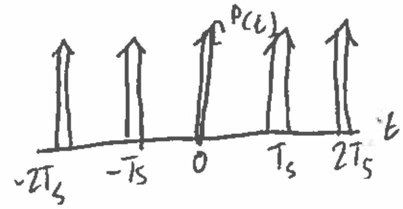
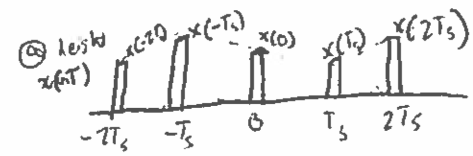
Alexander Crease

Sig Sys Problem Set 08

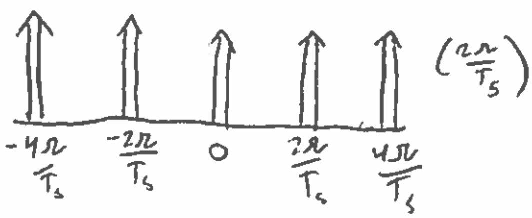
1\. a) ~~$x_p(\omega)$~~

$x(t)$ $\quad * \quad$ ~~$X(\omega)$~~ $p(t)$ $\quad = \quad$ $x_p(t)$

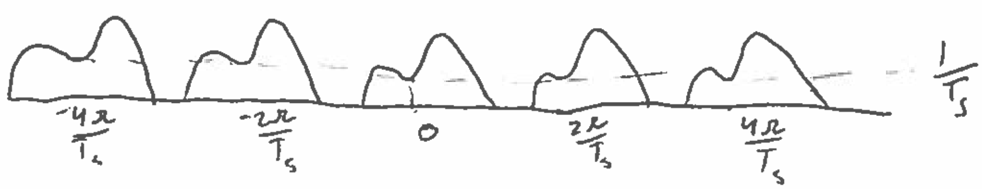




b) $P(\omega)$



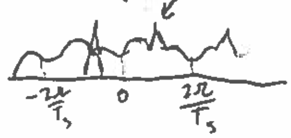
c) $X_p(\omega)$



d) $\frac{2\pi}{T_s} > 2\omega_m \qquad \frac{2\pi}{T_s} = \omega_s \rightarrow$ sampling freq.

$\omega_s > 2\omega_m$ or else the signals will interfere and combine, and the information won't be able to be recovered

sum of overlapping signals



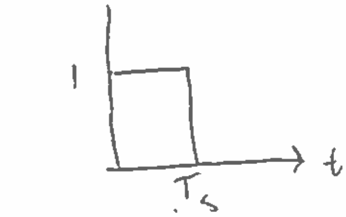
when $2\omega_m < \omega_s$

e) You run a ~~band~~ low pass filter with a cutoff of ω_m to recover the original signal.

~~e) To recover $x(t)$ from $x_p(t)$, you need to infinitely decrease T_s, infinitely increasing our sample size.~~

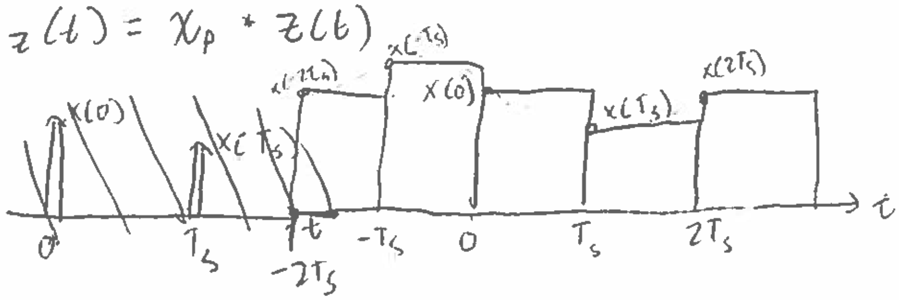
$x_p(t) = x(t) \lim \sum^{\infty} \delta(t - nT_s)$

f) $z(t)$



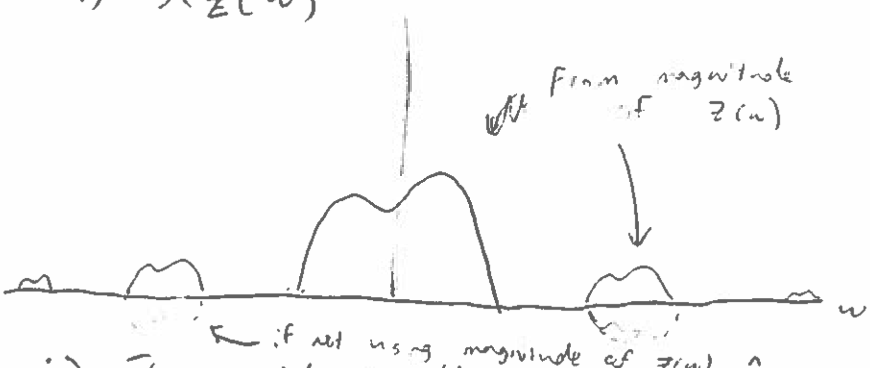
This rectangular pulse has been considered.

