Low-Cost Inertial Sensors Modeling Using Allan Variance

A. A. Hussen, I. N. Jleta

***Abstract*—**Micro-electromechanical system (MEMS) accelerometers and gyroscopes are suitable for the inertial navigation system (INS) of many applications due to low price, small dimensions and light weight. The main disadvantage in a comparison with classic sensors is a worse long term stability. The estimation accuracy is mostly affected by the time-dependent growth of inertial sensor errors, especially the stochastic errors. In order to eliminate negative effects of these random errors, they must be accurately modeled. In this paper, the Allan variance technique will be used in modeling the stochastic errors of the inertial sensors. By performing a simple operation on the entire length of data, a characteristic curve is obtained whose inspection provides a systematic characterization of various random errors contained in the inertial-sensor output data.

***Keywords*—**Allan variance, accelerometer, gyroscope, stochastic errors.

I. Introduction

MICROMACHINING and micro-electromechanical system (MEMS) technologies can be used to produce complex structures, devices and systems on the scale of micrometers. Many unique MEMS-specific micromachining processes are being developed, where can be used to convert real-world signals from one form of energy to another (physical signals into electrical signals and vice versa) [4].

Advances in the Micro-Electromechanical Systems (MEMS) technology combined with the miniaturization of electronics, have made possible to introduce light-weight, low-cost and low-power chip based inertial sensors for use in measuring of angular velocity and acceleration [2], as a substitute for more expensive conventional INS sensors.

MEME based Inertial sensors have several applications in low-cost navigation and control systems. Common disadvantages of these sensors are the significant errors which accompany the corresponding measurements. These errors consist of deterministic and stochastic parts. The deterministic part includes constant biases, scale factors, axis nonorthogonality, axis misalignment and so on, which are removed from row measurements by the corresponding calibration techniques. The stochastic part contains random errors which cannot be removed from the measurements and should be modeled as stochastic processes [4].

The requirements for accurate estimation of navigation information require modeling of the sensors, noise components. In order to improve the performance of the inertial sensors, must know more details about the noise components for a better modeling of the stochastic part to improve the navigation solution [1]. Several methods have been devised for stochastic modeling of inertial sensors noise (adaptive Kalman filtering, power spectral density, autocorrelation function). Variance techniques are basically very similar, and primarily differ only in that various signal processing, by way of weighting functions, window functions, etc.

II. Allan Variance

David Allan proposed a simple variance analysis method for the study of oscillator stability that is the Allan variance method, it's representing the root means square (RMS) random drift error as a function of averaging time. It is simple to compute and relatively simple to interpret and understand. The Allan variance method can be used to determine the characteristics of the underlying random processes that give rise to the data noise. This technique can be used to characterize various types of error terms in the inertial-sensor data by performing certain operations on the entire length of data [6].

A characteristic curve is obtained whose inspection provides a systematic characterization of various random errors contained in the inertial-sensor output data. Being a directly measurable quantity, the Allan variance can provide information on the types and magnitude of the various error terms.

III. Methodology

Assume there are N consecutive data points, each having a sample time of t_0. Forming a group of n consecutive data points (with $n<N/2$), each member of the group is a cluster, as shown in Fig. 1.

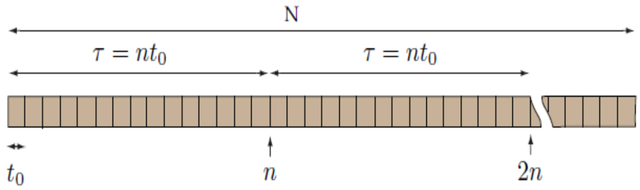

Fig. 1 Scheme of data structure used in Allan variance algorithm

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Associated with each Cluster is a time, T, which is equal to nt_0. If the instantaneous output rate of inertial sensor is $\Omega(t)$, the cluster average is defined as [6]:

