Topic 5: Noise in Images

Contents:

Introduction

Consider all processes effecting the image **NOT** related to the object as being *Noise*

Range of origins:

1. Discrete nature of radiation
2. Detector sensitivity
3. Electrical noise
4. Film grain
5. Data transmission errors
6. Air turbulence
7. Image Quantisation

In this section we will consider some of the simple noise models, including data transmissin errors, and intrinsic noise resulting from the disctere nature of radiation.

Data Drop-Out Noise

In many data transmission systems, random bits “corrupted” or “lost” on a data channel.

Typically appears as “*snow*” on images.


1 in 100 bits 1 in 20 bits

Corruption very common in satellite images and video systems where bits are set wrong,

Type of corruption is not correlated with the image data, and can be signif cantly reduced by;

- Threshold Average Filter
- Median Filter

With f ltering error rate of about 1.5% can be removed without signif - cant degradation of the image.

These f lters will be discussed in detail in Lectures 9-10.

Fixed Pattern Noise

2-D CCD systems, variable sensitivity of detectors. Typical problem on CCD sensors due to variability in manufacture.

"Noise" can be corrected on a "point by point" basis by calibration of each sensor.

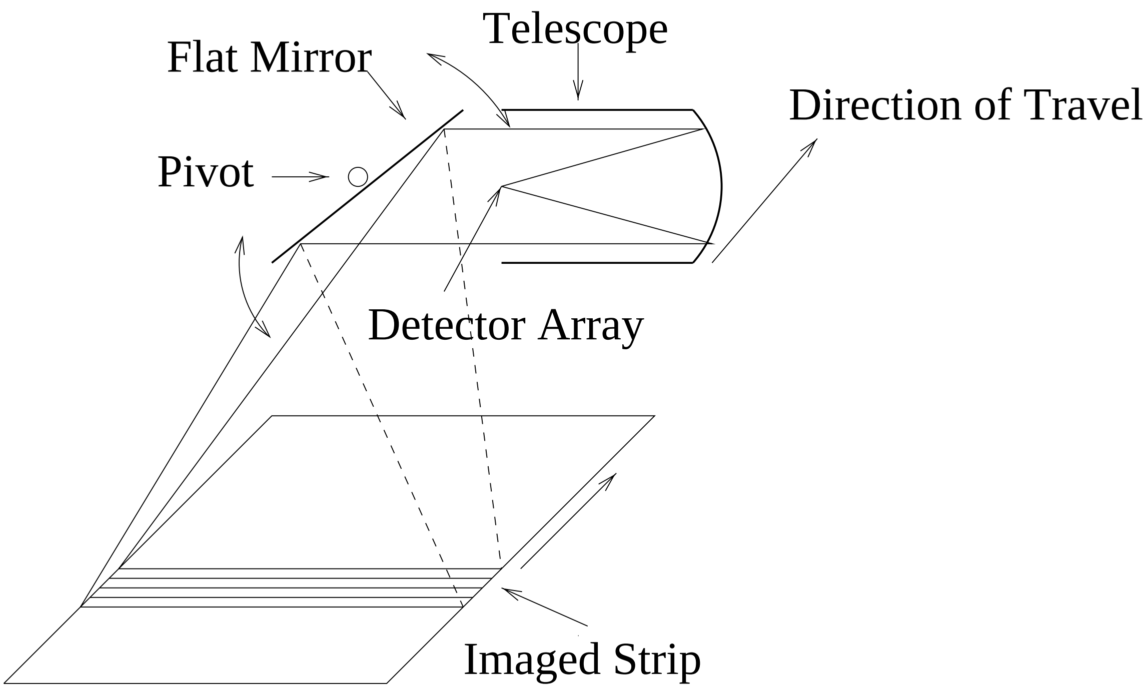
Typical f xed pattern noise from a CCD camera, (image stretch to about 5 grey levels).

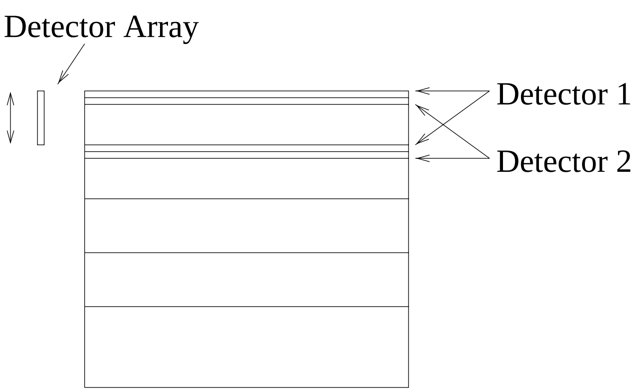
Take measures at a range of light intensities, build-up sensitivity prof le for each pixel point.

