# Sensory robotics

**Lecture 05.**

i.) Sensor characteristics

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# Calibration

- If sensor's manufacturer tolerances and tolerances of the interface circuit are broader than the required system accuracy, a calibration of the sensor and an interface circuit is required to minimize errors.
- Unique transfer function should be found to fit the real sensor's response or the specific transfer function parameters should be adjusted.
- Usually, it is sufficient to calibrate a sensor only at a few sample points (stimuli) that are generated by a known reference source.

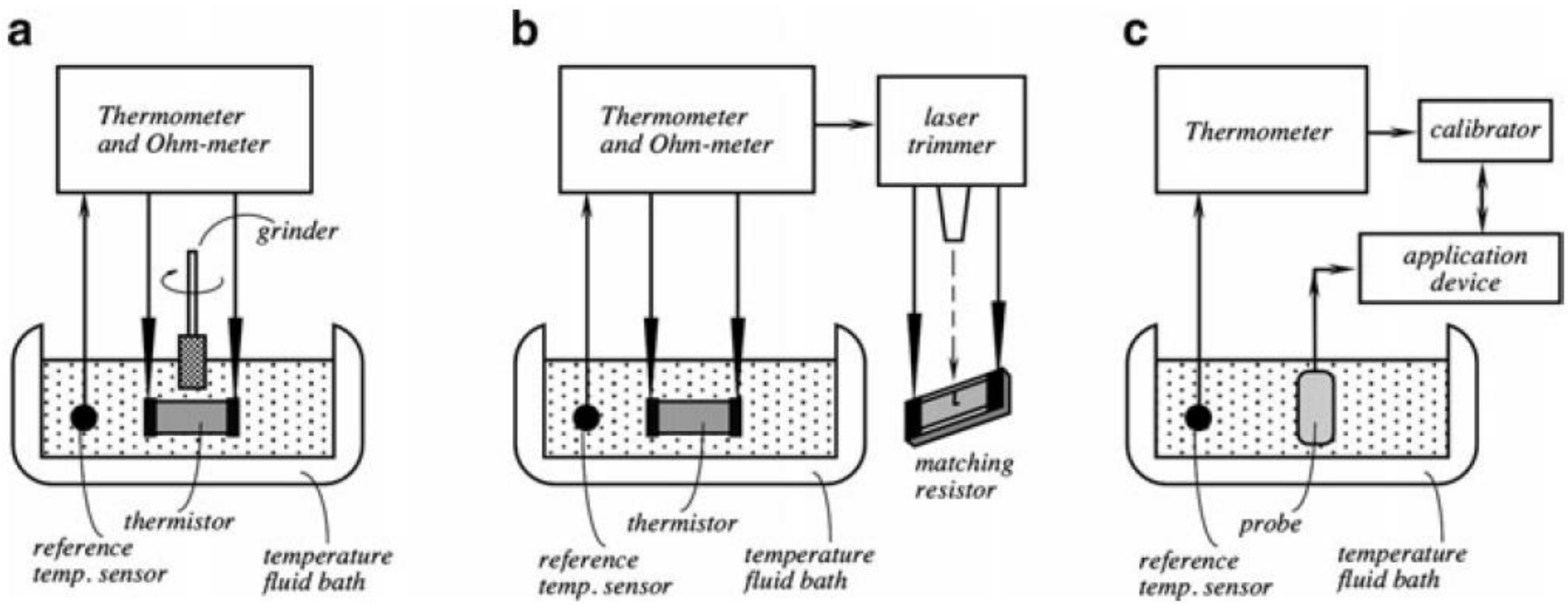
# Calibration

- In calibration, input stimuli are paired with the corresponding output electric responses. These pairs are used with the inverted transfer function to compute its parameters (coefficients).
- Either a mathematical model of a transfer function has to be known before calibration or a good approximation of the sensor's response over the entire span must be found.

# Calibration methods

- 1.) Calculation of the transfer function or its approximation to fit the selected calibration points (curve fitting by computing coefficients of a selected approximation).
- 2. Adjustment of the data acquisition system to trim (modify) the measured data by making them to fit into a normalized or “ideal” transfer function.
- 3. Modification (trimming) of the sensor’ properties to fit the predetermined transfer function.
- 4. Creating a sensor-specific reference device with matching properties at particular calibrating points.

# Calibration methods




- Calibrations of a thermistor: grinding (a), trimming of a reference resistor (b), calculating the transfer function (c)

# Computation of Linear Transfer Function Parameters

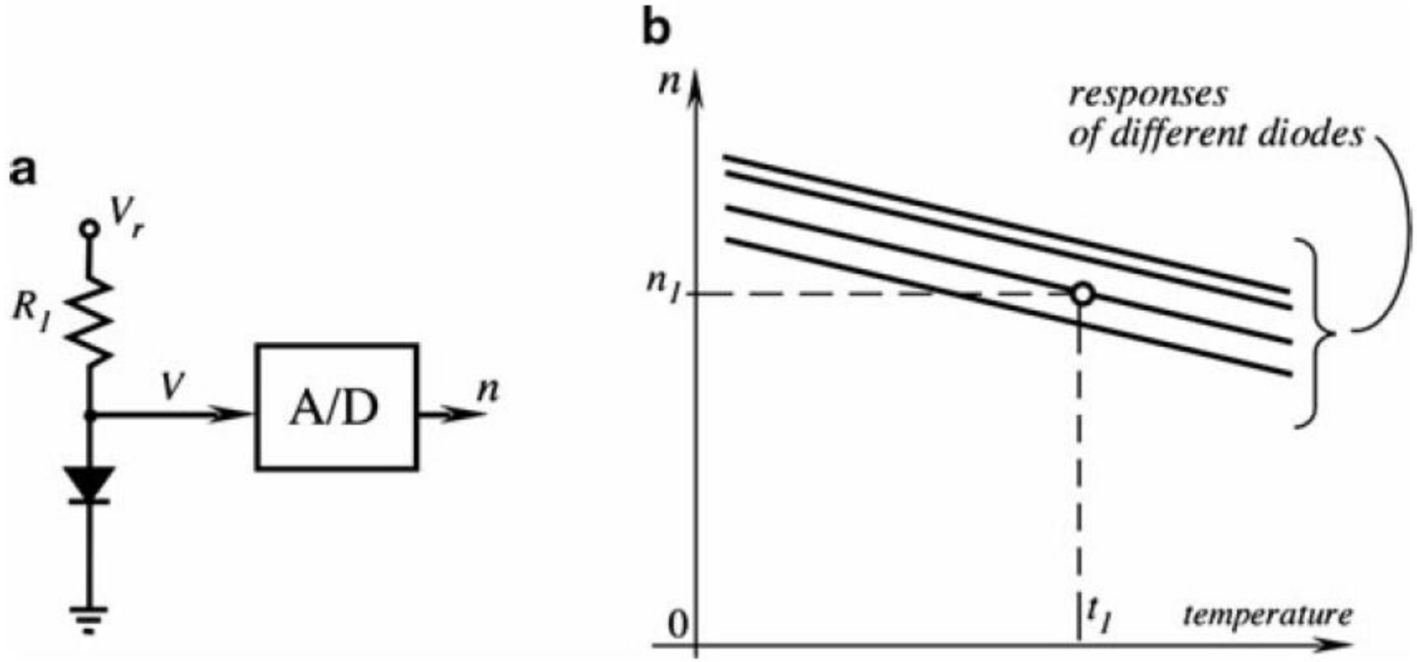



**Fig. 2.5** A p–n junction temperature sensor (**a**); calibration (**b**). Each diode will produce different $n_1$ at the same temperature $t_1$. The slopes $B$ are considered the same for all diodes

