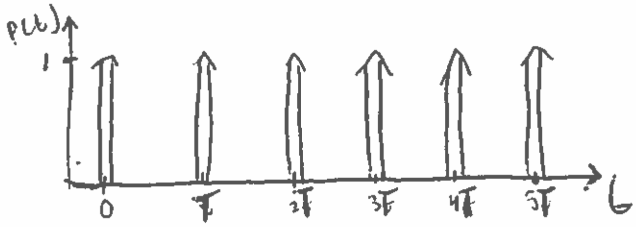
Alexander Crease

Sig Sys PS07

1. a.) $p(t)=\sum_{k=-\infty}^{\infty}\delta(t-kT)$



b.) $\tilde{p}(t)=\sum_{k=-\infty}^{k=\infty} c_k e^{j\frac{2\pi}{T}kt}$

via picking property

$$c_k=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} p(t)\, e^{-j\frac{2\pi}{T}kt}\,dt = \frac{1}{T}e^{\frac{-j2\pi k(kT)}{T}} = \frac{1}{T}e^{-j2\pi k^2}$$

$$\tilde{p}(t)=\frac{1}{T}\sum_{k=-\infty}^{\infty} e^{-j2\pi k^2} e^{j\frac{2\pi}{T}kt} = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{-j2\pi k^2 + j\frac{2\pi}{T}kt}$$

$$=\frac{1}{T}\sum_{k=-\infty}^{\infty} e^{j2\pi k(-k+\frac{t}{T})} = \boxed{\frac{1}{T}\sum_{k=-\infty}^{\infty} e^{-j\frac{2\pi k}{T}(t-kT)} = \tilde{p}(t)}$$

c.) $x(t)=\sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{T}kt}$

$$c_k=\frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi}{T}kt}\,dt = \frac{\omega_0}{2\pi}\int_{-\infty}^{\infty} x(t)\, e^{-j\omega_0 t} e^{k}\,dt$$

$$c_k=\frac{\omega_0}{2\pi}e^{k}X(\omega_0) \qquad T=\frac{2\pi}{\omega_0}$$

$$X(\omega_0)=\frac{c_k 2\pi}{\omega_0 e^{k}}$$

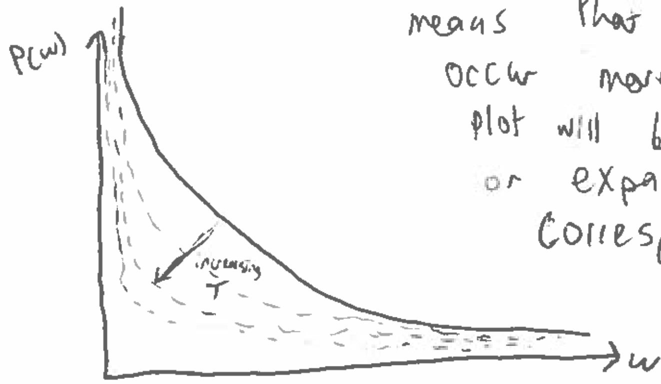
$$\boxed{X(\omega)=\frac{c_k 2\pi e^{-k}}{\omega}}$$

d) $P(w) = \frac{C_K 2\pi e^{-K}}{w} = \frac{w_0}{2\pi} e^{-j2\pi K^2}\left(\frac{2\pi e^{-K}}{w}\right) = \frac{w_0}{w} e^{-j2\pi K^2 - K} = \boxed{\frac{w_0}{w} e^{-K(j2\pi K+1)}} = P$

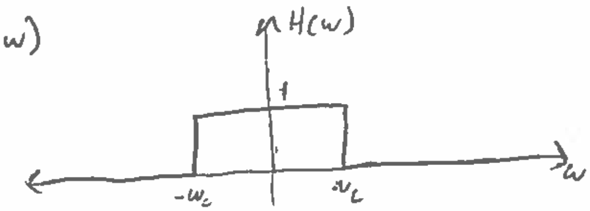
$C_K = \frac{1}{T} e^{-j2\pi K^2}$

e) $P(w) = \frac{w_0}{w} e^{-K(j2\pi K+1)}$

Increasing T decreases the fundamental frequency (w_0) of $p(t)$, and decreases the amplitude of $P(w)$. This means that in $p(t)$, the impulses will occur more frequently, and the frequency plot will be smaller in $P(w)$. Compressing or expanding in the time domain corresponds to scaling in the frequency domain.



