Euclidiad Introduction to Number Theory (Short Course)
Final Test

Answer all questions. Each problem is worth 5 marks.

Time allowed: 1 hour 15 minutes   Date: 28.4.2022

Problem 1. A number is called flippy if the digits alternate between two distinct digits. For example, 2020 and 37373 are flippy, but 3883 and 123123 are not. How many five-digit flippy numbers are divisible by 15?

Problem 2. Find the smallest positive integer k such that $2^{69} + k$ is divisible by 127.

Problem 3. The measure of each interior angle of a regular n-sided polygon is given by the formula $\frac{180(n-2)}{n}$. How many regular polygons (different values of n) have angles of integer measure?

Problem 4. There are 10 horses, named Horse 1, Horse 2, …, Horse 10. They are to run around a circular race track. Horse 1 runs one lap in exactly 1 minute; horse 2 runs one lap in exactly 2 minutes; …; horse 10 runs one lap in exactly 10 minutes. At time 0, all the horses are together at the starting point and they start running in the same direction at their own constant speeds. Calculate the time taken for at least 5 of the horses to meet again at the starting point for the first time.

Problem 5. What is the sum of all positive integers less than 100 that have exactly twelve divisors?

Problem 6. Find the units digit of $43^{2021} \times 7^{2022} \times 17^{2023}$.

Problem 7. Given that $\overline{20A17B}$ is a 6-digit number that is divisible by 56, find the value of $A + B$.

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