“How is this math?”

Math Club

September 2017

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Is this math?

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{y}{x^2+y^2}\right) = -\frac{2xy}{(x^2+y^2)^2},$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(-\frac{x}{x^2+y^2}\right) = \frac{2xy}{(x^2+y^2)^2}.$$

$$\frac{\partial^2 z}{\partial x\,\partial y} = \frac{\partial}{\partial y}\left(\frac{y}{x^2+y^2}\right) = \frac{1\cdot(x^2+y^2)-2y\cdot y}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}.$$

And this?

$$\boxed{\sin 2A} = \sin(A + A) = \sin A\cos A + \cos A\sin A$$
$$= 2\sin A\cos A$$
$$= 2\sin A\sqrt{1 - \sin^2 A}$$

$$\boxed{\cos 2A} = \cos(A + A) = \cos A\cos A - \sin A\sin A$$
$$= \cos^2 A - \sin^2 A = \cos^2 A - (1 - \cos^2 A)$$
$$= 2\cos^2 A - 1$$

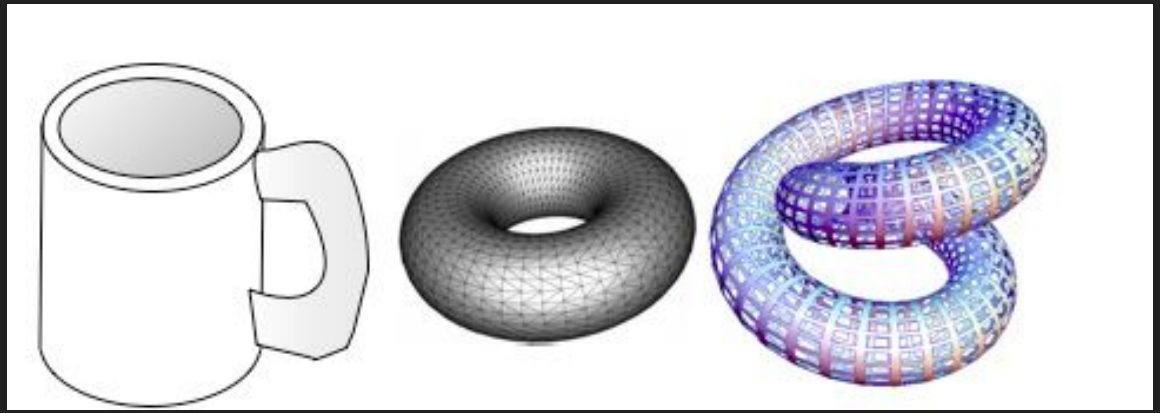
$$\boxed{\tan 2A} = \tan(A + A) = \frac{\tan A + \tan A}{1 - \tan A\tan A} = \frac{2\tan A}{1 - \tan^2 A}$$

or

$$\boxed{\tan 2A} = \frac{\sin 2A}{\cos 2A} = \frac{2\sin A\cos A}{\cos^2 A - \sin^2 A} = \frac{2\dfrac{\sin A\cos A}{\cos^2 A}}{\dfrac{\cos^2 A}{\cos^2 A} - \dfrac{\sin^2 A}{\cos^2 A}}$$

