm{s}) = 660\,\mathrm{m}$
- But the thickness of the Earth's atmosphere as observed from the muon's frame of reference is *length contracted* as:
  $$L = \frac{L_0}{\gamma}$$
- Thus even with an atmosphere 20km thick, it can appear less than 660 m if $\gamma \geq 30$
  - Thus muon can reach surface of Earth

Lorentz Transformation Derivation





- Suppose that there is a single flash of isotropic light at the origin of an inertial frame S at time $t = 0$
  - $r^2 = (ct)^2 = x^2 + y^2 + z^2$
- Now suppose that the same light flash is observed by another frame S′, traveling along the horizontal direction, that happens to coincide with S at $t' = t = 0$
- Outgoing sphere of light in frame S′ should be observed spherical in that frame also
  - $r'^2 = (ct')^2 = x'^2 + y'^2 + z'^2$

Lorentz Transformation

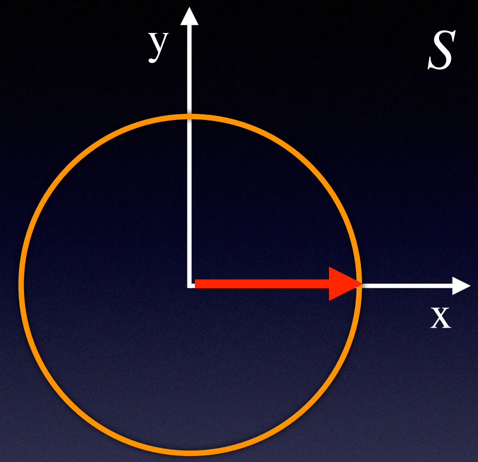
- Expect that coordinates transverse to the direction of ***v*** will not be affected by the horizontal velocity. Namely:
  - $y' = y$
  - $z' = z$

Recall that they were not affected by length contraction

- The coordinate parallel to the direction of ***v*** will be affected. Assume it will be a linear transformation:
  - $x' = k(x - vt)$ $\qquad x = k'(x' + vt')$
- But the transformation equations should have the same form, as there is no special reference frame. Only the relative velocity matters.
  - $k' = k$

Lorentz Transformation

- Consider the outgoing light propagation from the flash at t = 0 along the x-axis ($y = z = 0$)
  - $x' = ct'$ in frame S′ $\qquad x = ct$ in frame S
- Now plug these into the transformation equations:
  - $x' = k(x - vt) = k(ct - vt) = kct(1 - v/c)$
  - $x = k(x' + vt') = k(ct' + vt') = kct'(1 + v/c)$
- Plug these into light propagation equations:
  - $ct' = x' = kct(1 - v/c) \longrightarrow t' = kt(1 - v/c)$
  - $ct = x = kct'(1 + v/c) \longrightarrow t = kt'(1 + v/c)$
- Plug t' into t:
  - $t = k^2t(1 + v/c)(1 - v/c) = tk^2(1 - v^2/c^2)$



$$\boxed{k = \frac{1}{\sqrt{1 - v^2/c^2}} \equiv \gamma}$$

Lorentz Transformation

- So the modified transformation equations for the spatial coordinates are:
  - $x' = \gamma(x - vt)$ $\qquad$ $x = \gamma(x' + vt')$
  - $y' = y$
  - $z' = z$
- What about the transformation of time?
  - Plug x′ into x:
  - $x = \gamma(\gamma x - \gamma vt + vt')$
  - Solve for t′

$$x - \gamma^2 x + \gamma^2 vt = \gamma vt'$$

$$x(1 - \gamma^2) + \gamma^2 vt = \gamma vt'$$

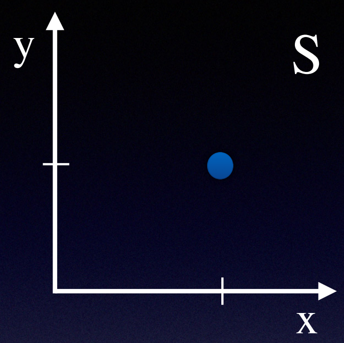
$$x\frac{1 - v^2/c^2 - 1}{1 - v^2/c^2} + \gamma^2 vt = \gamma vt'$$

