ex 1

$$G \begin{cases} S' \xrightarrow{0} S\$^k \\ S \xrightarrow{1} aSbS \\ S \xrightarrow{2} \lambda \end{cases}$$

• Is G LL(0) ? No. 2 rules for S.

• Is G LL(1) ? Yes if we prove

$1\text{-lookahead}(S \xrightarrow{1} aSbS) \cap 1\text{-lookahead}(S \xrightarrow{2} \lambda) = \emptyset$

$first_1(S) = first_1(aSbS) \cup first_1(\lambda)$

$= \{a, \lambda\}$

$first_1(S') = first_1(S\$^k) = first_1(\{a,\lambda\}.\$^k)$

$\{a, \$\}$

if $first_2(\{a,\lambda\}.\$^k)$

$= \{a\$, \$^2\}$

$follow_1(S') = \lambda$

$follow_1(S) = first_1(\$.follow_1(S')) \longrightarrow \$$

$\cup\ first_1(bS.follow_1(S)) \longrightarrow b$

$\cup\ first_1(\lambda follow_1(S)) \longrightarrow$ ignore it! recursive definit°

$1\text{-lookahead}(S \to aSbS) = first_1(aSbS.follow_1(S))$

$= a = E_1$

$$1\text{-lookahead}(\underbrace{S}\rightarrow\underbrace{\lambda}) = \text{first}_1(\underbrace{\lambda}.\text{follow}_1(\underbrace{S}))$$
$$= \text{follow}_1(S)$$
$$= \{\$, b\} = E_2$$

$$E_1 \cap E_2 = \emptyset \implies G \text{ is } LL(1)$$

- Building the analysis table.

k-lookahead

Non terminals

	a	b	$
S'	$S' \xrightarrow{0} S\$$	/	$S' \xrightarrow{0} S\$$
S	$S \xrightarrow{1} aSbS$	$S \xrightarrow{2} \lambda$	$S \xrightarrow{2} \lambda$
a	pop		
b		pop	
$			accept/pop

$$1\text{-lookahead}(S' \xrightarrow{0} S\$) = \text{first}_1(S\$.\text{follow}_1(S')) = \{a, \$\}$$

$\boxed{\text{if a symbol is on ruban and on stack: pop}}$

- analysis of "abab$"

ruban	stack	action
abab$	S'	$S' \xrightarrow{0} S\$$

abab$	S $	$S \xrightarrow{1} aSbS$
~~a~~bab$	~~a~~SbS$	pop
bab$	SbS $	$S \xrightarrow{2} \lambda$
~~b~~ab $	~~b~~S $	pop
ab$	S$	$S \xrightarrow{1} aSbS$
~~a~~b$	~~a~~SbS$	pop
b$	SbS$	$S \xrightarrow{2} \lambda$
~~b~~$	~~b~~S$	pop
$	S $	$S \xrightarrow{2} \lambda$
$	$	pop/accept.

- analysis of "abb$"

ruban	stack	action
abb $	S'	$S' \to S\$$
abb$	S$	$S \to aSbS$
~~a~~bb$	~~a~~SbS$	pop
bb$	SbS$	$S \to \lambda$
~~b~~b$	~~b~~S $	pop
b$	S$	$S \to \lambda$
b$	$	reject

Ex 2

- Which k ?
  - k = 0 ? No, several rules for S, R
  - k = 1 ?

    Several ways to explain

    * counter example

ruban	stack	action
b...	S	?

either $S \to bRS$

or $S \to RcSa$ + $R \to b$

    * compute 1-lookahead$(S \to bRS) = E_1$

      and 1-lookahead$(S \to RcSa) = E_2$

    and prove $E_1 \cap E_2 \neq \emptyset$

  - k = 2 ?

    S' only one rule, no need to compute 2-lookahead to prove LL(2), but necessary to build the analysis table.

    For R, it is straight forward that 2-lookahead$(R \to acR) \cap$ 2lookahead$(R \to b)$ $= \emptyset$, it will be necessary only for the analysis table

For S:

$2\text{-lookahead}(S \xrightarrow{1} bRS) = \{ba, bb\} = E_1$

$2\text{-lookahead}(S \xrightarrow{2} RcSa) = \{ac, bc\} = E_2$

$2\text{-lookahead}(S \xrightarrow{3} \lambda) = follow_2(S)$

$\underline{follow_2(S)} = first_2(\$^2 \cdot follow_2(S')) \longrightarrow \$^2$

$\cup\ first_2(\lambda \cdot follow_2(S)) \longrightarrow$ ignore recursive

$\cup\ \boxed{first_2(a\ follow_2(S))}$

$S' \to S\$^2 \ldots$

$S \to bRS \ldots$

$S \to RcSa \ldots$

$S \to \lambda$

$R \to acR$

$R \to b$

$first_2(\$^2 \cdot follow(S'))$

$first_2(\lambda \cdot follow(S))$

$first_2(a \cdot follow(S))$

$first_2(a \cdot follow_2(S)) = a \cdot first_1(follow_1(S))$

$follow_1(S) = first_1(\$^2 \cdot follow(S')) \longrightarrow \$$

$\cup\ first_1(\lambda\ follow_1(S)) \longrightarrow$ ignored recursive

$\cup\ first_1(a \cdot follow_1(S)) \longrightarrow a$

$= \{a, \$\}$

$follow_2(S) = \{\$^2, aa, a\$\}$

$2\text{-lookahead}(S \xrightarrow{3} \lambda) = \{\$^2, aa, a\$\} = E_3$

$E_1 \cap E_2 = \emptyset \qquad E_2 \cap E_3 = \emptyset$

$E_1 \cap E_3 = \emptyset$

$\Rightarrow G$ is $LL(2)$