Euclidiad Introduction to Number Theory (Short Course)
Lecture 3 – Divisibility

“Divisible By” means “when you divide one number by another, the result is a whole number”.
“$a|b$” = “a divides b” = “b is divisible by a”

Divisibility Tests

A number is divisible by

4	if the last 2 digits are divisible by 4.
25	if the last 2 digits are divisible by 25.
8	if the last 3 digits are divisible by 8.
3	if the sum of the digits is divisible by 3.
9	if the sum of the digits is divisible by 9.
11	if the difference of alternative digits of a number is divisible by 11.

Example 1. Given that $\overline{A2018B}$ is a 6-digit number which is divisible by 72, find the value of A.

Example 2. Find the sum of all four-digit natural numbers of the form $\overline{4AB8}$ which are divisible by $2, 3, 4, 6, 8$, and 9.

Example 3. Maung Suu Sann was playing with numbers. He chose a 5-digit positive integer, calculated the sum of its digits, and subtracted the sum from the original 5-digit integer. Finally, he deleted one of the digits of the resulting number. If his final result was 2022, then what is the digit that he deleted?

Example 4. The integer n is the smallest positive multiple of 15 such that every digit of n is either 0 or 8. Compute $\frac{n}{15}$.

Example 5. Find all positive integers n such that
(i) $2n - 1| - 6n + 5$.
(ii) $3n - 1|9n + 5$.

Example 6. Let x and y be positive integers such that

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{7}.$$

Find the sum of all possible values of x.

Example 7. Find all integers $x \neq 3$ such that $x-3|x^3-3$.

Lecture 3 - Homework Problems

Homework code : HW3
Issued on : 6^{th} March 2022
Due date : 12^{th} April 2022

Submit the solutions to at least 4 of the homework problems before the due date. Problems 1 to 7 are each worth 5 points. Challenge problem is 10 points worth.

Problem 1. A store sold 72 decks of cards for $\$\overline{a67.9b}$ and each deck costs a whole number of cents. Find $a + b$.

Problem 2. Find the ordered pair(s) of digits (A, B) that make $\overline{67A7B}$ a multiple of 225.

Problem 3. For all integer values of $n \geq 2$, k will divide $n^3 - n$. What is the greatest possible integer value of k?

(**Note that** there are two things you need to validate in this problem. First, you must show why your value of k divides all values of $n^3 - n$. Next, you still have to show, or explain, why any value larger than k doesn't satisfy the condition.)

Problem 4. Find all integers x such that $\frac{x^3-3x+2}{2x+1}$ is an integer.

Problem 5. 11 girls and n boys pick up mushrooms. All the children pick up $n^2 + 9n - 2$ mushrooms in total, and every child picks up the same number of mushrooms. Are girls more than boys or boys more than girls among these children? Give reasons for your answer.

Problem 6. Let m and n be positive integers such that

$$8m + 9n = mn + 6.$$

Find the greatest and smallest possible values of m.

Problem 7. Given that $5|n + 2$, which of the following are divisible by 5:

$$n^2 - 4, n^2 + 8n + 7, n^4 - 1, n^2 - 2n?$$

Challenge Problem

Problem 8. Let S be the set of positive integers N with the property that the last four digits of N are 2020, and when the last four digits are removed, the result is a divisor of N. For example, 42020 is in S because 4 is a divisor of 42020. Find the sum of all the digits of all the numbers in S. For example, the number 42020 contributes $4 + 2 + 0 + 2 + 0 = 8$ to this total.