# Computation of Linear Transfer Function Parameters

- Temperature is the input and the A/D count n is the output: $n = n_1 + B(t - t_1)$
- Calibrating temperatures: $t_1$ and $t_2$
- Second measurement: $n_2 = n_1 + B(t_2 - t_1)$
- The sensitivity (sloope): $B = \frac{n_2 - n_1}{t_2 - t_1}$
- These parameters are unique for a particular sensor
- After calibration, temperature can be computed by use of the inversed transfer function: $t = t_1 + \frac{(n - n_1)}{B}$
- Because of offset of diodes, at least a single poing calibration is needed to find our $n_1$ at $t_1$ temperature

# Computation of Non-Linear Transfer Function Parameters

- Often two and more input–output pairs would be required for calibration. When a 2nd or a 3rd degree polynomial transfer functions are employed, respectively 3 and 4 calibrating pairs are required. $S = as^3 + bs^2 + cs + d$
- to find four parameters a, b, c, and d, four calibrating input–output pairs are required: s1 and S1, s2 and S2, s3 and S3, s4 and S4.
- It should be solved:

$$S_1 = as_1^3 + bs_1^2 + cs_1 + d$$

$$S_2 = as_2^3 + bs_2^2 + cs_2 + d$$

$$S_3 = as_3^3 + bs_3^2 + cs_3 + d$$

$$S_4 = as_4^3 + bs_4^2 + cs_4 + d$$

# Computation of Non-Linear Transfer Function Parameters

$$\Delta = \left(\frac{s_1^2 - s_2^2}{s_1 - s_2} - \frac{s_1^2 - s_4^2}{s_1 - s_4}\right)\left(\frac{s_1^3 - s_2^3}{s_1 - s_2} - \frac{s_1^3 - s_3^3}{s_1 - s_3}\right)$$

$$- \left(\frac{s_1^2 - s_2^2}{s_1 - s_2} - \frac{s_1^2 - s_3^2}{s_1 - s_3}\right)\left(\frac{s_1^3 - s_2^3}{s_1 - s_2} - \frac{s_1^3 - s_4^3}{s_1 - s_4}\right)$$

$$\Delta_a = \left(\frac{s_1^2 - s_2^2}{s_1 - s_2} - \frac{s_1^2 - s_4^2}{s_1 - s_4}\right)\left(\frac{S_1 - S_2}{s_1 - s_2} - \frac{S_1 - S_3}{s_1 - s_3}\right)$$

$$- \left(\frac{s_1^2 - s_2^2}{s_1 - s_2} - \frac{s_1^2 - s_3^2}{s_1 - s_3}\right)\left(\frac{S_1 - S_2}{s_1 - s_2} - \frac{S_1 - S_4}{s_1 - s_4}\right)$$

$$\Delta_b = \left(\frac{s_1^3 - s_2^3}{s_1 - s_2} - \frac{s_1^3 - s_3^3}{s_1 - s_3}\right)\left(\frac{S_1 - S_2}{s_1 - s_2} - \frac{S_1 - S_4}{s_1 - s_4}\right)$$

$$- \left(\frac{s_1^3 - s_2^3}{s_1 - s_2} - \frac{s_1^3 - s_4^3}{s_1 - s_4}\right)\left(\frac{S_1 - S_2}{s_1 - s_2} - \frac{S_1 - S_3}{s_1 - s_3}\right)$$

# Computation of Non-Linear Transfer Function Parameters

- The polynomial coefficients are calculated in the following fashion:

$$a = \frac{\Delta_a}{\Delta}; \quad b = \frac{\Delta_b}{\Delta};$$

$$c = \frac{1}{s_1 - s_4}\left[S_1 - S_4 - a\left(s_1^3 - s_4^3\right) - b\left(s_1^2 - s_4^2\right)\right];$$

$$d = S_1 - as_1^3 - bs_1^2 - cs_1$$

# Calibration

- Since calibration may be a slow process, to reduce the manufacturing cost, it is important to minimize the number of calibration points.
- To calibrate sensors, it is essential to have and properly maintain precision and accurate references – physical standards of the appropriate stimuli.
- Accurate references are the most critical parts of calibration equipment.
- The calibration accuracy is directly linked to the accuracy of a reference sensor that is part of the calibration equipment.
- A value of uncertainty of the reference sensor should be included in the statement of the overall uncertainty,

