



NTR2 – Bezout's Theorema and Fundamental Theorem of Arithmetic

Problem 1. For natural numbers, a, m, n , prove that $\gcd(a^m - 1, a^n - 1) = a^{\gcd(m, n)} - 1$

Problem 2. (1998 IMO, P4) Determine all pairs (x, y) of positive integers such that $x^2y + x + y$ is divisible by $xy^2 + y + 7$.

Problem 3. (1992 IMO, P1) Find all integers a, b, c with $1 < a < b < c$ such that

$$(a - 1)(b - 1)(c - 1)$$

divides $abc - 1$.

Problem 4. (India Practice TST 2017 P3) Let a, b, c, d be pairwise distinct positive integers such that

$$\frac{a}{a+b} + \frac{b}{b+c} + \frac{c}{c+d} + \frac{d}{d+a}$$

is an integer. Prove that $a + b + c + d$ is not a prime number.