Sensory robotics

Lecture 04.

i.) Sensor characteristics

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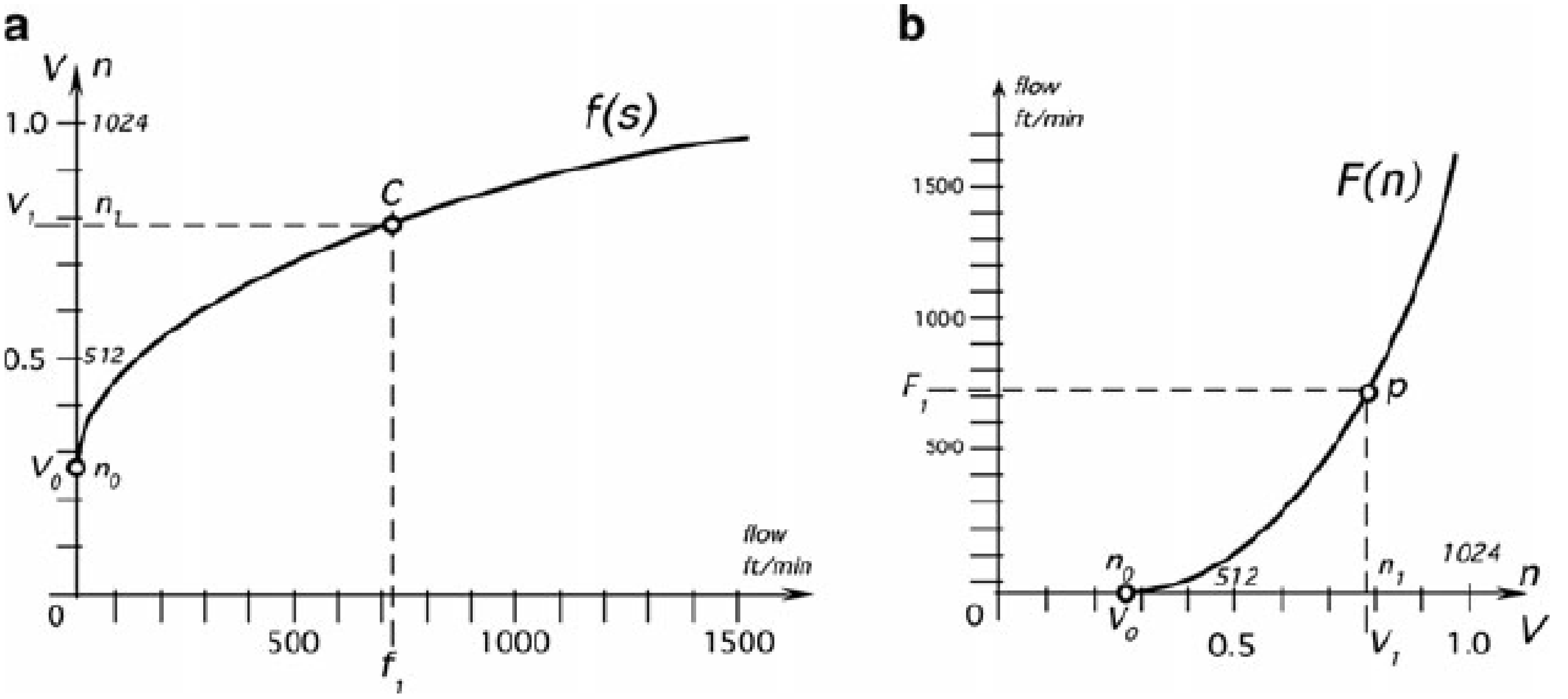
4. Sensor characteristics

- Sensor characteristics; basic principles through examples; sensitivity; accuracy; dynamic range; hysteresis; nonlinearity; resolution; environmental factors; special properties; transfer function; approximations; interpolation; calibration;
- Fraden, Jacob. *Handbook of modern sensors: physics, designs, and applications*. Springer Science & Business Media, 2010.