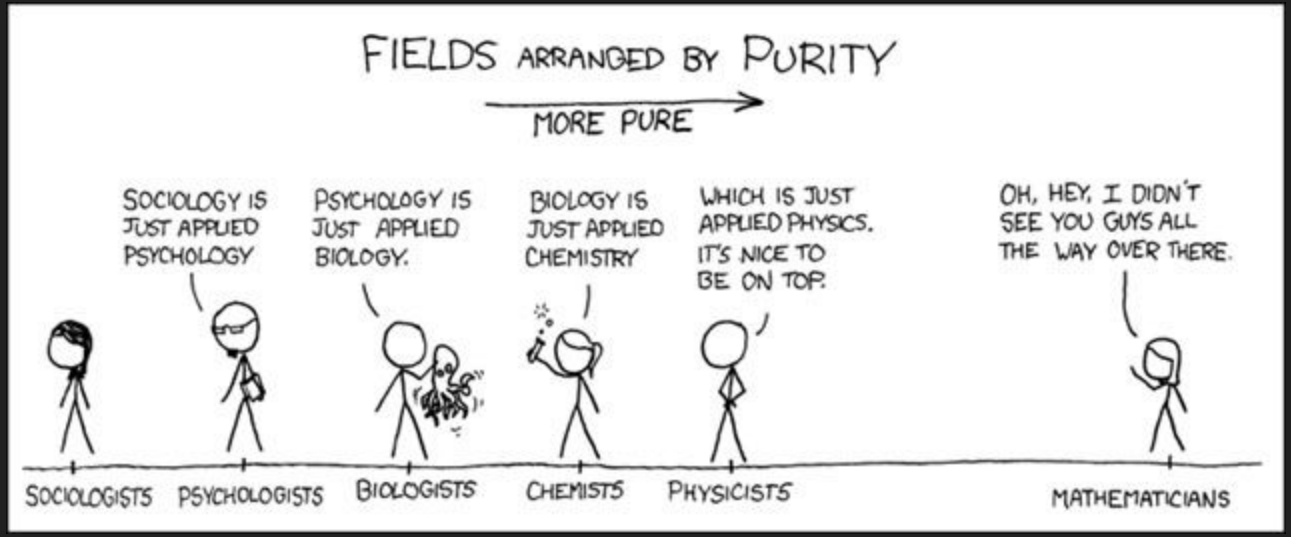
$$= \frac{2\tan A}{1 - \tan^2 A}$$

How about this?



Topology & the Dark Side of Math

- What is pure mathematics?
  - “the study of the basic concepts and structures that underlie mathematics.” [1]
- How is it different from high school mathematics?
  - Proofs!
  - Creative, stimulating, fun!
  - Difficult!
  - Most importantly, **beauty > application**