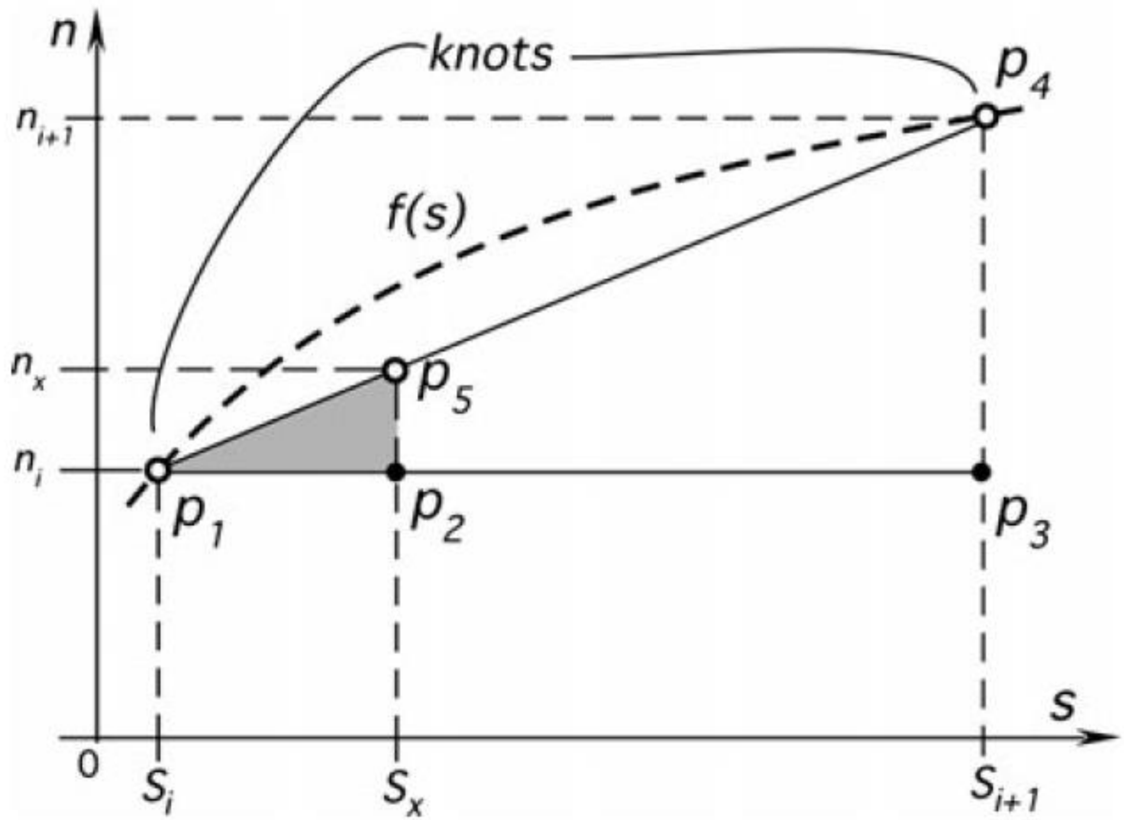
# Linear Regression

- If measurements of the input stimuli during calibration cannot be made with high accuracy and large random errors are expected, the minimal number of measurements will not yield a sufficient accuracy.
- To cope with random errors in calibration process a method of least squares could be employed.
- Measure multiple (k) output values S at input values s over a substantially broad range, preferably over the entire span. Use the following formulas for a linear regression to determine intercept A and slope B of the best fitting straight line:

$$A = \frac{\Sigma S \Sigma s^2 - \Sigma s \Sigma s S}{k \Sigma s^2 - (\Sigma s)^2}, \quad B = \frac{k \Sigma s S - \Sigma s \Sigma S}{k \Sigma s^2 - (\Sigma s)^2},$$

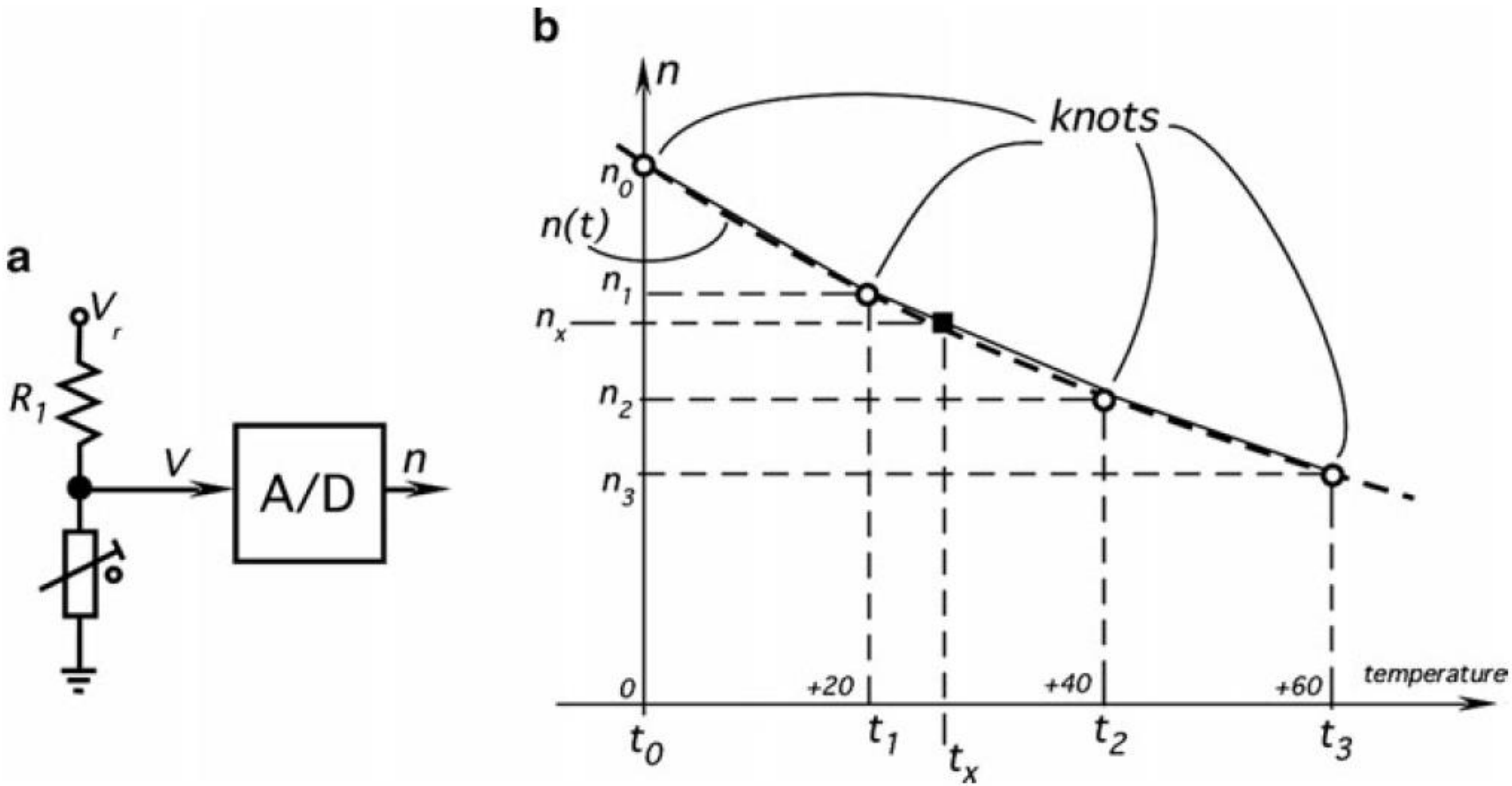
where Σ is the summation over all k pairs.

# Computation from Linear Piecewise Approximation




$$s_x = s_i + \frac{n_x - n_i}{n_{i+1} - n_i}\left(s_{i+1} - s_i\right)$$

# Computation from Linear Piecewise Approximation



# Computation from Linear Piecewise Approximation

- We assume that the thermistor is used to measure temperature from 0C to +60C.
- The output is modeled: $$n_x = N_0 \frac{R_0 e^{\beta(T^{-1}-T_0^{-1})}}{R_1 + R_0 e^{\beta(T^{-1}-T_0^{-1})}},$$ where
  T is the measured temperature in K, $T_0$ is the reference temperature in K, $R_0$ is the resistance of the thermistor at $T_0$
- After manipulating, the inverted transfer function enables us to compute analytically the input temperature in K:

$$T_x = \left(\frac{1}{T_0} + \frac{1}{\beta}\ln\left(\frac{n_x}{N_0 - n_x}\frac{R_1}{R_0}\right)\right)^{-1}$$

- We need to calibrate the sensor at two temperatures $T_x=T_{c1}$ and $T_x=T_{c2}$ in order to find out values of constants $R_0$ and β.

