Linnéuniversitetet
Kalmar Växjö

Examensarbete

DOA estimation based on MUSIC algorithm

Författare: Honghao Tang
Handledare: Sven Nordebo
Examinator: Pieternella Cijvat
Datum: 2014-05-16
Kurskod: 2ED14E, 15 hp
Ämne: Electrical Engineering
Nivå: Bachelor degree
Institutionen för Fysik och Elektroteknik

Summary

DOA estimation plays an important role in array signal processing, and has a wide range of application. In this thesis, I will describe what DOA (Direction of arrival) estimation is, and give a mathematical model of DOA estimation. Then estimate DOA based on the MUSIC algorithm, and also give some simulations with MATLAB to simulate what factors can affect the accuracy and resolution of DOA estimation when using the MUSIC algorithm.

Sammanfattning

DOA uppskattning spelar en viktig roll i array signalbehandling, och har ett brett användningsområde. I denna uppsats kommer jag att beskriva vad DOA (Direction of Arrival) uppskattning är, och ge en matematisk modell av DOA uppskattning. Efter det uppskattar jag DOA baserad på MUSIC-algoritmen, och ger några simuleringar på MATLAB för att simulera vilka faktorer som kan påverka noggrannheten och upplösningen av DOA uppskattning när MUSIC algoritmen används.

Abstract

Array signal processing is an important branch in the field of signal processing. In recent years, it has developed dramatically. It can be applied in such fields as radio detection and ranging, communication, sonar, earthquake, exploration, astronomy and biomedicine.

The field of direction of array signal processing can be classified into self-adaption array signal processing and spatial spectrum, in which spatial spectrum estimation theory and technology is still in the ascendant status, and become a main aspect in the course of array signal processing. Spatial spectrum estimation is focused on investigating the system of spatial multiple sensor arrays, with the main purpose of estimating the signal's spatial parameters and the location of the signal source.

The spatial spectrum expresses signal distribution in the space from all directions to the receiver. Hence, if one can get the signal's spatial spectrum, then the direction of arrival (DOA) can be obtained. As thus, spatial spectrum estimation is also called DOA estimation.

DOA technology research is important in array signal processing, which is an interdisciplinary technology that develops rapidly in recent years, especially the direction of arrival with multiple signal sources, the estimation of coherent signal sources, and the DOA estimation of broadband signals. DOA estimation has a wide application prospect in radar, sonar, communication, seismology measurement and biomedicine.

Over the past few years, all kinds of algorithms which can be used in DOA estimation have made great achievements, the most classic algorithm among which is Multiple Signal Classification (MUSIC).

In this thesis I will give an overview of the DOA estimation based on MUSIC algorithm.

Keywords: DOA estimation, spatial spectrum, MUSIC algorithm.

Preface

The weeks passed, and these two months have been really meaningful for my thesis. During this period, I had to face many challenges but finally I found some way to overcome them.

Here I should thank my supervisors Ellie and Sven, they helped me a lot and taught me a lot.

Honghao Tang

Växjö, June 4th 2014

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Chapter 1 An Introduction to DOA Estimation

1.1 Background and significance

Array signal processing has wide applications, such as radar, sonar, medicine, earthquake, satellite, and communication system. It becomes a hotspot and difficult point in the signal processing domain [1]. Array signal processing aims at processing signals received by array antenna, strengthening useful signals, restraining the interference and noise, while at the same time collecting useful signal parameters. Compared with traditional signal orientation sensor, sensor array can control the beam flexibly, with a high signal gain and strong ability for interference. That is the reason why array signal processing theory can boom in recent decade.

There are two research directions for DOA estimation, self-adaption array signal processing and spatial spectrum estimation. Self-adaption occurs earlier in literature than spatial spectrum and has already been used in many practical engineering systems. On the other hand, though spatial spectrum estimation has developed rapidly and had abundant references, it is rarely found in practical systems. At present, it is still being developed.

Spatial spectrum is an important concept in array signal processing theory. It presents the distribution of signals in every direction in the space. Hence, if one can get the signal's spatial spectrum, one can get the direction of arrival (DOA). Consequently, spatial spectrum estimation can be also called as DOA estimation.

DOA estimation is a key research area in array signal processing and many engineering applications, such as wireless communications, radar, radio astronomy, sonar, navigation, tracking of various objects, earthquake, medicine and other emergency assistance devices that need to be supported by direction of arrival estimation [2]. In modern society, DOA estimation is normally researched as a part in the field of array processing, so many works highlight radio direction finding. Over the past ten years, Wireless Local Area Networks (WLANs) have increased quickly because of its flexibility and convenience. In order to satisfy the requirements of advanced services, a high-speed data rate is necessary. Owing to the excessive use of the low end of the spectrum, people begin to search for the higher frequency bands for more applications. With higher user density, higher frequency and higher data rate, multipath fading and cross interference become the main issues. In order to solve these problems and get higher communication capacity, smart antenna systems are proved to be very effective in suppression of the interference and multipath signals [1]-[3].

Signal processing in smart antenna systems concentrates on the development of efficient algorithms for Direction of Arrival (DOA) estimation and adaptive beam forming. However, there are many limitations if DOA estimation uses a fixed antenna. Antenna main-lobe beam width is inversely proportional to its physical shape. It is not a practical option to improve the accuracy of angle measurement in accordance with an increase in the physical aperture of the receiving antenna. Some systems such as missile seeker or aircraft antenna have limited physical size, so they are sufficiently wide in beam width of the main lobe to correspond. They do not have a good resolution and if there are multiple signals falling in the antenna's main lobe, it becomes too difficult to distinguish between them.

Using an array antenna system with innovative signal processing instead of a single antenna can enhance the resolution of the DOA estimation. This array structure provides spatial samplings of the received waveform. In signal reception and parameter estimation, a sensor array has better performance than a single sensor.

There are many kinds of super resolution algorithms such as spectral estimation, Bartlett, Capon, ESPRIT, Min-norm and MUSIC [4]. One of the most popular and widely used subspace-based techniques to estimate the DOA of multiple signal sources is the MUSIC algorithm. Large numbers of computations are needed to search for the spectral angle when using the MUSIC algorithm, so in real applications its implementation can be difficult. Compared with spectral MUSIC algorithm, the Root-MUSIC method has better performance with reduced complexity computation. It can only be used in uniform linear array (ULA) or non-uniform linear array whose arrays are restricted to a uniform grid [6] [7]. In this thesis, I will focus on the MUSIC algorithm.

1.2 Overview of the development of Direction of Arrival (DOA)

Initially, the Direction of Arrival Estimation estimated the linear spectrum based on the method of Fourier transform. It mainly included the periodogram method. Because it was affected by the Rayleigh limit, it could not acquire a high-resolution performance, or resist the noise, so it did not obtain satisfied performance.

Afterwards, based on the statistical analysis of maximum likelihood spectrum estimation, which has a high-resolution performance and robust character, people began to pay attention to this method. However, maximum likelihood estimation needs to search for a high-dimensional parameter space, which means that abundant calculations are required. Therefore, it is hard to be put into practice [8]-[10].

In 1967, Burg proposed the maximum entropy estimation method, which opened a modern research area on spectrum. This method includes maximum entropy, AR（Autoregressive model）, MA（Moving Average Model）, ARMA（Auto-Regressive and Moving Average Model）parameter method. All those methods have a high resolution. Nevertheless, they all need a large amount of calculation and a low robustness. [9, 10].

When it came to the 1980s, the academic community put forward a series of spectrum estimations based on decomposition of eigenvalues. All those estimations were represented by Multiple Signal Classification (MUSIC) and Estimation Signal Parameters via Rotational Invariance Technique (ESPRIT) [11, 12]. In certain conditions, MUSIC is a one-dimensional implementation of maximum entropy, which shares the same character with maximum likelihood method [13, 14]. At that point, MUSIC was better than any other methods and received more and more appreciation. However, it has a weakness of heavy computation. ESPRIT arithmetic and its improved arithmetic such as TLS-ESPRTI, VIA-ESPRIT and GEESE have a high resolution. The most important thing is that this kind of arithmetic avoids large computation in search of the spectrum, so it can accelerate the speed of Direction of arrival estimation. However, ESPRIT arithmetic and its improved arithmetic can be achieved only under some special array structures, so its application is relatively narrow [15].

In recent years, some normal methods on DOA, like ML, MUSIC, ESPRIT, have ignored the time characteristic of the signal. Along with the wide application of array signal processing, one signal interferes with other signals in many occasions like in the field of communications. Therefore, when processing the spatial problem, one should consider the time-domain problem at the same time. Using useful information in the signal more sufficiently, researchers think signals can be sampled in the spatial domain and time domain at the same time. The surplus one dimension to replenish the shortage of spatial domain information, namely use two dimensions to process array signal and reduce the restraint of the array structure and improve the arithmetic ability of resistance to noise.

In practice, there is often a coloured noise environment. In recent years people try to use the array signal processing based on higher-order cumulant, since higher-order cumulant has a natural blind feature for any Gaussian noise. Based on the cumulative amount, the algorithm makes the original DOA estimation algorithm expand to Gaussian spatial coloured noise or non-Gaussian noise spatial coloured and white noise [16].

In array signal processing, when antenna array receives multiple signals which form the signal source, the signal source may be completely unknown and the transmission channel is also unknown and time-varying. This unknown feature for the transmission channel is the main reason to limit the high resolution of DOA. So scholars have put forward the concept of blind DOA estimation [17]. Blind DOA estimation can estimate the channel’s characteristics under unknown circumstances, and has broad application prospects. Adaptive blind signal separation began at the pioneering working by Heruah and Juttne in 1991. Since then, people have proposed many different algorithms in recent years. In principle, all these blind separation algorithms can be used for DOA estimation.

Many natural and artificial signals, such as voice, biomedical signals, radar and sonar signals are typical of non-stationary signals, whose characteristics are limited by duration and time variation. Considering the nonlinear and non-stationary characteristics in the actual system, use of artificial neural networks in DOA is the research direction in recent years [18].

All these methods basically stay in the theoretical and experimental simulation stage, which is far from actual applications. At present, interferometry is mainly used to estimate DOA. In a variety of DOA estimations which are based on spatial spectrum estimation, the MUSIC method has a higher resolution, a moderate amount of computation, better robustness, and wide application in array structure. In the practical engineering process, people often choose the MUSIC method in experimental studies, and thus develop a number of hardware devices. Certain results have been achieved in the practical process.

Chapter 2 Basic Knowledge of DOA Estimation

2.1 DOA estimation

2.1.1 The structure of spatial spectrum estimation system

Spatial spectrum estimation is a specialized signal estimation technology that uses space arrays to achieve a space signal parameter. The entire spatial spectrum system should be composed of three parts: the incident signal space, spatial array receiver and parameter estimation. The space can be divided into three corresponding spaces, namely target stage, observation stage, and estimation stage [19].

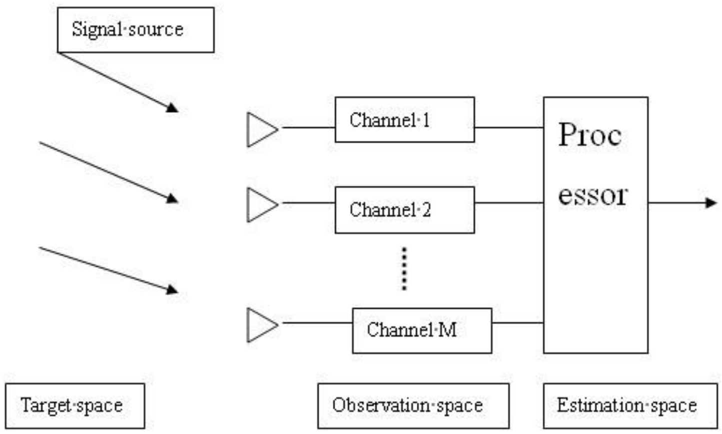

Fig 2.1 The system structure of DOA estimation

The above system architecture is described as follows:

1) Target stage is a stage that consists of signal source parameters and a complex environment. For the spatial spectrum estimation system, it uses some particular methods to estimate the unknown parameters of signals which come from this complex target stage.

2) Observation stage is a stage which receives the radiation signals from the target stage. Due to the complexity of the environment, the received data may contain some signal characteristics (azimuth, distance, polarization, etc.) and the space environment characteristics (noise, miscellaneous waves, interference, etc.). In addition, due to the influence of spatial array elements, the data received also contain some features of space array element (mutual coupling, channel inconsistent, frequency band inconsistency, etc.). This observation stage is a multidimensional stage which means that the system receiving dates are composed of plurality of channels, and the traditional time domain processing method is usually only used for one channel. Of particularly note is that the channel does not correspond to the array elements; a spatial channel is formed by several or all of the synthetic array elements. There is no doubt that certain array elements in the stage may be contained within different channels.