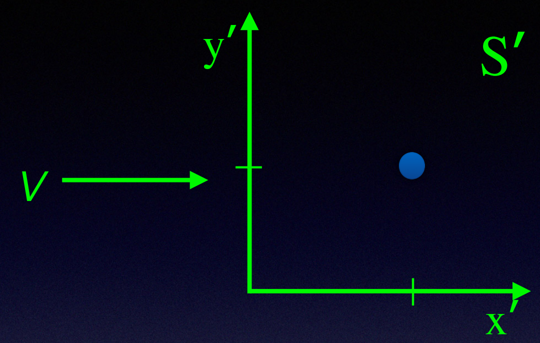
$$-\gamma^2 xv^2/c^2 + \gamma^2 vt = \gamma vt'$$

$$t' = \frac{1}{\gamma v}\left(- \gamma^2 xv^2/c^2 + \gamma^2 vt\right)$$

$$\boxed{t' = \gamma(t - vx/c^2)}$$

Lorentz Transformation Summary





- Transform from frame S′ to S
  - $x = \gamma(x' + vt')$
  - $y = y'$
  - $z = z'$
  - $t = \gamma(t' + vx'/c^2)$

We sometimes call the transformation from one frame to another a “Lorentz boost”

- Transform from frame S to S′
  - $x' = \gamma(x - vt)$
  - $y' = y$
  - $z' = z$
  - $t' = \gamma(t - vx/c^2)$

$$\boxed{\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}}$$

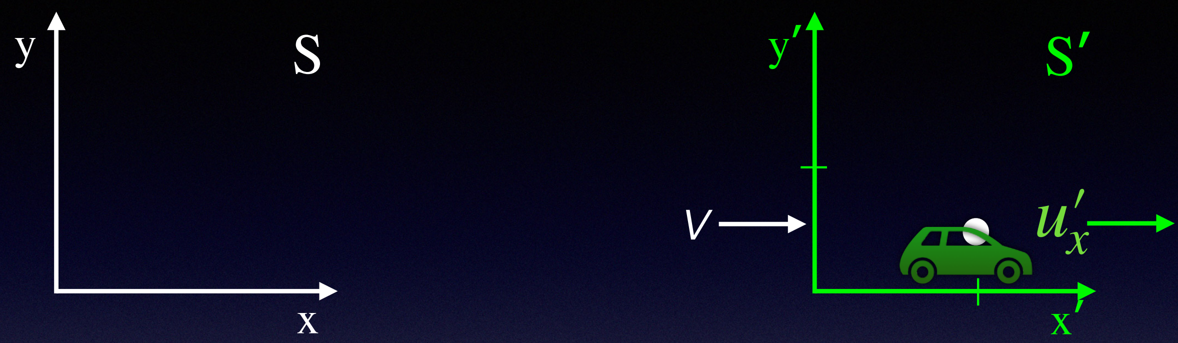
Lorentz Transformation Features

- $x = \gamma(x' + vt')$
- $y = y'$
- $z = z'$
- $t = \gamma(t' + vx'/c^2)$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

- We recover the Galilean transformation if $c \rightarrow \infty$ or $v \rightarrow 0$ so that $\gamma \rightarrow 1$
- Space and time coordinates are mixed (x,t)
  - Two events that are simultaneous in frame S′ (say at time t′=0 and at positions x_1' and x_2') are not simultaneous in frame S ($t_1 \neq t_2$)
- No change in form of equations from one frame to another (Einstein's 1st postulate)
- Only relative velocities matter
- One can derive time dilation and length contraction from this transformation
  - If a clock of period t' sits at rest in frame S′ at position x′=0, then an observer in frame S measures the period of the clock to be $t = \gamma t'$

