Advanced algorithmic : Practical sessions #3

Flows and Linear Programming
2 supervised practical sessions + 2 Questionnaires

How to use a Flow Solver

Setup

— You should be on a machine with a C compiler. You will need a terminal and an editor.
— Load the archive Hochbaum.tar
— Extract the files :
`tar xvf Hochbaum.tar`

Hochbaum algorithm (see the pdf of the article written by Chandran and Hochbaum in 2009) is one of the best current algorithm for computing a maximum flow. It is based on the notion of pseudo-flow (i.e. what enters is not equal to what gets out), the pseudo-flow is improved gradually until a maximum flow is obtained.

This program deals with transport networks described in files with a specific format called DIMACS explained there (you should read it).

Deduce a drawing of the graph described in file ex7_9.max of Hochbaum directory.

1. In order to create an executable program, you should enter the following command from Hochbaum directory :
`make pseudo`
2. Run "pseudo" algorithm on ex7_9.max file :
`./pseudo < ex7_9.max`
3. The program returns :
— the maximum flow value,
— the value of the maximum flow in each arc,
— the minimum capacity cut.

How to use a LP Solver

The following exercises should be solved with the solver available at the following address : `http://www.zweigmedia.com/simplex/simplex.php?lang=en`. First click on "Example" to see an example of the syntax used to encode linear programs then click on "Solve" in order to solve the problem (you can choose that solutions are expressed either with decimals or with fractions).

There is also a graphical solver for linear programs with only two variables : `http://www.zweigmedia.com/utilities/lpg/index.html?lang=en`.

Work to do before the next supervised session

You have to finish the representations in terms of flows or linear programs of all the exercises proposed in this document.

In order to answer the Quiz you will have to run the flow solver on a simple example to give the maximal flow obtained, describe the minimal cut associated, then represent a more complex problem and give its solution.

For the Quiz, you will also have to run the linear program solver on a simple example and give the optimum and the values of the variables that allow to get it, then represent a more complex problem and give its solution.