$$\overline{\Omega}_k(T) = \frac{1}{T}\int_{t_k}^{t_k+T} \Omega(t)dt \tag{1}$$

where $\overline{\Omega}_k(T)$ represents the cluster average of the output rate for a cluster which starts from the $\boldsymbol{k^{th}}$ data point and contains n data points. The definition of the subsequent cluster average is [6]:

$$\overline{\Omega}_{next}(T) = \frac{1}{T}\int_{t_{k+1}}^{t_{k+1}+T} \Omega(t)dt \tag{2}$$

where $t_{k+1}=t_k + T$

Consequently, the Allan variance of length T is defined as [5]

$$\sigma^2(T) = \frac{1}{2(N-2n)}\sum_{k=1}^{N-2n}\left[\overline{\Omega}_{next}(T) - \overline{\Omega}_k(T)\right]^2 \tag{3}$$

Clearly, for any finite number of data points N, a finite number of Clusters of a fixed length T can be formed. Hence, Equation represents an estimation of the quantity $\sigma^2(T)$ whose quality of estimate depends on the number of independent clusters of a fixed length that can be formed [3].

The Allan variance can also be defined in terms of the output angle or velocity as

$$\theta(t) = \int^t \Omega(t)dt \tag{4}$$

The lower integration limit is not specified, as only angle or velocity differences are employed in the definitions.

Angle or Velocity measurements are made at discrete times given by $t = kt_0$, $k = 1,2,3,\ldots, N$. Accordingly, the notation is simplified by writing $\theta_k = \theta(kt_0)$ [3]

$$\overline{\Omega}_k(T) = \frac{\theta_{k+n} - \theta_k}{T} \tag{5}$$

$$\overline{\Omega}_{next}(T) = \frac{\theta_{k+2n} - \theta_{k+n}}{T} \tag{6}$$

Allan variance is estimated as [6]:

$$\sigma^2(T) = \frac{1}{2T^2(N-2n)}\sum_{k=1}^{N-2n}\left(\theta_{k+2n} - 2\theta_{k+n} + \theta_k\right)^2 \tag{7}$$

The Allan variance is a measure of the stability of sensor output. As such it must be related to the statistical properties of the intrinsic random processes, which affect the sensor performance.

The Allan variance obtained by performing the described operations, is related to the power spectral density PSD of the noise terms in the original data. The relationship between Allan variance and the two-sided PSD, $S_\Omega(f)$ is given by [3]:

$$\sigma^2(T) = 4\int_0^\infty S_\Omega(f)\frac{\sin^4(\pi f T)}{(\pi f T)^2}df \tag{8}$$

where $S_\Omega(f)$ is the power spectral density of the random process $\Omega(T)$.

For non-stationary processes, such as flicker noise, the time average power spectral density should be used.

The power spectral density of any physically meaningful random process can be substituted in the integral, and an expression for the Allan variance $\sigma^2(T)$ as a function of cluster length is identified.

A log-log plot of the square root of the Allan variance, $\sigma(T)$ versus T provides a means of identifying and quantifying various noise terms that exist in the inertial sensor data.

IV. Representation of Noise Terms in Allan Variance

The key attribute of the method is that it allows for a finer, easier characterization and identification of error sources and their contribution to the overall noise statistics The five basic noise terms are angle random walk, rate random walk, bias instability, quantization noise, and drift rate ramp. In addition, the sinusoidal noise and exponentially correlated (Markov) noise can also be identified through the Allan variance method.

In general, any number of random noise components may be present in the data depending on the type of device and the environment in which the data is obtained. If the noise sources are statically independent, then the computed Allan variance is sum of the squares of each error type [6].

A. Quantization Noise

Quantization noise is one of the types of error introduced into an analog signal that results from encoding it in digital form. Quantization noise is caused by the small differences between the actual amplitudes of the points being sampled and the bit resolution of the analog-to-digital converter.

For a gyro output, for example, the angle PSD for such a process, as given in [6], is:

$$S_\theta(f) = TQ_z^2\left(\frac{\sin^2(\pi f T)}{(\pi f T)^2}\right) \approx T_s Q_z^2 \; f < \frac{1}{2T_s} \tag{9}$$

where Q_z is the quantization-noise coefficient and T_S is the sample interval.