Relativistic Addition of Velocities



- $x = \gamma(x' + vt') \rightarrow dx = \gamma(dx' + vdt')$
- $t = \gamma(t' + vx'/c^2) \rightarrow dt = \gamma(dt' + vdx'/c^2)$
- $u_x = \dfrac{dx}{dt} = \dfrac{dx' + vdt'}{dt' + vdx'/c^2}$
- $u_x = \dfrac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}}$

$$\boxed{u_x = \frac{u'_x + v}{1 + vu'_x/c^2}}$$

For velocities along direction of motion

Relativistic Addition of Velocities, Summary

- $u_x = \dfrac{u'_x + v}{1 + vu'_x/c^2}$ parallel component
- $u_y = \dfrac{u'_y}{\gamma(1 + vu'_x/c^2)}$ where $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$
- $u_z = \dfrac{u'_z}{\gamma(1 + vu'_x/c^2)}$ perpendicular components

Addition of Velocities Example

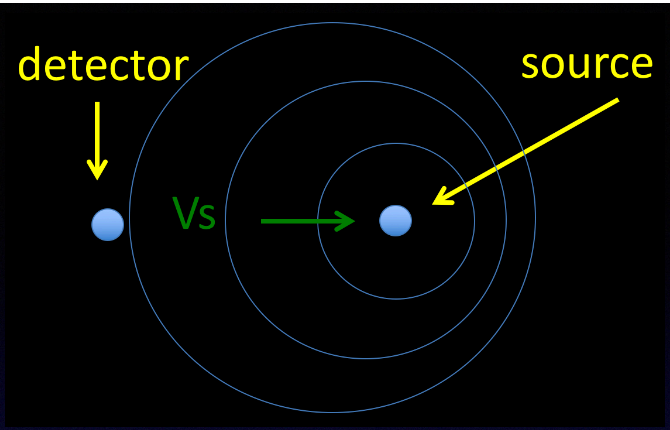
- Consider a spacecraft that travels at 0.6c from Earth and that launches a projectile forward with a relative velocity of 0.8c in the same direction. What is the velocity of the projectile from Earth?

  - Galilean: $u_x = 0.8c + 0.6c = 1.4c > c\,!$

  - Lorentz: $u_x = \dfrac{0.8c + 0.6c}{1 + (0.6)(0.8)} = 0.946c < c$

The final velocity will always be < *c* in the relativistic case. Try it!

Sound Doppler Effect

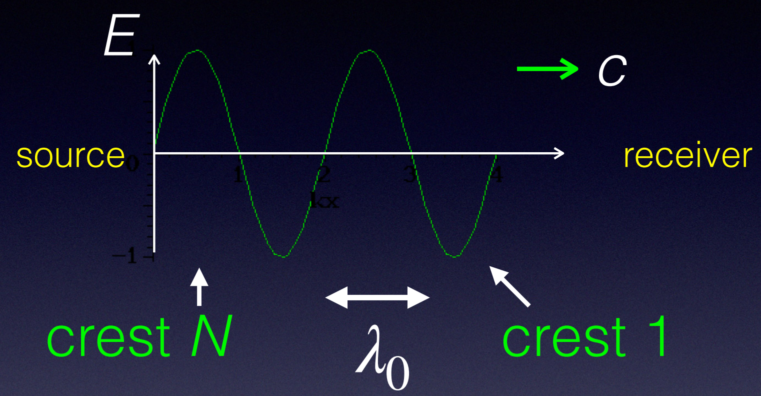


- The Doppler effect is a change in pitch (frequency) of a sound wave due to the motion of the source emitting the sound waves and/or the detector receiving them
- Sound waves propagate in the reference frame of the air at the speed of sound
- The perceived frequency depends on the motion of the source and the receiver
- $f' = f\dfrac{v \pm v_D}{v \pm v_S}$
- Increase the frequency if the source or detector move toward the other
- Decrease the frequency if the source or detector moves away from the other

Doppler Effect for Light (EM Waves)?

- Is there a Doppler effect for EM waves in the absence of an ether?