3) Estimation stage is a stage which uses spatial spectrum estimation techniques (including array signal processing techniques such as array correction and spatial filtering techniques)

to extract the signal character parameters from the complex environment.

Estimation stage is equivalent to the reconstruction of the target stage. The accuracy of reconstruction is determined by many factors, such as the complexity of the environment, the mutual coupling of spatial array, different channels, frequency band inconsistency, etc.

Spatial spectrum expresses the energy distribution of signals in all spatial directions. If one can get the spatial spectrum of the signal, the direction of arrival (DOA) of the signal can be obtained, so spatial spectrum estimation is also known as DOA estimation.

2.1.2 The basic principle of DOA estimation

DOA is for the direction of array antenna of the radio wave. If the radio wave received meets the condition of far field narrowband, it can take the front of the radio wave as a plane. The angle between the array normal and the direction vector of the plane wave is the Direction of arrival (DOA).

The estimated target of DOA gives N snapshots data: X (1)…X (N), using an algorithm to estimate the value of multiple signals' DOA (θ).

For generally far and wide signals, a wave-way difference exists when the same signal reaches different array elements. This wave-way difference leads to a phase difference between the arrival array elements. Using the phase difference between the array elements of the signal one can estimate the signal azimuth, which is the basic principle of DOA estimation [20].

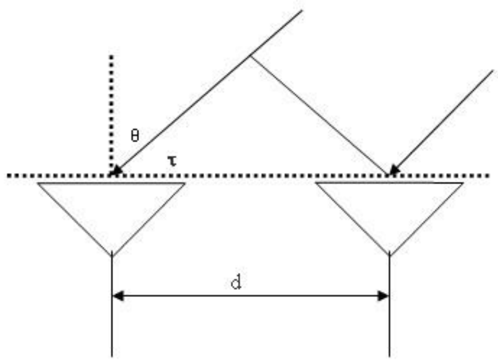

Figure 2.2 The principle of DOA estimation

For instance, Fig. 2.2 considers two array elements,

d is the distance between the array elements,

c is the speed of light,

θ is the incident angle of the far field signal,

τ is the time delay of the array element.

The signal received by the antenna due to the path difference is

$$\tau=\frac{d\sin\theta}{c}, \quad (2.1)$$

thus one can obtain the phase difference between the array elements as

$$\varphi = e^{-j\omega\tau}=e^{-j\omega\frac{d\sin\theta}{C}}=e^{-j2\pi\frac{d\sin\theta}{\lambda f_0}f}, \quad (2.2)$$

where f_o is the centre frequency. For narrow band signals, the phase difference is

$$\varphi = e^{-j2\pi\frac{d\sin\theta}{\lambda}}, \quad (2.3)$$

where λ is the wavelength of the signal. Therefore, if the time delay of the signal is known, the direction of the signal can be gained according to Formula (2.1), which is the basic principle of spatial spectrum estimation techniques.

In this thesis, the following assumptions are used:

1) Point source assumption. Assume that the signal source is a point source, when looking from the array signal source, the opening angle is zero, and thus the signal source relative to the direction of the array is determined uniquely.

2) Narrowband signal hypothesis. That means that the signal bandwidth is far less than the reciprocal of the signal wave propagation across the largest diameter time. Meeting the narrowband assumption is to ensure that all array elements in the array can capture a signal at the same time.

3) Array assumptions. Assuming the array is located in the far field region of the source, the wave is projected to the plane wave. Assuming each element is the same lattice element and the position is accurate, the array element channel and amplitude and phase are consistent. This assumption guarantees that the array elements and their channel have no error.

4) Noise assumptions. Assuming the noise between each array element is zero, variance σ^2 is Gaussian white noise, statistical independently between each array noise and statistically independent between signal and noise.

2.2 Common methods for the array signal DOA

This section describes some common methods for the DOA estimation.

1. Conventional beam forming method

The DOA estimation method was first used in conventional beam forming algorithm. Its main idea is: In a certain time, make all arrays estimate a certain direction and measure the output power; for the output power, produce a maximum power of direction that is needed by DOA estimation [21].

The main shortcoming of the conventional beam forming method is that: all the freedom degrees in the array are used to form a beam in the desired direction of observation. When multiple signal sources are incident, the method is limited to the height of the beam width and the side lobe, so the resolution is low.

2. Capon minimum variance method

Capon minimum variance method is a beam forming technique for the purpose of enhancing the effect of conventional methods [22]. Conventional beam forming methods have a defect: when there are multiple signal sources, spatial spectrum estimation includes the signal source power not only in the estimation direction but also in other directions. And Capon method reduces the influence of interference by minimizing the total output power, and thus estimates the direction of the wave.

Compared with conventional beam forming algorithm, the Capon method has greatly improved resolution [23]. However, the Capon method has obvious shortcomings: if the other signal's incident direction is close to the interest signal's incident direction, the Capon method will make many errors. It needs to calculate the matrix inversion. When the number of array elements is large, it needs correlation calculation. The ability to distinguish is decided by the array geometry and SNR.

3. Eigenspace algorithm

Although the classical beam forming method is usually very effective and frequently used, these methods have essential limitation in terms of resolution, by the array aperture limit. Most of these limitations are due to the model structure of input signal. Schmitt derived the completely geometric solution of DOA estimation without considering the noise situation, and promoted this geometric solution, finally obtaining a reasonable approximate solution when noise existed and creating a precedent for the eigenspace algorithm. This algorithm is later developed into MUSIC algorithm. Except for MUSIC algorithm, the formation of eigenspace algorithm is mainly due to rotational invariance techniques by means of signal parameters estimation, which was proposed by Roy. It is called the ESPRIT algorithm [24].

There are two properties when eigenspace algorithm mainly uses an array of received data covariance matrix R.

1) Expansion space of feature vectors can be decomposed into two orthogonal subspaces, the signal subspace (expansion by the larger eigenvector corresponding to the larger eigenvalue) and the noise subspace (expansion by the smaller eigenvector corresponding to the smaller eigenvalue)

2) The direction vector from the signal source is orthogonal with the noise subspace.

2.3 Factors affecting DOA estimation results

DOA estimation results can be affected by many factors, not only related to the source of the incoming signal, but also related to the actual application environment [25]. Here are a few important influential factors, and in Chapter 4, some simulation experiments will be conducted to study how to affect the DOA estimation results.

1 Number of array elements

The number of array elements in basic arrays can affect the estimation performance for super resolution algorithm. Generally speaking, if array parameters are the same, the more number of array elements, the better estimation performance for super resolution algorithm.

2 Snapshots

In the time domain, the number of snapshots is defined as the number of samples. In the frequency domain, the number of snapshots is defined as the number of time sub-segments of discrete Fourier transform (DFT).

3 SNR

Assuming the signal and noise have a flat pass band power spectral density, and the power of signal source is σ^2_p, noise power isσ^2_n, then in this case SNR can be defined as

$$\text{SNR}=20 \log \left(\frac{\sigma_P}{\sigma_n}\right), \tag{2.4}$$

SNR directly affects the performance of super-resolution DOA estimation algorithm. At a low SNR, super-resolution algorithm performance would drop dramatically. As thus, how to improve the algorithm under a low SNR is the research focus for the sup-resolution DOA algorithm [26].

4 The coherence of the signal source

The problem involving coherent sources is a fatal problem for subspace algorithms. When there is a coherent signal in the signal source, the signal covariance matrix is no longer for the non-singular matrix. In this case, the original super-resolution algorithm will fail. Therefore, it will greatly affect the performance of DOA estimation. In addition to the factors mentioned above, many other factors can affect the performance of DOA estimation in practical applications, such as the array element amplitude and phase inconsistencies, mutual coupling between array elements, and the wrong position of sensors.

2.5 Other relevant knowledge

1 Resolution

In the direction of array, the resolution of the signal source on one direction is directly related to the rate of change in the vicinity of the array direction vector. In the vicinity of the rapid changes direction vector, with the change of signal source angle, the snapshots also change; the corresponding resolution is high. Here a sensitivity characterization D(θ) is defined,

$$D(\theta)= \left\|\frac{da(\theta)}{d\theta}\right\| \propto \left\|\frac{d\tau}{d\theta}\right\|, \tag{2.5}$$

where $a(\theta)$ is the incident angle of array elements. The larger D(θ) indicates the higher resolution in this direction.

For uniform linear array (ULA)

$$D(\theta) \propto \cos(\theta). \tag{2.6}$$

It shows that signals have a sensitivity that is reduced to half in 60°, so the scope of the general linear measurement is from -60°to 60°.

2 Hermitian matrixes

Definition: If the complex matrix A satisfies $A^H=A$ (H denotes the conjugate transpose), then A is a Hermitian matrix [30].

Let A^T be the transpose matrix and $\bar{A}$ be conjugate matrix respectively, obviously the necessary and sufficient condition is $A^T=\bar{A}$ for the n-order matrix A= $[a_{ij}]$ and Hermitian matrix, namely

$$\bar{a}_{ij}=a_{ij} \quad (i,j=1,2,\ldots n). \tag{2.7}$$

As can be seen from Formula (2.7), the diagonal elements of the Hermitian matrix must be real numbers.

A Hermitian matrix has the following features：

If A is a Hermitian matrix, then det A is a real number.

If A is a Hermitian matrix, then A^T, $\overline{A}$, A^H are all Hermitian matrixes. When A can be reversible, A^{-1} is also the Hermitian.

If A is a Hermitian matrix and k is any real number, then kA is still a Hermitian matrix.

If A, B are all the n-order Hermitian matrixes, then A+B is still a Hermitian matrix.

3 Covariance and covariance matrix

For two-dimensional random variables (X, Y), if E[(X-E(X))(Y-E(Y))] exists, it is called the covariance of X and Y [31], denoted as COV(X, Y), namely

$$COV(X, Y)= E[(X-E(X))(Y-E(Y))]=E(XY)-E(X)E(Y). \tag{2.8}$$

The nature of the covariance is

COV(X, Y)=COV(Y, X);

COV(aX, bY)=abCOV(X, Y), (a, b are constants);

$COV(X_1+X_2, Y)=COV(X_1, Y)+COV(X_2, Y)$.

From the definition of covariance, it can be seen that

$$COV(X, X)=D(X),\ COV(Y, Y)=D(Y). \tag{2.9}$$

For n-dimensional random variables $(X_1,X_2,\ldots,X_n)$, denoted as

$$\begin{aligned} C_{ij}&=COV(X_i,X_j)\\ &=E[(X_i-E(X_i))(X_j-E(X_j))](i=1,2,\ldots,n), \end{aligned} \tag{2.10}$$

$C=\{C_{ij}\}$ is called covariance matrix for the C $(X_1, X_2, \ldots, X_n)$.

Covariance matrix C is positive definite (negative definite) symmetric, i.e. $C^T=C$, $C \geq 0$.

Chapter 3 The MUSIC algorithm

3.1 An introduction to the MUSIC algorithm

Multiple Signal Classification (MUSIC) algorithm was proposed by Schmidt and his colleagues in 1979 [32]. It has created a new era for spatial spectrum estimation algorithms. The promotion of the structure algorithm characterized rise and development, and it has become a crucial algorithm for theoretical system of spatial spectrum. Before this algorithm was presented, some relevant algorithms directly processed data received from array covariance matrices. The basic idea of MUSIC algorithm is to conduct characteristic decomposition for the covariance matrix of any array output data, resulting in a signal subspace orthogonal with a noise subspace corresponding to the signal components. Then these two orthogonal subspaces are used to constitute a spectrum function, be got though by spectral peak search and detect DOA signals.

It is because MUSIC algorithm has a high resolution, accuracy and stability under certain conditions that it attracts a large number of scholars to conduct in-depth research and analyses. In general, it has the following advantages when it is used to estimate a signal's DOA.

1) The ability to simultaneously measure multiple signals.

2) High precision measurement.

3) High resolution for antenna beam signals.

4) Applicable to short data circumstances.

5) It can achieve real-time processing after using high-speed processing technology.

3.2 The mathematical model of DOA estimation

In order to analyse and derive more conveniently, now assume the following conditions for the ideal mathematical model of DOA problems.

1) Each test signal source has the same but unrelated polarization. Generally consider that the signal sources are narrow bands, and each source has the same centre frequency ω_0. The number of testing signal source is D.

2) Antenna array is a spaced linear array which consists of M(M>D) array elements; each element has the same characteristics, and it is isotropic in each direction.

3) The spacing is d, and the array element interval is not larger than half the wavelength of the highest signal frequency.