Transfer function

- An ideal or theoretical input–output (stimulus–response) relationship exists for every sensor.
- This may be expressed in the form of a table of values, a graph, a mathematical formula, or as a solution of a mathematical equation.
- If it is time invariant it is commonly called transfer function.
- Stimulus s and response S: $S=f(s)$
- Stimulus s is unknown while the output signal S is measured. The value of S that becomes known during the measurement is just a number (voltage, current, digital count, etc.) that represents the value of stimulus s. We need the inverse transfer function $f^{-1}(S)$.

Transfer function



Transfer function (a) and inverse transfer function (b) of a thermo-anemometer

Matematical model

- A physical or chemical law can be expressed in form of a mathematical formula -> it can be used to calculate the sensor’s inversed transfer function
- In practice, readily solvable formulas for many transfer functions, especially for complex sensors, do not exist
- Applying various approximations of the direct and inverse transfer functions (functional approximations, polynomial approximations, linear piecewise approximations, spline approximation)

Functional approximations

- A curve-fitting of experimentally observed values
- The simplest transfer function is linear:

  $$S=A+Bs,$$

  corresponding to the straight line with intercept A, and slope B, which is sometimes called sensitivity (since the larger this coefficient the greater the influence of the stimulus)
- Very few sensors are truly linear, in many cases, nonlinearity cannot be ignored, the transfer function can be approximated by a multitude of linear mathematical functions

Functional approximations

- Logarithmic function:

$$S = A + B \ln(s)$$
$$s = e^{(S-A)/B}$$

- Exponential function:

$$S = Ae^{ks}$$
$$s = 1/k \ln(S/A)$$

- Power function and its inverse:

$$S = A + Bs^k$$
$$s = \sqrt[k]{(S-A)/B}$$

- Parameters must be determined during calibration

Polynomial Approximations

- In case of more complex transfer functions other approximation methods can be used
- One method is a polynomial approximation
- Any continuous function can be approximated by a
- power series, eg.: $S = Ae^{ks} \approx A\left(1 + ks + \frac{k^2}{2!}s^2 + \frac{k^3}{3!}s^3\right)$
- In many cases it is sufficient to investigate approximation of a sensor's response
  by the 2nd and 3rd degree polynomials:
  $$S = a_2 s^2 + b_2 s + c_2$$
  $$S = a_3 s^3 + b_3 s^2 + c_3 s + d_3$$
- The same technique can be applied to the inverse transfer function:
  $$s = A_2 S^2 + B_2 S + C_2$$
  $$s = A_3 S^3 + B_3 S^2 + C_3 S + D_3$$
- When a high accuracy is required, the higher order polynomials should be considered

