Euclidiad Number Theory I
Course End Test

Date: November 6, 2023

Instructions

- The exam period is from 7:00 pm to 8:30 pm.
- The full score for this final exam is 50 points.
- To submit the final exam answer sheet, you write down your solutions on paper, take photos, combine them into a single pdf file, and finally upload that pdf at the assigned place in google classroom.

Answer All Questions. Each problem is worth 5 points.

1. What is the smallest counting number that is divisible by all of first ten counting numbers?

2. How many three-digit integers are divisible by both 12, 14 and 21?

3. Let a, b and c be natural numbers whose remainders when divided by 13 are 4, 7, and 9. Find the remainder when their sum is divided by 13.

4. Find the units digit of $6^{11} + 2023^{2023}$.

5. Phwar Sein has three grandchildren, who call her regularly. One calls her every three days, one calls her every four days, and one calls her every five days. All three called her on December 31, 2023. On how many days during the next year did she not receive a phone call from any of her grandchildren?

6. The number $\overline{8173ABC}$ is divisible by 165. What is the average of all such 3 digit numbers $\overline{ABC}$ that satisfy this condition?

7. What is the greatest integer k such that 80! is divisible by 45^k?

8. Find the product of positive divisors of 2400 that are multiples of 6.

9. Let $N = 34 \cdot 34 \cdot 63 \cdot 270$. What is the ratio of the sum of the odd divisors of N to the sum of the even divisors of N?

10. A teacher wrote three numbers on the white board: 4875, 4563 and N, where N is a positive integer. He then asked the class to compute the lowest common multiple of the three numbers. One student misread 4875 as 4275. However, the student still got the answer correct. What is the least possible value of N?