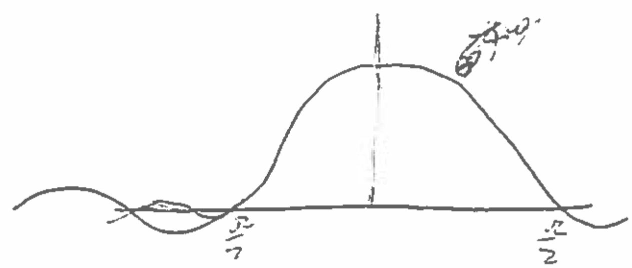
g) $x_z(t) = x_p * z(t)$



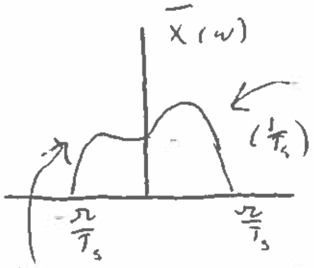
$Z(w) = e^{-jw\frac{T_s}{2}}\,\text{sinc}(w)$

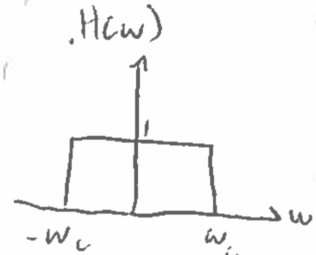
h) $X_z(w)$

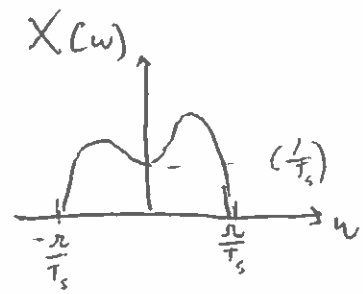




i) $\bar{X}(w) = X_z(w)\,H(w)$ $\qquad$ $\hat{X}(w) = X_p(w)\,H(w)$







j) The sides are slightly different because of the sinc function.

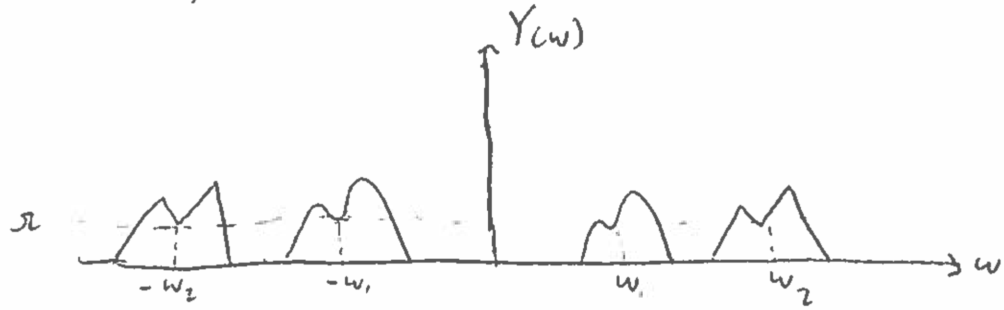
k) when $w_m = \frac{\pi}{T_s}$, $\quad \dfrac{\bar{X}(w_m)}{\hat{X}(w_m)} = e^{-jw_m\frac{T_s}{2}} = \boxed{e^{-j\frac{\pi}{2}}}$

2) a) $y(t) = x_1(t)\cos(w_1 t) + x_2(t)\cos(w_2 t)$

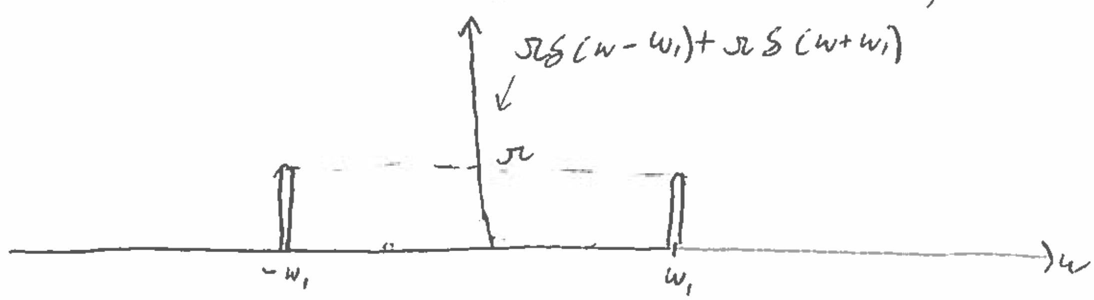
$$Y(w) = X_1(w) * \left(\pi\delta(w-w_1) + \pi\delta(w+w_1)\right) + X_2(w) * \left(\pi\delta(w-w_2) + \pi\delta(w+w_2)\right)$$