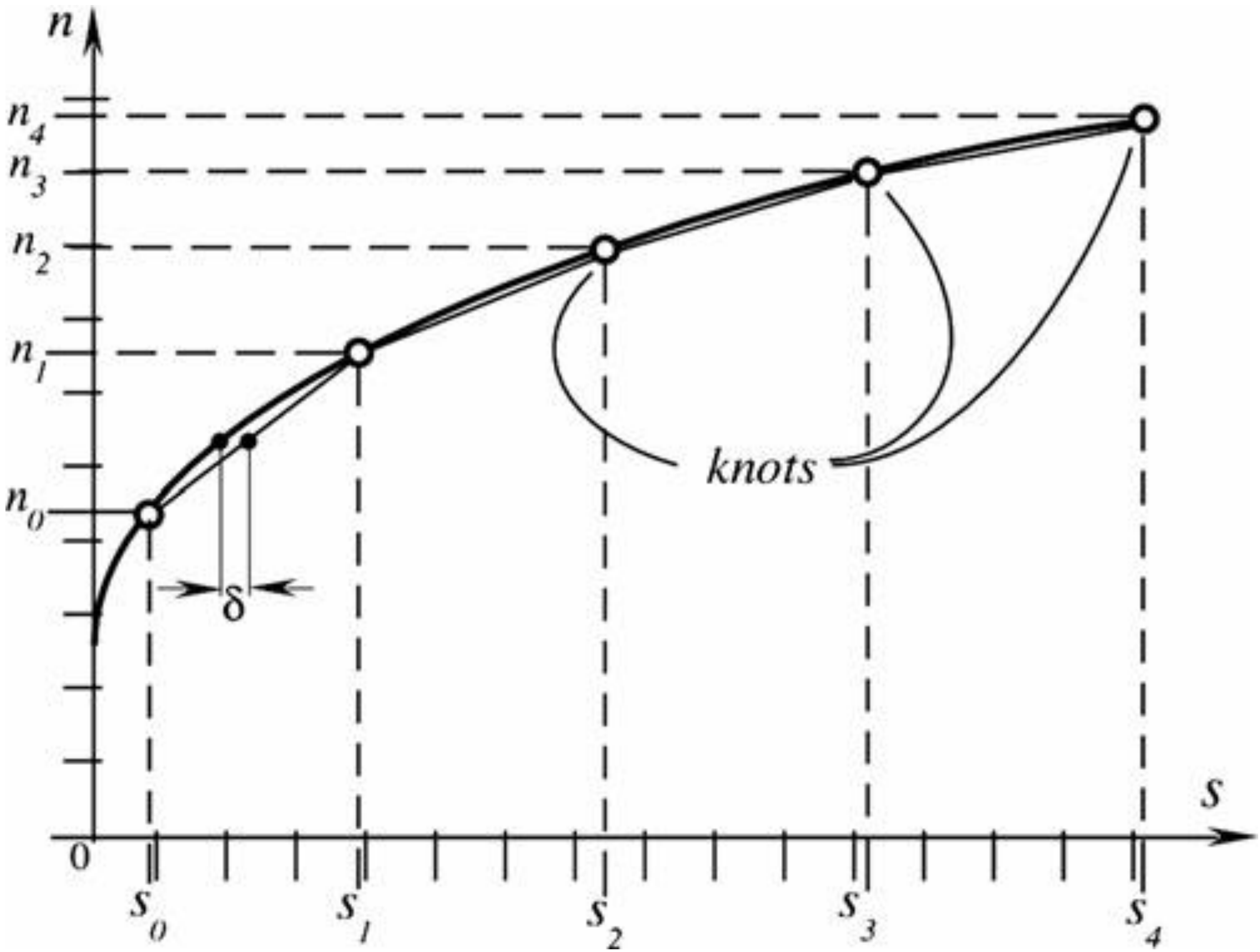
Sensitivity

- In case of linear transfer function the coefficient (slope) B is called sensitivity and it is a fixed number.
- For a nonlinear transfer function, sensitivity B is not a fixed number.
- A nonlinear transfer function exhibits different sensitivities at different points in intervals of stimuli.
- In case of nonlinear transfer functions, the sensitivity is defined as a first derivative of the transfer function:

$$b_i(s_i) = \frac{dS(s_i)}{ds} \approx \frac{\Delta S_i}{\Delta s_i}$$

where, traditionally Δs_i is a small increment of the input stimulus and ΔS_i is the corresponding change in the output S of the transfer function.

Linear Piecewise Approximation



Linear Piecewise Approximation

- To break up a nonlinear transfer function of any shape into linear sections.
- Knots separate the sections.
- Select knots only for the input range of interest.
- An error of a piecewise approximation can be characterized by a maximum deviation δ of the approximation lines from the real curve.
- The more sections are, the smaller the error is.
- The knots do not need to be equally spaced. They should be closer to each other where a nonlinearity is high and farther apart where a nonlinearity is small.

