

## CSE 330

1-9-12

\*errata list update in book

### Documentation

In each file: `/*x . . . . .x`  
Name  
date, started and stopped.  
filename  
Problem description  
Algorithm used and data structure used.  
`xxx . . . . .x/`

Each function: `/*x . . . . .x`  
Short description.  
`xxx . . . . .x/`

### indentation format

4 spaces for indent (Don't use tabs)

examples.

`if (i < 0) {  
 . . . .  
 . . . .  
} else {  
 . . . .  
}`

`for (i = 0; i < 10; i++) {  
 . . . .  
}`

`}`

### end of functions

```
main()
{
} //main
```

- user defined class names start with uppercase letters.
- objects should start with lower case letters.
- Do not abbreviate names!

### include files

test.h

```
#ifndef TEST_H
#define TEST_H
```

```
=====
#endif
```

#endif

- now test.h can be included  
without causing multiple definition  
error

main.cpp

```
#include "test.h"
main()
{
    void f();
    f();
} //main
```

f.cpp

```
#include <iostream>
#include "test.h"
f()
{
    cout << "f() \n";
} //f
```

## make files

make -f main.make

main: main.o f.o

$\xrightarrow{\text{tab}}$  g++ -o main main.o f.o

main.o: main.cpp test.h

$\xrightarrow{\text{tab}}$  g++ -c main.cpp

f.o: f.cpp

$\xrightarrow{\text{tab}}$  g++ -c f.cpp

main.cpp

f.cpp

int i;

extern int i;

main()

f()

- extern is an example of a  
Storage class

extern

automatic

static

register

## Lab notes - infix postfix Lab1

$$a+b*c = a+(b*c)$$

ab+

- Stack is an adapter

$$a*b+c = ab*c+$$

$$a+b*c = abc*+$$

$$(a+b)*(c-d) = ab+cd-*$$

$$((a+b)/c)*d = ab+c/d*$$

$$a+(b/c)*d = abc/d*+$$

$v.back = v.size() - 1$

vector<int> v;

v.push\_back(2);

v.push\_back(10);

v.back - top of stack

v.front - bottom of stack

v.empty - Boolean, true if empty.

stack in STL

#include <stack>

stack<int> first;

stack<char> second;

-input one character at a time.

-type out pop()

-use # for end of expression.

if (ch >= 'a' and ch <= 'z')

1-11-12

# Homework 1 : Ex 8 pg 21

Variance

main()

{ input until eof

call variance

}

double variance (array input)

{  
return variance.  
}

## Ascii codes

<u>char</u>	<u>dec</u>	<u>hex</u>	<u>binary</u>
0	48	30	0011 0000
1	49	31	
9	57	39	
A	65	41	0100 0001
Z	90	5A	
a	97	61	
z	122	7A	

Sample run

\$ script

At A D control d

puts everything in  
a file called typescript

```

#include <iostream>
#include <cstdio>
using namespace std;
main()
{
    char ch = 'a';
    int i;
    i = ch;

    cout << ch << " " << i << endl;
    printf ("%c %d", ch, ch);
}

```

output =    a   97  
                  a   97

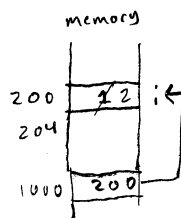
- dereference means content
- scaling  $p = a + 1 = 1004$

### Pointers, Arrays, strings, Parameters

```

main()
{
    int i = 1, *p;
    p = &i;
    *p = 2;
    cout << i; // 2
}

```



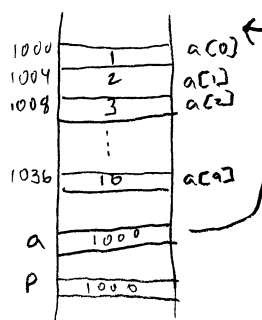
- pointers are always 4 bytes
- array is basically a pointer

array names are (constant) pointers

```

main()
{
    int a[10], *p;
    p = a;
    *p = 1;
}

```



```

for (p = a + 1; p < a + 10; p++)
{
    *p = *(p - 1) + 1;
}
for (p = a; p < a + 10; p++)
{
    cout << *p << endl;
}

```

pass by value.  
↑

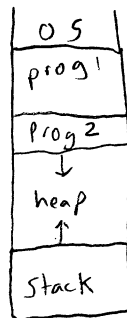
pass by reference  
↑

Parameters

<pre>f(int x) { x=3; }</pre>	<pre>f(int *x) { *x=3; }</pre>
------------------------------	--------------------------------

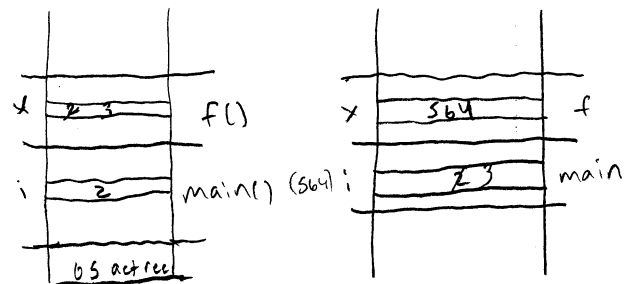
<pre>main() { int i=2;   f(i);   cout &lt;&lt; i; }</pre>	<pre>main() { int i=2;   f(&amp;i);   cout &lt;&lt; i; }</pre>
---	--

Stack of activation records.



-dynamic memory  
"new"

-default return type is  
int,

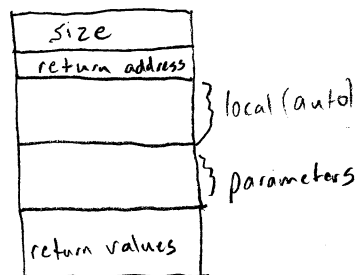


c++ version of above

```
f(int &x)
{ x=3;
}

main()
{ int i=2;
  f(i);
}
```

template of any activation record

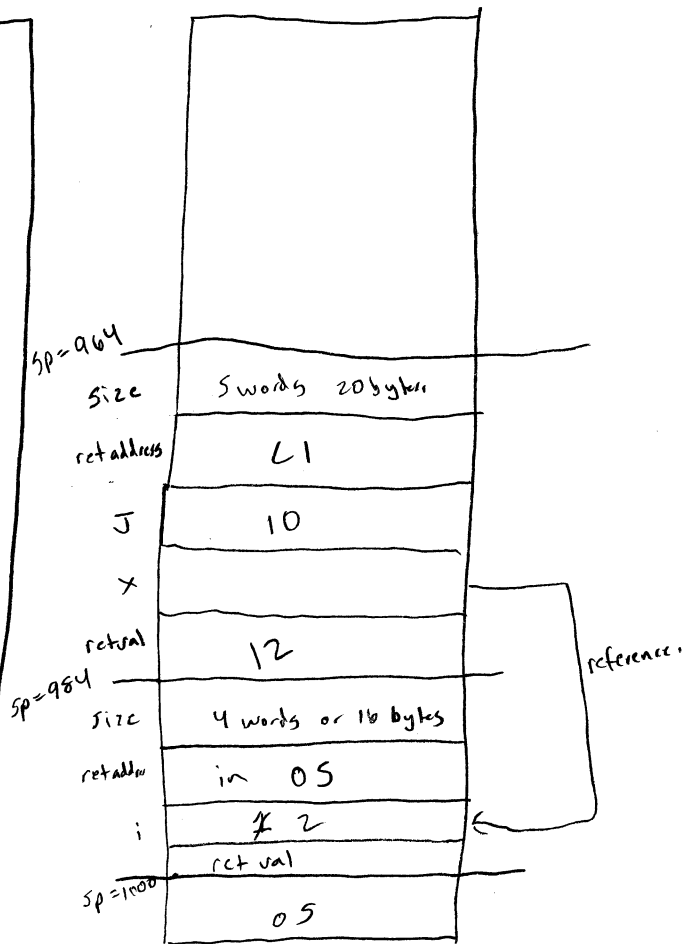


Prog.cpp

```
int y
int f(int &x)
{
    int j=10
    x++
    return x+j
}
```

```
int main()
{
    int i = 1
    L1: y = f(i)
    L2: cout << i << y
    L3: f(i)
    L4: cout << i << y;
    L5: return 0
}
```

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\$a.out

\$echo \$?, looks at return value of main program.



