NTR1 – Euclidean and Division Algorithm

Problem 1. Let k be an even number. Is it possible to write 1 as the sum of the reciprocals of k odd integers?

Problem 2. Let a, b, c be integers such that $a^6 + 2b^6 = 4c^6$. Show that $a = b = c = 0$.

Problem 3. A positive integer k greater than 1 is given. Prove that there exist a prime p and a strictly increasing sequence of positive integers $a_1, a_2, \ldots, a_n, \ldots$ such that the terms of the sequence

$$p + ka_1, p + ka_2, \ldots, p + ka_n, \ldots$$

are all primes.

Problem 4. Find all pairs (a, b) of positive integers such that $a^{2017} + b$ is a multiple of ab.

Problem 5. Let $k \geq 1$ be odd. Prove that for any positive integer n, $1^k + 2^k + \cdots + n^k$ is not divisible by $n + 2$.

Problem 6. Show that for all prime numbers p,

$$Q(p) = \prod_{k=1}^{p-1} k^{2k-p-1}$$

is an integer.