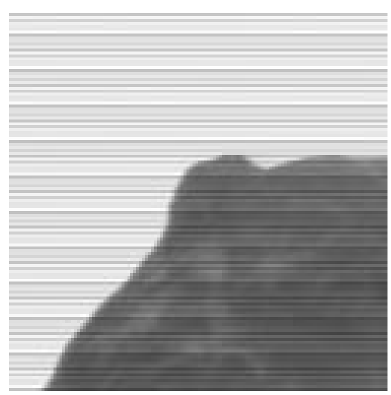
Typically only needed for "critical" applications, such as astronomy, or when sensor is very poor (Infra-red detector arrays).

Destriping

Many scanning systems systems use a one-dimensional sensor.





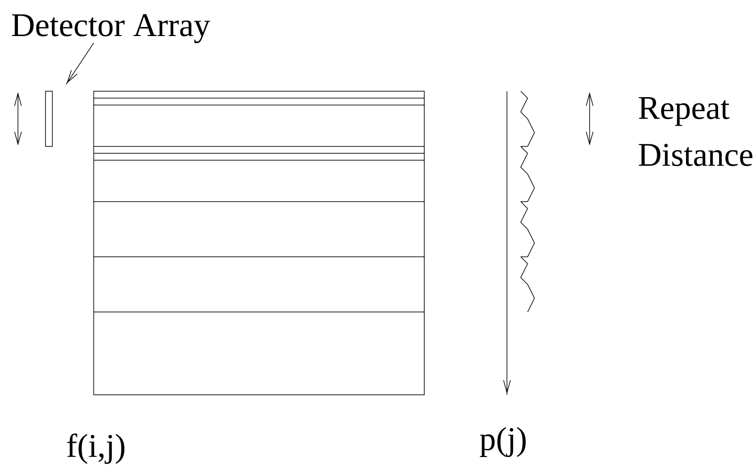


1-D scanning system show "striping" due to varying detector sensitivity.

Form a projection,

$$p(j) = \sum_{i=0}^{N-1} f(i,j)$$

sensor of K pixels long, periodic structure of period K associated with detector sensitivity, so can be calculated and corrected for.



If this is repeated on many images taken with the same sensor, able to obtain a good estimate of sensor variability.

- Additive error (charge leakage from CCD)
- Multiplicative error (gain variability in amplifi r)
- General non-linear errors, need look-up-table for each detector.

Fourier transform f ltering, information causing striping at particular spatial frequency, so can be removed by suitable Fourier f lters.

Basis of Project work for Course

Detector or Shot Noise

Noise process *intrinsic* to the process of measurement, not related to an imaging system.

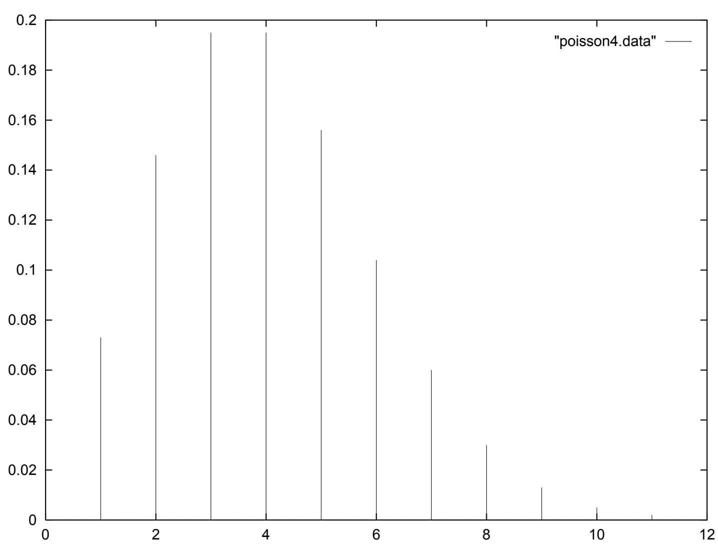
All imaging systems actually count *particles*, (electrons or photons), which are governed by statistical and physical laws.

For a "source" of average brightness $\langle \mu \rangle$ *expected* observed value is

$$\langle f \rangle = \Delta t \langle \mu \rangle$$

However a single observation is random variable from the PDF

$$p(f) = \frac{\langle f \rangle^{f} \exp(-\langle f \rangle)}{f!}$$



PDF of a poisson distribution with $\langle f \rangle = 4$

Simulated Example

Digital simulation with average number of photons per pixel specif ed.