1-18-12

In lab report, typescript or screen shot  
every function, time and storage complexity  
email actual program to Ari

### Time Complexity

- we are interested in the maximum number of times an algorithm  
"loops", not how much time it takes.

↓ while loop

calls itself

calls another alg that calls it back

### Ex Alg 1

input  $n$

sum = 0

for  $i = 1$  to  $n$

sum +=  $i$

output sum

#times loop:  $n$

set - all functions related to  $f(n) = n$

time complexity is in  $O(n)$

Ex Alg 2

input  $n$

sum = 0

for  $i = 1$  to  $n$

sum += i      work  
output sum

loops  $n$  times  $\Rightarrow O(n)$

Ex Alg 3

input  $n$

sum = 0

for  $i = 1$  to  $n$

sum += i

output sum

sum = 0

for  $i = 1$  to  $n$

for  $j = 1$  to  $n$

sum += j

output sum

loops  $n + n^2$  times  $\Rightarrow$  time complex  $O(n^2)$

$O(n^2) = \{n + n^2, n^2, 5n^2, 0.0001n^2 + 202, \dots\}$

everything is based on the size of  $n$ .

- we ignore constants and smaller functions.

In ex1 we have  $C_1 n^6 + C_2 n + C_3 \Rightarrow O(n)$

In ex2 we have  $C_1 + C_4 n \Rightarrow O(n)$

In ex3 we have  $C_1 + C_2 n + C_3 + C_6 n^2 + C_3 \Rightarrow O(n^2)$

Definition -

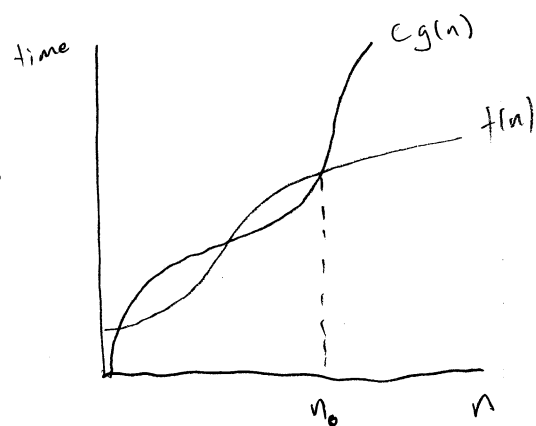
$f(n) \in O(g(n))$  iff  $f(n) \leq C g(n)$

$\uparrow$

time taken

by an algo

where  $C > 0$  &  $n \geq n_0$



Ex Alg 1

Suppose  $c_1 = 4$

$c_2 = 2$

$c_3 = 3$

$$4 + 2n + 3 = 2n + 7 \text{ } \mu_{\text{sec}}$$

$$2n + 7 \leq 3n \text{ for } n \geq 7$$

$$2n + 7 \in O(n)$$

$n$	$2n+7$	$3n$
-----	--------	------

1	9	3
---	---	---

2	11	6
---	----	---

$\vdots$	$\vdots$	$\vdots$
----------	----------	----------

from 7 on

7	21	21
---	----	----

8	23	24
---	----	----

Ex Alg 4

input  $n$

sum = 0

$i = 1$

while  $i \leq n$

sum +=  $i$

$i = 2 \times i$

output sum

time  $O(\log n)$

$$c_1 + c_2 \log_2 n + c_3 \leq c \log_2 n \Rightarrow O(\log_2 n) \Rightarrow O(\log n)$$

function time space

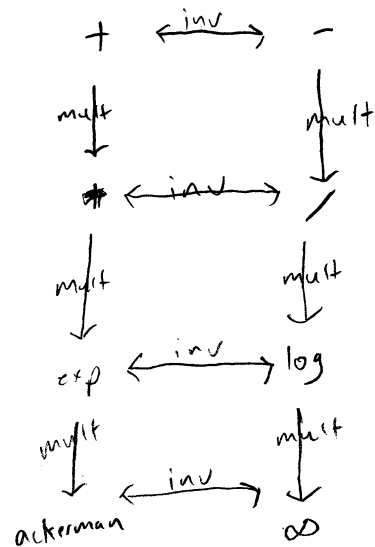
main()  $O(n)$   $O(n)$

variance()  $O(n)$   $O(n)$

inv = inverse  
mult = multiply

Ex Alg 5

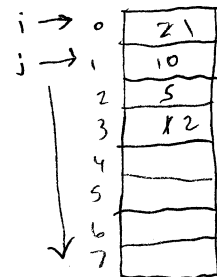
	<u>n</u>	<u>i</u>
input n	32	32
sum = 0		16
i = n		8
while i > 1		4
sum += i		2
i = i / 2		1
output sum		
time = O(log n)		



Selection Sort (Array [0, ..., n-1])

```

for i = 0 to n-1
    for j = i+1 to n
        if A[i] > A[j]
            swap(A[i], A[j])
    
```



$$\text{time} = (n-1) + (n-2) + (n-3) + \dots + 2 + 1 = \frac{(n-1)n}{2} \Rightarrow n^2 - n \Rightarrow O(n^2)$$

$$1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

```

f(int n)
{ if (n <= 1) return 1
  return n * f(n-1)
}

```

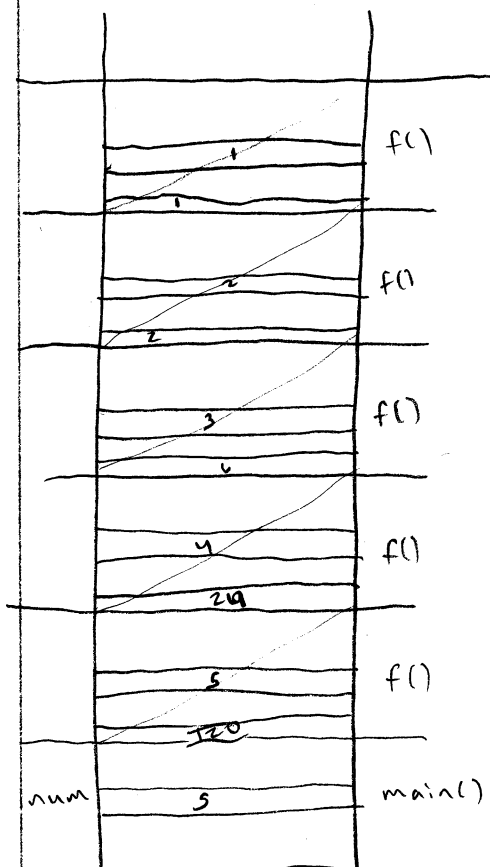
```

main()
{ int num
  cout << "enter num"
  cin >> num
  cout << "fact( " << num << ") = " << f(num)
}

```

3

Stack of activation records



time =  $O(n)$

fact(5) = 120

char = 1 byte  
int = 4 bytes

1-23-12

## Pointers & Operator Overloading

### C char strings

null terminated array of characters.

char str[10] = "test"

'\0' = 0 in ascii

'0' = 48 in ascii

'0' ≠ '\0'

0 is false, everything else is true

int strlen (char \* s)