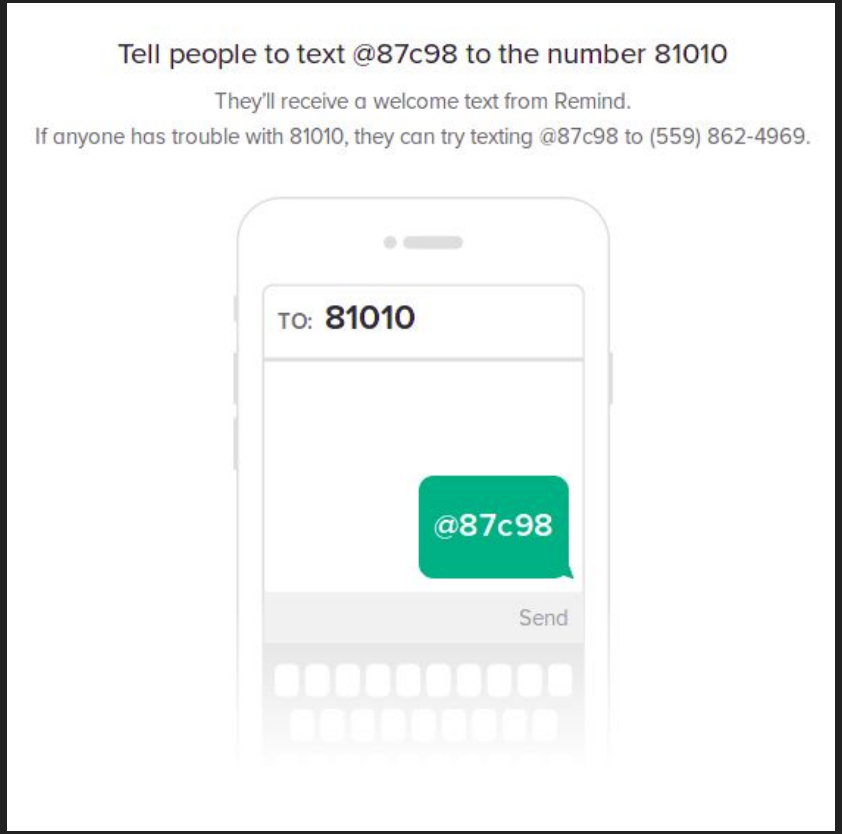

FIELDS ARRANGED BY PURITY
MORE PURE
SOCIOLOGY IS JUST APPLIED PSYCHOLOGY
PSYCHOLOGY IS JUST APPLIED BIOLOGY.
BIOLOGY IS JUST APPLIED CHEMISTRY
WHICH IS JUST APPLIED PHYSICS. IT'S NICE TO BE ON TOP.
OH, HEY, I DIDN'T SEE YOU GUYS ALL THE WAY OVER THERE.
SOCIOLOGISTS
PSYCHOLOGISTS
BIOLOGISTS
CHEMISTS
PHYSICISTS
MATHEMATICIANS

WHY AM I HERE, WHAT IS THIS CLUB ABOUT!

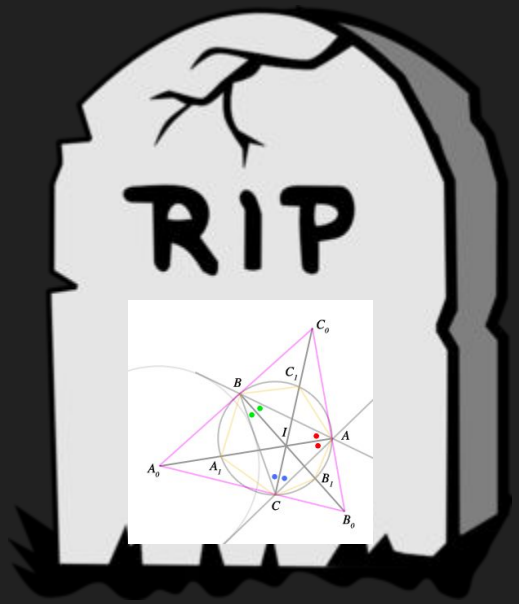
- Peer teaching
  - Why? People are interested in a lot of different things. Plus, it’s fun.
  - How?
    - Simply, volunteer to present a topic for the next meeting
    - Research that topic and create a small presentation
    - Provide some fun, collaborative problems to work on
- Math Competitions
  - Less emphasis than previous years
- Guest Speakers

- Once a week meetings
- Each meeting should last no more than an hour and a half
- Questions??
- Suggestions??