# Iterative Computation (Newton Method)

- The transfer function S=f(s) can be rewritten as S-f(s)=0
- Using numerical iterative methods for finding roots
  Initial guess – to define initial reasonable value of $s=s_0$ and F(s):

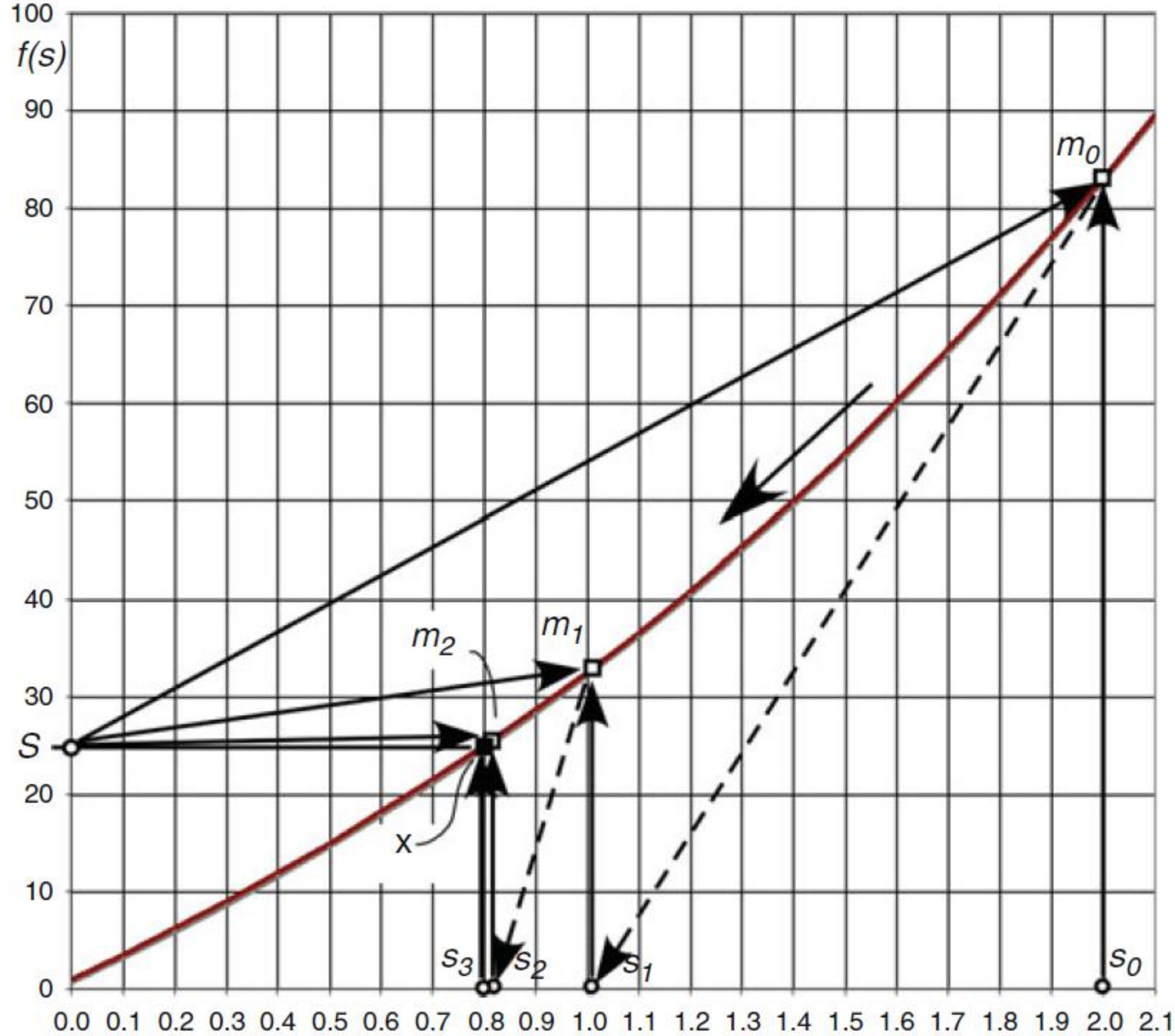
$$s_{i+1} = s_i - \frac{f(s_i) - S}{f'(s_i)},$$

- Eg. $f(s) = as^3 + bs^2 + cs + d$
  where a=1.5, b=5, c=25, d=1

$$s_{i+1} = s_i - \frac{as_i^3 + bs_i^2 + cs_i + d - S}{3as_i^2 + 2bs_i + c} = \frac{2as_i^3 + bs_i^2 - d + S}{3as_i^2 + 2bs_i + c}.$$

  let S=22, $s_0$=2, will result the following $s_{i+1}$. HW
- It should be noted that the Newton method results in large errors when the sensor's sensitivity becomes low.

