Euclidiad Olympiad Training LEVEL 1
Day 23 - Divisibility

We say that a number a is *divisible by* a number b, if dividing a by b gives a whole number. In the following, we will use $\overline{abcde}$ to denote a five digit number with unit digit e, ten's digit d, and so on. This is to avoid confusion between $abcde$ which would be the product $a \times b \times c \times d \times e$.

1 Divisibility Rules of 4, 8, 25 and 125

- If the last 2 digits of a number are divisible by 4, then that number is divisible by 4.
- If the last 2 digits of a number are divisible by 25, then that number is divisible by 25.
- If the last 3 digits of a number are divisible by 8, then that number is divisible by 8.
- If the last 3 digits of a number are divisible by 125, then that number is divisible by 125.

2 Divisibility Rules of 3 and 9

- If the sum of the digits of a number are divisible by 3, then that number is divisible by 3.
- If the sum of the digits of a number are divisible by 9, then that number is divisible by 9.

3 Divisibility Rule of 11

- If the difference of alternative digits of a number is divisible by 11, then that number is divisible by 11.

4 Divisibility Rules of 7 and 13

- If the difference between the number formed by the last three digits and the number formed by the preceding digits is divisible by 7, then that number is divisible by 7.
- If the difference between the number formed by the last three digits and the number formed by the preceding digits is divisible by 13, then that number is divisible by 13.

Examples Given in Class

Example 1. Show that a five-digit number $\overline{abcde}$ is divisible by 4 if $4 \mid \overline{de}$.

Example 2. Show that a five-digit number $\overline{abcde}$ is divisible by 3 if $3 \mid (a+b+c+d+e)$.

Example 3. Show that a five-digit number $\overline{abcde}$ is divisible by 11 if $11 \mid (a+c+e-b-d)$.

Example 4. Show that a five-digit number $\overline{abcde}$ is divisible by 13 if $13 \mid (\overline{cde} - \overline{ab})$.

Example 5. Let $x = 10a + b$ be a two-digit number. If $y = 5b + a$ is divisible by 7, show that x is also divisible by 7.

Example 6. If $\overline{4567m}$ is divisible by 11, find the possible values of m.

Example 7. How many numbers from 1 to 2020 are divisible by either 2, 5 or 7?

Example 8. A 6-digit number begins with the digit 7. The number is divisible by 9. All six digits of the number are different. Find the smallest possible value of this number.

Example 9. Find the digits represented by a and b in $\overline{a7889b}$ and the possible values of the number so that it is divisible by 15.

Example 10. Find the possible values of a in $\overline{333333a888888}$ so that the number is divisible by 7.

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Day 23 - Homework Problems

Homework code : **HWN103**
Issued on : 20th July 2021
Due date : 3rd August 2021

Submit the solutions to at least 6 of the homework problems before the due date. Problems 1-10 are each worth 5 points. Challenge problems are worth 10 points each.

1. Show that a five-digit number $\overline{abcde}$ is divisible by 8 if $8 \mid \overline{cde}$.

2. Show that a five-digit number $\overline{abcde}$ is divisible by 9 if $9 \mid (a+b+c+d+e)$.

3. Show that a five-digit number $\overline{abcde}$ is divisible by 7 if $7 \mid (\overline{cde} - \overline{ab})$.

4. a and b are both integers and $n = 10a + b$, show that $23 \mid n$ if $23 \mid (a + 7b)$.

5. a and b are both integers, show that $13 \mid (7a + 3b)$ if $13 \mid (3a + 5b)$.

6. Find the value of $a + b + c$ if $\overline{173a}$ is divisible by 9, $\overline{173b}$ is divisible by 11 and $\overline{173c}$ is divisible by 6.

7. A palindromic number is a number that reads the same forward and backwards. For example, 4567654 is a palindromic number. Check whether 9 and 11 divide 4567654.

8. A 6-digit number begins with the digit 7. The number is divisible by 9 and all six digits of the number are different. Find the smallest possible value of this number.

9. How many numbers from 1 to 2020 are divisible by either 3, 5 or 11?

10. A 6-digit number, $\overline{15abcd}$ is divisible by 36. Find the values of a, b and c, so that the number has the least value of the quotient when it is divided by 36.

Challenge Problems

11. The integer n is the smallest possible multiple of 15 such that every digit of n is either 0 or 9. Compute $\frac{n}{15}$.

12. If $\overline{7ab}$ is written 2020 times in this manner, $\underbrace{\overline{7ab7ab\ldots7ab}}_{2020\ \overline{7ab}\text{'s}}$, it will become a multiple of 143. Find the values of a and b.
(**Hint:** Show that $\overline{7ab7ab}$ is divisible by 143.)

13. The following number has 41 digits and is divisible by 7:

$$\overline{\underbrace{333\ldots333}_{20\ 3\text{'s}}a\underbrace{555\ldots555}_{20\ 5\text{'s}}}.$$

Find the value of a.