Sign Up for Remind Notifications

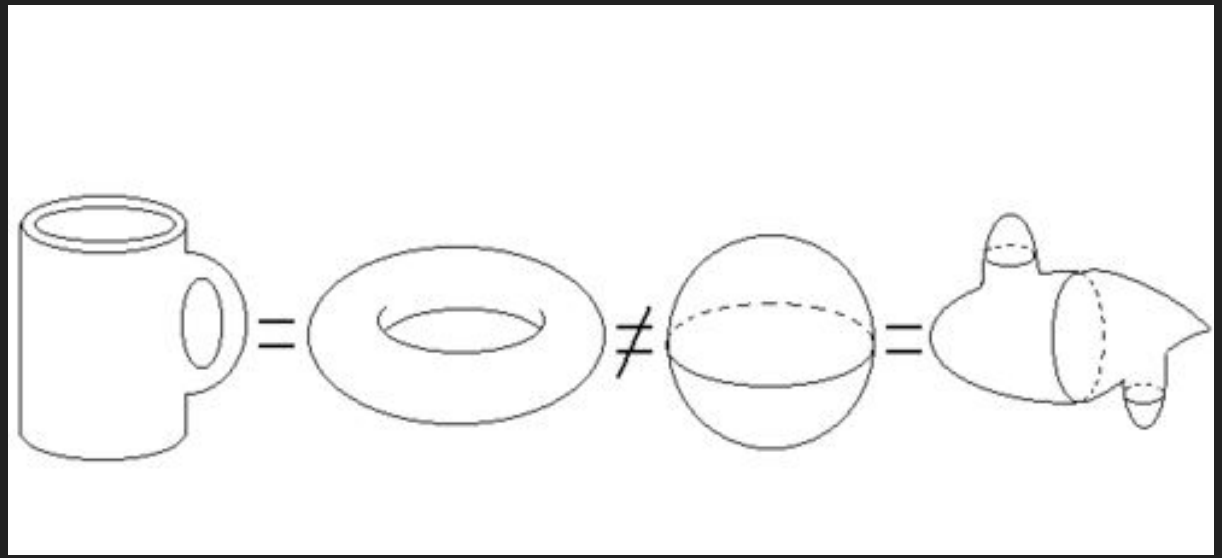


A Quick Moment of Silence for Euclidian Geometry



Topology

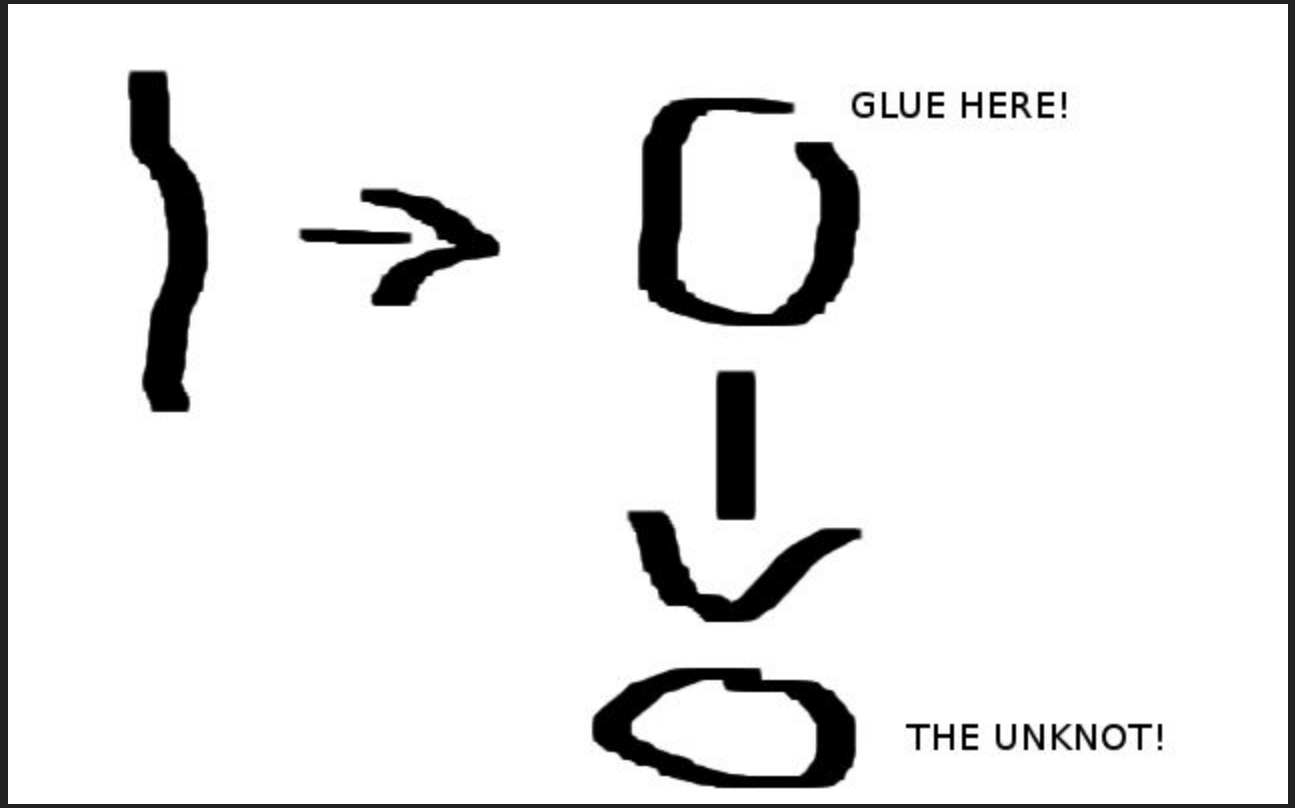
- What is it?
  - Topology is “the study of the properties that are preserved through deformations, twistings, and stretchings of objects. Tearing, however, is not allowed” [2]
- What does it mean to be topologically equivalent?
  - Basically, two objects are topologically equivalent if one can be deformed into the other.