The theoretical limit for Q_z is equal to $S/\sqrt{12}$, where S is the gyro scaling coefficient for the tests with fixed and uniform sampling times. The gyro rate PSD, on the other hand, is related to the angle PSD through the following relationship:

$$S_\Omega(2\pi f) = (2\pi f)^2 S_\theta(2\pi f) \tag{10}$$

and is

$$S_\Omega(f) = \frac{4Q_Z^2}{T_S}\sin^2(\pi f T_S) \approx (2\pi f)^2 T_S Q_Z^2, f < \frac{1}{2T_S} \quad (11)$$

Substituting (11) into (8) and performing the integration yields

$$\sigma^2(T) = \frac{3Q_Z^2}{T^2} \quad (12)$$

then

$$\sigma(T) = Q_Z\frac{\sqrt{3}}{T} \quad (13)$$

This indicates that the quantization noise is represented by a slope of -1 in a log-log plot of *σ(T)* versus *T*, as shown in Fig. 2. The magnitude of this noise can be read off the slope line at $T = \sqrt{3}$ [3].

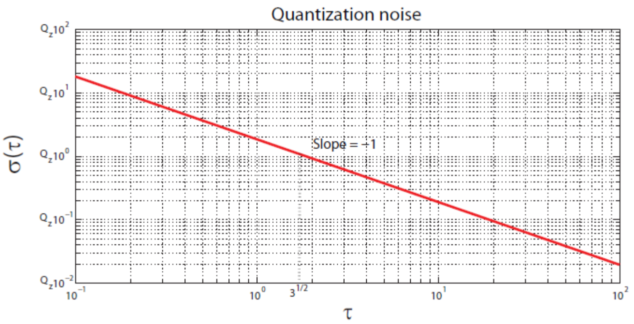

Fig. 2 σ(*T*) Plot for quantization noise

It should be noted that there are other noise terms with different spectral characteristics, such as flicker angle noise and white angle noise, that lead to the same Allan variance *T* dependence [6].

B. Angle (velocity) Random Walk

The high frequency noise term that have correlation time much shorter than the sample time can contribute to the gyro angle (or accelerometer velocity) random walk. These noise terms are all characterized by a white-noise spectrum on the gyro (or accelerometer) output rate. The associated rate noise PSD is represented by [7]:

$$S_\Omega(f) = N^2 \quad (14)$$

where *N* is the angle (velocity) random walk coefficient.

Substitution of (14) in (8) and performing the integration yields:

$$\sigma^2(T) = \frac{N^2}{T} \quad (15)$$

Then

$$\sigma(T) = \frac{N}{\sqrt{T}} \quad (16)$$

Equation (16) indicated a log-log plot of σ(*T*) versus *T* has a slope of -1/2, As shown in Fig. 3. Furthermore, the numerical valued of *N* can be obtained directly by reading the slope at *T*=1.

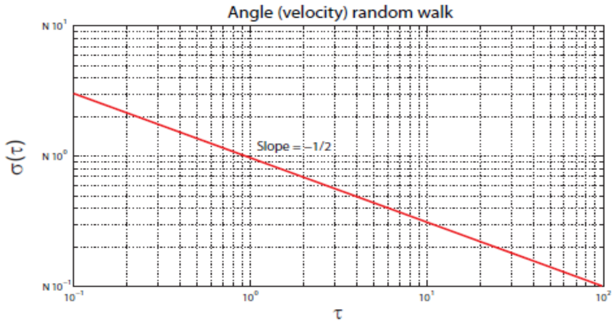

Fig. 3 σ(*T*) Plot for angle (velocity) random walk

C. Bias Instability

Bias Instability is also known as “flicker noise”. This is a low frequency bias fluctuation in the measured rate data. The origin of this noise is the electronics, or other components susceptible to random flickering [5]. Because of its low-frequency nature it shows as the bias fluctuations in the data. The rate PSD associated with this noise is [6]:

$$S_\Omega(f) = \begin{cases} \left(\frac{B^2}{2\pi}\right)\frac{1}{f}\ f \le f_0 \\ 0\ f > f_0 \end{cases} \quad (17)$$

where *B* is the bias instability coefficient and f_0 is the cutoff frequency.