2. $H(w)$



a.)

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(w) e^{jwt}\, dw$$

$$h(t) = \frac{1}{2\pi}\left(\int_{-\infty}^{-w_c} \underbrace{H(w) e^{jwt}}_{0}\, dw + \int_{-w_c}^{w_c} H(w) e^{jwt}\, dw + \int_{w_c}^{\infty} \underbrace{H(w) e^{jwt}}_{0}\, dw\right)$$

$$h(t) = \frac{1}{2\pi}\int_{-w_c}^{w_c} e^{jwt}\, dw = \frac{1}{2\pi}\left[\frac{e^{jwt}}{jt}\right]_{-w_c}^{w_c} = \cancel{\frac{-j}{2\pi t} e^{jw_c t}} + \cancel{\frac{j}{2\pi t} e^{-jw_c t}}$$

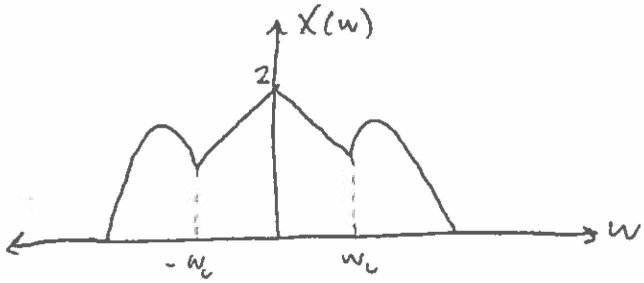
$$h(t) = \cancel{\frac{j}{2\pi t}\left(e^{-jw_c t} - e^{jw_c t}\right)} \frac{1}{2\pi j t} e^{jw_c t} - \frac{1}{2\pi j t} e^{-jw_c t}$$

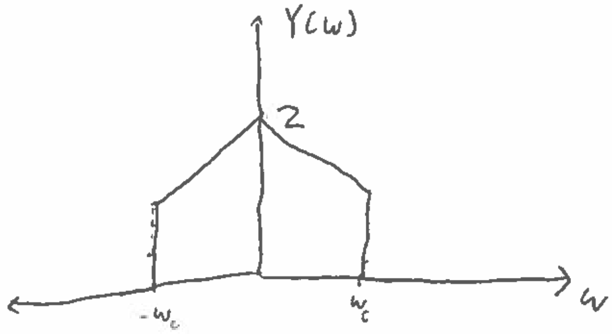
$\sin(\theta) = \frac{1}{2j} e^{j\theta} - \frac{1}{2j} e^{-j\theta}$

