Euclidiad Olympiad Training LEVEL 1
Day 17 - Arithmetic Progressions, Sum of Squares and Cubes

1 Arithmetic Progressions

- A sequence with constant common difference is called an *arithemetic progression (A.P)*.
- An arithemetic progressions has the form

$$a, a+d, a+2d, a+3d, \dots, a+(n-1)d, \dots$$

- In an A.P,

$$u_n = a + (n-1)d$$
$$n = \frac{l-a}{d} + 1$$
$$S_n = \frac{n}{2}(a+l)$$

Examples Given in Class

Example 1. Compute $1 + 2 + 3 + \cdots + 99 + 100 + 99 + 98 + \cdots + 3 + 2 + 1$.

Example 2. Find the sum of all two-digit integers that are divisble by 3.

Example 3. Find the sum of all odd numbers from 1 to 100 that are not divisble by 11.

Example 4. Evaluate $1^2 - 2^2 + 3^2 - 4^2 + \cdots + 2019^2 - 2020^2$.

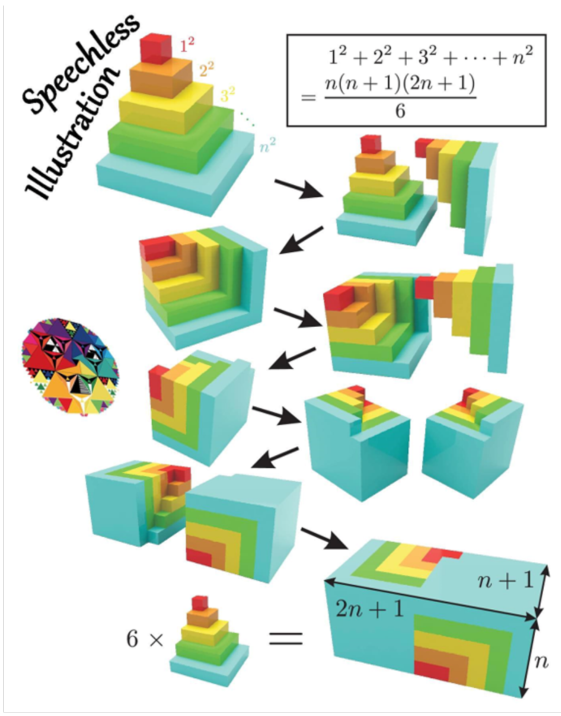
Example 5. Find the 20th term of the number sequence $1, 6, 11, 16, 21, \ldots$. Which term is number 136?

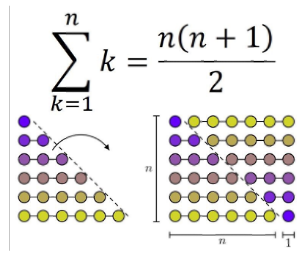
Example 6. There are 30 rows of seats in the North Wing of a stadium. Each row has 2 seats more than the row in front. The last row has 132 seats. How many seats does the first row have? How many seats are there altogether in the North Wing of the stadium?

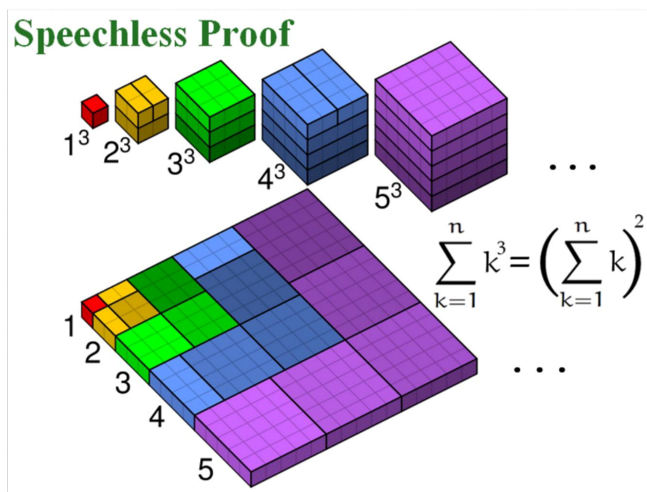
2 Sum of Squares and Cubes

$$
\begin{aligned}
1+2+3+\cdots+n &= \frac{n(n+1)}{2} \\
1^2+2^2+3^2+\cdots+n^2 &= \frac{n(n+1)(2n+1)}{6} \\
1^3+2^3+3^3+\cdots+n^3 &= (1+2+3+\cdots+n)^2
\end{aligned}
$$

3 Visual Geometric Proofs







Example 7. Find the value of $3^2+4^2+5^2+\cdots+9^2+10^2$.

Example 8. Prove that $1\times 2+2\times 3+3\times 4+\cdots+n(n+1) = \frac{n(n+1)(n+2)}{3}$.

Euclidiad Olympiad Training LEVEL 1
Day 17 - Homework

Homework code : **HWA107**
Issued on : 28th June 2021
Due date : 12th July 2021

Submit the solutions to at least 6 of the homework problems before the due date. Problems 1-10 are each worth 5 points. Challenge problems are worth 10 points each.

1. Find the value of $7 + 15 + 23 + \cdots + 767 + 775 + 783$.

2. Find the value of $1 + 3 + 5 + \cdots + 2019 + 2021 + 2019 + \cdots + 5 + 3 + 1$.

3. Twenty school teams took part in Euclidiad Monsoon Table Tennis Tournament. Each team was to play exactly one match with every other team. How many matches were played all together?

4. Find the sum of all three-digit integers that are divisible by 6.

5. Find the sum of all even numbers from 500 to 1000 that are not divisible by 6.

6. Evaluate
$$\frac{1}{2021} + \frac{2}{2021} + \frac{3}{2021} + \cdots + \frac{2019}{2021} + \frac{2020}{2021}.$$

7. Find the 32nd term of the number sequence $3, 7, 11, 15, 19, \ldots$. Which term is number 239?

8. Find the value of $2021^2 - 2020^2 + 2019^2 - 2018^2 + 2017^2 - \cdots + 3^2 - 2^2 + 1^2$.

9. Find the value of $1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + 20 \times 21$.

10. Find the last digit of A if $A = 1 + 4 + 9 + 16 + 25 + \cdots + 529 + 576 + 625$.

Challenge Problems

11. The four angles of a quadrilateral are in A.P. Given that the value of the largest angle is three times the value of the smallest angle, find the values of all four angles.

12. It is given that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Using this formula, find the value of

$$1^2 + 3^2 + 5^2 + 7^2 + \cdots + 19^2.$$

13. Prove that

$$(1 \times 2 \times 3) + (2 \times 3 \times 4) + (3 \times 4 \times 5) + \cdots + (n-1)n(n+1) = \frac{(n-1)n(n+1)(n+2)}{4}.$$

Using this formula, find the value of

$$(1 \times 2 \times 3) + (2 \times 3 \times 4) + (3 \times 4 \times 5) + \cdots + (15 \times 16 \times 17).$$