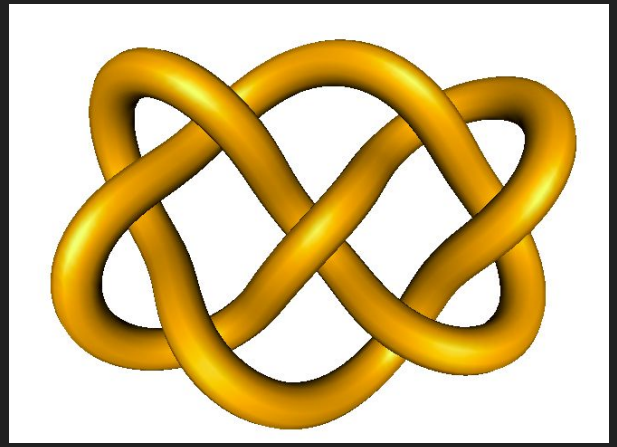


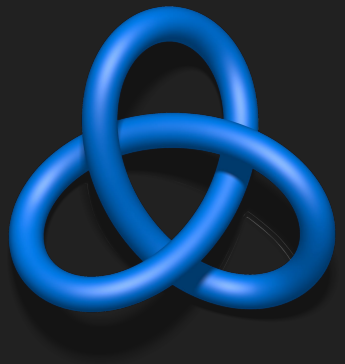
Imagine that geometric objects were made out of playdough

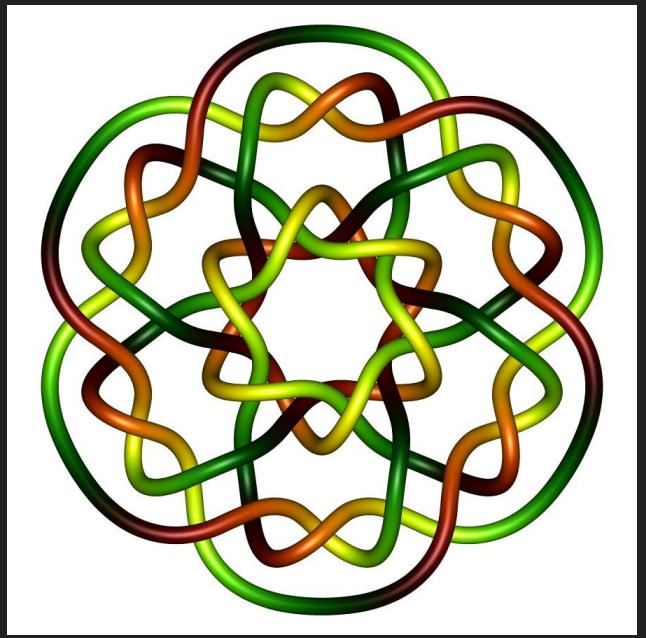
Knot Theory

- Imagine that you took a piece of string and glued the two ends of it together.
- This forms -> A knot
  - Once the knot is constructed, however, the ends must remain “glued” together.
  - The final construction of a knot is based upon how you tie, twist, and deform the string before the two ends are glued together.
  - More formally, we say that a knot is a simple closed curve in space. A knot is, topologically, a one-dimensional object


GLUE HERE!
THE UNKNOT!