- Consider an EM wave traveling along x-axis



- A source emits *N* wave crests in a time T_0 in the inertial frame of the source emitter

- Wavelength is λ_0 , wave speed is *c*

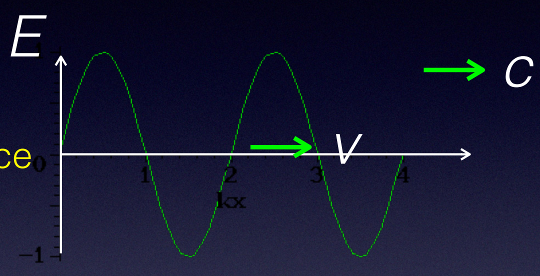
- Length of wave train is $L_0 = N\lambda_0 = cT_0$

- $\lambda_0 = \dfrac{cT_0}{N}$, and since $\lambda f = c$ for EM waves

- $f_0 = \dfrac{c}{\lambda_0} = \dfrac{N}{T_0}$ is frequency of light in the rest frame of the source

Doppler Effect for EM Waves

- Now consider a receiver in a different inertial frame from the source
- Again compute the frequency received from the length of the wave train:
- $L = N\lambda = cT \mp vT$
- $\lambda = \dfrac{c \mp v}{N} T$
- $f = \dfrac{c}{\lambda} = \dfrac{cN}{(c \mp v)T}$



choose : "−" for source approaching receiver,
"+" for receding

- From time dilation, $T = \gamma T_0$
- $f = \dfrac{c}{(c \mp v)\gamma}\left(\dfrac{N}{T_0}\right) = \dfrac{f_0}{\gamma(1 \mp v/c)}$
- $f = \dfrac{f_0\sqrt{1 - v^2/c^2}}{(1 \mp v/c)} = \dfrac{f_0\sqrt{(1 - v/c)(1 + v/c)}}{(1 \mp v/c)}$

Doppler Effect for EM Waves

- Source and receiver approaching each other:
  - $f = f_0 \sqrt{\dfrac{1 + v/c}{1 - v/c}}$
  - $f \geq f_0$ higher frequency received $\Rightarrow$ "**blue shifted**"
- Source and receiver receding from each other:
  - $f = f_0 \sqrt{\dfrac{1 - v/c}{1 + v/c}}$
  - $f \leq f_0$ lower frequency received $\Rightarrow$ "**red shifted**"

Doppler Effect for EM Waves

- Modern electronics allows for precise measurements of frequencies
- Thus can solve for the velocity of an object from the change in frequency between emitted EM waves and those waves received after reflection off an object
  - Police radar
  - Doppler weather radar

Note that a reflection actually corresponds to the Doppler effect applied twice

- Also can solve for the velocity of galaxies emitting known atomic light frequencies
  - Expansion of universe (Big Bang)

Example

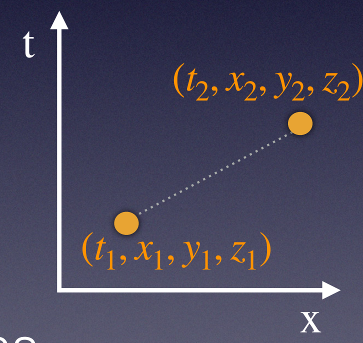
- How fast do you need to travel to make a red traffic light appear green?
  - $\lambda_{\text{red}} = 700$ nm, $\lambda_{\text{green}} = 550$ nm, $c = f\lambda$
  - Traveling toward light ⇒ blue shifted
  - $$\frac{f_{\text{green}}}{f_{\text{red}}} = \frac{c\lambda_{\text{red}}}{c\lambda_{\text{green}}} = \sqrt{\frac{1 + v/c}{1 - v/c}}$$
  - $$\frac{v}{c} = \frac{\left(\frac{\lambda_{\text{red}}}{\lambda_{\text{green}}}\right)^2 - 1}{\left(\frac{\lambda_{\text{red}}}{\lambda_{\text{green}}}\right)^2 + 1} = 0.24$$

Sorry officer, it looked green to me!