Substitution of (17) in (8) and performing the integration yields:

$$\sigma^2(T) = \frac{2B^2}{\pi}\left[\ln 2 - \frac{\sin^3 x}{2x^2}(\sin x + 4x\cos x) + Ci(2x) - Ci(4x)\right] \quad (18)$$

where *x* is $\pi f_0 T$ and *Ci*( ) is the cosine-integral function [3].

Refer to (18), it is shown

$$\sigma^2(T) \Rightarrow 0\ for\ T \ll \frac{1}{f_0} \quad (19)$$

and

$$\sigma(T) = \sqrt{\frac{2\ln 2}{\pi}}B \cong 0.664B\ for\ T \gg \frac{1}{f_0} \quad (20)$$

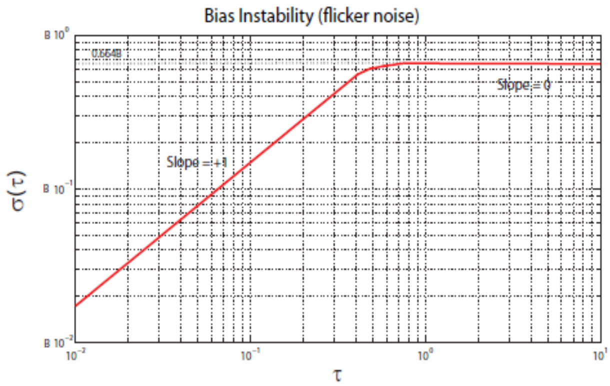

Fig. 4 σ(*T*) Plot for bias instability (for f_0 = 1)

Fig. 4 represents a log-log plot that shows the Allan variance for bias instability. Coefficient B can be determined from the region with zero slope.

D. Rate Random Walk

This is a random process of uncertain origin, possibly a limiting case of an exponentially correlated noise with a very long correlation time. The rate PSD associated with this noise is [3]:

$$S_{\Omega}(f) = \left(\frac{K}{2\pi}\right)^2 \frac{1}{f^2} \quad (21)$$

where K is the rate random walk coefficient.

Substitution of (21) in (8) and performing the integration yields:

$$\sigma^2(T) = \frac{K^2 T}{3} \quad (22)$$

Then

$$\sigma(T) = K\sqrt{\frac{T}{3}} \quad (23)$$

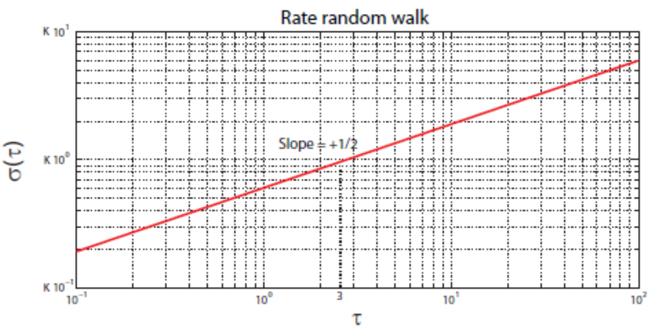

Fig. 5 σ(T) Plot for rate random walk

This indicates that the random walk is represented by a slope of +1/2 on a log-log plot of σ(T) versus T, as shown in Fig. 5. The magnitude of this noise, K, can be read off the slope at T=3 [3].

E. Drift Rate Ramp

This error belongs to deterministic errors. It is slow monotonic change of output over a long time period [3]. It can be described as:

$$\Omega(t) = Rt \quad (24)$$

where R is the drift-rate-ramp coefficient.

