Mécanique

Exercice 2 TD1 (suite)

① $\vec{F} = \int_S d\vec{F}$

$= \frac{-b\rho_e g}{\cos\alpha} \frac{H^2}{2} \vec{y_1}$ $\qquad S = b \times L = b\frac{H}{\cos\alpha}$

$\qquad \cos\alpha = \frac{H}{L} \Leftrightarrow L = H\cos\alpha$

$= -\rho_e g S \frac{H}{2} \vec{y_1}$

② $\left.\begin{array}{l}\overrightarrow{M_O(\vec{F})} = \overrightarrow{OP} \wedge \vec{F} \\ \overrightarrow{M_O(d\vec{F})} = \int_S \overrightarrow{OM} \wedge \overrightarrow{dF}\end{array}\right\}$ On veut que $\overrightarrow{OP} \wedge \vec{F} = \int_S \overrightarrow{OM} \wedge \overrightarrow{dF}$

On cherche $\overrightarrow{OP}\begin{pmatrix} x_{1P} \\ \cancel{y_{1P}}\ 0 \\ \cancel{z_{1P}}\ 0 \end{pmatrix} \Rightarrow \overrightarrow{OP}\begin{pmatrix} x_{1P} \\ 0 \\ 0 \end{pmatrix}_{(O,\vec{x_1},\vec{y_1},\vec{z_1})}$ $\qquad \overrightarrow{OM}\begin{pmatrix} x_1 \\ 0 \\ 0 \end{pmatrix}_{(O,\vec{x},\vec{y},\vec{z})}$

$\overrightarrow{OP} \wedge \vec{F} = \begin{pmatrix} x_1 P \\ 0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ -\rho_e g S \frac{H}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ x_{1P}\left(-\rho_e g S \frac{H}{2}\right) \end{pmatrix}$

$\int_S \overrightarrow{OM} \wedge \overrightarrow{d_F} = \begin{pmatrix} x_1 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ -\rho_e g x dS \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -x_1 \rho_e g x dS \end{pmatrix}$ $\qquad L = \frac{H}{\cos\alpha}$

$\int_S \overrightarrow{OM} \wedge \overrightarrow{dF} = \int_S -x_1 \rho_e g x dS \cdot \vec{z} = \int_0^L -x_1 \rho_e g x b dx_1 \cdot \vec{z}$

$= -\rho_e g b \int_0^H \frac{x}{\cos\alpha} \frac{x\, dx}{\cos\alpha} \cdot \vec{z} = \cancel{\int_0^H \ldots}$

$$= -\rho_e g b \frac{1}{\cos^2\alpha} \int_0^H x^2 \, dx \, \vec{z}$$

$$= -\rho_e g b \frac{1}{\cos\alpha} \frac{1}{3}\left[x^3\right]_0^H \vec{z}$$

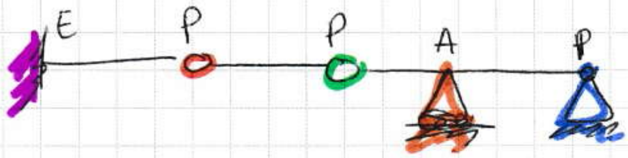
$$= -\rho_e g b \frac{1}{\cos\alpha} \frac{1}{3} H^3 \vec{z}$$

On voudrait $\vec{OP} \wedge \vec{F} = \int \vec{OM} \wedge d\vec{F}$ selon $\vec{z}$ $\quad \frac{-x_1 p\, b\, \rho_e g H^2}{2\cos\alpha} = \frac{-\rho_e g b}{\cos^2\alpha}\frac{H^3}{3}$

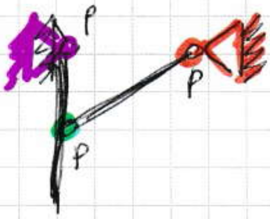
$$\boxed{x_1 p = \frac{2}{3}\frac{H}{\cos\alpha}}$$

Exercice 3 — formule $\boxed{H = n_i - n_e}$

$n_i = 3 + 1 + 1 = 5$
$n_e = 3$
$H = 5 - 3 = 2$ ⇒ hyperstatique



$n_i = 3 + 2 + 2 + 1 = 10$
$n_e = 9$
$H = 10 - 9 = 1$ ⇒ hyperstatique



$n_i = 2 + 2 + 2 = 6$
$n_e = 3 + 3 = 6$
$H = 6 - 6 = 0$ ⇒ Isostatique

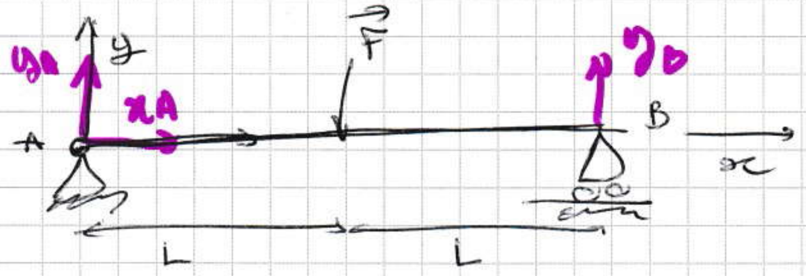
$n_i = 3 + 2 + 2 = 7$
$n_e = 3 + 3 = 6$
$H = 7 - 6 = 1$ ⇒ hyperstatique