4) Each antenna element in the far field source, namely, an antenna array receiving the signals coming from the signal source is a plane wave.

5) Both array elements and test signals are uncorrelated; variance σ^2 is zero-mean Gaussian noise $n_m(t)$.

6) Each receiving branch has the same characteristics.

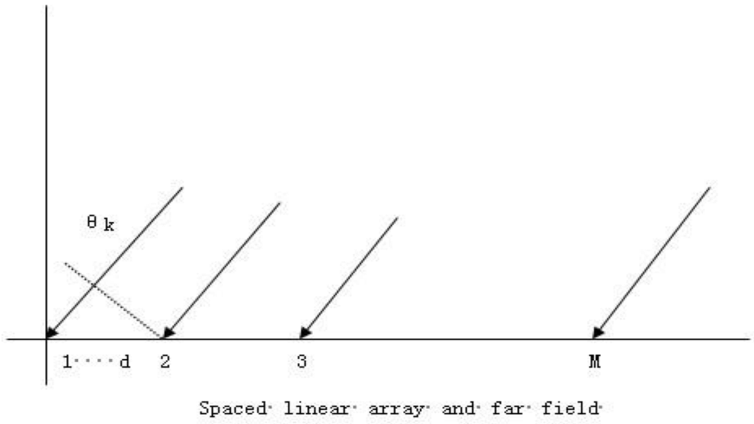

Figure 3.1 The model of DOA estimation

Let the number of signal sources k (k=1,2,…,D) to the antenna array, the wavefront signal be $S_k(t)$, as previously assumed, $S_k(t)$ is a narrowband signal, and $S_k(t)$ can be expressed in the following form

$$S_k(t) = s_k(t)\exp\{j\omega_k(t)\}, \tag{3.1}$$

where $s_k(t)$ is the complex envelope of $S_k(t)$ and $\omega_k(t)$ is the angular frequency of $S_k(t)$. As assumed before, all signals have the same centre frequency. So

$$\omega_k=\omega_0=\frac{2\pi c}{\lambda}, \tag{3.2}$$

where c is electromagnetic wave velocity, λ is wave length.

Let the time required by the electromagnetic antenna array dimension be t_1. According to the narrowband assumption, the following approximation is valid,

$$S_k(t- t_1)\approx S_k(t). \tag{3.3}$$

Therefore, the delayed wave front signal is

$$\tilde{s}_k(t-t_1) = s_k(t-t_1)\exp[j\omega_0(t-t_1)] = s_k(t)exp[j\omega_0(t-t_1)]. \tag{3.4}$$

So use the first array element as reference points. At the moment t, the induction signal of the array element m (m=1,2,…M) to the k-th signal source in the spaced linear array is

$$a_k s_k(t)\exp[-j(m-1)\frac{2\pi\,\mathrm{d}\sin\theta_k}{\lambda}], \tag{3.5}$$

where a_k is the impact of array element m on the signal source k-th. As assumed before, each array element has no direction, so let a_k=1. Θ_k is direction angle of signal source k, $(m-1)\frac{\mathrm{d}\sin\theta_k}{\lambda}$ is signal phase difference which is caused by the path difference between m-th array element and the first array element.

Record and measure the noise and all waves from the source, the output signal of the m-th element is

$$x_m(t) = \sum_{k=1}^{D} s_k(t) \exp\left[-j(m-1)\frac{2\pi \mathrm{d} \sin\theta_k}{\lambda}\right] + n_m(t), \tag{3.6}$$

where $n_m(t)$ is measurement noise; all quantities of labelled m belong to the m-th array element; all quantities of label k belong to the signal source k. Let

$$a_m(\theta_k) = \exp[-j(m-1)\frac{2\pi \mathrm{d} \sin\theta_k}{\lambda}], \tag{3.7}$$

be the response function of array element m to signal source k.

Then the output signal of array element m is

$$x_m(t) = \sum_{k=1}^{D} a_m(\theta_k) s_k(t) + n_m(t), \tag{3.8}$$

where $s_k(t)$is signal strength of signal source k.

This expression can be described by matrices:

$$X=AS+N, \tag{3.9}$$

where

$$X=[x_1(t),x_2(t),\ldots,x_M(t)]^T, \tag{3.10}$$

$$S=[S_1(t),S_2(t),\ldots,S_D(t)]^T, \tag{3.11}$$

$$A=[a(\theta_1), a(\theta_2), \ldots, a(\theta_D)]^T$$

$$=\begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j\varphi_1} & e^{-j\varphi_2} & \cdots & e^{-j\varphi_D} \\ \cdots & \cdots & \cdots & \cdots \\ e^{-j(M-1)\varphi_1} & e^{-j(M-1)\varphi_2} & \cdots & e^{-j(M-1)\varphi_D} \end{bmatrix}, \tag{3.12}$$

$$\text{with } \varphi_k = \frac{2\pi d}{\lambda}\sin\theta_k, \tag{3.13}$$

$$N=[n_1(t),n_2(t),\ldots,n_M(t)]^T. \tag{3.14}$$

To conduct N sampling point for $x_m(t)$, the issue becomes to sample the output signal $x_m(t)$, then estimate the angle $\theta_1, \theta_2, \ldots, \theta_D$ of signal source DOA from $\{x_m(i), i=1,2,\ldots,M\}$.

Thus, it can take the array signal as a superimposed spatial harmonic noise.

3.3 Eigen decomposition of array covariance

For array output x, corresponding calculations can get its covariance matrix R_x:

$$R_x=E[XX^H], \tag{3.15}$$

where H is the conjugate transpose matrix.

As assumed before, signal and noise is uncorrelated, and the noise is zero mean white noise, so Formula (3.9) can be substituted into Formula (3.5). As thus, the following can be obtained:

$$R_x=E[(AS+N)](AS+N)^H] \tag{3.16}$$

$$=AE[SS^H]A^H+E[NN^H]$$

$$=AR_sA^H+RN,$$

where $R_s=E[SS^H]$ (3.17) is called the signal correlation matrix.

$$RN=\sigma^2 I, \tag{3.18}$$

is the noise correlation matrix, $\sigma^2$is the power of noise, I is the unit matrix of M*M.

In practical applications, R_x usually cannot be directly obtained and only sample covariance $\tilde{R}_x$ can be used:

$$\tilde{R}_x = \frac{1}{N}\sum_{i=1}^{N} x(i)x^H(i), \tag{3.19}$$

where $\tilde{R}_x$ is the maximum likelihood estimation of R_x. When the number of samples $N\rightarrow\infty$, they are the same, but in actual situations there are some errors because of the limitation of the samples number.

According to the theory that matrix can conduct eigenvalue decomposition, conduct eigenvalue decomposition to the array covariance matrix. First consider the ideal case, where noise doesn't exist:

$$R_x=AR_SA^H. \tag{3.20}$$

For uniform linear array (ULA), the matrix A is a Vandermond matrix which is defined by Formula (3.12), as long as:

$$\theta_i \neq \theta_j,\ i\neq j, \tag{3.21}$$

then, each column is independent. Hence, if R_s is non-singular matrix (Rank(R_s)=D, each signal source is independent), then:

$$\text{Rank}(AR_SA^H)=D, \tag{3.22}$$

since $R_x=E[XX^H]$, so:

$$R_x^H=R_x, \tag{3.23}$$

i.e. R_x is a Hermitian matrix, whose eigenvalue is real. Because R_s is positive definite, AR_sA^H is semi-positive definite, it has positive eigenvalues D and zero eigenvalues M-D.

Next, consider there is noise

$$R_x = AR_S A^H + \sigma^2 I, \tag{3.24}$$

since $\sigma^2 > 0$, R_x is a full rank matrix, R_x has M positive real eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_M$, respectively corresponding to the M eigenvectors $v_1, v_2, \ldots, v_M$. For R_x is a Hermitian matrix, each eigenvector is orthogonal.

$$\text{i.e. } v_i^H v_j = 0 \quad i \neq j, \tag{3.25}$$

only D eigenvalue is relevant to signals. They are equal to the sum of matrix AR_sA^H and every eigenvalue σ^2 respectively. That is to say, σ^2 is the smallest eigenvalue of R, which is M-D dimension. For the corresponding eigenvalue v_i, i=1,2,…, M, there is still D related to the signals. In addition, M-D is related to the noise. In the next section, the nature of characteristic decomposition will be used to determine the source of DOA.

3.4 The principle and implementation of MUSIC algorithm

Characterized by an array of covariance decomposition, the following conclusions can be drawn:

The eigenvalues of the matrix R_x are sorted in accordance with size, which is

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_M > 0, \tag{3.26}$$

where larger eigenvalues D are corresponding to signal while M-D smaller eigenvalues are corresponding to noise.

The eigenvalues and eigenvectors which belong to matrix R_x are corresponding to signal and noise respectively. Therefore, the eigenvalue (eigenvector) of R_x to signal eigenvalue (eigenvector) and noise eigenvalue (eigenvector) can be divided.

Let λ_i be the i-th eigenvalues of the matrix R_x, v_i is eigenvector corresponding to λ_i, then:

$$R_x v_i = \lambda_i v_i, \tag{3.27}$$

let $\lambda_i = \sigma^2$ be the minimum of R_x

$$R_x v_i = \sigma^2 v_i \qquad i = D+1, D+2, \ldots, M, \tag{3.28}$$

place (3.24) into (3.28), the following can be acquired

$$\sigma^2 v_i = (AR_S A^H + \sigma^2 I)\, v_i, \tag{3.29}$$

expand the right side and compare to the left, the following can be obtained

$$AR_S A^H v_i = 0. \tag{3.30}$$

Because A^HA is D*D dimensionally full rank matrix and $(A^HA)^{-1}$ exists, R_s^{-1} also exists. From the above, multiply $R_s^{-1}(A^HA)^{-1}A^H$ on both sides at the same time, then:

$$R_s^{-1}(A^HA)^{-1}A^H AR_S A^H v_i = 0, \tag{3.31}$$

so,

$$A^H v_i = 0 \qquad i = D+1, D+2, \ldots, M. \tag{3.32}$$

The above equation indicates that the eigenvector corresponding to the noise eigenvalue (the noise eigenvector) v_i is perpendicular with the column vector of the matrix A. Each row of A is corresponding to the direction of a signal source. That is the starting point of using noise eigenvector to get the direction of signal source.

Using noise characteristic value as each column, construct a noise matrix E_n can be constructed:

$$E_n=[V_{D+1},V_{D+2},\ldots,V_M], \tag{3.33}$$

to define spatial spectrum $P_{mu}(\theta)$ can be defined:

$$P_{mu}(\theta)=\frac{1}{a^H(\theta)E_nE_n^H a(\theta)}=\frac{1}{\|E_n^H a(\theta)\|^2}, \tag{3.34}$$

where the denominator of the formula is an inner product of the signal vector and the noise matrix. When $a(\theta)$ is orthogonal with each column of E_n, the value of this denominator is zero, but because of the existence of the noise, it is actually a minimum. $P_{mu}(\theta)$ has a peak. By this formula, make θ change and estimate the arrival angle by finding the peak.

The implementation steps of MUSIC algorithm are shown below.

Obtain the following estimation of the covariance matrix based on the N received signal vector:

$$R_x=\frac{1}{N}\sum_{i=1}^{N}X(i)X^H(i), \tag{3.35}$$

to eigenvalue decompose the covariance matrix above

$$R_x=AR_sA^H+\sigma^2 I. \tag{3.36}$$

According to the order of eigenvalues, take eigenvalue and eigenvector which are equal to the number of signal D as signal part of space; take the rest, M-D eigenvalues and eigenvectors, as noise part of space. Get the noise matrix E_n:

$$A^H v_i=0 \quad i=D+1, D+2,\ldots,M, \tag{3.37}$$

$$E_n=[V_{D+1},V_{D+2},\ldots,V_M], \tag{3.38}$$

vary θ; according to the formula

$$P_{mu}(\theta)=\frac{1}{a^H(\theta)E_nE_n^H a(\theta)}. \tag{3.39}$$

Calculate the spectrum function; then obtain the estimated value of DOA by searching the peak.

3.5 Improved MUSIC Algorithm

Under the premise of a precise model, MUSIC algorithm can theoretically achieve an arbitrarily high resolution to DOA [34]. However, for MUSIC algorithm signals, it is limited to uncorrelated signals. When the source is a correlated signal or a signal with low SNR, the estimated performance of the MUSIC algorithm deteriorates or even completely loses. This section provides a brief introduction to an improved MUSIC algorithm, which is proposed by conjugate reconstruction of the data matrix of the MUSIC algorithm [33].