{ int i

for(i=0; s[i] ≠ '\0'; i++)<sup>null</sup> → for(i=0; s[i]; i++) → for(i=0, \*s; i++, s++)

return i

}

↓  
for(int i=0; \*s++; i++)  
↑ comma  
post increment

memory {  
 strcat  
 strcpy  
 strlen  
 strncpy

<string>  
 c++ string

```
void strcpy(char *t, char *s)
{
  for(; *t = *s; t++, s++);
}
  \
  for(; *t++ = *s++;);
```

main()

```
char str1[10] = "test"
char str2[5] = "ab"
```

String a = "abc", b = "1234"

a = a + b

operator overloading

strcpy

↓  
 strcat

Chapter 7 - string

Chapter 4 - time complexity

Modulo 3 Arithmetic - only numbers 0, 1, 2

0	1	2
0	1	2
3	4	5
6	7	8
9	10	11
12	13	14

2 + 1 = 3

1

14 / 3 = 4 rem 2

not required  
default private

modulo m(5); // 0,1,2,3,4

modulo m(3)

m=3 n=8

m = m + 1 → 4

m++ → 0

m = m + n → 2

m = ++n

m = n++

cout << m << n

things we  
want to do

class Modulo {

private:

int n; // base

int value; // value

public:

Modulo(); // default constructor

Modulo(int m); // modulo m(5);

Modulo operator = (int) // assignment n=8

Modulo operator+ (int) // m = m + 1

int getValue(); // returns value

Modulo operator++ (); // pre increment

modulo operator++ (int); // post increment

};

Modulo::Modulo() {

n = 2

value = 0

}

Modulo::Modulo(int m) {

n = m

value = 0

assert(n > 0)

}

#include  
cassert

modulo modulo::operator = (int op) {

value = op;

value = value % n;

return \*this; // pointer to implicit parameter

}

modulo::modulo(int m, int v) {

n = m

value = v % m

}



Why time complexity,

## Lab 2

### selection sort

$n$	time	$C = \text{time} / n^2$
1000	.008	.000000008
2000	.032	.000000008
3000	.066	.000000007
4000	.108	.000000007
5000	.160	.000000006
6000	.220	.000000006
7000	.295	.000000006
8000	.370	.000000006
9000	.455	.000000006
10000	.551	.000000006

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modulo continued

public:

modulo (const modulo & m)

modulo operator = (modulo op)

modulo operator++ () // prefix

modulo operator++ (int) // prefix

modulo modulo::operator = (modulo op) {

value = op.value;

n = op.n

value %= n;

return \*this

}

Example

m1 = ++m2

modulo modulo::operator++ () {

value++;

value %= n;

return \*this

example.

m1 = m2++

modulo modulo::operator++ (int) {

modulo temp = \*this; // copy of implicit

++ (\*this);

return temp;

outside of class CC belongs to <sup>ostream</sup> ostream & operator CC (ostream & out, Modulo n):  
{ out << n.getvalue();  
return out;  
}

## Strings

end points to past end()

In STL:

```
String s = "abc"
String s ("abc");
cout << s.length();
cout << s[1];
s += "dc"; // abc dc
s.insert (2, "123") // ab123dc
s.replace()
s.find()
```

### generic functions

```
count (s.begin(), s.end(), 'i')
i = find ( " " " " )
```

## String

```
class String {
```

```
    int size;
```

```
    char * buffer; // allocate memory from heap
```

```
public:
```

```
    String ()
```

```
    String (const char *)
```

```
    String (const String &)
```

```
    ~String (); // deallocate memory
```

Big 3  
 copy constructor  
 destructor  
 assignment operator

strlen need <cstring>

```
String::String()      String::String(const char * p) // String s("abc");
{ size = 0           { size = strlen(p);
  buffer = 0          buffer = new char[size];
}                    for (int i = 0; i < size; i++)
                    { buffer[i] = p[i];
                    }
                    }
```

```
String::String(const String & source)
{ size = source.size;
  buffer = new char[size]
  for (int i = 0; i < size; i++)
  { buffer[i] = source.buffer[i];
  }
}
```

} → memcpy(buffer, source.buffer, size)

```
String String::operator=(const char * p)
{ delete [] buffer;
  size = strlen(p);
  buffer = new char[size];
  for (int i = 0; i < size; i++)
  { buffer[i] = p[i];
  }
}
```

S      0 1 2 3 4  
 2      5 6

7 - 2 = 5

S

1-30-12

## War - the game

Two players

Deck of cards

Each player 3 cards initially

Each player draws one of her cards

Highest rank card wins (2 pts)

a draw results in one pt for each player.

Players replace their drawn cards from the deck

game ends when player hands are empty (23 rounds)

Books version ends when deck is empty

player with highest score wins.

ace has lowest rank, king has highest

## Lets go over bottom up design

nouns are classes

verbs are member functions

enum Suites { diamonds, clubs, hearts, spades }  
0 1 2 3

enumeration

class Card {

private:

int rank

int suit → Suites suit;

public:

Card()

Card(suites, int.)

};

class Deck {

private:

Card cards[sz];

int topCard; // pointer to top card

public:

Deck()

Card Draw()

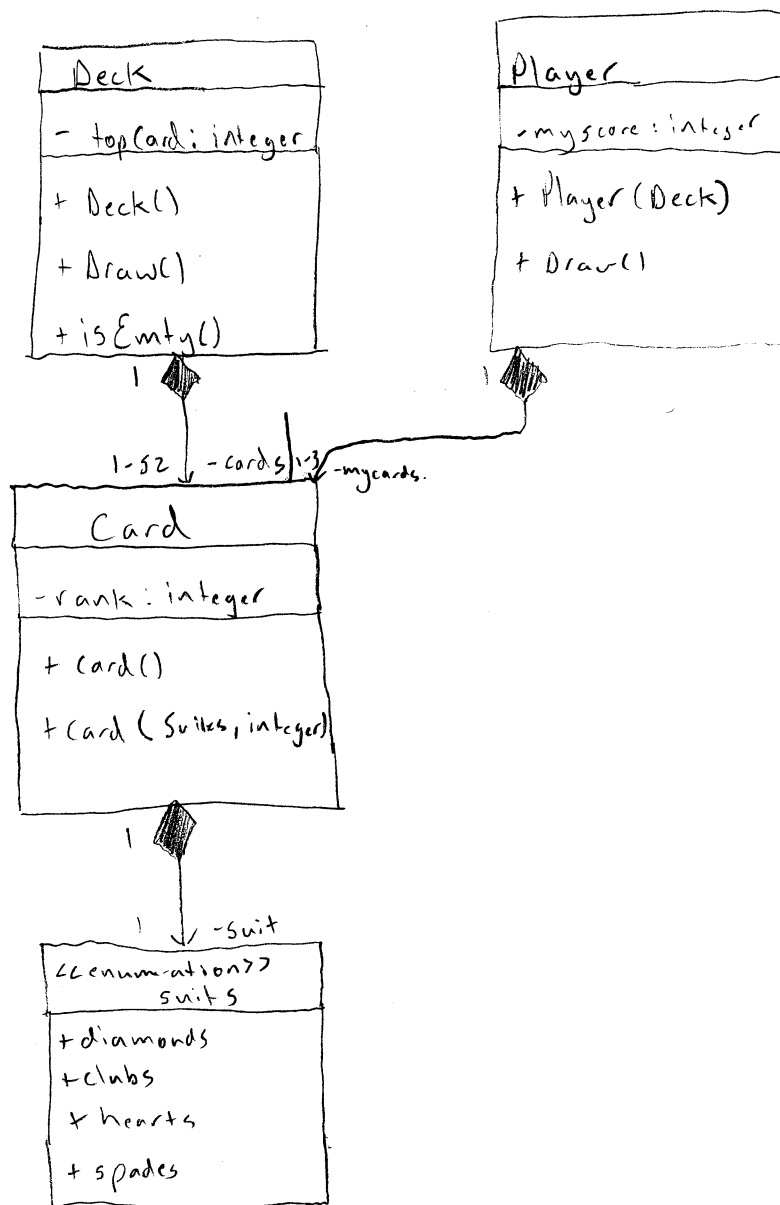
void shuffle()

bool isEmpty()

};

- means private.  
+ means public.

