Written homework 4

Math 187: Introduction to Applied Linear Algebra

Due in class: Wednesday, September 25

1. Consider the vectors $x = \begin{bmatrix} -2 \\ 2 \\ -2 \\ 0 \end{bmatrix}$, $y = \begin{bmatrix} -1 \\ 1 \\ -6 \\ 1 \end{bmatrix}$, $z = \begin{bmatrix} -1 \\ 0 \\ -1 \\ 2 \end{bmatrix}$ and $w = \begin{bmatrix} -1 \\ 1 \\ -3 \\ 1 \end{bmatrix}$.

   (a) Calculate the Euclidean norm of x, y, z and w.

   (b) Calculate the Manhattan (taxicab) norm of x, y, z and w.

   (c) Calculate the max norm of x, y, z and w.

   (d) Norms also allow us to calculate the distance between two vectors. Is x, y, or z closer to w when using the Euclidean norm? when using the Manhattan norm? when using the max norm? What does this indicate about comparing vectors? Show all of your calculations.

2. At the beginning of the semester, I collect data from my students about which from a list of characteristics they identify in themselves. This list of 29 items was developed by a mathematician at the University of Chicago as characteristics that could make one "good at math". It includes, for instance, being curious, relentless, careful, a communicator, an abstract thinker, thorough, clever, etc. In a separate analysis, data collected from eight award winning professional mathematicians showed that they were all "good at math" for very different reasons.

   The dataset `characteristics.csv` contains responses from 83 Math 18700 students over the past few years. The columns of the dataset list the characteristics and each row corresponds to a unique response, with an entry of "1" if the respondent listed the characteristic as one they have and a "0" if they did not list it.

   For each of the following, write down the linear algebra computation that will answer the question and use R to carry out that computation.

   (a) Out of 83 students, how many students listed "patient" as a characteristic? How about "fast learner"?

   (b) How many students reported being *both* patient and a fast learner?

(c) Consider respondents R1, R2, and R4 (i.e., rows 1, 2, and 4).

   i. How many characteristics do R1 and R2 share? How about R1 and R4? How about R2 and R4?

   ii. How many characteristics do R1 and R2 disagree on (i.e., one of them has it and the other does not)? How about R1 and R4? How about R2 and R4?

   iii. Which pair of students would have the most characteristics covered between the two of them? (Hint: you can get this from your previous work...)

3. You worked with a dictionary and an associated document earlier in the semester. Let's dig into that a bit more. Consider a table wherein each of the 300 rows represents a book that is set on or near an ocean. We use u_i to represent the i-th row. For example, the vector in row twelve (u_{12}) represents <u>Old Man and the Sea</u> by Ernest Hemingway and the vector in row thirty-nine (u_{39}) represents <u>The Pearl</u> by John Steinbeck. Each column represents a different word. We use v_j to represent the j-th column. For example, the vector in column 3 (v_3) represents the word "sea", and the vector in column 8 (v_8) represents the word "fish". We have a dictionary list that contains 1500 common words. Additionally, we have a weight vector w whose elements are the number of letters in each word in the dictionary. For example, since the third word in the dictionary is "sea", we have $w_3 = 3$. Since the eighth word in the dictionary is "fish", we have $w_8 = 4$.

   Use proper mathematical vector notation and vector operations to write down an expression that computes each of the following.

   (a) the number of times that the word "fish" shows up in <u>Old Man and the Sea</u>

   (b) the average number of times the word "fish" shows up in a book from the books in our table

   (c) the difference between the number of times that the word "fish" shows up in each of the books in our table and the average number of times the word "fish" shows up in these books (this is a "de-meaned" vector seen in Section 3.3)

   (d) the standard deviation of the length of the words in our dictionary

4. What a great skill to learn, to be able to take a mathematical idea and execute it on a computer! As with every translation skill, you learn it well when you can also translate in the other direction. Problem 2 of this assignment asks you to do some of this, but we're going to dig deeper. There are four different translations for you to do here. The first two will ask you to translate from the mathematical vector representation of an idea to R code and the final two ask you to translate from the R code implementation of the idea to the mathematical vector representation of that same idea. As an interesting example, if v and w are vectors in $\mathbb{R}^n$, and we create these vectors in R, the R code `v*w` will multiply the vectors

element-wise, and the product will be a vector in $\mathbb{R}^n$; yet, we don't have a mathematical representation of this type of vector multiplication.

When you translate into R code, you may only use parentheses, addition, subtraction, exponentiation, element-wise multiplication, and these additional functions: $c()$, $sum()$, $rep(1, n)$, and $sqrt()$. Note that $rep(1, n)$ creates a vector of 1's of length n. No additional functions are allowed.

(a) Consider

$$v = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \text{ and } w = \begin{bmatrix} -1 \\ 2 \\ -3 \\ 4 \\ -5 \end{bmatrix}.$$

- Write a sentence that explains what this mathematical expression computes:

$$\sqrt{(v-w)^T(v-w)}.$$

- Write R code that stores the vectors v and w and computes the expression above.

(b) Consider

$$u = \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}, x = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \text{ and } y = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}.$$

- Write a sentence that explains what this mathematical expression computes:

$$(\mathbf{1}/3)^T(u+x+y).$$

- Write R code that stores the vectors u, x, and y and computes the expression above.

(c) Translate the following R code into a single mathematical vector expression, using vector notation (do not simply write in the computed number). Then comment each line of the R code with what it does.

```
v <- c(0,1,2,3)
ones <- rep(1,4)
sum(ones*v)
```

(d) Translate the following R code into a single mathematical vector expression, using vector notation (do not simply write in the computed number). Then comment each line of the R code with what it does.

```
x <- c(1,1,2,3,5,8)
y <- c(-4.1,-2,4,0,-4,2)
sqrt(sum(x*x)+sum(y*y)+2*sum(x*y))
```