Lorentz Invariance

- We have seen that some quantities change from one inertial frame to another (length, time, velocity, frequency). Is there anything that is constant (aside from the speed of light)?

- Let's generalize the Cartesian 3D distance to a 4D space-time interval:
  - $(\Delta s)^2 = (c\Delta t)^2 - \left[(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2\right]$
  - where $\Delta t = t_2 - t_1$, $\Delta x = x_2 - x_1$, etc.



- Can show that this metric is invariant to Lorentz transformations
  - Therefore it is called a <u>Lorentz invariant</u> quantity

Lorentz Invariance

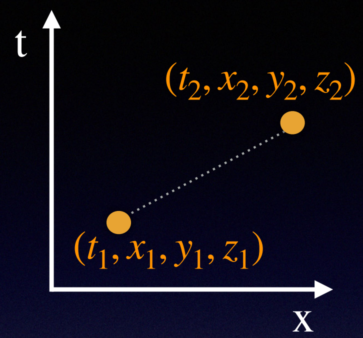
- Consider two events that happen in rest frame S' (e.g. particle creation and decay at origin)
  - Event 1: $t_1' = 0;\ x_1' = 0;\ y_1' = 0;\ z_1' = 0$
  - Event 2: $t_2' = T_0;\ x_2' = 0;\ y_2' = 0;\ z_2' = 0$
  - $(\Delta s')^2 = (c\Delta t')^2 - (\Delta x')^2 - (\Delta y')^2 - (\Delta z')^2 = c^2T_0^2 - 0 - 0 - 0 \ \Rightarrow\ \Delta s' = cT_0$
- Make Lorentz transformation to a frame S moving at velocity v:
  - $x = \gamma(x' + vt'),\ \ y = y' = 0; z = z' = 0,\ \ t = \gamma(t' + vx'/c^2)$
- Compute space-time interval Δs :
  - $x_1 = 0,\ x_2 = \gamma v T_0 \ \Rightarrow\ \Delta x = \gamma v T_0,\ \Delta y = \Delta z = 0 \qquad t_1 = 0,\ t_2 = \gamma T_0 \ \Rightarrow\ \Delta t = \gamma T_0$
  - $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 = c^2\gamma^2T_0^2 - \gamma^2v^2T_0^2 - 0 - 0$
  - $(\Delta s)^2 = c^2\gamma^2T_0^2(1 - v^2/c^2) \ \Rightarrow\ (\Delta s)^2 = \dfrac{c^2T_0^2(1 - v^2/c^2)}{1 - v^2/c^2} = c^2T_0^2$

$$\Rightarrow \Delta s = \Delta s' = cT_0$$

Interval is invariant!

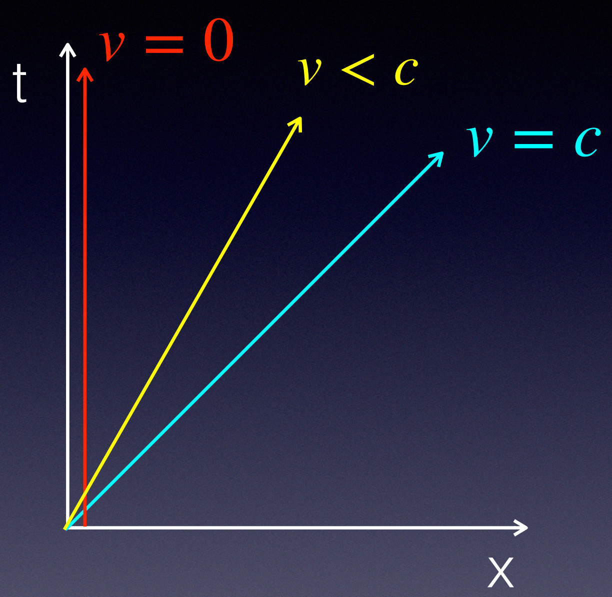
Lorentz Invariance

- $(\Delta s)^2 = (c\Delta t)^2 - \left[(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2\right]$