## UML



## Functors

an instance of random integer is a function object or functor.

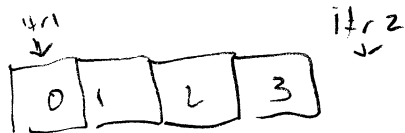
A functor is an object acts like a function (an object that can be called like a function).

A functor overloads a function called operator.

A number of functions in STL expect functors as a parameter.

Ex: random shuffle, for-each

random shuffle



Swap ( $obj, 0$ ,  $obj, (0+1)$ ) 0

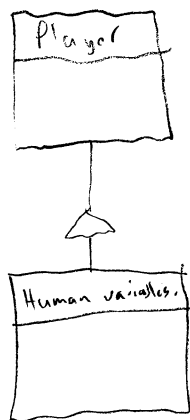
Swap ( $obj, 1$ ,  $obj, (1+1)$ ) 0, 1

Swap ( $obj, 2$ ,  $obj, (2+1)$ ) 0, 1, 2



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War game continued.



clean: in make file.

## Vector Lab

type def = define new types.

$T \times$  iterator

new name for  $T \times$  iterator

```
template <class T>
Vector<T>::Vector() {
    mysize = 0
    mycapacity = 0
    buffer = 0
}
```

`Vector<T>::Vector(int size)`

`my.`

`buffer = new T[size];`

`for(int i = 0; i < size; i++)`

`{ buffer[i] = T(); }`  $\searrow$  called constructor

Operator =

my size = v. my size

my capacity = v. my capacity

delete [] buffer

buffer = new T[my.capacity]

begin()      beginning of buffer

end()                      buffer + size

:: scope resolution operator

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mid term feb 15<sup>th</sup>

template

swap.h

T = template parameter

```
template < class T >
void swap ( T & x , T & y )
{
    T temp = x
    x = y
    y = temp
}
```

\$ g++ -c swap.cpp

```
String str ("abc")
String::iterator it = str.begin()
for ( ; it != str.end(); it++)
{
    cout << *it;
}
```

class String {

public:

typedef char \* iterator

iterator begin () { return buffer }

reverse order

```
String::reverse_iterator j
for ( j = str.rbegin(); j != str.rend(); j++)
    cout << *j
```

```
i = str.begin(), i++, i++
for (; i != str.end(); i++)
    cout << *i      prints "cb"
```

str.c\_str() → returns char \*

A	B	C	\0
---	---	---	----

friend classes can access everything, private & protected fields

Table 6.1 lookup

Lists - container class

list<int> alist

alist.push\_back(1)

alist → [1] → [2]

alist.push\_back(2)

alist.push\_front(3)

alist → [3] → [1] → [2] → [4]

alist.push\_back(4)

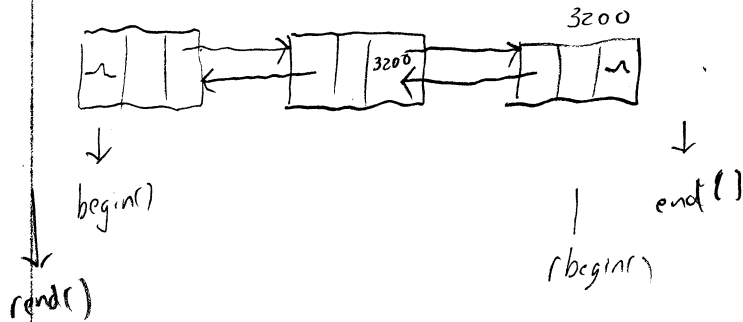
```
for (list<int>::iterator i = alist.begin(); i != alist.end(); i++)
```

```
    cout << *i
```

2-8-12

## Chapter 9 Lists

elements are linked together using pointers / addresses



use when number of elements is not known in advance  
and random access is not required.

operator[]

### List vs vector

$V.front() = V[0]$

<u>Name</u>	<u>operation</u>	<u>List</u>	<u>Vector</u>
Access {	front()	$O(1)$	$O(1)$
	back()	$O(1)$	$O(1)$
	*iter	$O(1)$	$O(1)$
	operator[]	X	$O(1)$
add new {	push front()	$O(1)$	X
	push back()	$O(1)$	$O(n)$
	insert(iter, va)	$O(1)$	$O(n)$

find generic

Binary search requires random access

Remove	{	pop-front()	$O(1)$	X
		pop-back	$O(1)$	$O(1)$
		erase(iter)	$O(1)$	$O(n)$ → generic algorithm
		remove(val) <small>-all occurrences.</small>	$O(n)$	remove(iter, val) $O(n)$
inclusion test	{	find(iter, iter, val)	$O(n)$	$O(n)$
		binary-search(iter, iter, val)	X	$O(\log n)$

v 

1	2	3	4
---	---	---	---

remove(v.begin, v.end, 2)

v 

1	3	4	4
---	---	---	---

vector<int>::iterator iter = v.begin

iter++

v.erase(iter)

v 

1	4	4
---	---	---

find returns null pointer if nothing found.

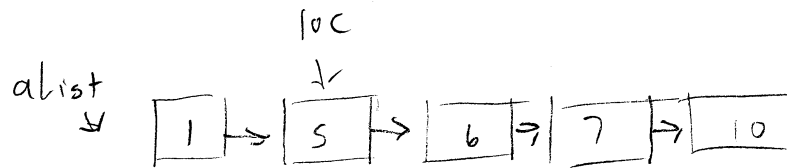
## Binary Search

0	10
1	15
2	20
3	25
4	30
5	40
6	50
7	70
8	75
9	76
10	77

20 is value

compare to middle element of 40. reduce search in half everytime.

↑  
sorted



list<int>::iterator loc = find(alist.begin(), alist.end(), 5)

loc = alist.insert(loc, 10)

→ 10 → 5 →

loc

list<int>::iterator stop = find(loc, alist.end(), 7)

10 → 5 → 6 → 7

loc stop

alist.erase(loc, stop)

alist 1 → 7 → 10

num = count(alist.begin(), alist.end(), 10)

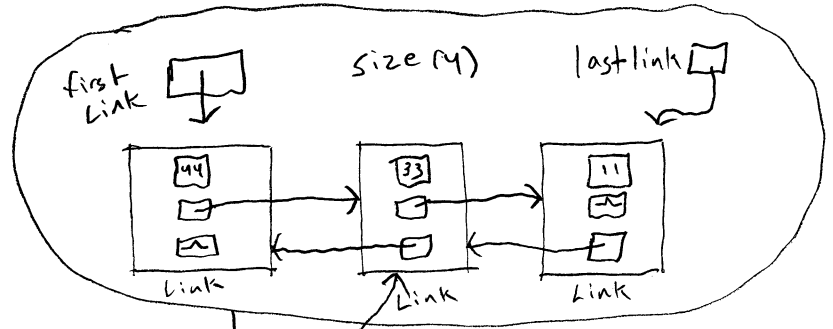
↓  
1

\*itr should get value

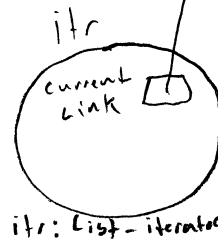
~ = null pointer

```
List<int> L;
L.push-back(44)
"      33
"      11
"      22
```

```
List<int>::iterator itr
itr = L.begin()
++itr
cout << itr
```



whole thing is L: List



itr: List-iterator

itr++ points to next link

itr++ current-link = current-link → next-link

\*

```
template <class T>
class Link {
    T value
    Link * next-link
    Link * previous-link
}
friend class List(T)
"      " List-iterator(T)
};
```

```
template <class T>
class List {
    int size;
    Link * first-link
    Link * last-link
public:
    push-back(T)
    pop-back()
    push-front(T)
    pop-front()
    insert(iterator, T)
```

};



```

template < class T>
class List_iterator {
    Link * current_link

public:
    T operator * ();
    List_iterator operator ++ ()
    " " " " (int)

friend class List (T)

```

prefix →

postfix →

### Link constructor

```

value = x
next_link = previous_link = 0

```

### constructor for List

```

size = 0
first_link = last_link = 0

```

### List Pushback

```

List :: push-back (x)
    new_link = new Link (x);
    if (first_link == 0) {
        first_link = last_link = new_link
    }
    else {
        newlink → previous link = last_link
        lastlink → next link = new-link
        lastlink = newlink
    }
    size++

```

### List begin

```

List::begin
    return List_iterator (first-link)

```

### List iterator \*

```

operator*
    return current_link → value.

```

```

return * currentlink, value.

