Assignment 5

Reading Assignment:

1. Chapter 6: Meeting Expectations.

Problems:

1. A prize is randomly placed in one of ten boxes, numbered from 1 to 10. You search for the prize by asking yes-no questions. Find the expected number of questions until you are sure about the location of the prize, under each of the following strategies.
   (a) An enumeration strategy: you ask questions of the form "is it in box k?"
   (b) A bisection strategy: you eliminate as close to half of the remaining boxes as possible by asking questions of the form "is it a box numbered less than or equal to k?"

2. **St. Petersburg paradox.** You toss independently a fair coin and you count the number of tosses until the first tail appears. If this number is n, you receive 2^n dollars. What is the expected amount that you will receive? How much would you be willing to pay to play this game?

3. A total of 4 buses carrying 148 students from the same school arrives at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying this randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on her bus.
   (a) Which of $\mathrm{E}[X]$ or $\mathrm{E}[Y]$ do you think is larger? Why?
   (b) Compute $\mathrm{E}[X]$ and $\mathrm{E}[Y]$.

4. An insurance company writes a policy to the effect that an amount of money A must be paid if some event E occurs within a year. If the company estimates that E will occur within a year with probability p, what should it charge the customer in order that its expected profit will be 10 percent of A.

5. A sample of 3 items is selected at random from a box containing 20 items of which 4 are defective. Find the expected number of defective items in the sample.

6. There are two possible causes for a breakdown of a machine. To check the first possibility would cost C_1 dollars, and, if that were the cause of the breakdown, the trouble could be repaired at a cost of R_1 dollars. Similarly, there are costs C_2 and R_2 associated with the second possibility. Let p and $1-p$ denote, respectively, the probabilities that the breakdown is caused by the first and second possibilities. Under what conditions on $p, C_i, R_i, i = 1, 2$, should we check the first possible cause of breakdown and then the second, as opposed to reversing the checking order, so as to minimize the expected cost involved in returning the machine to working order? *Note*: If the first check is negative, we must still check the other possibility.