Make a transformation matrix J, J is an M^{th}-order anti-matrix, known as the transition matrix, i.e.

$$J=\begin{bmatrix} 0 & 0 & ... & 1 \\ 0 & 0 & ... & 0 \\ ... & ... & ... & ... \\ 1 & 0 & ... & 0 \end{bmatrix}, \tag{3.40}$$

let Y=JX*, where X* is the complex conjugate of X, then the covariance of data matrix Y is

$$R_y=E[YY^H]=JRX^*J. \tag{3.41}$$

From the sum of R_x and R_y, the reconstructed conjugate matrix can be obtained.

$$R= R_x+ R_y=AR_sA^H+J[AR_sA^H]^*J+2\sigma^2I. \tag{3.42}$$

According to matrix theory, the matrices R_x, R_y and R have the same noise subspace. To conduct characteristic decomposition of R and get its eigenvalue and eigenvector, according to the estimated number of signal source, separate the noise subspace, and then use this new noise subspace to construct spatial spectrum and obtain the estimated DOA value by finding the peak.

Chapter 4 Simulation

An introduction to MATLAB

MATLAB is released by the U.S. MathWorks Company. It mainly faces scientific computing, visualization and interactive program designed for a high-tech computing environment [27]. It makes numerical analysis, matrix computation, scientific data visualization, modelling and simulation of nonlinear dynamic systems. Besides, many other powerful features are integrated in a windows environment which can be used easily. It provides a comprehensive solution for scientific research, engineering design, and an effective numerical solution for numerous scientific fields. It is out of the traditional non-interactive programming languages (such as C, FORTRAN) in some distance. It represents the highest level of the current international scientific computing software.

MATLAB is a kind of language, and is also a programming environment. MATLAB provides a lot of user-friendly tools to manage variables, input and output data, and generate and manage M files.

Users can type a command in the MATLAB command window. It can also write applications in the editor using the language it defines. After explaining the language, process them in a MATLAB environment, and finally returns the results.

The main features of MATLAB [28] are shown below.

1) MATLAB language is simple, compact, easy to use, flexible, and has extremely rich library functions. The form to write a MATLAB program is free. Using functions from the library can avoid complicated subroutines programming tasks and compress all unnecessary programming work. Because library functions are written by experts in this field, users need not to worry about the reliability of function. It can say that using MATLAB technology is like standing on the shoulders of experts.

2) Rich operators. Since MATLAB is written in C language, it has the same operators like C language. Using MATLAB operators flexibly can make the program extremely brief and easy to understand.

3) MATLAB has both structured control statements (like for loops, while loops, break statements and if statements) and object oriented programming features.

4) No strict limit for the program and free program design. For example, in MATLAB, users do not need to predefine matrix when they use it.

5) Well portability procedures. Basically one can run the program without any modification on various types of computers and operating systems.

6) Powerful graphics function. In both C and FORTRAN languages, graphics are not easy to draw, but in MATLAB, data visualization is very simple. MATLAB also has strong ability to edit graphical interface.

7) Powerful toolbox is another feature of MATLAB. MATLAB consists of two parts: a core part and a variety of optional toolboxes. There are hundreds of core internal functions in

the core part. Its toolbox is divided into two categories: functional toolbox and disciplinary toolbox. Functional toolbox is mainly used to expand the functionality of its symbolic computation capabilities, modelling and simulation capabilities, text processing and hardware real-time interactivity. Functional toolbox can be used for a variety of disciplines. All those toolboxes are written by some high-level experts in the field, so users do not need to write basic procedures within the scope of their discipline.

8) Open source program. Perhaps it is the most welcomed feature by people. In addition to the internal function, all MATLAB core files and toolboxes are readable and writeable source files, so users can modify the source files and add their own files to constitute a new toolbox.

The drawback of MATLAB is that, compared with other advanced procedures [28], the processing speed is slow. Because the procedures in MATLAB do not need to do the pre-treatment, and cannot generate an executable file, in the light of interpretation, the speed is slow.

In this chapter, I will use MATLAB to do some simulations on the which factors can affect the DOA estimation.

1 Basic simulation of the MUSIC algorithm for DOA estimation

The first simulation shows how two signals are recognized by the MUSIC algorithm. There are two independent narrow band signals, the incident angle is 20° and 60^0 respectively, those two signals are not correlated, the noise is ideal Gaussian white noise, the SNR is 20dB, the element spacing is half of the input signal wavelength, array element number is 10, the number of snapshots is 200. The simulation results are shown in Figure 4.1:

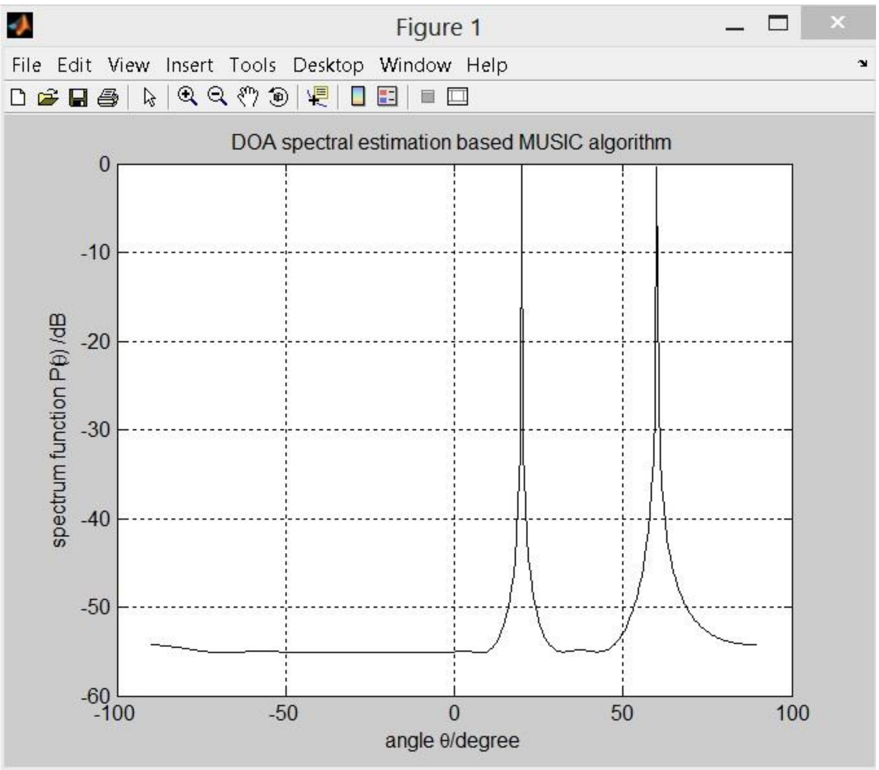

Figure 4.1 basic simulation for MUSIC algorithm

As can be seen from Figure 4.1, for the hypothetical situation with two independent signals, using MUSIC a needle spectrum peak algorithm can be constructed. It may well estimate the number and direction of the incidence signal, which can be used to estimate the independent signal source DOA effectively. Under the accurate model, the DOA estimation can reach any precision; by overcoming the traditional shortcomings of low precision, it can solve the direction problem with high resolution and high precision in a multiple signal environment. Hence, high resolution MUSIC algorithm may measure accuracy, high sensitivity features and have potentially capability with multi-resolution signals; with better performance and higher efficiency, it can provide high resolution and asymptotically unbiased DOA estimation, which has a great significance for practical application.

2 The relationship between DOA estimation and the number of array elements

The second simulation shows there are two independent narrow band signals, the incident angle is 20° and 60^0 respectively, those two signals are not correlated, the noise is ideal Gaussian white noise, the SNR is 20dB, the element spacing is half of the input signal wavelength, array element number is 10, 50 and 100, the number of snapshots is 200. The simulation results are shown in Figure 4.2:

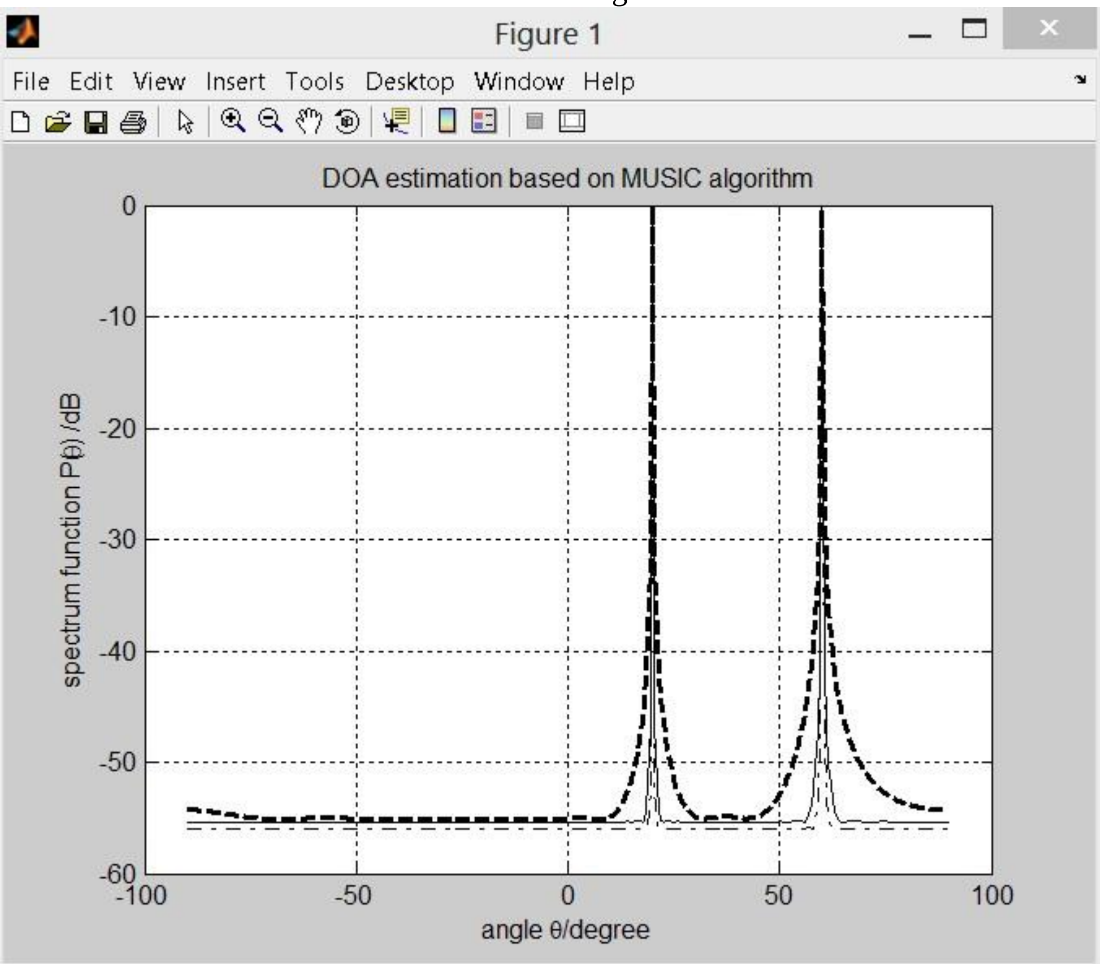

Figure 4.2 simulation for the relationship between MUSIC algorithm and the number of array elements

As can be seen from Figure 4.2, the dashed line shows the number of array elements are 10, the solid line shows the number of array elements are 50, and the dash-dotted line shows the number of array elements are 100. With other conditions remaining unchanged and with the increase in the number of array elements, DOA estimation spectral beam width becomes narrow, the directivity of the array becomes good; that is to say, the ability to distinguish spatial signals is enhanced. Hence, to get more accurate estimations of DOA it can increase the number of array elements, but the more the number of array elements the more the data needs processing; and the more amount of computation, the lower the speed. From the above figure, when the number of the array elements amounts to 50 and 100, their beam width is very similar. Therefore, in practice, the number of elements can be appropriately selected according to specific conditions, and we make sure of the accuracy of estimates spectrum. By minimizing the waste of resources and accelerating the speed of operation, work efficiency may improve.