# Iterative Computation (Newton Method)



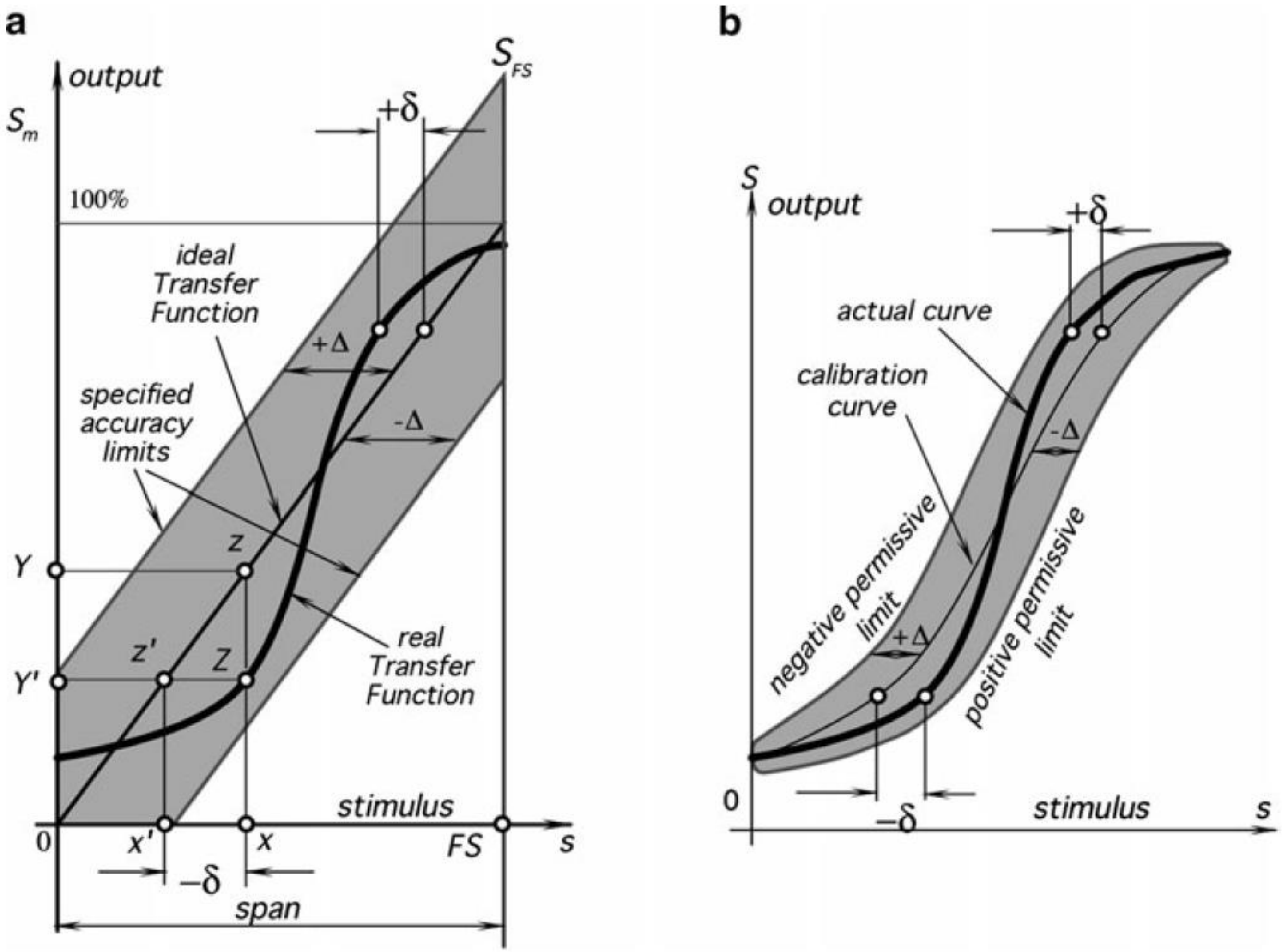
# Accuracy

- **Inaccuracy** is measured as a **highest deviation** of a value represented by the sensor from the ideal or true value of a stimulus at its input.
- Eg, a linear displacement sensor with sensitivity $B = 1$ mV/mm. A reference displacement of $s = 10$ mm produced an output of $S = 10.5$ mV. Converting this number back into the displacement value by using the inverted transfer function ($1/B = 1$ mm/mV), we calculate the displacement as $s_x = S/B = 10.5$ mm. The result overestimates the displacement by $s_x - s = 0.5$ mm. This is an erroneous deviation in the measurement, or error. Therefore, in a 10 mm range the sensor's absolute inaccuracy is 0.5 mm, or in relative terms the inaccuracy is 0.5 mm/10 mm times 100% = 5%.

# Accuracy

- For a larger displacement, the error may be larger.
- If we repeat this experiment over and over again without any random error and every time we observe an error of 0.5 mm we may say that the sensor has a **systematic inaccuracy** of 0.5 mm over a 10 mm span.
- Naturally, a random component is always present, so the systematic error may be represented as an average or mean value of multiple errors.

# Accuracy



a
output
$S_m$
$S_{FS}$
$+\delta$
100%
ideal Transfer Function
$+\Delta$
$-\Delta$
specified accuracy limits
z
Y
z'
Z
Y'
real Transfer Function
stimulus
0
x'
x
FS
s
$-\delta$
span
b
S
output
$+\delta$
actual curve
calibration curve
$-\Delta$
negative permissive limit
$+\Delta$
positive permissive limit
0
$-\delta$
stimulus
s

# Inaccuracy rating

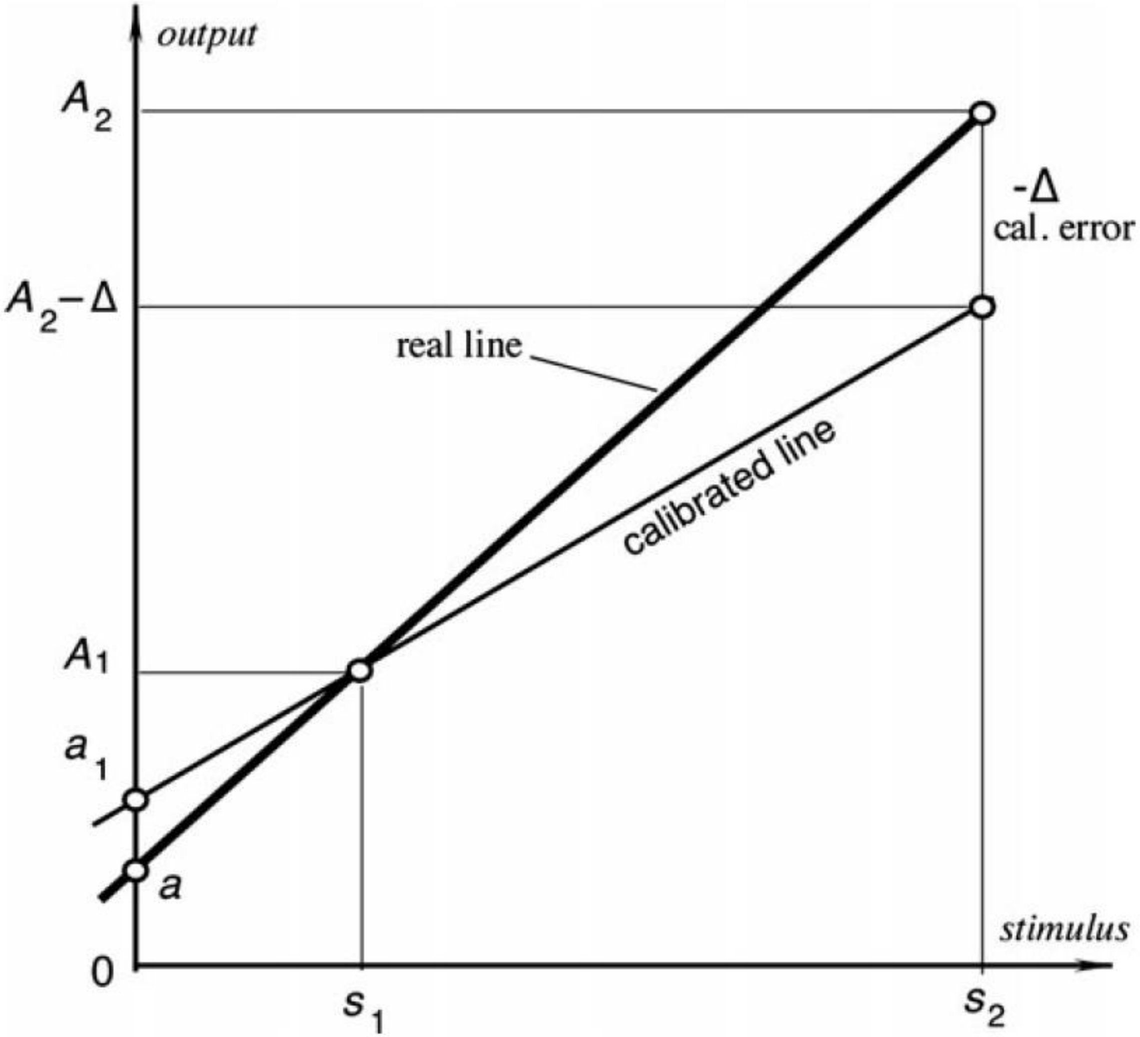
- Directly in terms of **measured value** (D): This form is used when error is independent on the input signal magnitude (it relates to an additive noise or systematic bias, calibration error).
- In **% of the input span** (full scale): This form is useful for a sensor with a linear transfer function
- In **% of the measured signal**: It is useful for a sensor with a highly nonlinear transfer function
- In terms of the output signal: This is useful for sensors with a digital output format so the error can be expressed, for example, in units of LSB