By forming and operating on the cluster of data containing an input given by (24), we obtain:

$$\sigma^2(T) = \frac{R^2 T^2}{2} \quad (25)$$

then

$$\sigma(T) = R\frac{T}{\sqrt{2}} \quad (26)$$

This indicates that the rate ramp noise has a slope of +1 in the log-log plot of σ(T) versus T, as shown in Fig. 6. The magnitude of drift rate ramp R can be obtained from the slope line at $T = \sqrt{2}$.

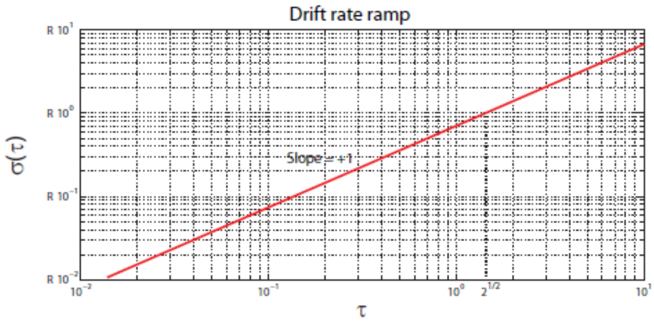

Fig. 6 σ(T) Plot for drift rate ramp

V. Combined Effects of All Processes

In general, any number of random processes discussed above (as well as others) can be present in the data. Thus, a typical Allan variance plot looks like the one shown in Fig. 7. Experience shows that in most cases, different terms appear in different regions of T. This allows easy identification of various random processes that exist in the data. If it can be assumed that the existing random processes are all statistically independent then it can be shown that the Allan variance at any given T is the sum of Allan variances due to the individual random processes at the same T [3].

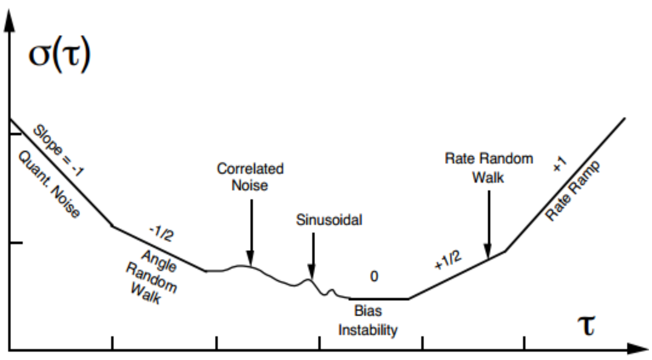

Fig. 7 σ(T) Sample plot of square root of Allan variance analysis results

VI. Measurement Description

The proposed Allan variance method was applied to the real data collected from Sparkfun 9DOF Razor IMU sensor. This IMU sensor is shown in Fig. 8 which includes IvenSense ITG-3200 triple-axis digital output gyroscope, Analog Devices ADXL345 triple-axis accelerometer and HMC5883L triple-axis digital magnetometer. This IMU has ATmega328 processor on board to process the outputs [8]. The IMU was placed on a flat surface stationary for 8 hours without external environmental disturbance to the system, at stable room temperature. Sensor outputs are recorded to file at sampling rate of 50Hz, then the outputs is processed in Matlab, the

Allan Variance method was implemented to determine Angle / Velocity Random Walk, Bias Instability, Quantization noise, Drift Rate Ramp and Rate Random Walk errors.

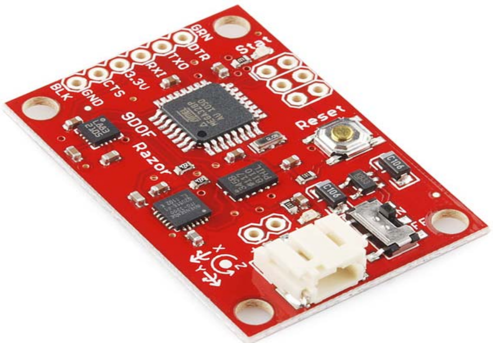

Fig. 8 Sparkfun 9DOF Razor IMU

VII. Results

The data is collected in 8 hours from fixed position IMU at room temperature. By applying the Allan-variance method to the whole data set, a log–log plot of the Allan standard deviation versus the cluster time is shown in Fig. 9 for the Gyroscope data and Fig. 10 for the accelerometer data.