3 The relationship between DOA estimation and the array element spacing

The third simulation shows there are two independent narrow band signals, the incident angle is $20°$ and 60^0 respectively, those two signals are not correlated, the noise is ideal Gaussian white noise, the SNR is 20dB, array element number is 10, the number of snapshots is 200, the array spacing is $\lambda/6$, $\lambda/2$, λ. The simulation results are shown in Figure 4.3:

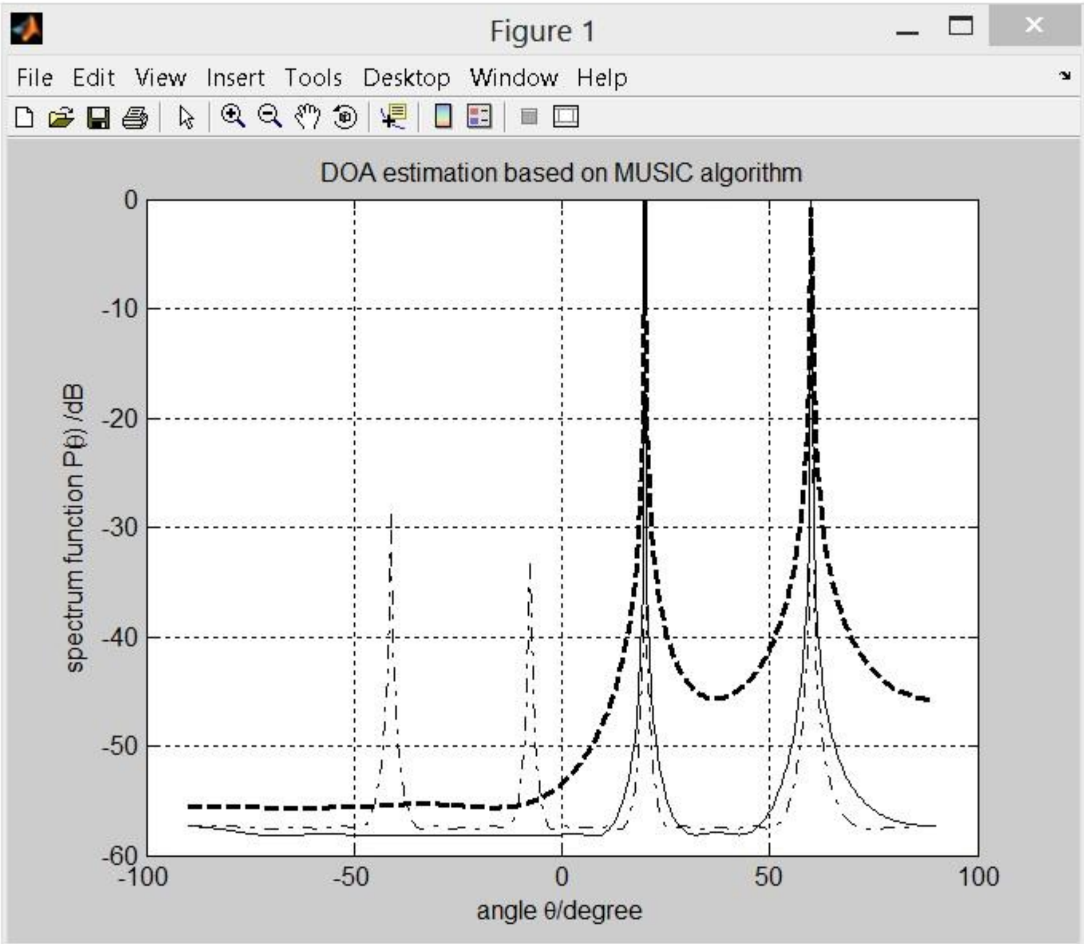

Figure 4.3 simulation for the relationship between MUSIC algorithm and array element spacing

As can be seen from Figure 4.3, the dashed line shows the array elements spacing is $\lambda/6$, the solid line shows the array elements spacing is $\lambda/2$, and the dash-dotted line shows the array elements spacing is λ. With the other conditions remaining the same, when the array element spacing is not more than half the wavelength, with increasing array element spacing, the beam width of DOA estimation spectrum becomes narrow, the direction of the array elements becomes good; that is to say, the resolution of MUSIC algorithm improves with the increase in the spacing of array element, but, when the spacing of the array elements is larger than half the wavelength, the estimated spectrum, except for the signal source direction, shows false peaks, so it has lost the estimation accuracy. Hence, in practical applications, more attention should be paid to the spacing of the array elements; element spacing can be increased but must not exceed half the wavelength, which is a very important point. It is best to set half the wavelength element spacing.

4 The relationship between DOA estimation and the number of snapshots

The fourth simulation shows how two signals are recognized by the MUSIC algorithm. There are two independent narrow band signals, the incident angle is $20°$ and 60^0 respectively, those two signals are not correlated, the noise is ideal Gaussian white noise, the SNR is 20dB, the element spacing is half of the input signal wavelength, array element number is 10, the number of snapshots is5, 50 and 200. The simulation results are shown in Figure 4.4:

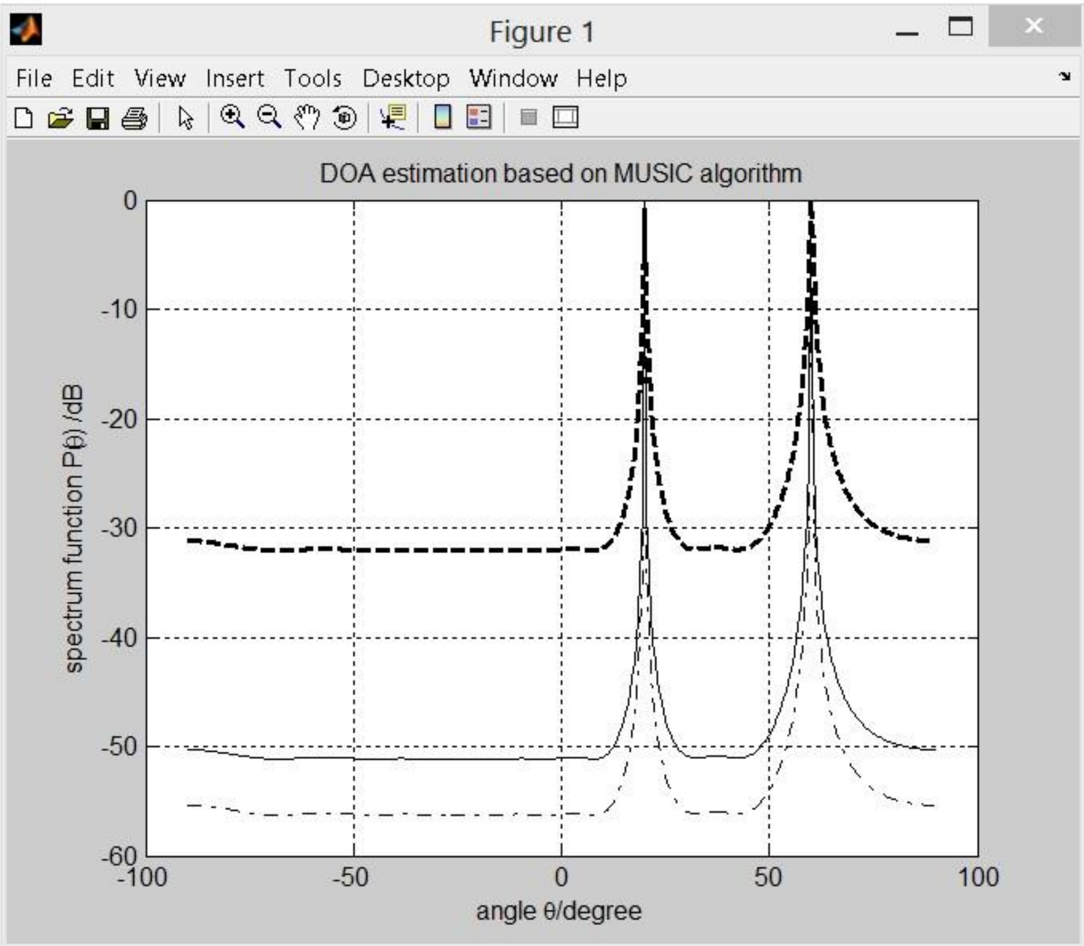

Figure 4.4 simulation for relationship between MUSIC algorithm and the number of snapshots

As can be seen from Figure 4.4, the dashed line shows the number of snapshots are 5, the solid line shows the number of snapshots are 50 and the dash-dotted line shows the number of snapshots are 200. With the other conditions remaining unchanged and with the increase in the number of snapshots, the beam width of DOA estimation spectrum becomes narrow, the direction of the array element becomes good and the accuracy of MUSIC algorithm is also increased. Hence, the number of sample snapshots can be expanded to multiply the accuracy of DOA estimation, but the more sample snapshots, the more the data needs to be processed; the more amount of calculation of MUSIC algorithm, the lower the speed. So in practical application, we select reasonable sampling snapshots which ensure the accuracy of DOA estimation, minimize the amount of computation and accelerating the speed of work and saving resources.

5 The relationship between DOA estimation and SNR

The fifth simulation shows how two signals are recognized by the MUSIC algorithm. There are two independent narrow band signals, the incident angle is 20° and 60^0 respectively, those two signals are not correlated, the noise is ideal Gaussian white noise, the element spacing is half of the input signal wavelength, array element number is 10, the number of snapshots is 200, the SNR is -20dB, 0dB and 20dB. The simulation results are shown in Figure 4.5:

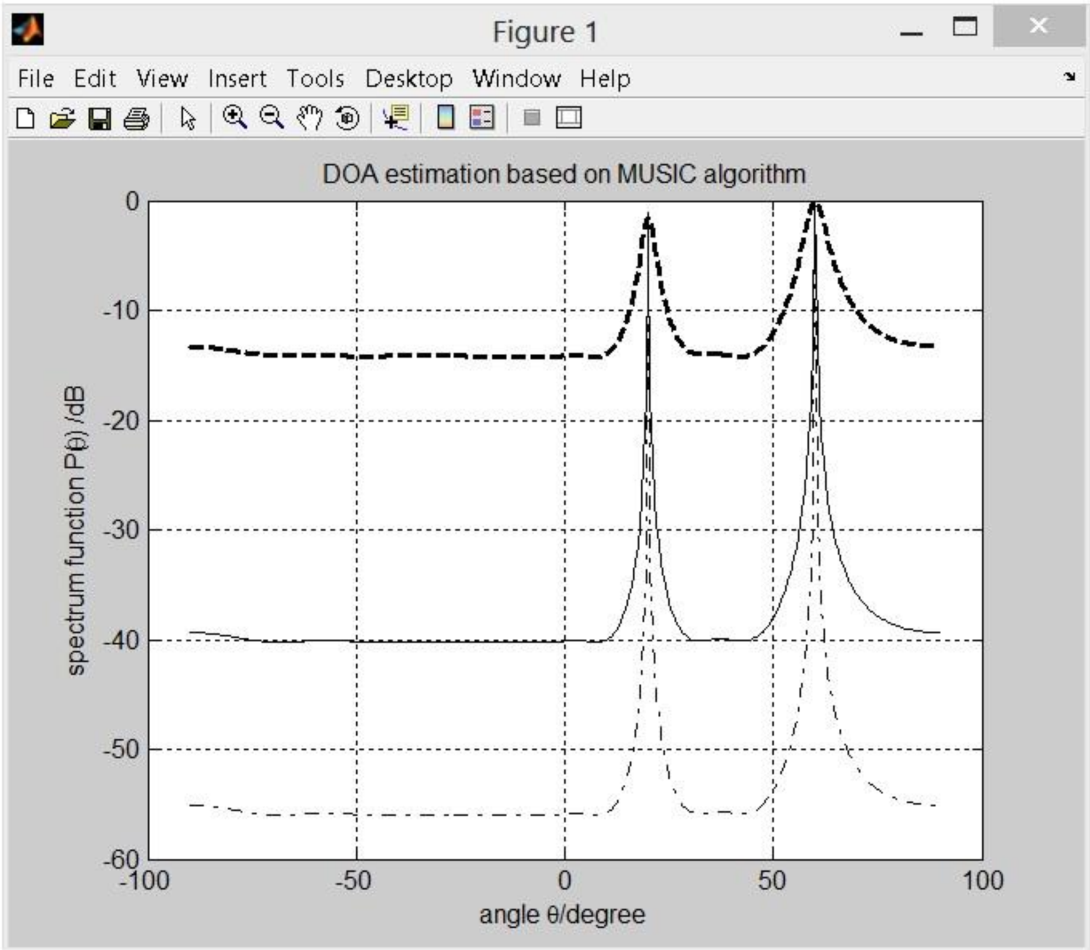

Figure 4.5 simulation for the relationship between MUSIC algorithm and SNR

As can be seen from Figure 4.5, the dashed line shows the SNR is -20dB, the solid line shows the SNR is 0dB and the dash-dotted line shows the SNR is 20dB. With the other conditions remaining unchanged, with the increase in the number of SNR, the beam width of DOA estimation spectrum becomes narrow, the direction of the signal becomes clearer, and the accuracy of MUSIC algorithm is also increased. The value of SNR can affect the performance of high resolution DOA estimation algorithm directly. At low SNR, the performance of MUSIC algorithm will sharply decline, thus, improving the estimation performance under low SNR is a main research topic for high resolution DOA estimation. Some scholars have proposed a DOA estimation algorithm based on Multistage Weiner filter (MSWF), which uses MSWF to estimate the signal subspace in the incident direction of the signal, then makes sure the estimate is valid through finding the minimize across-correlation function after decomposition of MSWF. The signal subspace and orthogonal noise subspace are estimated, and then the spatial spectrum is constructed to achieve DOA estimation. Experts have proved in low SNR conditions, compared with subspace algorithm, MSWF algorithm has better resolution and error performance. The accuracy of DOA estimation under low SNR has large room for development and improvement, pending further study.