```

### erase

two iterators

$p \rightarrow \text{next\_link} = n$

$n \rightarrow \text{prev\_link} = p$

delete iter; current\_link

### insert

insert before iterator

new Link

two iterators

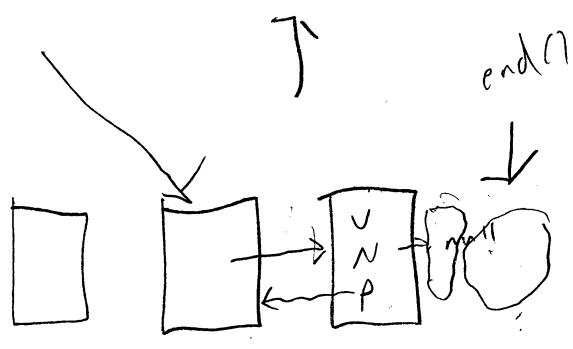
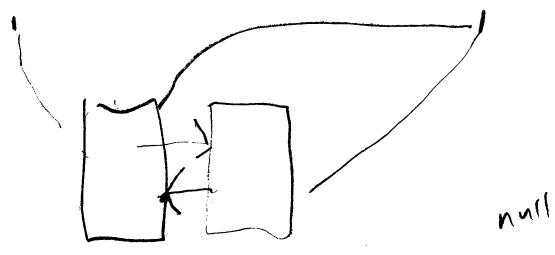
2-13-12

# Binary Search

must be sorted

$T(n)$  = time for bin search rec with  $n$  elements. = high-low

$$T(n) = T(n/2) + C$$



$$m = itr++$$
$$m = ++itr$$

## Stack & Queue Containers

2-20-12

### Stack

push	push-back
pop	pop-back
top	back
empty	empty
size	size

### Queue

push	push-front
pop	pop-back
top	back
empty	
size	

### Container Header

template < class T, template < class T > class Container >

Allocator manages memory efficiently

2-22-12

- Iterator traits can tell you the type of object you are pointing to.

why  $n \cdot \log n^2$

$n$  elements

$T(n)$  = time to sort  $n$  elements

$$T(n) = T(n/2) + T(n/2) + c n$$

everytime size divides by 2

recurrence

$$T(n) = 2T(n/2) + cn$$

$$= 2(2T(n/4) + c n/2) + cn$$

$$= 2^2 T(n/4) + cn + cn$$

$$= 2^2 T(n/4) + 2cn$$

$$= 2^2(2T(n/8) + c n/4) + 2cn$$

$$= 2^3 T(n/8) + cn + 2cn$$

$$= 2^3 T(n/8) + 3cn$$

Assume  $n=8$ ,  $\log_2 n = 3$

$$n \times T(1) + \log_2 n (cn)$$

$$cn + cn \log n$$

$$n \log n$$

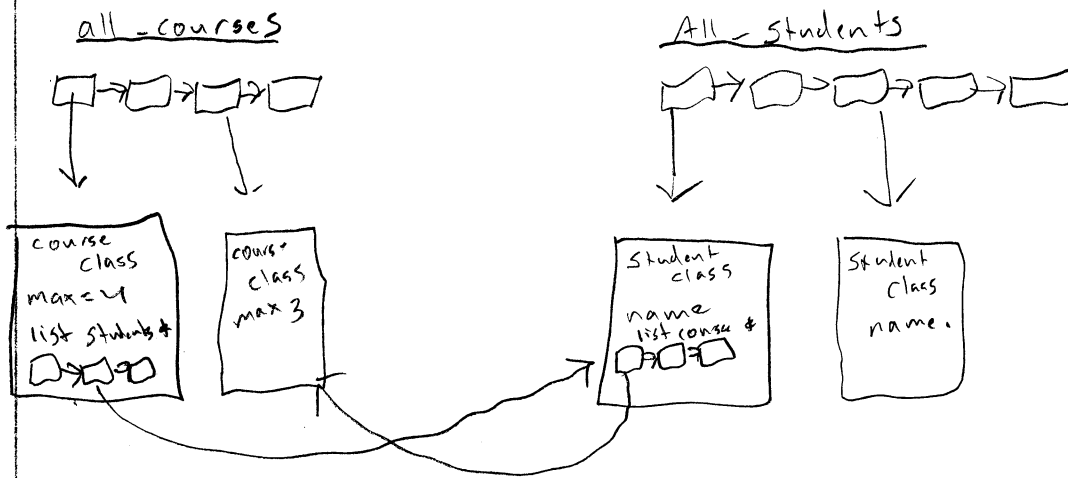
generic list

### Homework 3 - course registration system

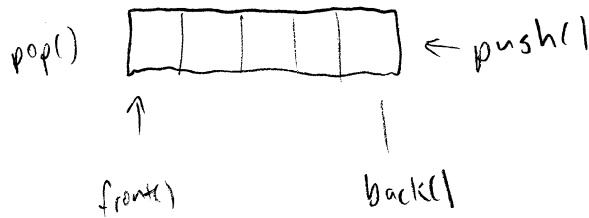
global / in main() lists:

list <Course > all-courses

list <Student > all-students



## Queue



#include <queue>

queue<int> q;

q.push(1)

q.push(2)

cout << queue.front() // 1

## Deque - double ended queue

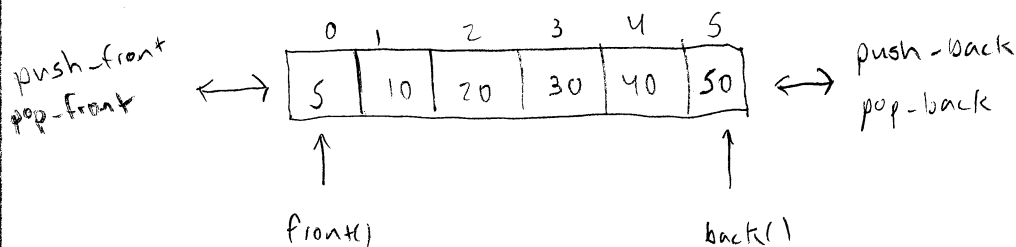
- one of most versatile data structures

Example: Deck of cards

STL defines it in #include <deque>

similar to queue

but has [] operator



↓  
takes  $O(\log n)$

deque<int> d

d.push-back(30)

d.push-back(40)