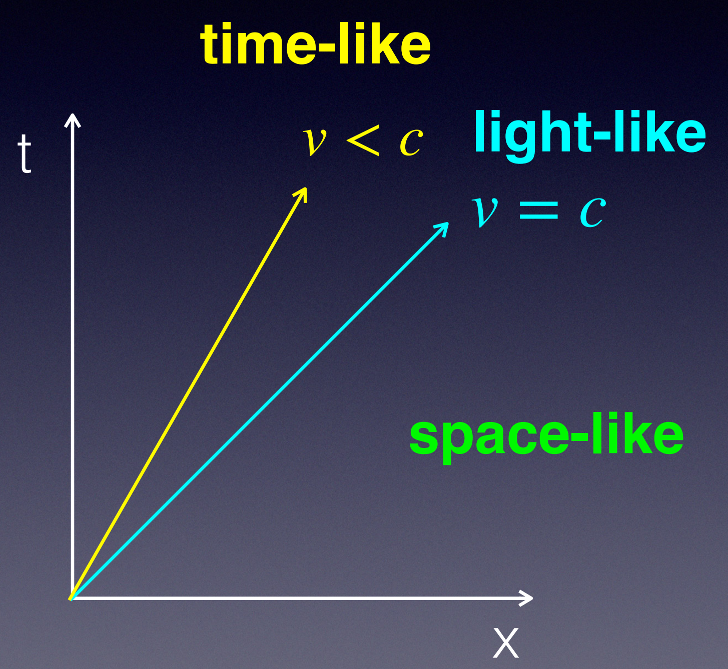
- Time-like: $(\Delta s)^2 > 0$
  - A frame exists where two events occur in one place, separated only by time. Causally connected: one event can influence the other.
- Light-like: $(\Delta s)^2 = 0$
  - Two events are separated by the speed of light
- Space-like: $(\Delta s)^2 < 0$
  - No light signal can connect the two events. Events cannot influence each other

Space-Time Diagrams



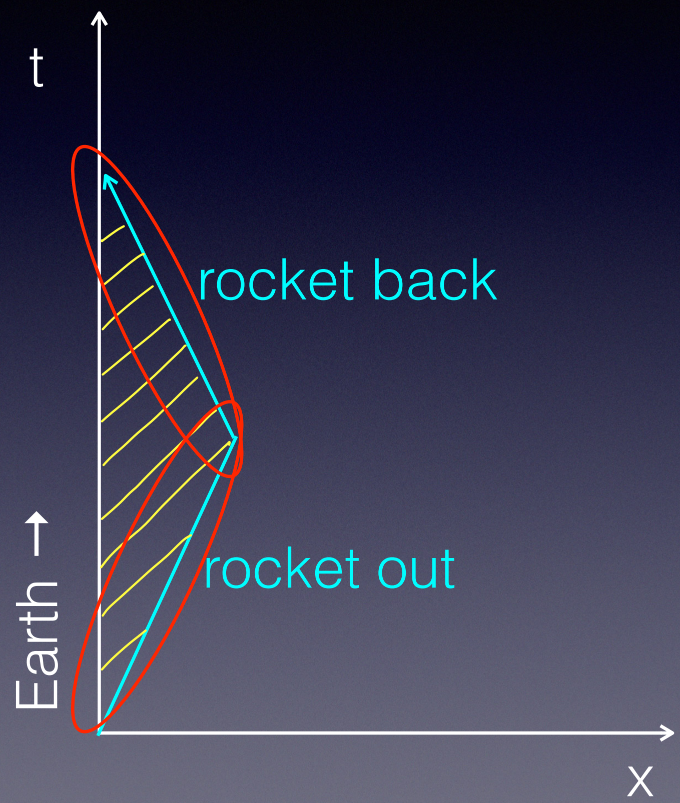
- Motion of object in position and time traces out a **world line**
- Slope is inverse of velocity
  - Typically $v = c$ is shown at a 45° angle
  - $v < c$ has angle > 45°
  - $v = 0$ is vertical

