



NTL7 – Diophantine Equations (1)

Parity (Even or Odd)

Example 1. (2005 JBMO P1) Find all positive integers x, y satisfying the equation

$$9(x^2 + y^2 + 1) + 2(3xy + 2) = 2005.$$

Factoring Equations

Example 2. (INMO) Determine all non-negative integral pairs (x, y) for which

$$(xy - 7)^2 = x^2 + y^2$$

Bounding (Using Inequalities)

Example 3. (Russia) Find all natural pairs of integers (x, y) such that $x^3 - y^3 = xy + 61$.

Example 4. (2010 IMO Shortlist 1) Find the least positive integer n for which there exists a set $\{s_1, s_2, \dots, s_n\}$ consisting of n distinct positive integers such that

$$\left(1 - \frac{1}{s_1}\right)\left(1 - \frac{1}{s_2}\right) \dots \left(1 - \frac{1}{s_n}\right) = \frac{51}{2010}.$$