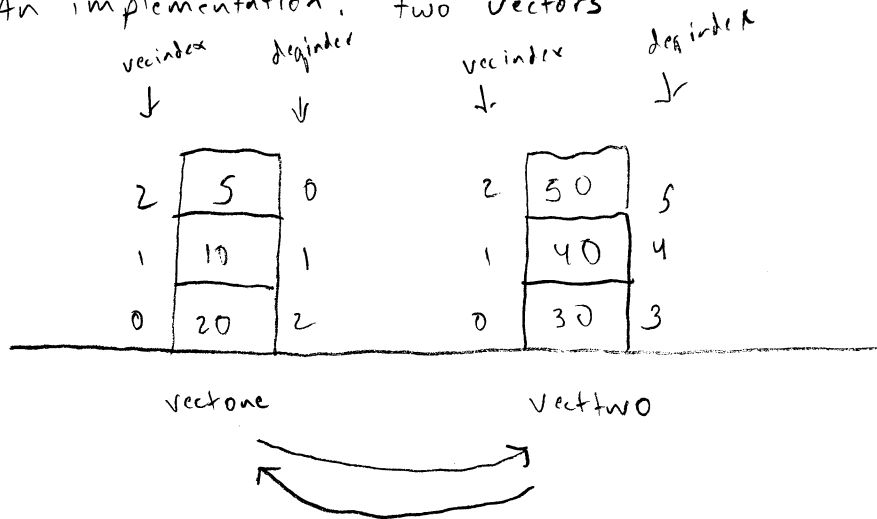
d.push-front(20)

d.push-front(10)

d.push-front(5)

d.push-back(50)

An implementation: two vectors



back() into deque = back() vectwo  
front() into deque = back() vec one

```
template <class T>
T deque::front() {
    if (vecOne.empty())
        return vectwo.front();
    else
        return vecOne.back();
}
```

```
T & deque::operator[](int index) {
    int n = vecOne.size();
    if (index < n)
        return vecOne[n - 1 - index];
    else
        return vectwo[index - n];
}
```

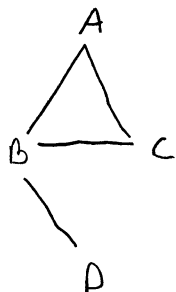


2-27-12 Trees & sets - Chapter 12 & 13

Trees

- a tree is an acyclic graph
- a graph consists of a set of nodes and set of edges.
- an edge connects a pair of nodes

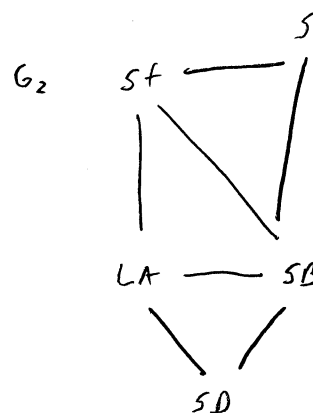
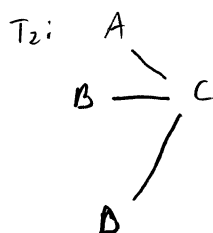
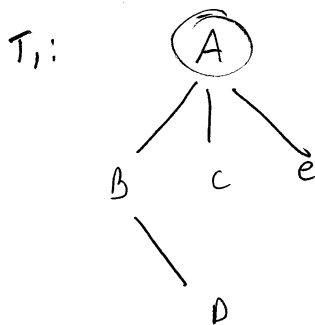
Ex.  $G_1$



$G_1: \langle N, E \rangle$

$N = \{A, B, C, D\}$

$E = \{(A, B), (A, C), (B, C), (B, D)\}$



- In a tree, one of the nodes is designated as the root.
- root can be any one of the nodes.

- in a binary tree, no node has more than two children

- longest path from root to farthest leaf is height

$\langle \text{stmt} \rangle \rightarrow \langle \text{select stmt} \rangle \mid \langle \text{expr} \rangle \mid \langle \text{while} \rangle \mid \langle \text{comp stmt} \rangle$

$\langle \text{comp stmt} \rangle \rightarrow \{ \langle \text{stmt} \rangle; \langle \text{stmt} \rangle^* \}$

$\langle \text{assign} \rangle \rightarrow \text{identifier} = \langle \text{expr} \rangle$

- complete binary trees, everything pushed up and to the left.

$$h = \lfloor \log_2 n \rfloor \quad h=2 \quad 4 \leq n < 8$$

$$h=3 \quad 8 \leq n < 16$$

$$h=4 \quad 16 \leq n < 32$$

truncates  
fraction  
portion

$$3.27 \rightarrow 3.00$$

- height balanced binary tree - height of left subtree and right subtree doesn't differ by more than 1.

- binary search tree - complete or height balanced

- root is only access point for a tree

- In a complete binary tree, all nodes are pushed up and then pushed left
- a complete binary tree of height  $h$  has between  $2^h$  and  $2^{h+1} - 1$  nodes

Or

Floor of

- a complete binary tree containing  $n$  nodes has height of  $\lfloor \log_2 n \rfloor$

- in a height balanced binary tree at every node, the difference between the right and left subtrees is not larger than one.  $\nearrow \nearrow$

- a complete binary tree is height balanced and this implies that the largest number of nodes in a height balanced tree of height  $h$  is  $2^{h+1} - 1$  (upper bound)

What is the smallest number of nodes in a height balanced binary tree?

for height = 0

= 1

= n



$$M_n = 1$$

$$M_n = 2$$

$$M_n = M_{n-2} + M_{n-1} + 1$$

$\log_2 n$  time complexity

2-29-12

max nodes based on height  $2^h \rightarrow 2^{h+1} - 1$

min nodes " " "  
 $M_n = M_{n-1} + M_{n-2} + 1$

close to fibonacci

$$M_n \approx \frac{1}{\sqrt{5}} \left( \frac{1+\sqrt{5}}{2} \right)^n + 1$$

height of tree  $\log_A M_n = \log_A \left( \frac{1}{\sqrt{5}} A^n + 1 \right)$   $A = \frac{1+\sqrt{5}}{2}$  global ratio

$$\approx \log_n \left( \frac{1}{\sqrt{5}} A^n \right)$$

$$\approx \log_a \frac{1}{\sqrt{5}} + \log_A A^n$$

$$\approx -1.673 + n$$

$$n \approx \frac{\log M_n}{\log A} = 1.44 (\log M_n)$$

$$n \in O(\log M_n)$$

Therefore, algorithms on height balanced binary trees that run proportional to the height  $n$  are in  $O(n)$

book fixes

pg. 302 - bottom of page.