Space-Time Diagrams



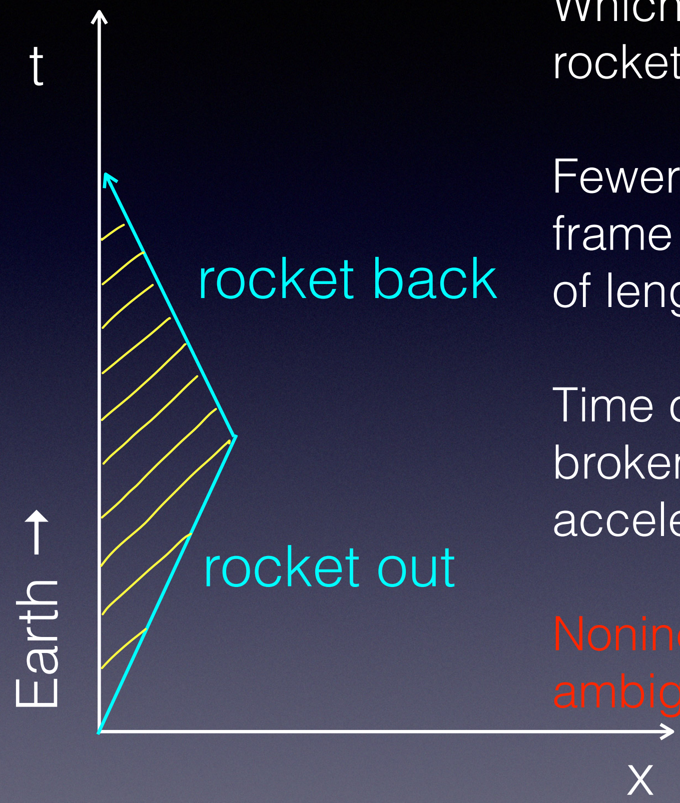
- Motion of object in position and time traces out a **world line**

Doppler Effect Revisited



- Rocket leaves Earth and travels away (blue world line)
- Earth sends signals at certain frequency (yellow lines)
- As rocket travels away, frequency of Earth's signals received at rocket is reduced
- As rocket travels toward Earth, frequency of received signals increases

Twin Paradox

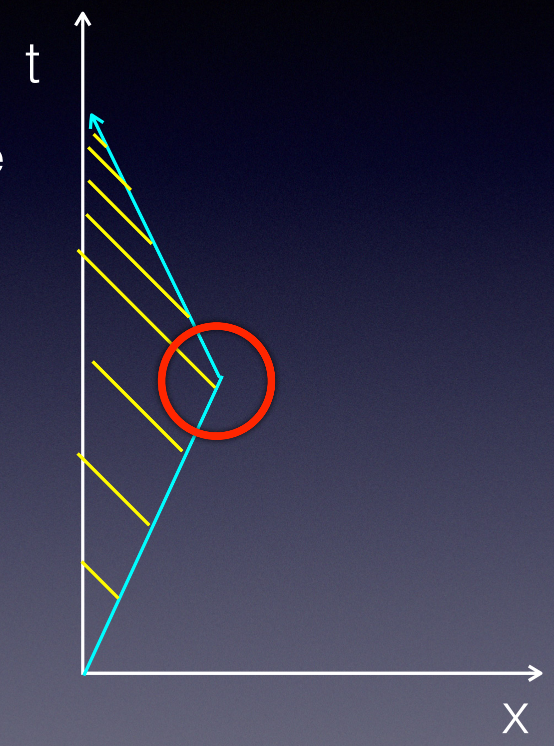


Which twin is younger when rocket returns?

Fewer years elapse in rocket frame each direction because of length contraction.

Time dilation symmetry is broken because rocket must accelerate to turn around.

Noninertial frame! Resolves ambiguity of who ages less!



round trip ~11 years in Earth's frame

~8 years in rocket's frame