# Calibration error

- Calibration error is inaccuracy permitted by a manufacturer when a sensor is calibrated in the factory
- This error is of a systematic nature, meaning that it is added to all possible real transfer functions.
- It shifts the accuracy of transduction for each stimulus point by a constant.
- This error is not necessarily uniform over the range and may change depending on the type of error in calibration
- Eg. linear TF, the first response was measured absolutely accurately, the other response was measured with error –Δ:

$$\delta_a = a_1 - a = \frac{\Delta}{s_2 - s_1}, \ \delta_b = -\frac{\Delta}{s_2 - s_1}$$

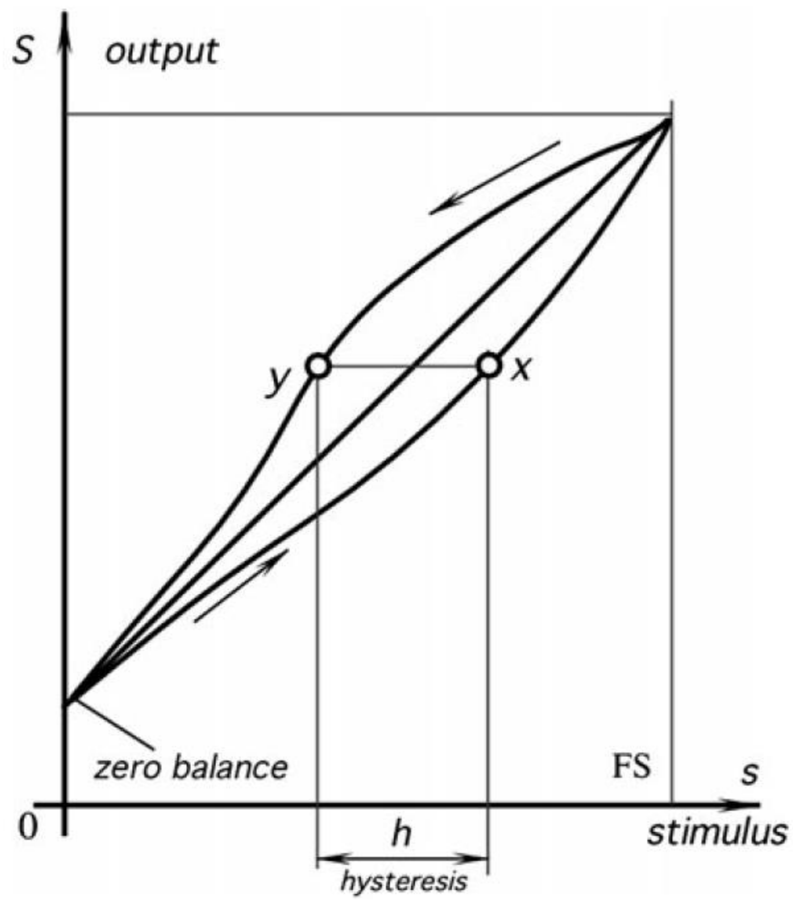
# Calibration error



# Hysteresis error

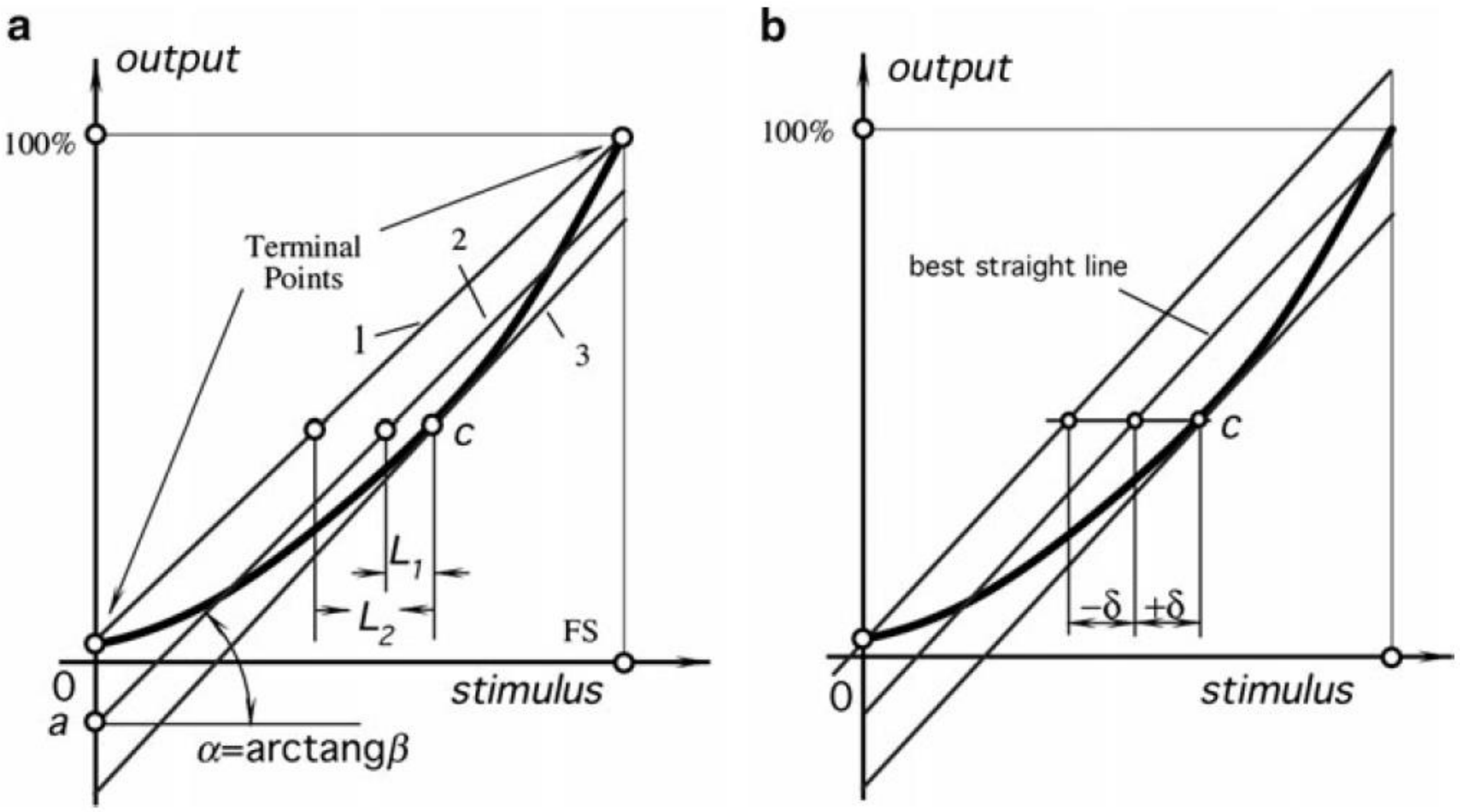
A hysteresis error is a deviation of the sensor's output at a specified point of the input signal when it is approached from the opposite directions. Pl. a displacement sensor when the object moves from left to right at a certain point produces voltage, which differs by 20 mV from that when the object moves from right to left.