I Flow Problems

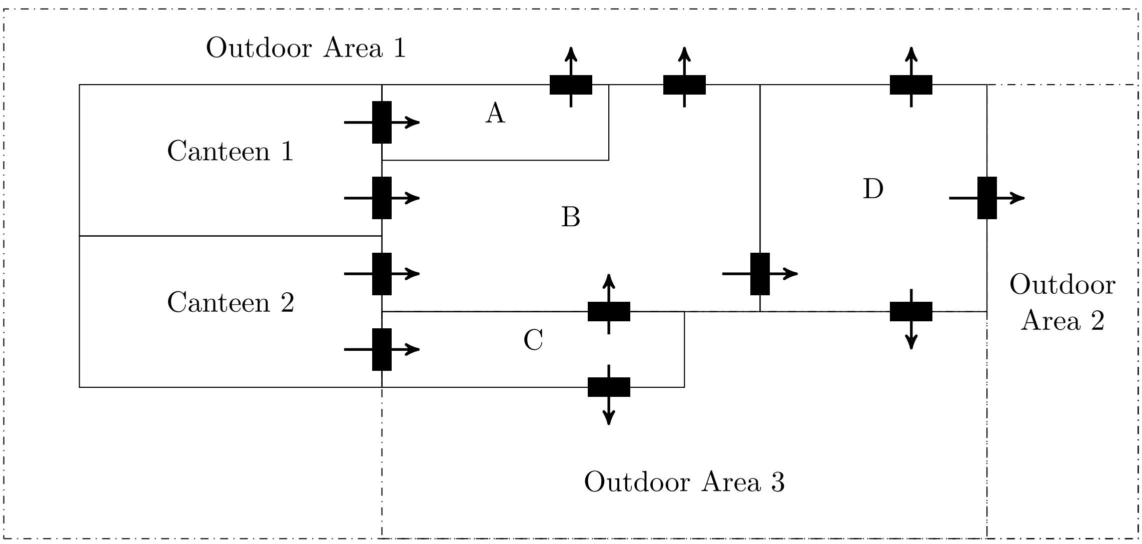
A) Transportation Problem

A company with two factories should transport the products to its three local warehouses in order to answer to local needs. It is a priori possible to transfer the products from any factory to any warehouse with no constraint about the quantity to transport. Each factory has a limited capacity. The capacity production of factory 1 (denoted F1) is 100 (in thousands of tons) and the one of factory F2 is 150. The warehouses have expressed a demand that should be satisfied : the warehouse 1, denoted W1, is demanding 50 (in thousands of tons), the second one 70 and the third 80. The warehouses cannot produce more than what is demanded.

What is the maximum quantity in total that could be transported from the factories to the warehouses ?

B) Canteen evacuation

The following figure is an **evacuation map for the canteens** of a school (the doors are the black rectangles). In case of fire, the school decides that it is more rational to use each door only in one way (indicated by an arrow).



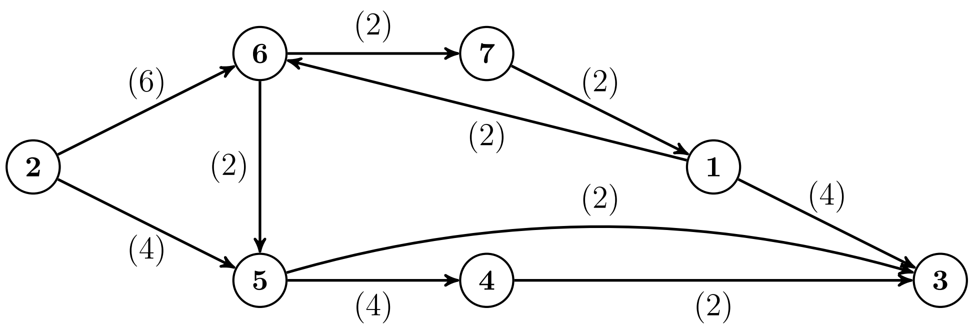
If a fire occurs then the children should cross the indoor areas (A, B, C or D) in order to go outside (in one of the area 1 2 or 3).
The doors have different size. The school has tested how many children can cross a door without pushing in one minute. The results are given on the right.
The school wants to know how long it takes to evacuate all the children that are in the canteens to the outdoor areas.

Door		Number of children
From	to	
Canteen 1	A	80
Canteen 1	B	120
Canteen 2	B	60
Canteen 2	C	70
A	Outdoor Area 1	70
B	Outdoor Area 1	80
B	D	70
C	B	20
C	Outdoor Area 3	60
D	Outdoor Area 1	40
D	Outdoor Area 2	50
D	Outdoor Area 3	60

9. Draw a transport network for which the maximum compatible flow would help to know how many children can be evacuated in 1 minute.
10. What is the value of the maximum flow ?
11. At peak times, there is at most 550 children that are eating in the canteen. By neglecting the travel time of each child, are the doors able to let go the 550 children in 2 minutes ?
12. Describe the cut separating s from t of minimum capacity (give the cut, its arcs, and its capacity).
13. We would like to have 600 pupils eating together, we can install new doors that allow 90 children to cross them per minutes. Is it possible to change only one door in order to have a possible evacuation (in less than 2mn) ? If it is possible which door should we change ?

C) River Pollution

We consider the following river network $R = (X, U, c)$ where capacities c are expressed between parentheses :



Given that the cost to build a dam is proportional (k times the capacity) to the capacity of each river, the problem is to determine what are the dams to build in order to isolate at the lowest cost, the city 2 from the city 3 in case of a pollution by a chemical industry in city 2.

1) Explain how this problem is linked to the search of a maximum flow.
2) Give the list of arcs on which the dams should be built and estimate the cost w.r.t. k.