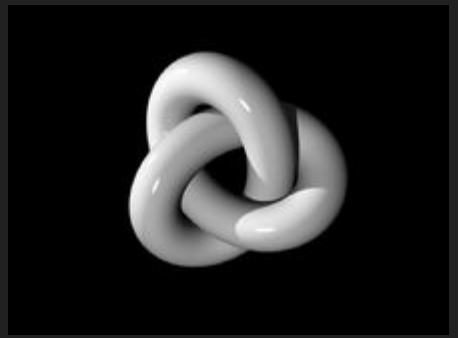






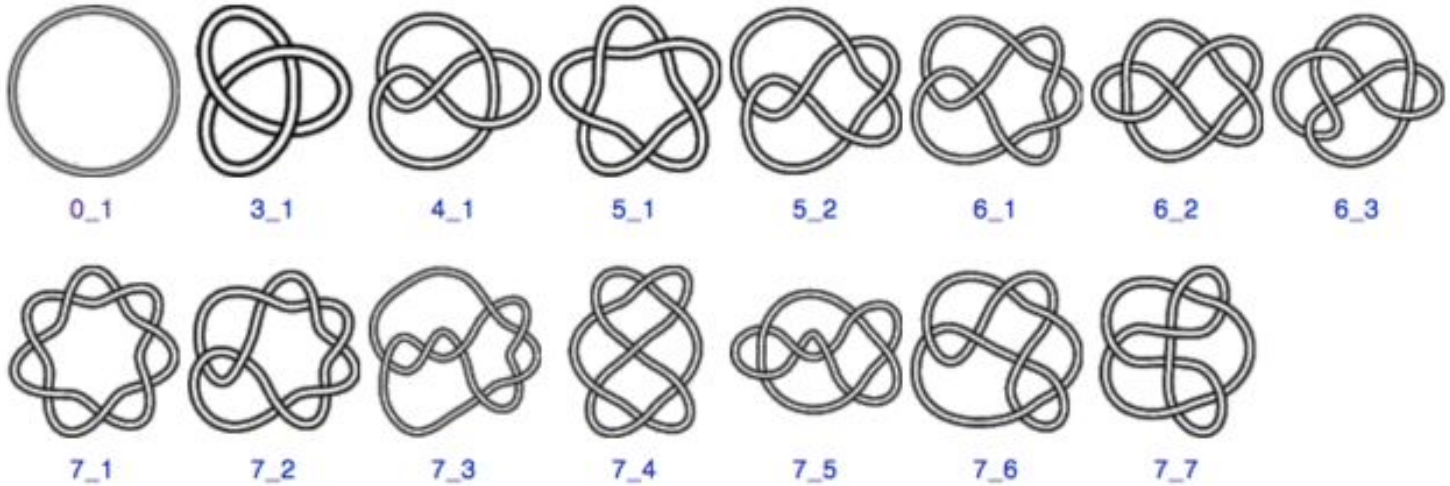
Let’s Take a breather

Any questions so far?



Let’s Build Some Knots

- Unknot
- Trefoil
- What’s the craziest knot you can make?

