NTR5 – p-adic Valuation

Problem 1. Prove that for all positive integers $n, n!$ divides

$$\prod_{k=0}^{n-1}(2^n - 2^k).$$

Problem 2. (2019 IMO P4) Find all pairs (k, n) of positive integers such that

$$k! = (2^n - 1)(2^n - 2)(2^n - 4) \dots (2^n - 2^{n-1}).$$

Problem 3. (China 2009) Find all pairs of primes p, q such that

$$pq \mid 5^p + 5^q.$$