Euclidiad Olympiad Training LEVEL 1
Day 13 - Notes

Linear equations are equations of first order (i.e. every term in the equation has degree one or less.) These equations define lines in the coordinate plane. In general, a system of two equations containing two variables can be expressed in the form

$$a_1x + b_1y = c_1,$$
$$a_2x + b_2y = c_2.$$

To solve such a system of equations, we can use *substitution* and *elimination.*

- When $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the system has unique solution

$$x = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1}, \quad y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}.$$

- When $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the two equations in the system are identical, so it has infinitely many solutions.
- When $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the two equations in the system are inconsistent, so it has no solution.

Examples Given in Class

Example 1. Solve the following systems of equations.

1. $$\begin{aligned} x + 3y &= 4, \\ -2x + 5y &= -30. \end{aligned}$$

2. $$\begin{aligned} 3r + \frac{s}{2} &= \frac{33}{2}, \\ -\frac{5r}{2} - 2s &= -\frac{37}{2}. \end{aligned}$$

3. $$\begin{aligned} \frac{3}{x} - \frac{2}{y} &= -\frac{7}{2}, \\ \frac{6}{x} + \frac{4}{y} &= 9. \end{aligned}$$

Example 2. Find r and s if $\sqrt[3]{r} + 9\sqrt{s} = 21$ and $10\sqrt[3]{r} - \sqrt{s} = 28$.

Example 3. Solve the following systems of equations.

1. $$\begin{aligned} \frac{x-y}{5}-\frac{x+y}{4}&=\frac{1}{2}, \\ 2(x-y)-3(x+y)+1&=0. \end{aligned}$$

2. $$\begin{aligned} 5.4x+4.6y&=104, \\ 4.6x+5.4y&=96. \end{aligned}$$

3. $$\begin{aligned} x+2(5x+y)&=16, \\ 5x+y&=7. \end{aligned}$$

4. $$\begin{aligned} \frac{x}{2}=\frac{y}{3}&=\frac{z}{5}, \\ x+3y+6z&=15. \end{aligned}$$

5. $$\begin{aligned} x+y&=5, \\ y+z&=6, \\ z+x&=7. \end{aligned}$$

6. $$\begin{aligned} x+2y&=5, \\ y+2z&=8, \\ z+2u&=11, \\ u+2x&=6. \end{aligned}$$

7. $$\begin{aligned} \frac{3x-4y}{xy}&=-8, \\ \frac{2x+7y}{xy}&=43. \end{aligned}$$

Example 4. Given that x, y and z satisfy the system of equations

$$\begin{aligned} 2000(x-y)+2001(y-z)+2002(z-x)&=0, \\ 2000^2(x-y)+2001^2(y-z)+2002^2(z-x)&=2001, \end{aligned}$$

find the value of $z-y$.

Euclidiad Olympiad Training LEVEL 1
Day 13 - Homework

Homework code : **HWA105**
Issued on : 14th June 2021
Due date : 28th June 2021

Submit the solutions to at least 6 of the homework problems before the due date. Problems 1-10 are each worth 5 points. Challenge problems are worth 10 points each.

1. Solve the following systems of equations.

   (a) $3x + 4y = 5,$
   $4x - 2y = 14.$

   (b) $\frac{6}{x} + \frac{7}{y} = 4,$
   $\frac{2}{x} - \frac{5}{y} = 16.$

2. If $3a + 7b + c = 103$, $4a + 10b + c = 143$, then find the value of $a + b + c$.

3. Find all possible values of a and b such that the sum of their square roots is 37 and the square root of a is 10 more than twice the square root of b.

4. If $3x + 2y = 12$, $3x - y = 3$, then find the value of a in $4x - 8y + 2a = 0$.

5. Solve the system of equations

$$\begin{aligned} 2r + s &= 4, \\ 2s + t &= 7, \\ 2t + u &= 10, \\ 2u + r &= 9. \end{aligned}$$

6. It is given that $2r - 4s + 2t = 0$ and $r - 3s + 4t = 0$. Find the value of $r : s : t$.

7. My father's age 5 years ago plus twice my age now gives 65, while my age 5 years ago plus three times my father's age now gives 130. What is my father's age?

8. You have almost found the treasure. Starting from this spot, you must walk a certain number of steps north and east. Three times the sum of the number of northerly steps and the number of easterly steps is four more than four times the number of northerly steps. You also know that when you multiply by five, the number two less than the number of northerly steps, you get the number that is two more than seven times the number of easterly steps.

   How many steps north and east should you take to get to the treasure?

9. Compute $\frac{x}{y}$ if $x + \frac{1}{y} = 4$ and $y + \frac{1}{x} = \frac{1}{4}$.

10. Solve the system of equations

$$\begin{aligned}
2x + y + z + u + v &= 16, \\
x + 2y + z + u + v &= 17, \\
x + y + 2z + u + v &= 19, \\
x + y + z + 2u + v &= 21, \\
x + y + z + u + 2v &= 23.
\end{aligned}$$

Challenge Problems

1. Given $\frac{ab}{a+b} = 2$, $\frac{ac}{a+c} = 5$, and $\frac{bc}{b+c} = 4$, find the value of $a + b + c$.

2. Given

$$\begin{aligned}
\frac{1}{x} + \frac{2}{y} + \frac{3}{z} &= 0, \\
\frac{1}{x} - \frac{6}{y} - \frac{5}{z} &= 0,
\end{aligned}$$

   Find the value of $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$.

3. Solve the system of equations

$$\begin{aligned}
\frac{1}{x} + \frac{1}{y+z} &= \frac{1}{2}, \\
\frac{1}{y} + \frac{1}{z+x} &= \frac{1}{3}, \\
\frac{1}{z} + \frac{1}{x+y} &= \frac{1}{4}.
\end{aligned}$$