6 The relationship between DOA estimation and the signal incident angle difference

The sixth simulation shows how two signals are recognized by the MUSIC algorithm. There are two independent narrow band signals, those two signals are not correlated, the noise is ideal Gaussian white noise, the SNR is 20dB, the element spacing is half of the input signal wavelength, array element number is 10, the number of snapshots is 200 and the incident angle is 5° and 10^0 and 20^0respectively. The simulation results are shown in Figure 4.6:

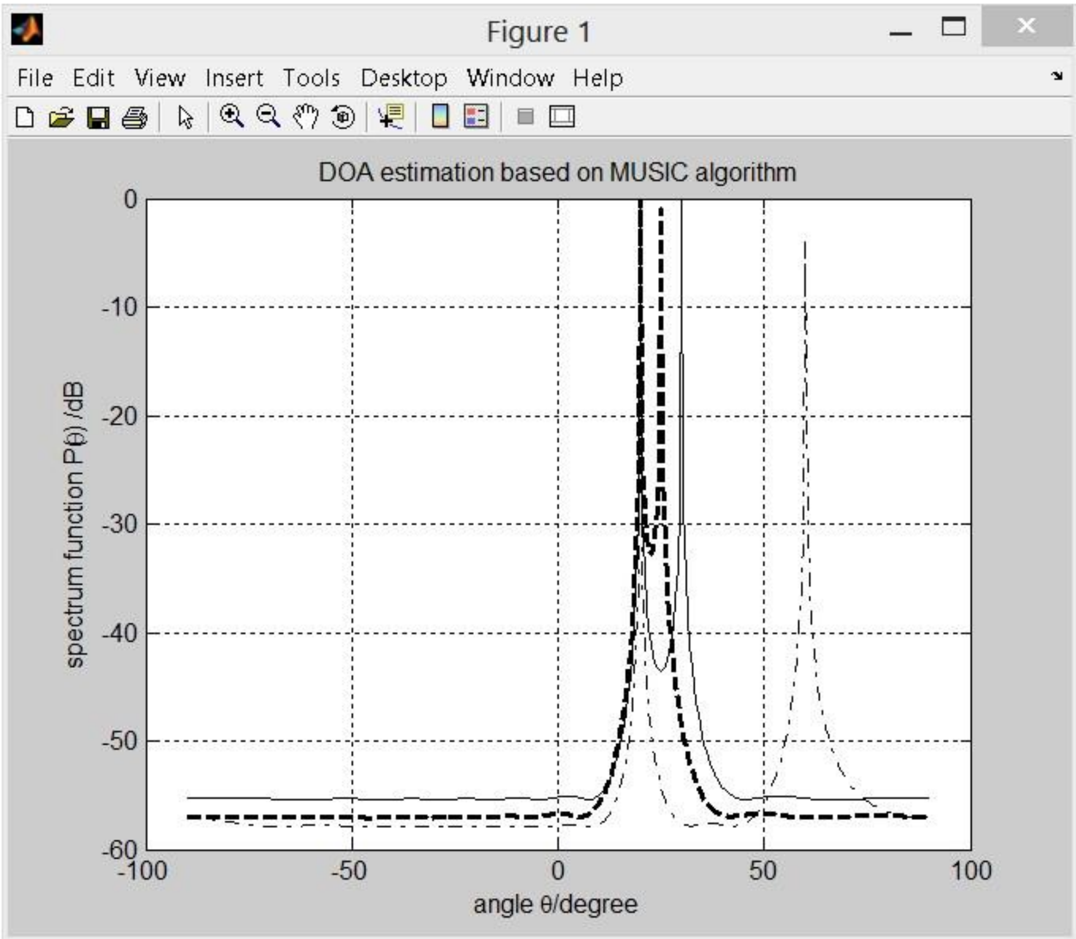

Figure 4.6 simulation for the relationship between MUSIC algorithm and the incident angle difference

As can be seen from Figure 4.6, the dashed line shows the incident angle is 5°, the solid line shows the incident angle is 10° and the dash-dotted line shows the incident angle is 20°. With the other conditions remaining unchanged and with the increase in incidence angle difference, the beam width of the DOA estimation spectrum becomes narrow, the direction of the signal becomes clear and the resolution of MUSIC algorithm is also increased. When the signal wave angle space is very small, the algorithm cannot estimate the number of signal sources.

The usual array signal source estimation method conducted under the condition of the incident angle being large, when the angle difference signal wave direction is relatively small, are estimated to be ineffective. Some scholars have proposed amendments to the square root of the Gerschgorin radius estimation method, since it can estimate the source well when the incident angle difference of signal direction is small. Some kind of method to estimate the number of signal sources, mostly have certain application conditions, therefore, to research the real time, steady number of signal sources in line with practical application and DOA estimation still has much significance.

7 The MUSIC algorithm and improved MUSIC algorithm for coherent signals

The seventh and eighth simulations show how two signals are recognized by the MUSIC algorithm and improved MUSIC algorithm. When the signals are coherent, let the incident angle is 20° and 60^0 respectively, those two signals are not correlated, the noise is ideal Gaussian white noise, the SNR is 20dB, the element spacing is half of the input signal wavelength, array element number is 10, and the number of snapshots is 200. The simulation results are shown in Figure 4.7 and Figure 4.8.

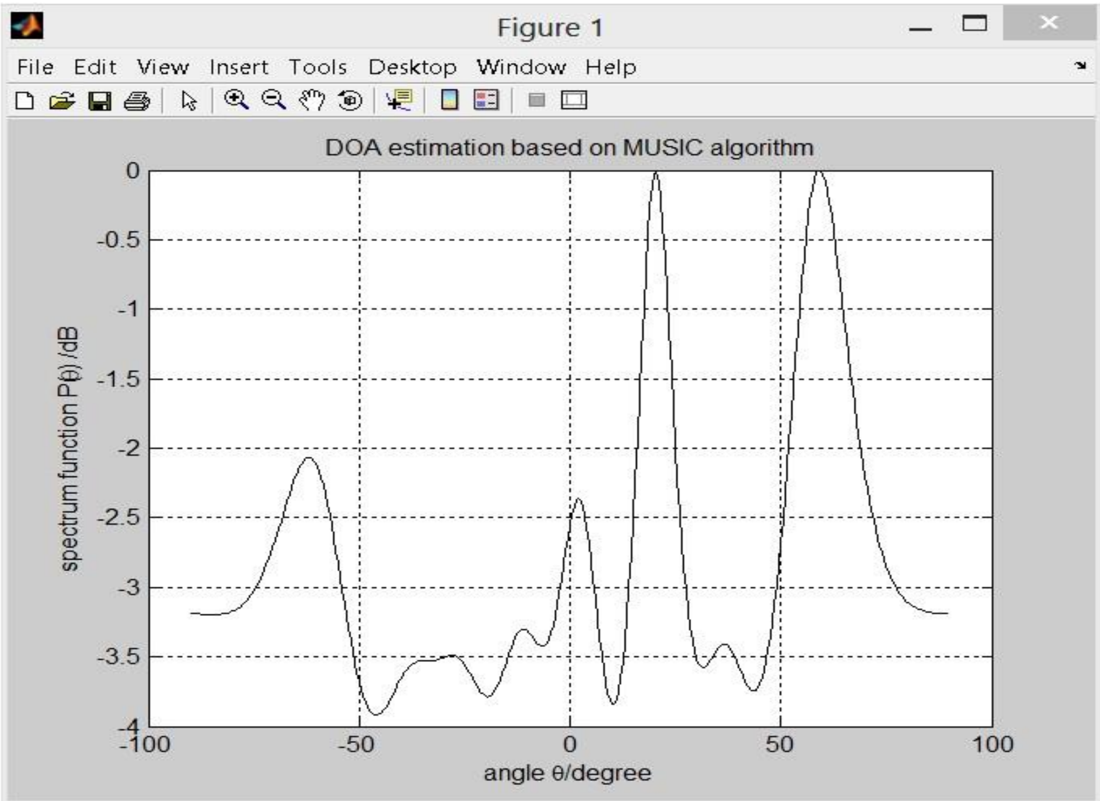

Figure 4.7 Simulation for MUSIC algorithm when the signals are coherent

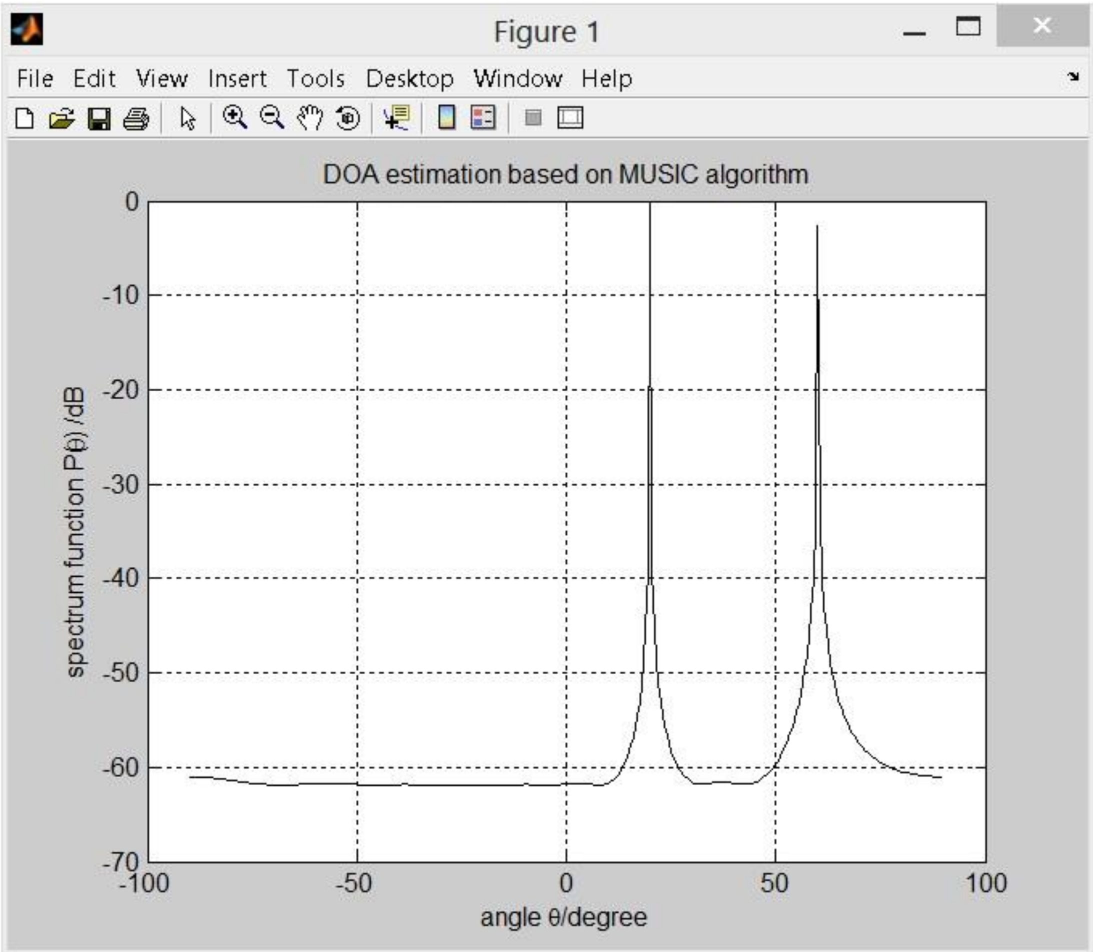

Figure 4.8 Simulation for the improved MUSIC algorithm when the signals are coherent

As can be seen from Figure 4.7 and Figure 4.8, for coherent signals, classic MUSIC algorithm has lost effectiveness, while improved MUSIC algorithm can be better applied to remove the signal correlation feature, which can distinguish the coherent signals, and estimate the angle of arrival more accurately. Under the right model, using MUSIC algorithm to estimate DOA can get any high resolution. But MUSIC algorithm only focuses on uncorrelated signals; when the signal source is correlation signal, the MUSIC algorithm estimation performance deteriorates or fails completely. This improved MUSIC algorithm can make DOA estimation more complete, and have a marked effect both on theoretical and practical study. For signals to stay coherent, there are many jobs needed to be done with the realization of DOA estimation, and thus further research is needed.