$$w_1 + 2w_m < w_2$$

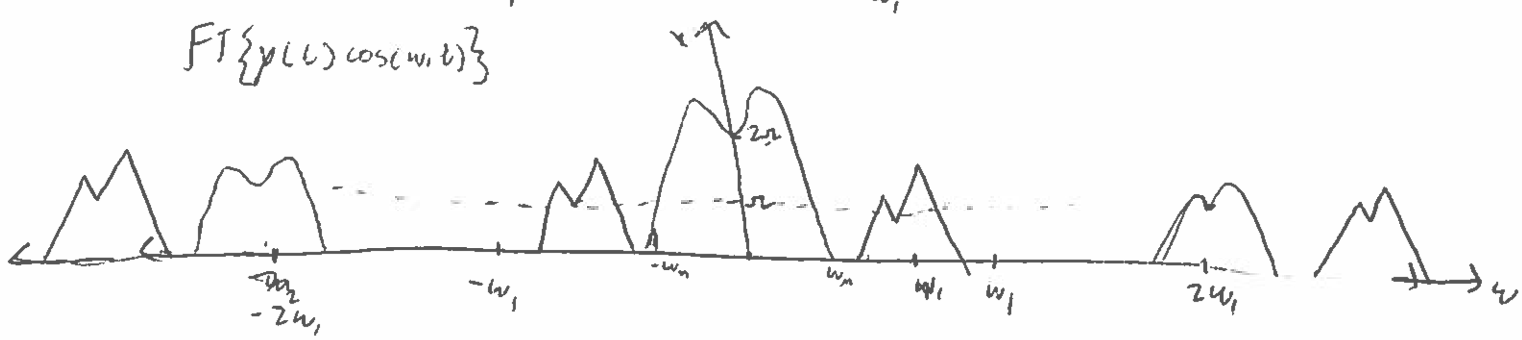
$$w_1 - w_2 < -2w_m$$



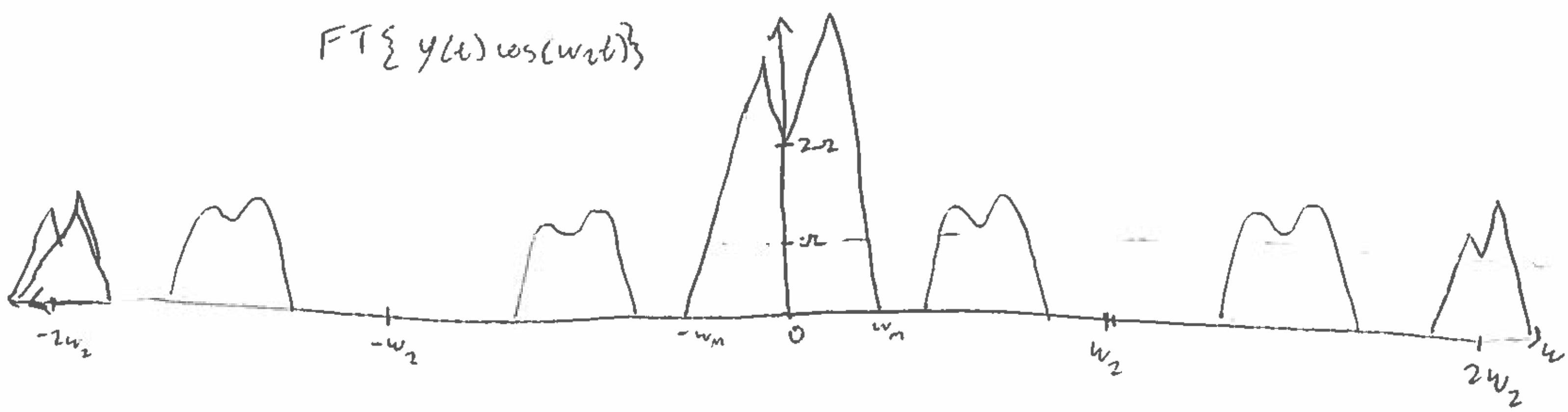
b) $y(t)\cos(w_1 t) = Y(w) * \left(\pi\delta(w-w_1) + \pi\delta(w+w_1)\right)$



$FT\{y(t)\cos(w_1 t)\}$



$FT\{y(t)\cos(w_2 t)\}$



c) You ~~A~~ would obtain $x_1(t)$ using a low pass filter at w_m ~~from~~ $-w_m$ ~~w_m to~~ on $X_1(w)$, and a low pass similarly ~~filter~~ ~~$w_2 - w_m$ to $w_2 + w_m$~~ on $X_2(w)$, and then do an IFT to convert back to the time domain. You then need to divide by 2π.