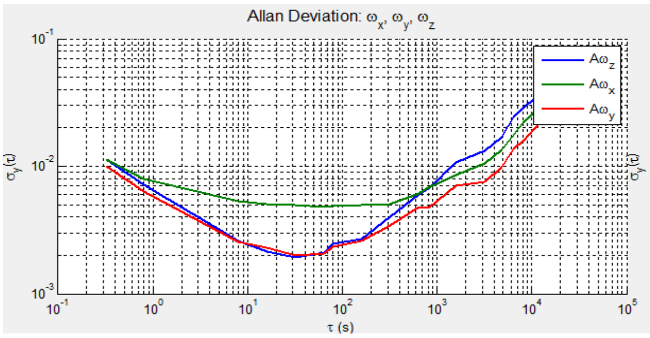

Fig. 9 Gyroscope Allan-variance results

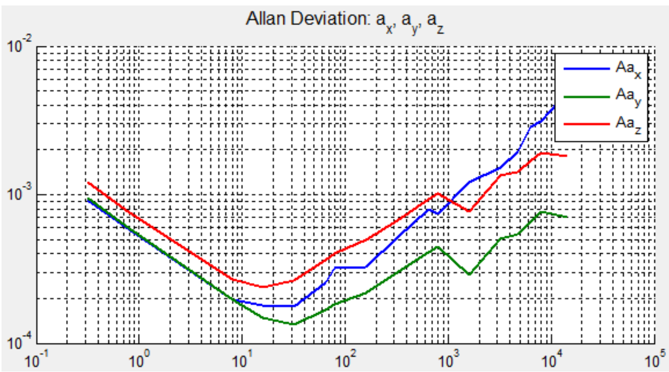

Fig. 10 Accelerometer Allan-variance results

Estimated parameters for gyroscope are in Table I and accelerometer in Table II.

TABLE I
GYROSCOPE PARAMETERS SPECIFICATION

Gyro	Quantization	Angle (velocity) Random Walk	Bias instability	Rate Random Walk	Drift Rate Ramp
	Deg	Deg/(h$^{0.5}$)	Deg/S	Deg/S^2	Deg/h
X	N/A	0.4055	0.0072	4.4027e-04	0.0141
Y	N/A	0.3387	0.0030	3.2020e-04	0.0104
Z	N/A	0.3830	0.0029	4.0484e-04	0.0183

TABLE II
ACCELEROMETER PARAMETERS SPECIFICATION

Acc.	Quantization	Angle (velocity) Random Walk	Bias instability	Rate Random Walk	Drift Rate Ramp
	m/S	m/S/(h$^{0.5}$)	m/S^2	m/S/S^2	m/S/h
X	N/A	0.0311	2.6556e-04	4.9391e-05	0.0021
Y	N/A	0.0319	2.0224e-04	2.8535e-05	9.6527e-04
Z	N/A	0.0409	3.6100e-04	5.1015e-05	0.0015

VIII. Conclusion

The Allan Variance is a simple and efficient method for identifying and characterizing different stochastic processes and their coefficients. Through some simple operations on the sensors output, a characteristic curve of the Allan deviation can be obtained, which can be further used to determine the types and magnitudes of errors residing in the data.

From the experiment results, There isn't a slope of -1 on plot of Allan Variance, Consequently the quantization errors are much less than other errors and can be ignored. This because the sensors output is 16-bit digital data measured using on-chip ADCs, as the number of bits increases the quantization error decreases.

The results clearly indicate that the random walk is the dominant error term in the short cluster time, whereas the bias instability and rate random walk are the dominant errors in the long cluster time.

This paper clearly shows that the Allan variance analysis is a powerful technique to investigate the sensor error behaviors on different time scales; this analysis is an effective method for error modeling and parameter estimation.

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