Système 6 ⇒ $\begin{cases} n_i = 3 + 3 + 2 + 2 = 10 \\ n_e = 3 + 3 + 3 = 9 \end{cases}$ $H = 10 - 9 = 1$ ⇒ hyperstatique

Système 7 ⇒ $\begin{cases} n_i = 2 + 2 + 2 + 2 + 2 + 2 = 12 \\ n_e = 3 + 3 + 3 + 3 = 12 \end{cases}$ $H = 12 - 12 = 0$ ⇒ ~~hyperstatique~~ Isostatique

Système 8 ⇒ $\begin{cases} n_i = 3 + 2 + 2 + 2 + 1 = 10 \\ n_e = 3 + 3 + 3 = 9 \end{cases}$ $H = 10 - 9 = 1$ ⇒ hyperstatique

Exercice 4



① nature du système

$h = n_i - n_e = 3 - 3 = 0$

⇒ Isostatique

↳ on peut calculer

②

PFS : • $\sum \vec{F} = \vec{0} \Rightarrow y_A \vec{y} + x_A \vec{x} + y_B \vec{y} - F\vec{y} = \vec{0}$

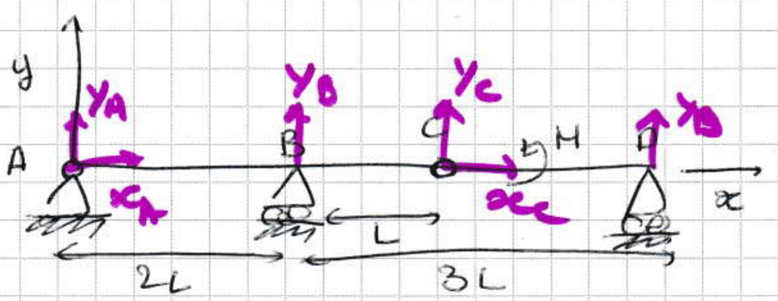
projection selon $\vec{x}$: $\boxed{x_A = 0}$

projection selon $\vec{y}$: $y_A + y_B - F = \vec{0}$

• $\sum \vec{M} = \vec{0} \Rightarrow \vec{M}_{A,\vec{y_B}} + \vec{M}_{A,\vec{F}} = \vec{0}$

$2Ly_B - LF = 0 \Leftrightarrow \boxed{y_B = \frac{F}{2} = y_A}$

Exercice 5



① nature du système

$h = n_i - n_e = 6 - (3+3) = 0$

→ Isostatique

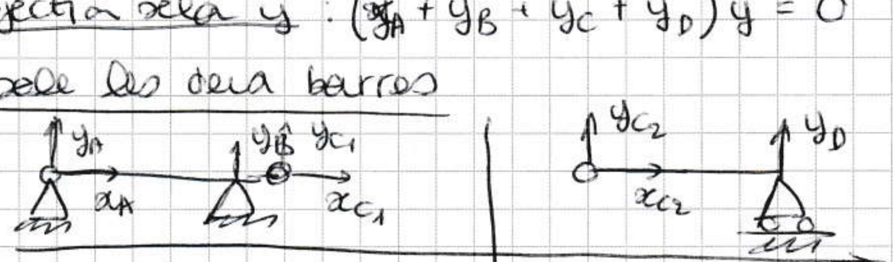
↳ on peut calculer

PFS : • $\sum \vec{F} = 0 \Rightarrow y_A \vec{y} + x_A \vec{x} + y_B \vec{y} + y_C \vec{y} + x_C \vec{x} + y_B \vec{y} = \vec{0}$

projection selon $\vec{x}$: $x_A \vec{x} + x_C \vec{x} = \vec{0}$

projection selon $\vec{y}$: $(y_A + y_B + y_C + y_D)\vec{y} = \vec{0}$

On isole les deux barres



$\underbrace{y_A \vec{y} + x_A \vec{x} + y_B \vec{y} + y_{C_1} \vec{y} + x_{C_1} \vec{x}}_{=\vec{0}}$ | $y_{C_2} \vec{y} + \underbrace{\cancel{x_{C_2} \vec{x}}}_{0} + y_D \vec{y} = \vec{0}$

$2Ly_B + 3Ly_{C_1} = 0$ | $y_D 2L + M = 0$

⇒ en utilisant le principe d'action réaction : $x_{C_1} = -x_{C_2}$

$y_{C_1} = -y_{C_2}$