Physical Mechanics Homework 10

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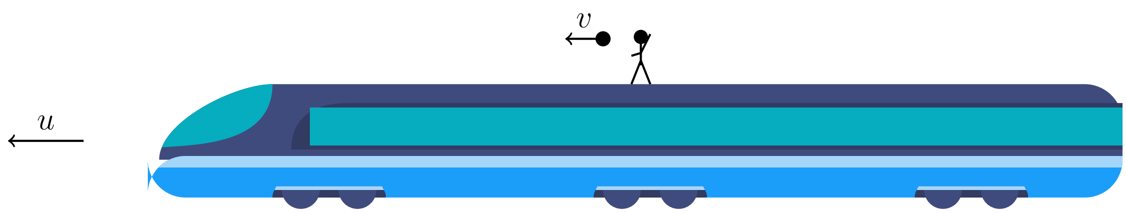
TM 2-41 A train moves along the tracks at a constant speed u. A woman on the train throws a ball of mass m straight ahead at a speed v with respect to herself.

(a) What is the kinetic energy gain of the ball as measured by a person on the train?

(b) What is the kinetic energy gain of the ball as measured by a person standing by the railroad tracks?

(c) How much work is done by the woman throwing the ball?

(d) How much work is done by the train?



$$\boldsymbol{T_{Person} = T_f - t_i = \frac{1}{2}mv^2}$$

$$T_{observer} = T_f - T_i = \frac{1}{2}m(v+u)^2 - \frac{1}{2}mu^2$$

$$\boldsymbol{T_{observer} = \frac{1}{2}mv^2 + mvu}$$

$$\boldsymbol{W_{woman} = \Delta T_{person} = \frac{1}{2}mv^2}$$

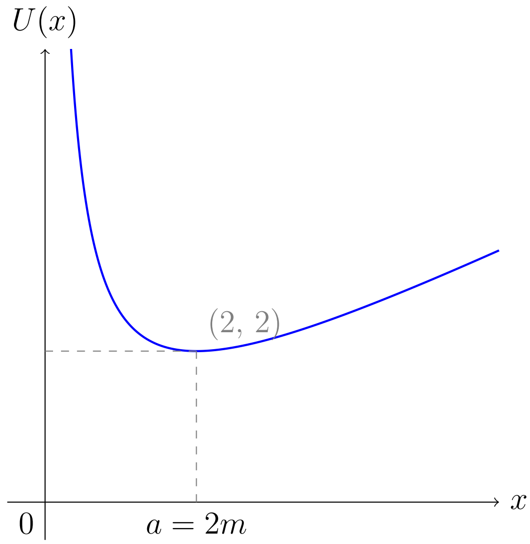
$$\boldsymbol{W_{train} = T_{observer} - T_{woman} = mvu}$$

TM 2-47 Consider a particle moving in the region $x > 0$ under the influence of the potential

$$\boldsymbol{U(x) = U_0(\frac{a}{x} + \frac{x}{a})}$$

Where $U_0 = 1J$ and $a = 2m$

(a) Sketch the potential.
(b) Find the equilibrium point(s).
(c) Determine whether each point found in part (b) represents a stable or an unstable equilibrium.



$$U(x) = U_0(\frac{a}{x} + \frac{x}{a})$$

$$\frac{d}{dx}U(x) = \frac{d}{dx}U_0(-\frac{a}{x^2} + \frac{1}{a}) = U_0(-\frac{a}{x^2} + \frac{1}{a}) = 0$$

Ignoring -2 because $x > 0$

$$\therefore x = \pm 2 \rightarrow \boldsymbol{x = 2}$$

$$\frac{d^2}{dx^2}U(x) \mid_{x=2} = \frac{d^2}{dx^2}U_0(\frac{a}{x} + \frac{x}{a}) \mid_{x=2} = U_0(\frac{2a}{x^3}) \mid_{x=2} = \frac{1}{4}U_0 a > 0$$

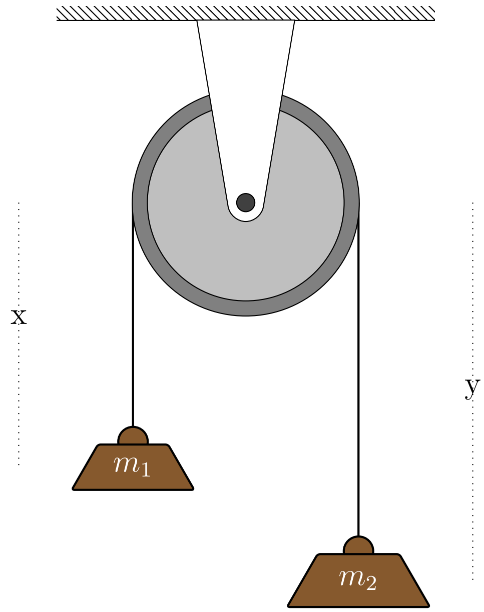
$$U_0, a > 0 \rightarrow \frac{d^2}{dx^2}U(x) > 0$$

$$\therefore \exists \textit{ a stable equilibrium at the point } \boldsymbol{x = 2}$$