II Counterintelligence

We consider a network of 8 persons A, ... H, who can transmit some information to each-other more or less easily. We know that the means of communication between the persons have a robustness degree between 0 and 10. (10 meaning that it is very difficult to forbid the communication, 0 means that it is very easy to do it)

spy i :	A	B	C	D	E	F	G	H
can contact spy j	C	A,C		A,B,G	B,G,H	C,E,H		G
robustness of the means of communication between i and j	3	4,5		5,6,3	4,3,7	8,2,4		6

The above table describes arcs (i, j) that relates two persons such that person i is able to join person j (but the converse is not necessarily true).

The persons D and F are suspected to know some secrete information and the counterintelligence service has to forbid them to transmit any piece of information to the persons C, G and H, what is the easiest way to achieve this task (i.e. by attacking the least robust communication links) ? We want to obtain that none of the three persons receive any information.

1) Draw a transport network in order to transform this problem in terms of search of a minimal cut in this network : you should precise the arc capacities.
2) Explain why the minimum cut is a solution to the problem.
3) Propose a solution to this problem and estimate the difficulty for the counterintelligence service.

III Linear Programming Problems

A) Transportation problem : (continued)

We consider the transport problem of section I.A) with supplementary information concerning the carriage cost [1].

The carriage cost depends on the path to follow and on the transport means, i.e. the start factory and the destination warehouse. We consider now that the warehouse have no storage problem, we just want to meet the demand.

The carriage cost per tons from factory Fi $(i = 1, 2)$ towards warehouse Wj $(j = 1, 2, 3)$ is given in the following table :

	W1	W2	W3
F1	4	3	6
F2	3	5	3

The problem is to compute the quantities to transport from each factory to each warehouse, in order to minimize the total transportation cost while respecting the capacity constraints of the factories and the warehouses demands.

B) Elections

During the last days of a campaign, a US candidate wants to convince the undecided voters of the swing states to vote for him. In order to do that, he's campaign team decides to use several media : television (T), radio (R) and press (P).

For efficiency reasons, this campaign should reach at least 65% of young people aged between 18 and 25 years, at least 45% of adults aged between 25 and 40 years and at least 10% of adults over the age of 40.

The following table gives an estimation in thousands of undecided people of the three categories that are influenced by a message according to the media. The last line gives the estimated total of undecided people in each category. The last column represents the cost in dollars of the diffusion of a message by each type of medium.

1. This problem could be solved thanks to a cost-Flow Algorithm but we are going to use linear programming.

Categories	18-25	25-40	40 and more	Cost
T	5	12	2	10000
R	2	15	2	7000
P	1	5	3	5000
Total Population	300	1300	2600	

Knowing that the number of messages diffused by the television should not be more than three times the number of messages diffused by the other media, write a linear program in order to compute the number of messages that each media should diffuse at the lowest cost. You can use t, r and p to denote the number of messages that should be diffused by the corresponding media T, R and P respectively.

C) Foundry and Workshop

In order to produce castings, a company uses a foundry and a mechanical workshop.

	Foundry	**Workshop**
Type 1 castings	6 tons per hour	12 tons per hour
Type 2 castings	5 tons per hour	15 tons per hour
Available time	100mn	45mn

The energy to use and the profits are given in the following table :

	Energy	**Profit per ton**
A ton of Type 1 castings	14kW/h	2000€
A ton of Type 2 castings	30kW/h	3000€
Available quantities	210kW/h	

Represent this problem in order to answer to the following question : how many tons of each type of castings should be produced in order to maximize the profit ?

D) Production Scheduling

We learn that the maximum production of the items of a factory in 1 month in normal conditions is 1200 items, we can produce a supplement of 400 items per month by paying some sub-contractors (the extra cost is 7€ per item).

On the other hand, the client demands for the next 3 months are described in the following table :

	month 1	month 2	month 3
Demands	900	1100	1700

We would like to schedule the production of the next 3 months in order to minimize the stock (which costs 3€ per item per month) and the sub-contractors cost. We want a zero stock at the end of the third month, at the beginning the initial stock is zero.

Propose a linear program optimizing the production of the items for the 3 months in order to meet the demand. Hence, the problem is to decide how many items we shall produce each month in normal conditions and how many items we shall produce by paying subcontractors.