Name: _____

ECE4131 — Mid-Term #2

(Monday March 23^{rd}, 2020 – 6:00PM to 7:30PM via HuskyCT.)

This is a 90-minute, at home exam (and to be fair to everyone, open book). Remember to check your answers wherever possible, and to express them in simplest form. Questions have weights given. Upload a readable copy of your exam at the end of the 90 min time period (there will be 15 additional minutes for you to upload your answers)

1. *(25 points.)* The 4-point sequence $x[n] = \{10,\ 6,\ \alpha,\ \beta\}$ has DFT $X[k] = \{14,\ 4+6j,\ 18,\ 4-6j\}$. What are α and β?

2. (*25 points.*) The sequence $x[n] = u[n] - u[n-20]$ has z-transform $X(z)$.

   (a) $X(z)$ is sampled uniformly around the unit circle to give the 16-point sequence $Y(k)$ such that

   $$Y(k) \;=\; X(z)|_{z=e^{j2\pi k/16}}$$

   and the inverse DFT (16 points) is taken to get $y[n]$. Give $y[n]$ explicitly.

   (b) $X(z)$ is sampled uniformly around the unit circle to give the 8-point sequence $V(k)$ such that

   $$V(k) \;=\; X(z)|_{z=e^{j2\pi k/8}}$$

   and the inverse DFT (8 points) is taken to get $v[n]$. Give $v[n]$ explicitly.

3. (*30 points.*) Develop a general expression for the output $y(n)$ of a LTI discrete-time system in terms of its input $x(n)$ and the **unit step response** $s(n)$ of the system

4. (*10 points.)* Let $X(w)$ denote the DTFT of a real sequence $x(n)$. Determine the DTFT $Y(w)$ of the sequence $y(n) = x(n) \otimes x(-n)$ in terms of $X(w)$ and show that it is a real valued function of w

5. (*10 points.)* A sequence $x(n)$ has a zero-phase DTFT $X(w)$ as sketched below. Sketch the DTFT of the sequence $x(n)e^{-j\pi n/3}$