TM 7-26 Determine the Hamiltonian and Hamilton's equations of motion for

(a) a simple pendulum of length and bob mass m (hint: do not assume small angles)

(b) a simple Atwood's machine with a single pulley and a massless string



$$\ell = x + y + 2\pi \to x = x, \quad y = \ell - x - 2\pi$$

$$T = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\dot{x}^2$$

$$U = -m_1gx - m_2g(\ell - x - 2\pi)$$

$$\mathscr{L} = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\dot{x}^2 + m_1gx + m_2g(\ell - x - 2\pi)$$

$$P_x = \frac{d\mathscr{L}}{d\dot{x}} = (m_1 + m_2)\dot{x}$$

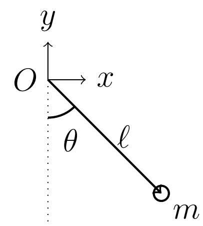
$$H = T + U = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\dot{x}^2 - m_1gx - m_2g(\ell - x - 2\pi)$$

$$H = \frac{P_x^2}{2(m_1 + m_2)} - m_1gx - m_2g(\ell - x - 2\pi)$$

$$\dot{x} = \frac{dH}{dP_x} \to P_x = (m_1 + m_2)\dot{x}$$

$$-\dot{P}_x = \frac{dH}{dx} \to (m_1 + m_2)\ddot{x} = (m_1 - m_2)g$$

$$\boldsymbol{\ddot{x} = g\frac{m_1 + m_2}{m_1 - m_2}}$$



$$x = \ell \sin\theta, \quad y = -\ell\cos\theta$$

$$\dot{x}^2 + \dot{y}^2 = \cancel{\dot{\ell}^2} + \ell^2\dot{\theta}^2$$

$$T = \frac{1}{2}m\ell^2\dot{\theta}^2, \quad U = -mg\ell\cos\theta$$

$$\mathcal{L} = T - U = \frac{1}{2}m\ell^2\dot{\theta}^2 + mg\ell\cos\theta$$

$$P_\theta = \frac{\partial\mathcal{L}}{\partial\dot{\theta}} = m\ell^2\dot{\theta}$$

$$\dot{P}_\theta = \frac{d}{dt}P_\theta = m\ell^2\ddot{\theta}$$

$$H = T + U = \frac{1}{2}m\ell^2\dot{\theta}^2 - mg\ell\cos\theta$$

$$P_\theta = m\ell^2\dot{\theta}$$

$$H = \frac{P_\theta^2}{2m\ell^2} - mg\ell\cos\theta$$

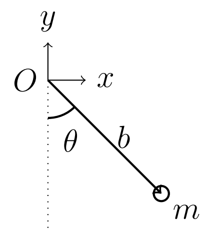
$$\dot{\theta} = \frac{dH}{dP_\theta} \to P_\theta = m\ell^2\dot{\theta}$$

$$-\dot{P}_\theta = \frac{dH}{d\theta} \to -m\ell^2\ddot{\theta} = mg\ell sin(\theta)$$

$$m\ell^2\ddot{\theta} + mg\ell\sin\theta = 0$$

$$\boldsymbol{\ddot{\theta} + \frac{g}{\ell}\sin\theta = 0}$$

TM 7-41 A pendulum of length b and bob mass m is oscillating at small angles when the length of the pendulum string is shortened at a velocity of α (i.e. $\frac{db}{dt} = \alpha$). Find Lagrange's equations of motion. (Hint: since the oscillation is very small, the string is very nearly vertical.)



$$T = \frac{1}{2}m\dot{b}^2 + \frac{1}{2}mb^2\dot{\theta}^2$$

$$U = -mgbcos(\theta)$$

$$\mathscr{L} = \frac{1}{2}m\dot{b}^2 + \frac{1}{2}mb^2(\dot{\theta})^2 + mgbcos(\theta)$$

$$\frac{\partial \mathscr{L}}{\partial b} - \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{b}} = 0$$

$$\boldsymbol{b\dot{\theta}^2 - gcos(\theta)\dot{\alpha} = 0}$$

$$\frac{\partial \mathscr{L}}{\partial \theta} - \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{\theta}} = 0$$

$$\boldsymbol{2\alpha\dot{\theta} + b\ddot{\theta} + \frac{g}{b}sin(\theta) = 0}$$