Assignment 2

Reading Assignment:

1. Chapter 3: Basic Concepts of Probability

Problems:

1. Out of the students in a class, 60% are geniuses, 70% love chocolate, and 40% fall into both categories. Determine the probability that a randomly selected student is neither a genius nor a chocolate lover.

2. A six-sided die is loaded in a way that each even face is twice as likely as each odd face. All even faces are equally likely, as are all odd faces. Construct a probabilistic model for a single roll of this die and find the probability that the outcome is less than 4.

3. A four-sided die is rolled repeatedly, until the first time (if ever) that an even number is obtained. What is the sample space for this experiement?

4. **Bonferroni's inequality.**

   (a) Prove that for any two events A and B, we have

   $$\Pr(A \cap B) \geq \Pr(A) + \Pr(B) - 1.$$

   (b) Generalize to the case of n events $A_1, A_2, \ldots, A_n$, by showing that

   $$\Pr(A_1 \cap A_2 \cap \cdots \cap A_n) \geq \Pr(A_1) + \Pr(A_2) + \cdots + \Pr(A_n) - (n-1).$$

5. We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely.

   (a) Find the probability that doubles are rolled.
   (b) Given that the roll results in a sum of 4 or less, find the conditional probability that doubles are rolled.
   (c) Find the probability that at least one die roll is a 6.
   (d) Given that the two dice land on different numbers, find the conditional probability that at least one die roll is a 6.

6. A die is rolled continually until a 6 appears, at which point the experiment stops. What is the sample space of this experiment?

   Let E_n denote the event that n rolls are necessary to complete the experiment. What points of the sample space are contained in E_n?

   What is $\left(\bigcup_{n=1}^{\infty} E_n\right)^c$?