---

$$n \approx 1.44 \log M_n$$

---

$$\textcircled{n} M_n$$

---

$$\textcircled{\log n} \text{ - minimum } \log M$$

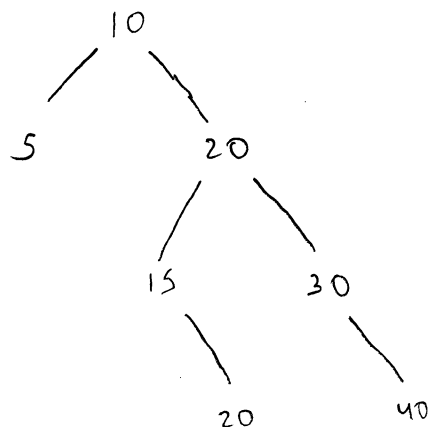
---

$$\textcircled{O(\log n)} \quad O(n)$$

10, 20, 30, 5, 15, 40, 20

< = left

> right

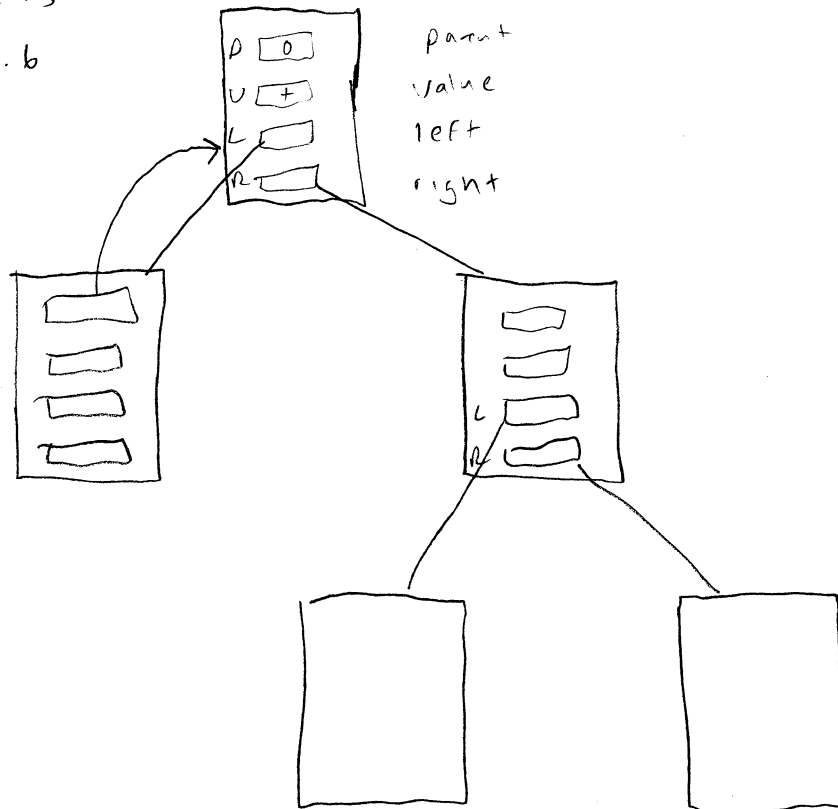


not height balanced.

Chapter 14 AVL TREES

## Binary Tree Representation

① Use pointers  
fig 13.6



## Node Class - template

template < class T >

class Node {

Public:

T value

Node<T> \* parent

Node<T> \* left-child.

Node<T> \* right-child.

Node (T x) { value = x; parent = leftchild = rightchild = 0; }

copy constructor

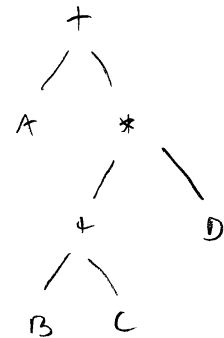
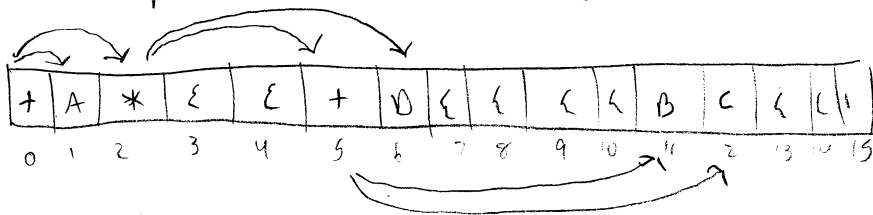
default constructor.

② use a vector  $\rightarrow$  vector <char> V

a. root is in V[0]

b. children of V[i] are in V[2i+1] & V[2i+2]

or parent of V[i] is at V[(i-1)/2]



{ = not used.

read chapter 12 & 13 & Deque

### Sets & Multisets -

- must be unique

- no order

$$A = \{30, 10, 40, 20\}$$

$$B = \{5, 40, 20, 15\}$$

not ordered but

unique elements

$$A \cup B = \{30, 10, 40, 20, 5, 15\}$$

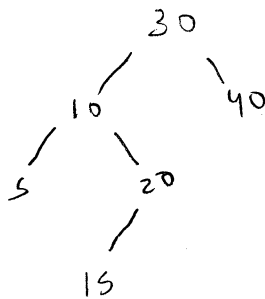
$$A \cap B = \{40, 20\}$$

In STL, both set and multiset are ordered.

Set has unique elements, multisets allows duplicates

↓

bag



LNR traversal, left node right

5, 10, 15, 20, 30, 40

pg. 109 operations on sets and their time complexities.  $O(\log n)$



```
#include <set>
```

```
set<int> s, t, u;
```

```
s.insert(30)
```

```
s.insert(10)
```

```
s.insert(40)
```

```
s.insert(20)
```

```
cout << s.count(10) // 1 occurrences
```

```
cout << s.erase(10) // 1 erased.
```

```
" " s.count(10) // 0 occurrences
```

```
s.insert(10)
```

```
t.insert(5)
```

```
" (40)
```

```
" (20)
```

```
" (15)
```

```
multiset<int> m
```

```
m.insert(30)
```

```
" " 10
```

```
" " 40
```

```
" " 20
```

```
" " 10
```

```
cout << m.count(10) // 2 occurrences
```

```
cout << m.erase(10) // 2 erased.
```

```
" " m.count(10) // 0 occurrences.
```

### Union of sets

insert-iterator < set<int> > <sup>space.</sup>  
↓  
inserter (u, u.begin)  
object

```
set_union (s.begin(), s.end(), t.begin(), t.end(), inserter)
```

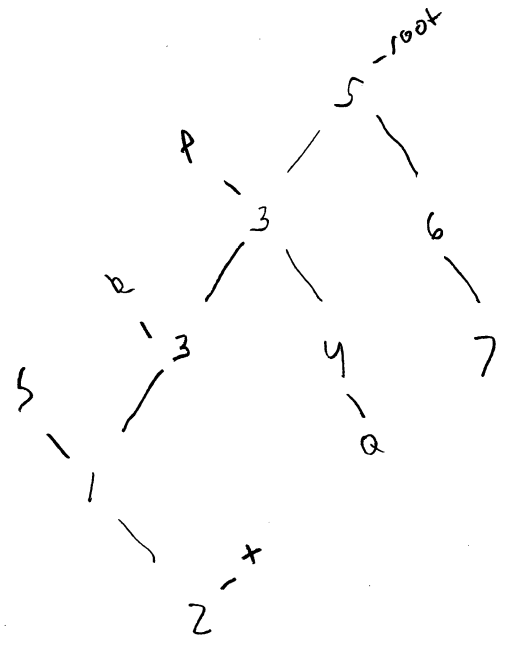
```
set<int>::iterator i
```

```
for (i = u.begin; i != u.end; i++)
```

```
cout << *i
```

3-5-12

Sets & Binary Trees continued...



LNR

1, 2, 3, 3<sub>2</sub>, 4, 5, 6, 7

lower bound

root  $\rightarrow lb(3, end)$  return result  
 $\downarrow$   $\uparrow$

p  $\rightarrow lb(3, end)$  return result  
 $\downarrow$   $\uparrow$

r  $\rightarrow lb(3, end)$  return set iterator (2)  
 $\downarrow$  end

s  $\rightarrow lb(3, end)$   $\uparrow$   
 $\downarrow$  end

+  $\rightarrow lb(3, end)$   $\uparrow$

upper bound

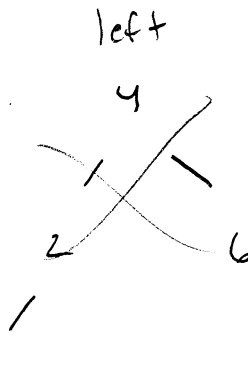
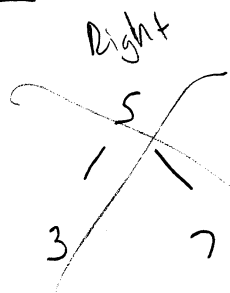
root  $\rightarrow ub(3, end)$   $\uparrow$   
 $\downarrow$

p  $\rightarrow ub(3, end)$   $\uparrow$   
 $\downarrow$

q  $\rightarrow ub(3, end)$   $\uparrow$

Since no left child  
return set iterator this

merge



right  $\rightarrow$  left child,  
goes down left  
side of right tree

In order traversal LNR

Post  
LRN

```
void inorder (node<T> *current)
```

{

```
    if (current != 0)
```

```
        inorder (current  $\rightarrow$  leftchild);
```

```
        cout << current  $\rightarrow$  value;
```

```
        inorder (current  $\rightarrow$  rightchild)
```

iterators

```
set<int> s;
```

```
set<int>::iterator i = s.begin()
```

s.begin() perform left side.

left slide

3-7-12

## Priority Queues

need complete binary trees.

need # include <queue>

priority - queue <int> pq

pq.push(2)

pq.push(4)

" (3)

(1):

cout << pq.top() // 4

pq.pop()

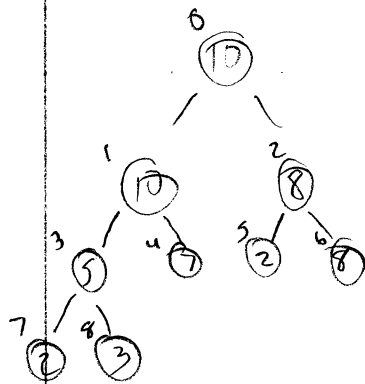
cout << pq.top() // 3

implementation: binary heap

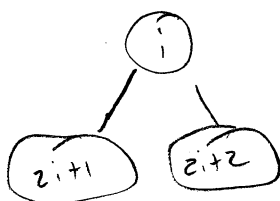


Complete binary trees in an array or vector

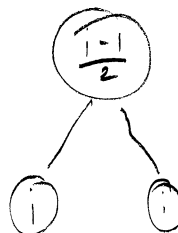
Every node greater than or equal to children.