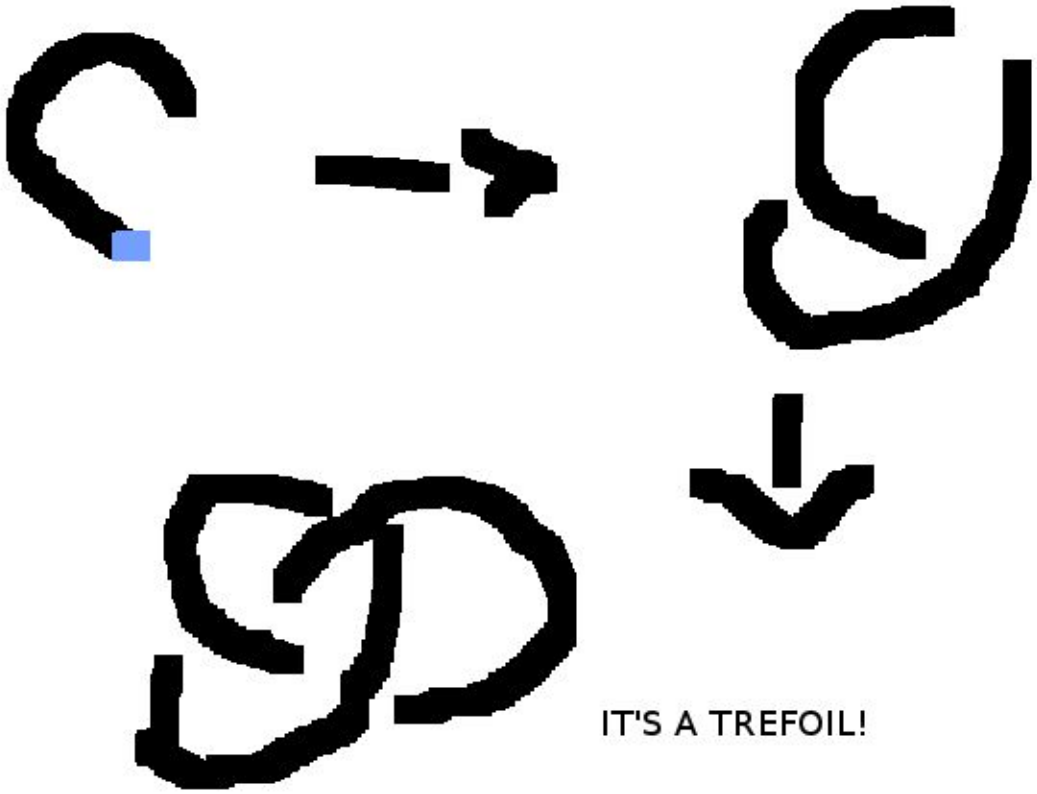

0_1
3_1
4_1
5_1
5_2
6_1
6_2
6_3
7_1
7_2
7_3
7_4
7_5
7_6
7_7

Knot Projections

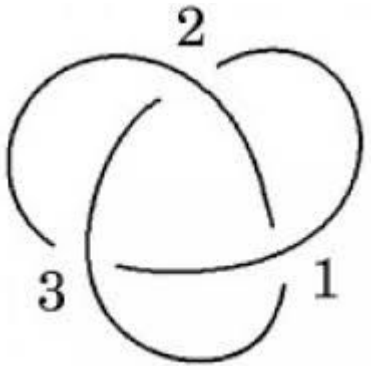
- How do you illustrate a knot on a flat, 2D surface such as paper?
  - Projections!!
  - Imagine you held up a piece of paper at some angle relative to the knot, then outlined the shape of knot, from that angle, onto the paper. That is called a projection.
  - Projections are commonly used to visualize higher dimensions (ex. 4D -> 3D).
  - Projections are also used in video games to display 3D Graphics on a 2D computer screen.

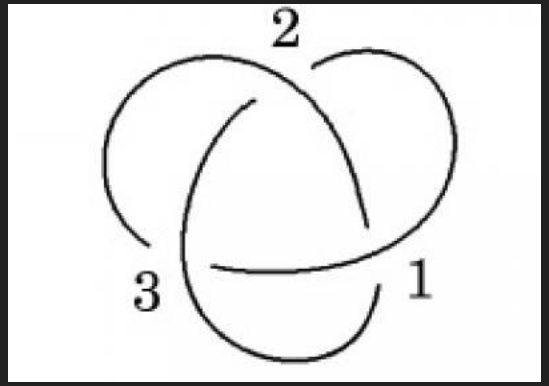
How do you draw a knot?

- First, pick a point on the knot to start from.
- Outline the shape of the knot until you reach a crossing (where two sections of the knot meet)
  - If the the strand you are drawing goes under the other put a break in your line and continue onwards.
  - This creates the appearance that one strand is crossing over the other
- If you drew the knot correctly your pencil should return to the point it started from.