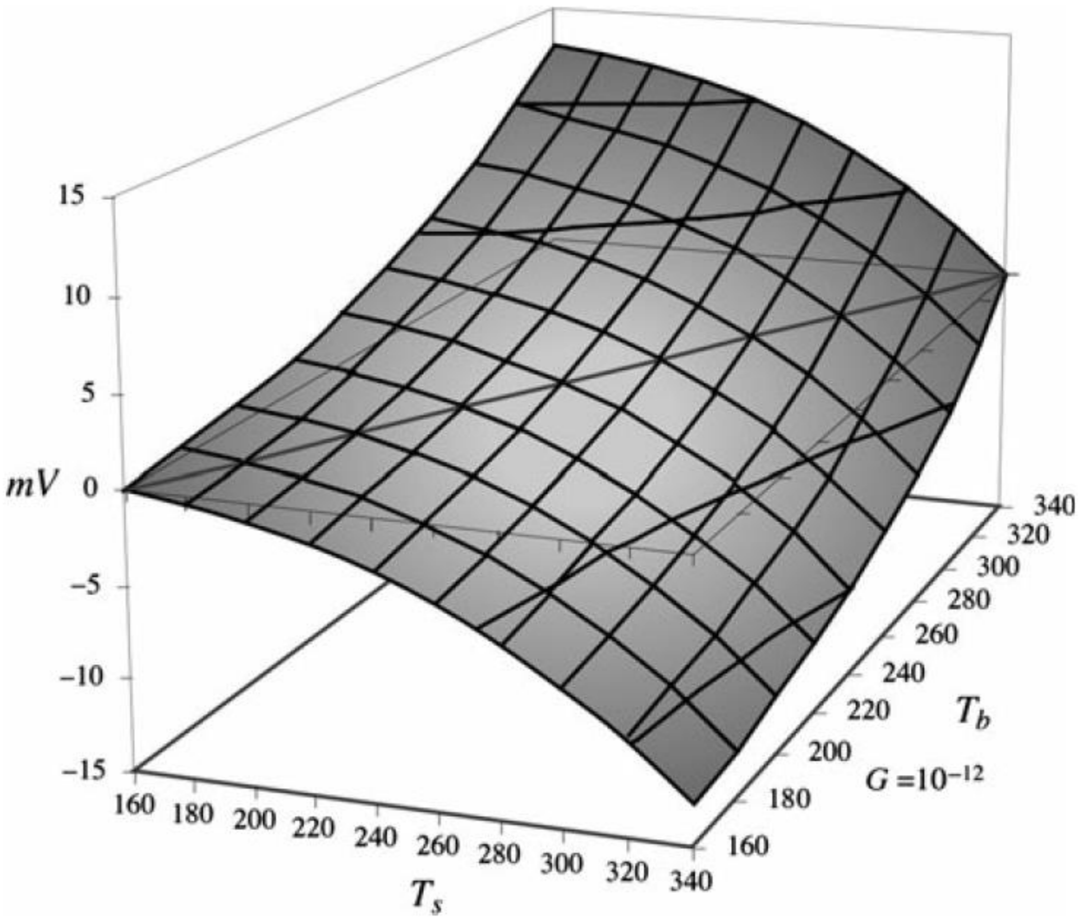
Spline Interpolation

- The approximation by higher order polynomials (3rd-order and higher) have some disadvantages: selected points at one side of the curve make strong influence on the remote parts of the curve.
- This deficiency is resolved by the spline method of approximation.
- A linear spline-interpolation (1st order) is the simplest form and is equivalent to a linear piecewise interpolation
- Curvature of a line at each point is defined by the 2nd derivative.
- The simplicity of the implementation and the computational costs of spline interpolation should be taken into account particularly in a tightly controlled microprocessor environment.

Multidimensional Transfer Functions

- A transfer function may be a function of more than one variable when the sensor's output is dependent on more than one input stimulus.
- Eg.: a humidity sensor whose output depends on two input variables – relative humidity and temperature
- Eg.: thermal radiation (infrared) sensor, has two arguments – two temperatures (T_b), the absolute temperature of an object of measurement and (T_s), the absolute temperature of the sensor's surface)
- Output voltage (V) is proportional to the difference: $V = G(T_b^4 - T_s^4)$ where G is a constant. The output voltage (transfer function) is not only nonlinear (it depends on the 4th order parabola) but also depends on the sensor's surface temperature T_s, which should be measured by a separate temperature sensor.

Multidimensional Transfer Functions



- Two-dimensional transfer function of a thermal radiation sensor

To be continued

End of Lecture 04.

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