Summary

The more the number of array elements, the more the number of snapshots; the more different between the incident angles, the higher resolution the MUSIC algorithm has. When the array element spacing is not more than half the wavelength, the resolution of MUSCI algorithm increases correspondingly with the increase of array element spacing; however, if the array element spacing is greater than half the wavelength, the spatial spectrum causes false peaks in other direction except the direction of signal source.

When moving low SNR and small difference of incident angle, the performance of the MUSIC algorithm will decline. Some scholars have proposed some improvements in algorithm, but these problems are still a hot research topic. Hence, the MUSIC algorithm still has much room for development, and it is also worth further study.

Chapter 5 Existing problems and solutions in MUSIC algorithm

1 Channel lost pairing

The MUSIC algorithm has many advantages which traditional methods cannot compare with, but there also exist many limitations when applied in some real systems. One major reason is that it is more sensitive to error in the system, when the system is in long course of work, making the ageing characteristics of each channel increasingly serious. With differences that cause channel mismatch, it will give the performance of MUSIC algorithm serious trouble.

Correction of inconsistencies in channels can be classified into two types, self-correction and active correction. Active correction is a relatively mature correction method, as long as it predicts the direction of the signal source, and it can perform channel amplitude correction or directly compensate for the algorithm through the general correction matrix. In terms of self-correction, it is by use of a priori knowledge of the structure of the array to conduct a pre-process to the received data. In addition, there have existed many algorithms to improve the robustness of the MUSIC algorithm, such as the Toeplitz algorithm based on maximum likelihood estimation, but there are special requirements for the array structure.

2 Underestimation and overestimation of the number of interference source

When the estimated interferers coincide with the number of actual interferers, the MUSIC algorithm to estimate the direction of interference is accurate. However, when the number of estimated interferers is more than the number of actual interferers (overestimation), then in dividing the signal subspace and noise subspace, the MUSIC space spectrum may possess more peaks than the actual number of interferers, namely false peaks. Similarly, if the estimated interferers are less than the number of actual interferers (underestimation), when in dividing the signal subspace and noise subspace, the dimensionalities in signal subspace will reduce, and some peak in the MUSIC space will disappear.

The method of source estimation is based on the Akaike[35] information criterion and minimum description length to determine the number of interference source; it also cannot consider the number of interference, but the MUSIC algorithm can be used with weighted modifications.

3 How the coherent interference source influences the algorithm

When the interference sources are coherent, MUSIC algorithms have problems when determining the number of interferer sources; they cannot divide the signal subspace and noise subspace, and thus will not be able to estimate the spatial spectrum.

To solve this problem we generally have the following categories: spatial smoothing technique, signal feature vector technique and frequency smoothing technique. The spatial smoothing method is the most commonly used method. Spatial smoothing method requires a specific spatial structure of the array that can be divided into several subarrays, then each sub-array can be calculated of the signal correlation matrix. Afterwards, we do the direction estimation for each sub-array correlation matrix. But its ability of decorrelation is weak, thus always making much more decrease of the estimated performance, reducing the effective aperture of the array, increasing the beam width of the array, and reducing the resolution of the array. It is a dimension reduction process, and because it requires a special antenna array structure, it is inapplicable for the reflecting surface of the multi-beam antenna.

The basic idea of frequency-domain smoothing [36] is taking the average of all covariance matrix of signal power in frequency components. We obtain a non-singular signal covariance matrix. This method is also applicable to broadband signal source processing.

Chapter 6 The future of DOA estimation

DOA estimation theory and technology have become more mature, but there are many directions which need further research.

1 DOA Estimation Theory

1) Signal model areas. From a rational mathematical model to the study of more complex and more realistic environment signal model, this would lay a solid foundation for the application of spatial spectrum theory and algorithm. For example, consideration of the array signal model causes all kinds of error in the system, the features of noise in an actual environment, noise and signal correlation and distributed signal models.

2) New theories and new methods for DOA estimation. On the one hand, focus on the research of super-resolution DOA estimation theory and algorithm under general background is still necessary; on the other hand, focus on the research of DOA algorithm under specific background does not just stay in the research of general algorithm.

3) Information utilization aspect. Spatial spectrum estimation techniques not only use the information signal to estimate the spatial orientation parameters of the signal, but also make full use of the information of the time-domain signal. The different statistical characteristics of signal and noise as well as other available information is to increase the signal separability to improve the DOS estimation performance. At present, this aspect of study concentrates on the use of information, for instance, the use of Doppler information signals to achieve a multidimensional parameter estimation dimensionality reduction; utilizing pulse echo signal to improve the signal to noise ratio; using high-order Cumulant to restrain Gaussian noise; using different signals Cyclostationarity to separate signals. Meanwhile, the time-domain information for DOA algorithm is not deep enough, for example, the use of Doppler information dimensionality reduction will have much impact on DOA estimation; utilizing pulse information will bring much extent to estimates the signal subspace and noise subspace; the requirement of high-order cumulants to the number of samples and all of this requires more in-depth study.

2 Robust DOA estimation

The present spatial spectrum research normally only considers the array amplitude and phase errors, mutual coupling and position error. The other factors impacting on the estimated errors are relatively small, such as near-field scattering, electromagnetic interference and channel bandwidth inconsistency (especially for broadband systems), non-linear channel amplifier, quantization error and I/Q quadrature sampling errors.

Since array calibration and angle joint estimation of the parameters is an important direction for robustness of the algorithm, it has a lot of potential to explore. This will greatly promote the application of spatial spectrum estimation techniques, such as correction path in multi-array conditions and error correction for broadband arrays.

DOA estimation and array calibration can all contribute to the parameters optimization problem, and therefore, optimizing the function constructor, a fast algorithm for solving optimization function is worthy of further study.

3 Fast algorithm for DOA estimation

In general, existing high-resolution DOA algorithm still has a relatively large amount of computation. How to reduce the computational resources, to improve SNR and the performance of DOA estimation in small snapshots condition and to enhance the real-time character, robustness and lower implementation complexity will be an important part of link for spatial spectrum estimation technology. Therefore, the software and hardware design issues for DOA estimation are bound to be a hot spot. Important aspects are rapid estimation feature subspace and VLSI implementation in DOA algorithm.

4 Array configuration setting problems

Currently, one-dimensional linear array setup issues has much in-depth research, but research on two-dimensional array has less work done. This aspect of study is more in accordance with real environments and targets. Array configuration settings in particular environments or platforms, the optimal DOA algorithm in particular array setting, and planar array error on DOA algorithm are important.

5 DOA estimation for the signal form

At present, although the multipath signals, cyclostationary signals, wideband signals, and distribution signals all have a large amount of research, the research is not mature enough; in regard to the DOA estimation when multipath signal azimuth is dense, and the DOA estimation for wideband signal bands is inconsistent. The problem with the joint between parameters of circulatory angle, and the problems about the parameter estimation under the condition of variety of signal coexist.

Chapter 7 Conclusion

DOA estimation plays an important role in array signal processing, and has a wide range of applications. In many areas, such as communication, radar, sonar, weather forecasting, ocean and geological exploration, seismic survey and biomedicine, DOA estimation problems may occur.

The key to DOA estimation is to use an antenna signal array which is located in different spatial regions to receive signals from signal sources in different directions. Then the use of modern signal processing methods may quickly and accurately estimate the direction of the signal sources. In recent years, a variety of DOA estimation algorithms has achieved fruitful results, which provides a solid theoretical foundation for practical application. In this thesis, I have done some research for multiple signal classification theoretical study and simulation. The main contents and conclusions made in this thesis are summarized as follows:

In this thesis, by describing DOA estimation, spatial spectrum estimation, and giving a mathematical model of DOA estimation, an understanding of DOA estimation was provided. And then the MUSIC algorithm (Multiply Signal Classification) was implemented in MATLAB, and simulations were performed. From the simulations, it could be seen that the MUSIC algorithm has a higher resolution the more the number of array elements, the more the number of snapshots, and the larger the difference between the incident angles. When the array element spacing is less than half the wavelength, the MUSIC algorithm resolution increases in accord with the increase of array element spacing, however, when the array element spacing is greater than the half of wavelength, except the direction of signal source, other directions appeared as false peaks in the spatial spectrum. When the signal is coherent, classical MUSIC algorithm has lost effectiveness, and improved MUSIC algorithm is able to effectively distinguish their DOA. I implemented the improved MUSIC algorithm for coherent signals. Finally, I puzzled out some problems by using the MUSIC algorithm in practical application and giving some solutions on those problems, then looked forward to the future of DOA estimation as I have mentioned in Chapter 6.

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APPENDIX

Appendix 1 MATLAB codes for the basic MUSIC algorithm

```
%By Honghao Tang
clc
clear all
format long %The data show that as long shaping scientific
doa=[20 60]/180*pi; %Direction of arrival
N=200;%Snapshots
w=[pi/4 pi/3]';%Frequency
M=10;%Number of array elements
P=length(w); %The number of signal
lambda=150;%Wavelength
d=lambda/2;%Element spacing
snr=20;%SNA
D=zeros(P,M); %To creat a matrix with P row and M column
for k=1:P
D(k,:)=exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M-1]); %Assignment matrix
end
D=D';
xx=2*exp(j*(w*[1:N])); %Simulate signal
x=D*xx;
x=x+awgn(x,snr);%Insert Gaussian white noise
R=x*x'; %Data covarivance matrix
[N,V]=eig(R); %Find the eigenvalues and eigenvectors of R
NN=N(:,1:M-P); %Estimate noise subspace
theta=-90:0.5:90; %Peak search
for ii=1:length(theta)
 SS=zeros(1,length(M));
 for jj=0:M-1
   SS(1+jj)=exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
end
PP=SS*NN*NN'*SS';
Pmusic(ii)=abs(1/ PP);
end
Pmusic=10*log10(Pmusic/max(Pmusic)); %Spatial spectrum function
plot(theta,Pmusic,'-k')
xlabel('angle \theta/degree')
ylabel('spectrum function P(\theta) /dB')
title('DOA estimation based on MUSIC algorithm ')
grid on
```

Appendix 2 MATLAB codes for the relationship between DOA estimation and the number of array elements

```
clc
clear all
format long %The data show that as long shaping science and technology
N=200; %Snapshots
doa=[20 60]/180*pi; %DOA
w=[pi/4 pi/3]'; %Frequency
M1=10; %Array elements number
M2=50;
M3=100;
P=length(w); %Number of signal
lambda=150; %Wavelength
d=lambda/2; %Array element spacing
snr=20; %SNR
D1=zeros(P,M1);
D2=zeros(P,M2);
D3=zeros(P,M3);
for k=1:P
D1(k,:)=exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M1-1]); %Assignment matrix
D2(k,:)=exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M2-1]);
D3(k,:)=exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M3-1]);
end
D1=D1';
D2=D2';
D3=D3';
xx=2*exp(j*(w*[1:N])); %Simulate the signal
x1=D1*xx;
x2=D2*xx;
x3=D3*xx;
x1=x1+awgn(x1,snr); %Add Gaussian white noise
x2=x2+awgn(x2,snr);
x3=x3+awgn(x3,snr);
R1=x1*x1'; %Data covariance matrix
R2=x2*x2';
R3=x3*x3';
[N1,V1]=eig(R1); %Find the eigenvalues and eigenvectors of R
[N2,V2]=eig(R2);
[N3,V3]=eig(R3);
NN1=N1(:,1:M1-P); ; %Estimate the noise subspace
NN2=N2(:,1:M2-P);
NN3=N3(:,1:M3-P);
theta=-90:0.5:90;
%%Search the peak
for ii=1:length(theta)
SS1=zeros(1,length(M1));
 for jj=0:M1-1
   SS1(1+jj)=exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
end
PP1=SS1*NN1*NN1'*SS1';
```

```
Pmusic1(ii)=abs(1/ PP1);
end
for ii=1:length(theta)
 SS2=zeros(1,length(M2));
 for jj=0:M2-1
   SS2(1+jj)=exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
end
PP2=SS2*NN2*NN2'*SS2';
Pmusic2(ii)=abs(1/ PP2);
end
for ii=1:length(theta)
 SS3=zeros(1,length(M3));
 for jj=0:M3-1
   SS3(1+jj)=exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
end
PP3=SS3*NN3*NN3'*SS3';
Pmusic3(ii)=abs(1/ PP3);
end
Pmusic1=10*log10(Pmusic1/max(Pmusic1)); %Spatial spectrum function
Pmusic2=10*log10(Pmusic2/max(Pmusic2));
Pmusic3=10*log10(Pmusic3/max(Pmusic3));
plot(theta,Pmusic1,'--k','LineWidth',2.0)
hold on
plot(theta,Pmusic2,'k','LineWidth',1.0)
hold on
plot(theta,Pmusic3,'-.k','LineWidth',0.1)
hold off
xlabel('angle \theta/degree')
ylabel('spectrum function P(\theta) /dB')
title('DOA estimation based on MUSIC algorithm')
grid on
```

