Assignment 6

Reading Assignment:

1. Chapter 7: Multiple Discrete Random Variables.

Problems:

1. A stock market trader buys 100 shares of stock A and 200 shares of stock B. Let X and Y be the price changes of A and B, respectively, over a certain time period, and assume that the joint PMF of X and Y is uniform over the set of integers x and y satisfying

$$-2 \leq x \leq 4, \quad -1 \leq y - x \leq 1.$$

   (a) Find the marginal PMF's and the means of X and Y.

   (b) Find the mean of the trader's profit.

2. Consider four independent rolls of a 6-sided die. Let X be the number of 1's and let Y be the number of 2's obtained. What is the joint PMF of X and Y?

3. Consider $2m$ persons forming m couples who live together at a given time. Suppose that at some later time, the probability of each person being alive is p, independently of other persons. At that later time, let A be the number of persons that are alive and let S be the number of couples in which both partners are alive. For any survivor number a, find $\mathrm{E}[S|A = a]$.

4. Alice passes through four traffic lights on her way to work, and each light is equally likely to be green or red, independently of the others.

   (a) What is the PMF, the mean, and the variance of the number of red lights that Alice encounters?

   (b) Suppose that each red light delays Alice by exactly two minutes. What is the variance of Alice's commuting time?

5. Each morning, Hungry Harry eats some eggs. On any given morning, the number of eggs he eats is equally likely to be 1, 2, 3, 4, 5, or 6, independently of what he has done in the past. Let X be the number of eggs that Harry eats in 10 days. Find the mean and variance of X.

6. **Computational problem.** Here is a probabilistic method for computing the area of a given subset of S of the unit square. The method uses a sequence of indpendent random selections of points in the unit square $[0, 1] \times [0, 1]$, according to a uniform probability law. If the ith point belongs to the subset S the value of a random variable X_i is set to 1, and otherwise it is set to 0. Let $X_1, X_2, \ldots$ be the sequence of random variables thus defined, and for any n, let

$$S_n = \frac{X_1 + X_2 + \cdots + X_n}{n}.$$

(a) Show that $\mathrm{E}[S_n]$ is equal to the area of the subset S, and that $\mathrm{Var}[S_n]$ diminishes to 0 as n increases.

(b) Show that to calculate S_n, it is sufficient to know S_{n-1} and X_n, so the past values of $X_k, k = 1, \ldots, n-1$, do not need to be remembered. Give a formula.

(c) Write a computer program to generate S_n for $n = 1, 2, \ldots, 10000$, using the computer's random number generator, for the case where the subset S is the circle inscribed within the unit square. How can you use your program to measure experimentally the value of π?

(d) Use a similar computer program to calculate approximately the area of the set of all (x, y) that lie within the unit square and satisfy $0 \leq \cos \pi x + \sin \pi y \leq 1$.