10	10	8	5	7	2	8	2	3
0	1	2	3	4	5	6	7	8



for any i



for any i

newElement = 30

C.push\_back(newElement)

if 30 > parent  
swap

larger of two children on percolate down.

last (stop-start) - 1

(heap size / 2) - 1

Starting position where might not be in  
heap property.

3-12-12

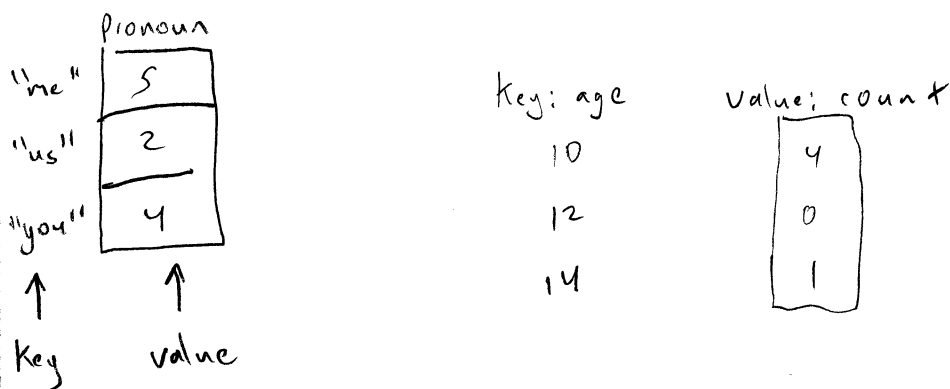
## Maps & Multimaps

- maps are built on top of sets, chapter 16

Ordered collection of key-value pairs ← data structure.

An index data structure

Other names are associative memory or dictionary



key is ordered.

map - key must be unique.

multimap - key can be duplicates.

space  
↓

Set < Pair < T1, T2 > >

cout << pronoun["you"]



```
#include <map>
```

```
map<string, int> m
```

```
int x = m["Henry"];
```

→ x=0

Henry doesn't exist when called

Henry



```
m["Harry"] = 7
```

```
m["Henry"] = 9
```

- if value doesn't exist, [] will create it with default value.

- map is not limited to binary search trees.

### hardware

nail	100
saw	3
nail	250
hammer	2
saw	4
hammer	7
nail	1000

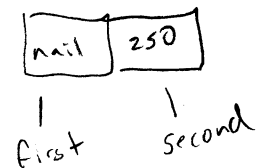
```
void readItems (map<string, int> &m)
```

```
String word;
```

```
int val;
```

```
while (cin >> word >> val)
```

```
m[word] += val;
```



```
int main()
```

```
map<string, int> tbl;
```

→ int total = 0

```
readItems (tbl);
```

```
map<string, int>::const_iterator p;
```

```
for (p = table.begin(); p != table.end(); p++) {
```

```
total += p->second
```

```
cout << p->first << "+" << p->second
```

```
}
```

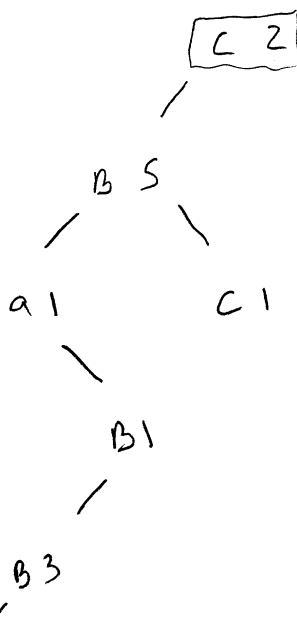
cout << "total " << total

3-14-12

# maps & multimaps continued

```
multimap<string, int> m;
m.insert(pair<string, int>("c", 2));
m.insert(pair<string, int>("b", 5));
```

a 1  
b 1  
b 3  
c 1  
b 1



```
cout << m.count("c"); // 2
```

// cout << m["c"]; not defined with multimap, is for map.

```
multimap<string, int>::iterator it
```

```
for (it = m.begin(); it != m.end(); it++)
```

```
cout << it->first << " " << it->second << endl;
```

a 1  
b 1  
b 3  
b 1  
b 5  
c 1  
c 2

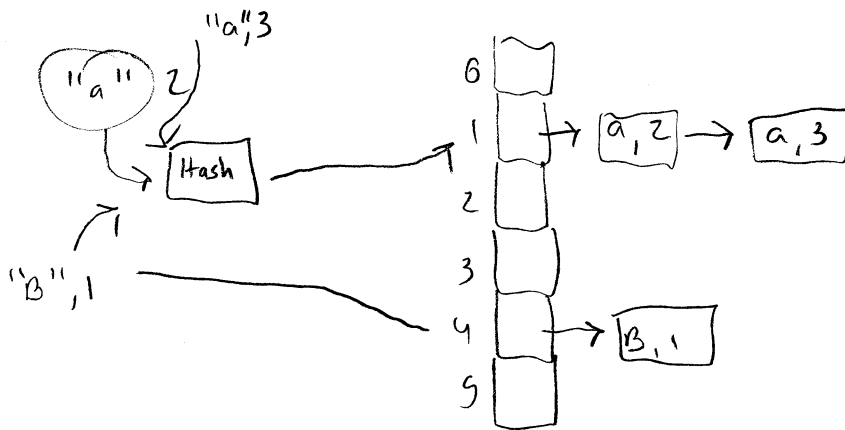
```
it = m.find("c")
```

```
cout << it->second; 1
```

b 1



## Hash Table



## map lab

find call find from set  
erase call erase from set.

parent::erase()

String friends.

## Chapter 17 Hash Tables

- use a hash function ( $O(1)$ ) to store and lookup values in a hash table.

Ex. Hash table  $\rightarrow$  `vector<string>` `hash-table(10);`

hash-function  $\rightarrow$  `int hfunc(const string &s)`  
`return (s[2] - 'a') % 10;`

### Problems:

- what if name didn't have the 3rd character?
- what if there are two names that "map" or hash to the same place? (same 3rd letter)
- what if more than 10 names?
- what if our hash function is not perfect?

This is always the case.

### Solutions

use buckets, every location can hold more than one name  
try to improve your hash function

Time complexity deteriorates to  $O(n)$

```
class item {
```

```
public:
```

```
    item(const string & v) : value(v), next(0) {}
```

```
    string value;
```

```
    item * next;
```

```
}
```

```
vector< item * > bucket(10);
```

3-19-12

Debugger / keeps symbols.

g++ -g test1.cpp

gdb a.out

find line number.

break test1.cpp:19

break Deque.h:77

run

p to print : p saved=size

Hash tables continued

use prime number, number of buckets.

hash with list.

hash function returns unsigned int.