- Typical causes for hysteresis are geometry of design, friction, and structural changes in the materials.



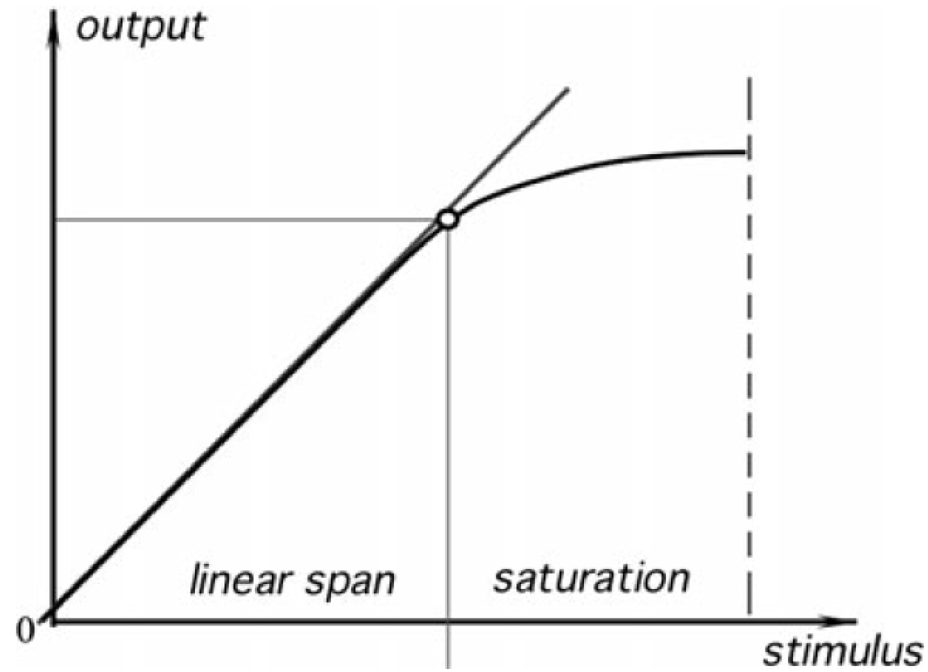
# Nonlinearity error

- transfer function may be approximated by a straight line



# Saturation error

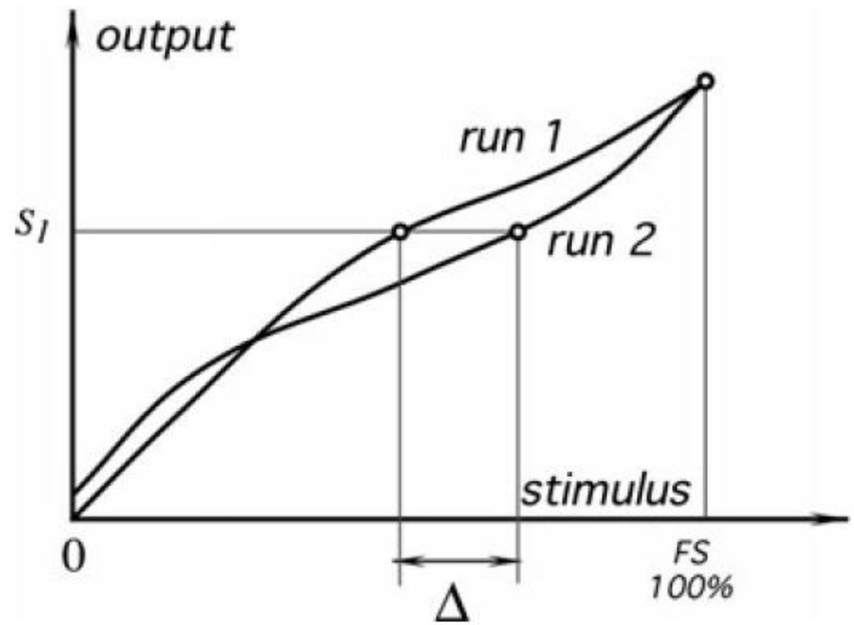
- Every sensor has its operating range and operating limits.
- Even if it is considered linear, at some levels (over) of the input stimuli, its output signal no longer will be responsive



# Repeatability error

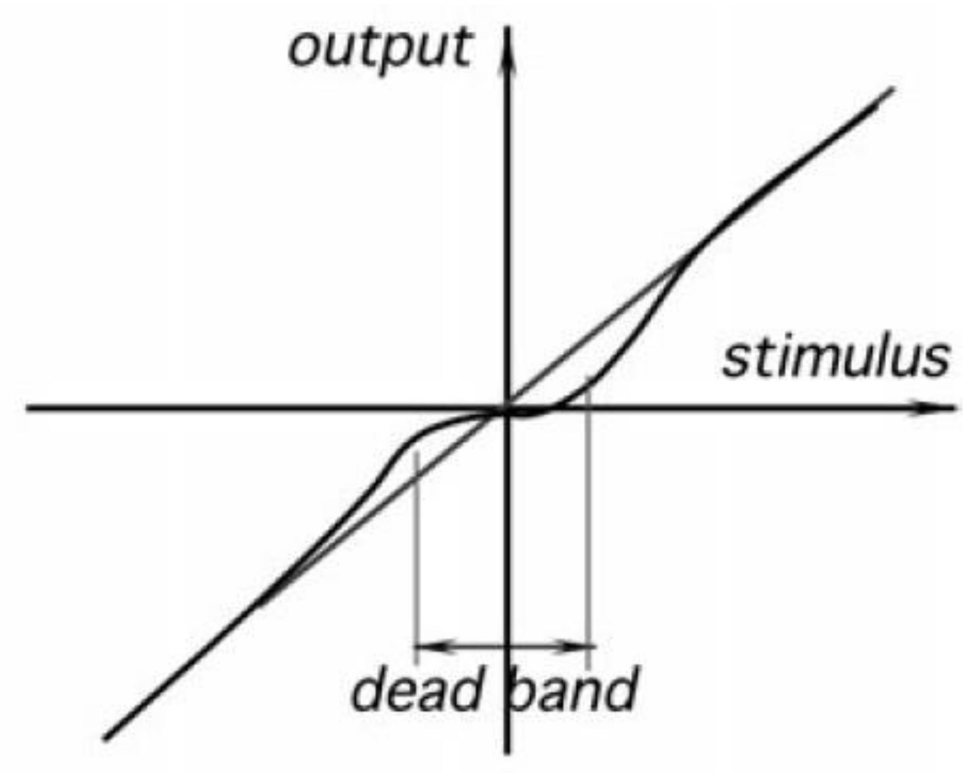
- The inability of a sensor to represent the same value under presumably identical conditions. Causes: thermal noise, build up charge,material plasticity, etc.