Appendix 3 MATLAB codes for the relationship between DOA estimation and the array element spacing

```
clc
clear all
format long
N=200;
doa=[20 60]/180*pi;
w=[pi/4 pi/3]';
M=10;
P=length(w);
lambda=150;
d=lambda/6;
snr=20;
D=zeros(P,M);
for k=1:P
D(k,:)=exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M-1]);
end
D=D';
xx=2*exp(j*(w*[1:N]));
x=D*xx;
x=x+awgn(x,snr);
R=x*x';
[N,V]=eig(R);
NN=N(:,1:M-P);
theta=-90:0.5:90;
for ii=1:length(theta)
 SS=zeros(1,length(M));
 for jj=0:M-1
   SS(1+jj)=exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
end
PP=SS*NN*NN'*SS';
Pmusic(ii)=abs(1/ PP);
end
Pmusic=10*log10(Pmusic/max(Pmusic));
plot(theta,Pmusic ,'--k','linewidth',2.0)
hold on
clc
clear all
format long
N=200;
doa=[20 60]/180*pi;
w=[pi/4 pi/3]';
M=10;
P=length(w);
lambda=150;
d=lambda/2;
snr=20;
D=zeros(P,M);
for k=1:P
```

```
D(k,:)=exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M-1]);
end
D=D';
xx=2*exp(j*(w*[1:N]));
x=D*xx;
x=x+awgn(x,snr);
R=x*x';
[N,V]=eig(R);
NN=N(:,1:M-P);
theta=-90:0.5:90;
for ii=1:length(theta)
 SS=zeros(1,length(M));
 for jj=0:M-1
   SS(1+jj)=exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
end
PP=SS*NN*NN'*SS';
Pmusic(ii)=abs(1/ PP);
end
Pmusic=10*log10(Pmusic/max(Pmusic));
plot(theta,Pmusic,'k','linewidth',1.0)
hold on
clc
clear all
format long
N=200;
doa=[20 60]/180*pi;
w=[pi/4 pi/3]';
M=10;
P=length(w);
lambda=150;
d=lambda;
snr=20;
D=zeros(P,M);
for k=1:P
D(k,:)=exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M-1]);
end
D=D';
xx=2*exp(j*(w*[1:N]));
x=D*xx;
x=x+awgn(x,snr);
R=x*x';
[N,V]=eig(R);
NN=N(:,1:M-P);
theta=-90:0.5:90;
for ii=1:length(theta)
 SS=zeros(1,length(M));
 for jj=0:M-1
   SS(1+jj)=exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
end
PP=SS*NN*NN'*SS';
```

```
Pmusic(ii)=abs(1/ WW);
end
Pmusic=10*log10(Pmusic/max(Pmusic));
plot(theta,Pmusic,'-.k','linewidth',0.1)
hold off
xlabel('angle \theta/degree')
ylabel('spectrum function P(\theta) /dB')
title('DOA estimation based on MUSIC algorithm')
grid on
```

Appendix 4 MATLAB codes for the relationship between DOA estimation and the number of snapshots

```
clc
clear all
format long
N1=5;
N2=50;
N3=200;
doa=[20 60]/180*pi;
w=[pi/4 pi/3]';
M=10;
P=length(w);
lambda=150;
d=lambda/2;
snr=20;
D=zeros(P,M);
for k=1:P
D(k,:)=exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M-1]);
end
D=D';
xx1=2*exp(j*(w*[1:N1]));
xx2=2*exp(j*(w*[1:N2]));
xx3=2*exp(j*(w*[1:N3]));
x1=D*xx1;
x2=D*xx2;
x3=D*xx3;
x1=x1+awgn(x1,snr);
x2=x2+awgn(x2,snr);
x3=x3+awgn(x3,snr);
R1=x1*x1';
R2=x2*x2';
R3=x3*x3';
[N1,V1]=eig(R1);
[N2,V2]=eig(R2);
[N3,V3]=eig(R3);
NN1=N1(:,1:M-P);
NN2=N2(:,1:M-P);
NN3=N3(:,1:M-P);
theta=-90:0.5:90;
for ii=1:length(theta)
 SS=zeros(1,length(M));
for jj=0:M-1
   SS(1+jj)=exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
end
PP1=SS*NN1*NN1'*SS';
PP2=SS*NN2*NN2'*SS';
PP3=SS*NN3*NN3'*SS';
Pmusic1(ii)=abs(1/ PP1);
Pmusic2(ii)=abs(1/ PP2);
```

```
Pmusic3(ii)=abs(1/ PP3);
end
Pmusic1=10*log10(Pmusic1/max(Pmusic1));
Pmusic2=10*log10(Pmusic2/max(Pmusic2));
Pmusic3=10*log10(Pmusic3/max(Pmusic3));
plot(theta,Pmusic1,'--k','linewidth',2.0)
hold on
plot(theta,Pmusic2,'-k','linewidth',1.0)
hold on
plot(theta,Pmusic3,'-.k','linewidth',0.1)
hold off
xlabel('angle \theta/degree')
ylabel('spectrum function P(\theta) /dB')
title('DOA estimation based on MUSIC algorithm')
grid on
```

Appendix 5 MATLAB codes for the relationship between DOA estimation and SNR

```
clc
clear all
format long
N=200;
doa=[20 60]/180*pi;
w=[pi/4 pi/3]';
M=10;
P=length(w);
lambda=150;
d=lambda/2;
snr1=-20;
snr2=0;
snr3=20;
D=zeros(P,M);
for k=1:P
D(k,:)=exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M-1]);
end
D=D';
xx=2*exp(j*(w*[1:N]));
x=D*xx;
x1=x+awgn(x,snr1);
x2=x+awgn(x,snr2);
x3=x+awgn(x,snr3);
R1=x1*x1';
R2=x2*x2';
R3=x3*x3';
[N1,V1]=eig(R1);
[N2,V2]=eig(R2);
[N3,V3]=eig(R3);
NN1=N1(:,1:M-P);
NN2=N2(:,1:M-P);
NN3=N3(:,1:M-P);
theta=-90:0.5:90;
for ii=1:length(theta)
 SS=zeros(1,length(M));
 for jj=0:M-1
  SS(1+jj)=exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
end
PP1=SS*NN1*NN1'*SS';
PP2=SS*NN2*NN2'*SS';
PP3=SS*NN3*NN3'*SS';
Pmusic1(ii)=abs(1/ PP1);
Pmusic2(ii)=abs(1/ PP2);
Pmusic3(ii)=abs(1/ PP3);
end
Pmusic1=10*log10(Pmusic1/max(Pmusic1));
Pmusic2=10*log10(Pmusic2/max(Pmusic2));
Pmusic3=10*log10(Pmusic3/max(Pmusic3));
```

```
plot(theta,Pmusic1,'--k','linewidth',2.0)
hold on
plot(theta,Pmusic2,'-k','linewidth',1.0)
hold on
plot(theta,Pmusic3,'-.k','linewidth',0.1)
hold off
xlabel('angle \theta/degree')
ylabel('spectrum function P(\theta) /dB')
title('DOA estimation based on MUSIC algorithm')
grid on
```

Appendix 6 MATLAB codes for the relationship between DOA estimation and the signal incident angle difference

```
clc
clear all
format long
N=200;
doa1=[20 25]/180*pi;
doa2=[20 30]/180*pi;
doa3=[20 60]/180*pi;
w=[pi/4 pi/3]';
M=10;
P=length(w);
lambda=150;
d=lambda/2;
snr=20;
D=zeros(P,M);
for k=1:P
D1(k,:)=exp(-j*2*pi*d*sin(doa1(k))/lambda*[0:M-1]);
D2(k,:)=exp(-j*2*pi*d*sin(doa2(k))/lambda*[0:M-1]);
D3(k,:)=exp(-j*2*pi*d*sin(doa3(k))/lambda*[0:M-1]);
end
D1=D1';
D2=D2';
D3=D3';
xx=2*exp(j*(w*[1:N]));
x1=D1*xx;
x2=D2*xx;
x3=D3*xx;
x1=x1+awgn(x1,snr);
x2=x2+awgn(x2,snr);
x3=x3+awgn(x3,snr);
R1=x1*x1';
R2=x2*x2';
R3=x3*x3';
[N1,V1]=eig(R1);
[N2,V2]=eig(R2);
[N3,V3]=eig(R3);
NN1=N1(:,1:M-P);
NN2=N2(:,1:M-P);
NN3=N3(:,1:M-P);
theta=-90:0.5:90;
for ii=1:length(theta)
 SS=zeros(1,length(M));
 for jj=0:M-1
   SS(1+jj)=exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
end
PP1=SS*NN1*NN1'*SS';
PP2=SS*NN2*NN2'*SS';
PP3=SS*NN3*NN3'*SS';
```

```
Pmusic1(ii)=abs(1/ PP1);
Pmusic2(ii)=abs(1/ PP2);
Pmusic3(ii)=abs(1/ PP3);
end
Pmusic1=10*log10(Pmusic1/max(Pmusic1));
Pmusic2=10*log10(Pmusic2/max(Pmusic2));
Pmusic3=10*log10(Pmusic3/max(Pmusic3));
plot(theta,Pmusic1,'--k','linewidth',2.0)
hold on
plot(theta,Pmusic2,'-k','linewidth',1.0)
hold on
plot(theta,Pmusic3,'-.k','linewidth',0.1)
hold off
xlabel('angle \theta/degree')
ylabel('spectrum function P(\theta) /dB')
title('DOA estimation based on MUSIC algorithm')
grid on
```

Appendix 7 MATLAB codes The MUSIC algorithm and the improved MUSIC algorithm for coherent signals

1 The MUSIC algorithm for coherent signals

```
clc
clear all
format long
N=200;
doa=[20 60]/180*pi;
w=[pi/4 pi/4]';
M=10;
P=length(w);
lambda=150;
d=lambda/2;
snr=20;
D=zeros(P,M);
for k=1:P
D(k,:)=exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M-1]);
end
D=D';
xx=2*exp(j*(w*[1:N]));
x=D*xx;
x=x+awgn(x,snr);
R=x*x';
[N,V]=eig(R);
NN=N(:,1:M-P);
theta=-90:0.5:90;

for ii=1:length(theta)
 SS=zeros(1,length(M));
 for jj=0:M-1
   SS(1+jj)=exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
end
PP=SS*NN*NN'*SS';
Pmusic(ii)=abs(1/ PP);
end
Pmusic=10*log10(Pmusic/max(Pmusic));
plot(theta,Pmusic,'-k')
xlabel('angle \theta/degree')
ylabel('spectrum function P(\theta) /dB')
title('DOA estimation based on MUSIC algorithm')
grid on
```

2 The improved MUSIC algorithm for coherent signals

```
clc
clear all
format long
N=200;
doa=[20 60]/180*pi;
w=[pi/4 pi/4]';
M=10;
P=length(w);
lambda=150;
d=lambda/2;
snr=20;
D=zeros(P,M);
for k=1:P
D(k,:)=exp(-j*2*pi*d*sin(doa(k))/lambda*[0:M-1]);
end
D=D';
xx=2*exp(j*(w*[1:N]));
x=D*xx;
x=x+awgn(x,snr);
R=x*x';
J=fliplr(eye(M));
R=R+J*conj(R)*J;
[N,V]=eig(R);
NN=N(:,1:M-P);
theta=-90:0.5:90;
for ii=1:length(theta)
 SS=zeros(1,length(M));
 for jj=0:M-1
   SS(1+jj)=exp(-j*2*jj*pi*d*sin(theta(ii)/180*pi)/lambda);
end
PP= SS*NN*NN'*SS';
Pmusic(ii)=abs(1/ PP);
end
Pmusic=10*log10(Pmusic/max(Pmusic));
plot(theta,Pmusic,'-k')
xlabel('angle \theta/degree')
ylabel('spectrum function P(\theta) /dB')
title('DOA estimation based on MUSIC algorithm')
grid on
```

Lnu.se

Linnéuniversitetet
Kalmar Växjö
Fakulteten för teknik
391 82 Kalmar | 351 95 Växjö
Tel 0772-28 80 00
teknik@lnu.se
Lnu.se/fakulteten-for-teknik