#3

3) $i(t) = C \frac{d}{dt} V_{out}(t)$

$V_L(t) = L \frac{d}{dt} i(t)$

a) $V_{in} = V_R + V_L + V_{out}$

$V_{in} = i(t)R + L \frac{d}{dt} i(t) + V_{out}$

$V_{in} = RC \frac{d}{dt} V_{out}(t) + LC \frac{d^2}{dt^2} V_{out} + V_{out}$

$$\boxed{V_{in} = LCV_{out}'' + RCV_{out}' + V_{out}}$$

b) $V_{in}(\omega) = j^2 \omega^2 LC V_{out}(\omega) + j\omega RCV_{out}(\omega) + V_{out}(\omega)$

$V_{in}(\omega) = V_{out}(\omega)\left[j^2\omega^2 LC + j\omega RC + 1\right]$

$j^2 = -1$

$\cancel{\frac{V_{in}(\omega)}{V_{out}(\omega)} = H(\omega) = j^2\omega^2}$

$$\boxed{\frac{V_{out}(\omega)}{V_{in}(\omega)} = H(\omega) = \frac{1}{-\omega^2 LC + j\omega RC + 1}}$$

c) $|H(\omega)| = \frac{1}{\sqrt{(j\omega RC)^2 + (-\omega^2 LC + 1)^2}} = \frac{1}{\sqrt{j^2\omega^2(RC)^2 + (\omega^4(LC)^2 - 2\omega^2 LC + 1)}}$

$$\boxed{|H(\omega)| = \frac{1}{\sqrt{-\omega^2 R^2 C^2 + \omega^4 L^2 C^2 - 2\omega^2 LC + 1}}}$$

$\cancel{\frac{1}{\sqrt{\omega^2(-R^2C^2 + \omega^2 L^2C^2 - 2LC) + 1}}}$

d) $|H(w)| = \dfrac{1}{\sqrt{-w^2R^2C^2 + w^4L^2C^2 - 2w^2LC + 1}}$

$$|H(w)| = (-w^2R^2C^2 + w^4L^2C^2 - 2w^2LC + 1)^{-1/2}$$

$$\frac{d|H(w)|}{dw} = -\frac{(-w^2R^2C^2 + w^4L^2C^2 - 2w^2LC + 1)^{-3/2}}{2}(-2wR^2C^2 + 4w^3L^2C^2 - 4wLC)$$

$$\frac{d|H(w)|}{dw} = \frac{(2wR^2C^2 - 4w^3L^2C^2 + 4wLC)}{2(-w^2R^2C^2 + w^4L^2C^2 - 2w^2LC + 1)^{3/2}}$$

$$= \frac{wR^2C^2 - 2w^3L^2C^2 + 2wLC}{(-w^2R^2C^2 + w^4L^2C^2 - 2w^2LC + 1)^{3/2}}$$

$$\frac{wR^2C^2 - 2w^3L^2C^2 + 2wLC = 0}{w}$$

$$R^2C^2 - 2w^2L^2C^2 + 2LC = 0$$

$$\frac{R^2C^{\cancel{2}} + 2L\cancel{C}}{2L^2C^{\cancel{2}}} = w^2$$

$$\frac{R^2C + 2L}{2L^2} = w^2$$

Because of the w^2 & w^4 terms in the original magnitude function, both the + & - of this solution give equal values of $|H(w)|$

$$\boxed{w = \pm\sqrt{\frac{R^2C + 2L}{2L^2}}}$$

e) $H(w) = \dfrac{1}{-w^2LC+1+jwRC} = \dfrac{1}{1-w^2LC+jwRC}$

$$= \frac{1}{(1-w^2LC)+jwRC}\left(\frac{(1-w^2LC)-jwRC)}{((1-w^2LC)-jwRC)}\right)$$

$$\frac{1-w^2LC-jwRC}{1-2w^2LC+w^4L^2C^2+w^2R^2C^2} = \frac{1-w^2LC-jwRC}{1-2w^2LC+w^4L^2C^2+w^2R^2C^2}$$

$$\phi = \arctan\left(\frac{jwRC}{1-w^2LC}\right)$$

graphs
down
here

e) **Matlab Code:**

```
C = 10^-7;
L = 10^-2;
R = 400;
J = sqrt(-1);
H = tf([1],[-L*C J*R*C 1]);
bode(H)
```