$$\delta_r = \frac{\Delta}{FS} 100\%$$



# „Dead band”

- Dead band is insensitivity of a sensor in a specific range of the input signals the output may remain near a certain value (often zero) over an entire dead band zone

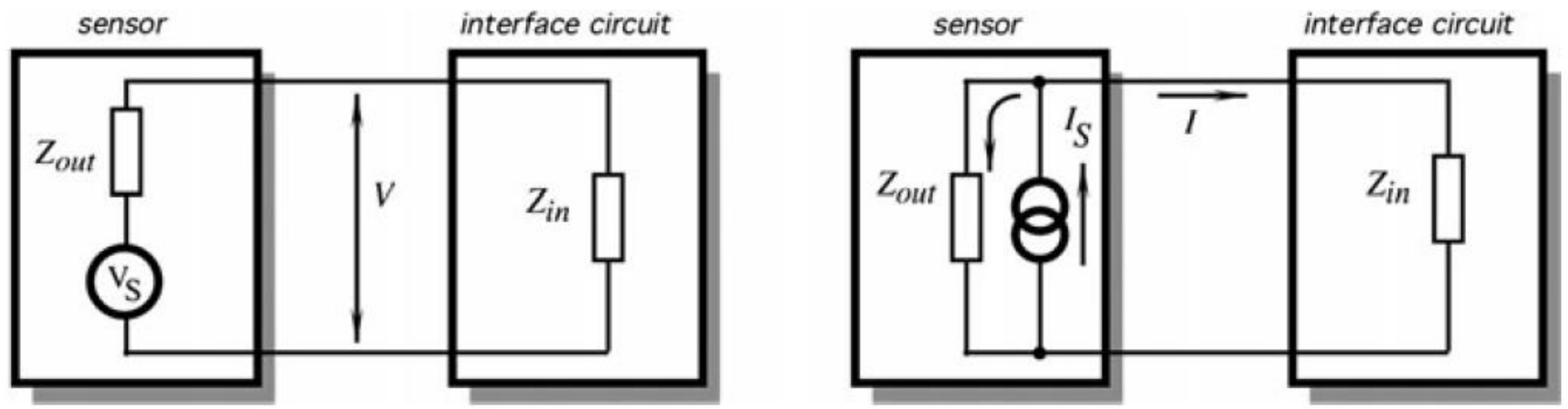


# Resolution

- Resolution describes smallest increments of stimulus, which can be sensed.
- When a stimulus continuously varies over the range, the output signals of some sensors will not be perfectly smooth. The output may change in small steps.
- Any signal that is converted into a digital format is broken into small steps
- It may be specified as percents (%) of full scale (FS).
- The step size may vary over the range, hence, the resolution may be specified as typical, average, or “worst”.
- X-bit resolution, LSB (least significant bit)
- When there are no measurable steps in the output signal, it is said that the sensor has continuous or infinitesimal resolution (vs. infinite resolution)

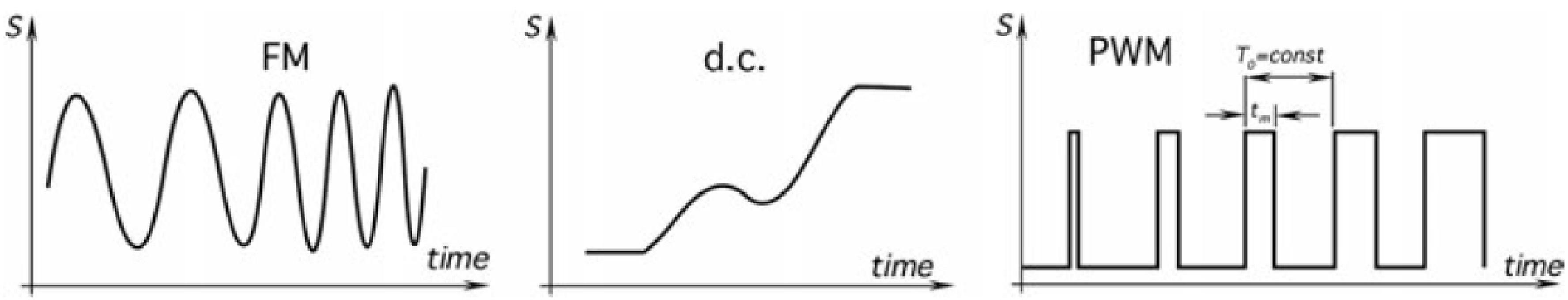
# Output Impedance

- Important to know to better interface a sensor with the electronic circuit.
- To minimize the output signal distortions
- For the voltage connection (A), a sensor is preferable with lower $z_{out}$ while the circuit should have $z_{in}$ as high as practical. In case of a current generating sensor (B) should have an output impedance as high as possible while the circuit's input impedance should be low.



# Output format

- Output format is a set of the output electrical characteristics that is produced by the sensor alone or together with the excitation circuit: voltage, current, charge, frequency, amplitude, phase, polarity, shape of a signal, time delay, and digital code.



# Excitation

- Excitation is the electrical signal needed for operation of an active sensor.
- Excitation is specified as a range of voltage and/or current.
- The frequency and shape of the excitation signal and its stability must also be specified.
- Spurious variations in the excitation may alter the sensor transfer function and cause output errors.
- Various media (eg. air, water) has different value

# Dynamic Characteristics

- When an input stimulus varies with an appreciable rate, a sensor response generally does not follow with perfect fidelity.
- A sensor may be characterized with a time-dependent characteristic
- The sensor responds with a dynamic error
- Warm-up time
- In a control system theory, it is common to describe the input-output relationship through a constant-coefficient linear differential equation.

$$b_1 \frac{\mathrm{d}S(t)}{\mathrm{d}t} + b_0 S(t) = s(t)$$

# Environmental Factors

- Defining the operational environment
- Temperatures of air and surrounding components, pressure, humidity, vibration, ionizing radiation, electromagnetic fields, gravitational forces, etc.
- Short- and long-term stabilities, aging
- Irreverisible, reversible changes
- Self-heating error

# Reliability

- is the ability of a sensor to perform a required function under stated conditions for a stated period.
- It is expressed in statistical terms as a probability that the device will function without failure over a specified time or a number of uses.
- It is rarely specified, MTBF (mean-time-between-failure)
- Emulated tests, compressing years into weeks.
- Highest and lowest ranges (temperature, humidity, and pressure), vibration, extrem conditions, heatshock, simulating sealife
- Autimation and space industries etc.

# Uncertainty

- Nothing is perfect in this world, at least in a sense that we perceive it. All materials are not exactly as we think they are.
- A sensor can be very accurate, but the measurement always has error and uncertainty
- The result of measurement should be considered complete only when accompanied by a quantitative statement of its uncertainty
- Two classes of uncertainty:
  - A) those, which are evaluated by statistical methods (arise from random effects)
  - B) those, which are evaluated by other means. (arise from systematic effects)

# End of Lecture 05.

i.) Sensor characteristics

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