$$h(t) = \frac{1}{\pi t}\sin(w_c t)$$

$$\boxed{h(t) = \frac{\sin(w_c t)}{\pi t}}$$

b) Suppose $X(\omega)$ is the following. Sketch $Y(\omega)$



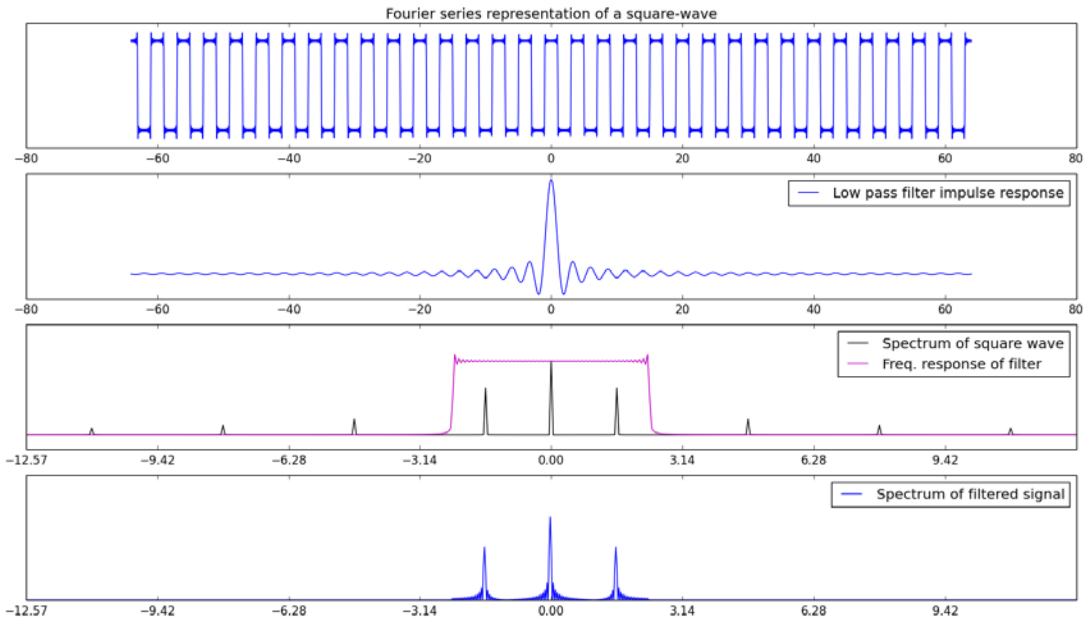


c) In the frequency domain, convolution corresponds to multiplication, so wherever $H(\omega) = 0$, the ~~resulting~~ output spectrum would also be equal to 0. This means that all ~~frequencies~~ $|\omega| > \omega_c$, the frequency goes to zero.

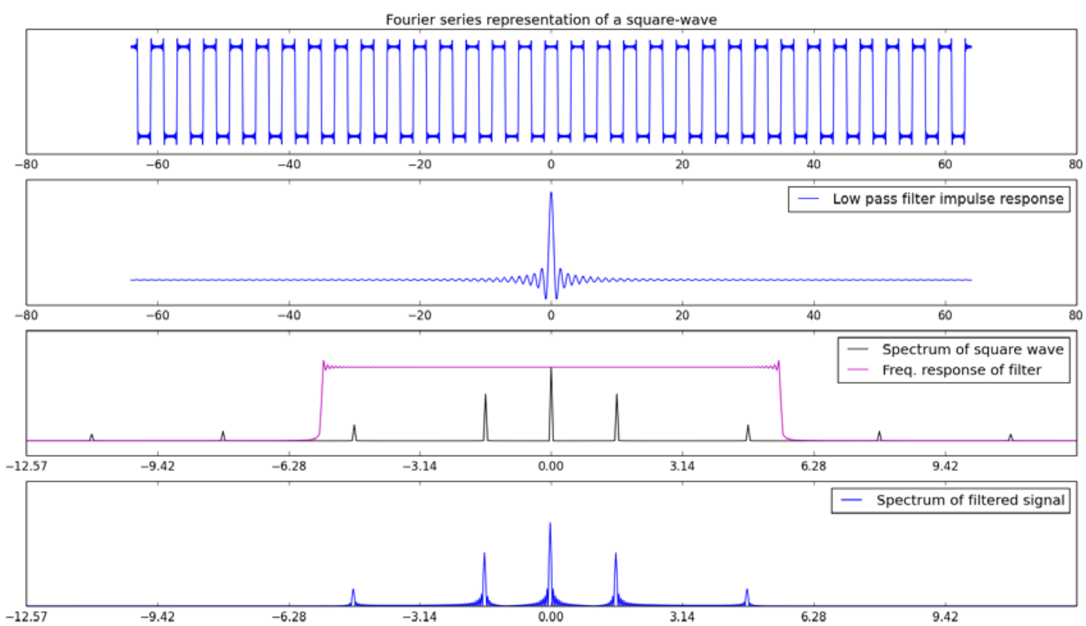
d)

Both filters are of the form $h(t)=\sin(w_c t)/\pi t$

For when $w_c=.75\pi$



For when $w_c=1.75\pi$



3. $y(t) = x(t)\cos(\omega_c t)$

Multiplication in the time domain

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convolution in the frequency domain