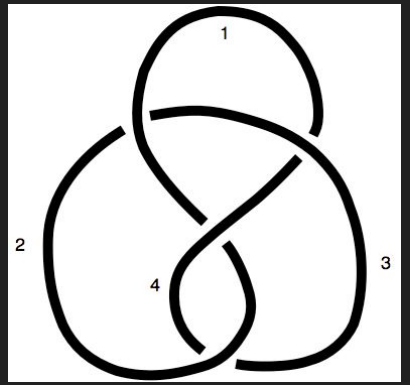


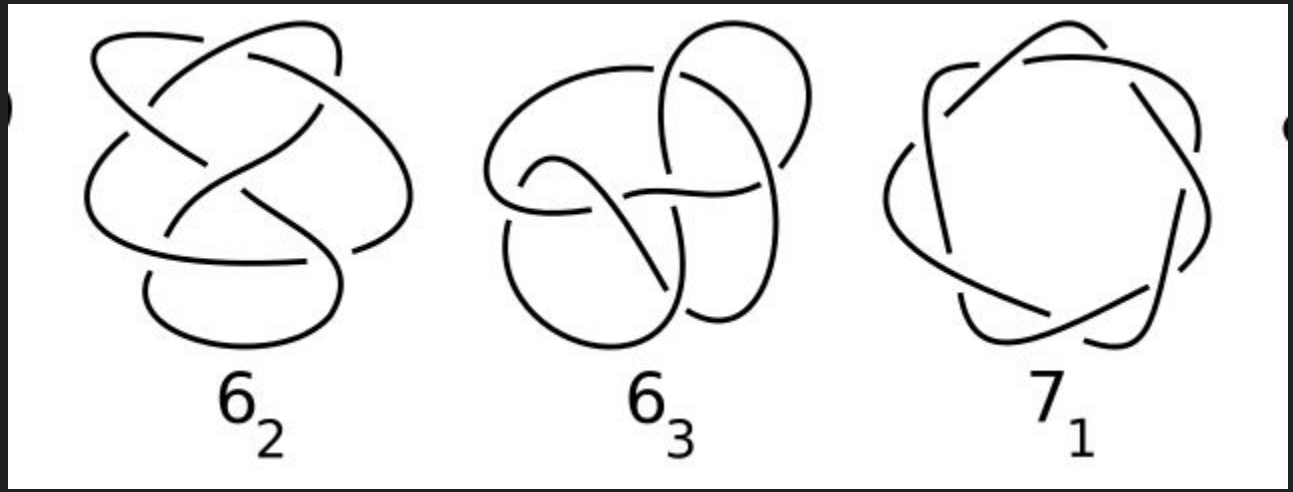
IT'S A TREFOIL!





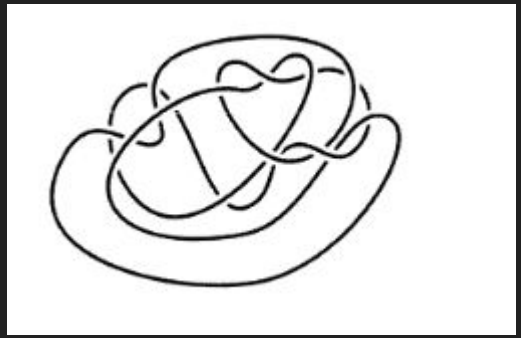
Practice Drawing These Knots





Knot Equivalence

- How do we determine whether two knots are equivalent to one another?
  - Crossing Number?
  - Observation?
- Is this the unknot?

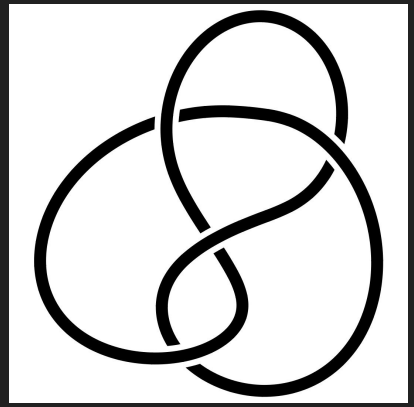


Yes!

But the answer is out of the scope of this lecture.

About Crossings...

- Crossings are the most important part of the knot
- Alternating knot
  - In the projection of the knot crossings alternate between over and under when traveling in a fixed direction



Why?

- Fun
- Chemistry
- Biology
  - DNA Molecules

Have Fun Knotting!

Sources

1. http://math.mit.edu/academics/undergrad/major/course18/pure.php
2. http://mathworld.wolfram.com/Topology.html
3.