Problem I9: Partial Product 2

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For the rational class of infinite products defined by

$$\prod_{n=1}^{\infty}\left[1+\frac{f(n)}{g(n)}\right]$$

,

1. If $f(n) = n^2$ and $g(n) = n^5 + 6$, then the partial product $\prod_{n=1}^{\infty}\left[1+\frac{n^2}{n^5+6}\right]$ seems to converge. This makes sense, because for $f(n)/g(n)$, if the degree of the polynomial is larger in $g(n)$, the term $f(n)/g(n)$ will approach zero, so eventually the sequence will converge. Based on the output of my code, the specific sequence I mentioned above seems to converge to 1.3618.

   Additionally, I found that the larger the difference is between the degree of the polynomial of $f(n)$ and $g(n)$, the faster the partial product sequence will converge.

   I also looked at some partial product sequences where $f(n)$ was a fraction with a higher degree polynomial in the denominator (for example $f(n) = 1/n^2$), and found that the results I saw were similar to the results I mentioned above- the partial product sequence will converge because the $f(n)$ term will continue to get smaller as n increases.

2. If $f(n) = n^5 + 7$ and $g(n) = n^3$, the partial product $\prod_{n=1}^{\infty}\left[1+\frac{n^5+6}{n^3}\right]$ seems to diverge. This makes sense because the degree of the polynomial in $f(n)$ is larger than the degree of the polynomial in $g(n)$, so as n increases, the term $f(n)/g(n)$ will continue to increase, so the terms of the partial product sequence will get larger and larger without bound. Additionally, similar to the example in part (1), the larger the difference is between the degrees of $f(n)$ and $g(n)$, the faster the sequence will appear to diverge.

For the exponential class of infinite products defined by

$$\prod_{n=1}^{\infty}(1+b^n)$$

1. When $b = 0.5$, the partial product $\prod_{n=1}^{\infty}(1+0.5^n)$ seems to converge, and based on the output of my code, it seems to converge to about 2.38423103. It makes sense that if $0 < b < 1$ the partial product sequence will converge, because a number in the interval $(0, 1)$ raised to any positive power will return a number that is smaller than itself and also in the interval $(0, 1)$. Therefore, moving through the sequence, the 0.5^n term will become smaller and smaller, approaching zero, so eventually the sequence will converge.

   Based on this conclusion, it also seems reasonable to assume that the smaller the value of b is (in the interval $0 < b < 1$), the faster the partial product sequence will converge.

2. When $b = 3$, the partial product $\prod_{n=1}^{\infty}(1+3^n)$ diverges. When I ran this through my code with $N = 250$, where N is the length of the partial product sequence, the last fifteen terms of the sequence were all infinity. This makes sense to me, because as n increases, the term 3^n will increase also, which will then cause the terms of the sequence to increase without bound.

   Again, similar to example (1), based on the pattern that I saw when running some trials with my code, the larger the value of b is, the faster the partial product sequence will appear to diverge.