Physical Mechanics Homework 4

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(1). A simple harmonic oscillator consists of a 100-g mass attached to a spring whose force constant is 10^4 dyne/cm. The mass is displaced 3 cm and released from rest. Calculate

(a) The natural frequency ν_0
(b) The natural period τ_0
(c) The total energy
(d) The maximum speed of the mass

$$\omega = \sqrt{\frac{k}{m}} \rightarrow \omega = 2\pi f \implies \nu_0 \Leftrightarrow f = \frac{\sqrt{k/m}}{2\pi}$$

$$\boldsymbol{f} = \frac{\sqrt{(10^4 dyne/cm)/(100g)}}{2\pi} = \boldsymbol{1.6Hz}$$

$$\boldsymbol{T} = \frac{1}{f} = \boldsymbol{0.63s}$$

$$From\ A^2 = x_0^2 + (\frac{V_0}{\omega_0})^2,\ V_0 = 0 \therefore A = x_0 = 3cm$$

$$\boldsymbol{E} = \frac{1}{2}kA^2 = \frac{1}{2}(10^4 dyne/cm)(3cm)^2 = \boldsymbol{4.5 \times 10^4 ergs}$$

$$E = T + U \implies T_max = E \therefore E = \frac{1}{2}mv^2$$

$$\boldsymbol{v} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(10^4 ergs)}{100g}} = \boldsymbol{30cm/s}$$

(2). The oscillator of Problem 1 is set into motion by giving it an initial velocity of 1 cm/s at its equilibrium position. Calculate

(a) The maximum displacement of the mass

(b) The maximum potential energy

$$\Delta E = 0 \implies T + U = T_0 + U_0$$

Initial position is zero, and the final velocity is zero, therefore

$$U = T_0 \implies \frac{1}{2}kx^2 = \frac{1}{2}mv_0^2$$

$$\boldsymbol{x} = \sqrt{\frac{mv_0^2}{k}} = \sqrt{\frac{(100g) * (1\frac{cm}{s})^2}{10^4 dyne/cm}} = \boldsymbol{0.1\ cm}$$

$$\boldsymbol{U} = \frac{1}{2}kx^2 = \frac{1}{2}(10^4 dyne/cm)(0.1cm)^2 = \boldsymbol{50\ ergs}$$