1 Photons/Pixel

4 Photons/Pixel

16 Photons/Pixel

64 Photons/Pixel

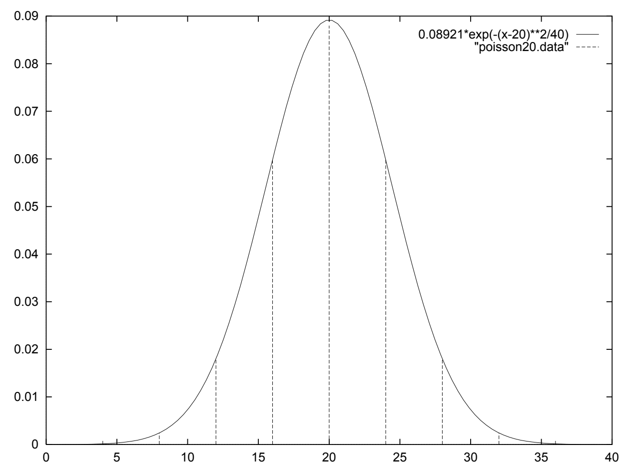
Low numbers of photons, noise dominates, but as number increase the image become usable. Most “normal” images has many hundred photons per pixel.

Astrononical images down to 0.1 photons per pixel possible.

Gaussian Approximation

The Poisson distribution is mathematically diffic lt, (discrete distribution). However for large *expected* values, this may be approximated by a Gaussian of **mean** u and **halfwidth** $\sqrt{2u}$ (or $\sigma^2 = 2u$).

$$p(n) = \frac{u^n \exp(-u)}{n!} \rightarrow \frac{1}{(2\pi u)^{1/2}} \exp\left(\frac{-(n-u)^2}{2u}\right)$$



This has an error of $\approx 1\%$ for $u > 20$.

So for mean of $\langle f \rangle$, approximate $p(f)$ by

$$p(f) = \frac{1}{(2\pi\langle f \rangle)^{1/2}} \exp\left(\frac{-(f - \langle f \rangle)^2}{2\langle f \rangle}\right)$$

So if we Regard the measured value f as

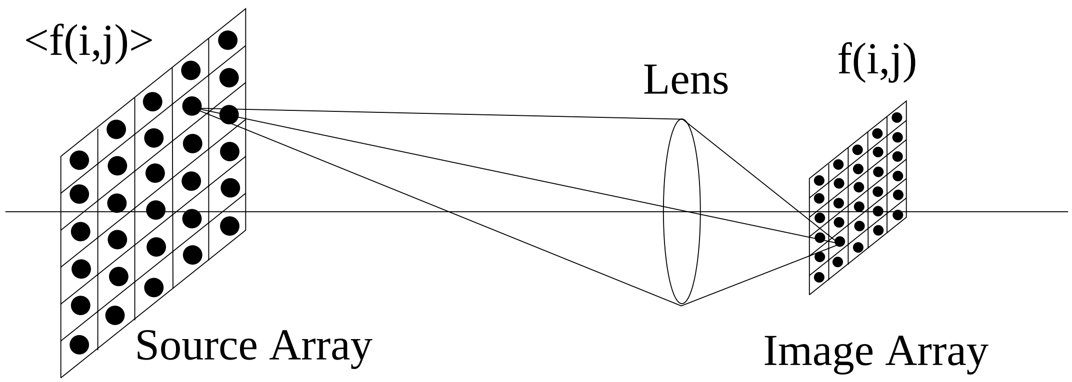
$$f = \langle f \rangle + n$$

where n is the “noise” so we have that the PDF of the “noise” is,

$$p(n) = \frac{1}{(2\pi\langle f \rangle)^{1/2}} \exp\left(\frac{-n^2}{2\langle f \rangle}\right)$$

Two-Dimensional Image

For the 2-D case we have, we assume that we have a two-dimensional array of “sources”.



Source brightness of: $\langle f\rangle(i,j)$ then we measure,

$$f(i,j) = \langle f\rangle(i,j) + n(i,j)$$

where the PDF of the “noise” is given by.

$$p(n(i,j)) = \frac{1}{(2\pi\langle f\rangle(i,j))^{1/2}} \exp\left(\frac{-n(i,j)^2}{2\langle f\rangle(i,j)}\right)$$

This is an “additive” noise model, where the PDF of the noise **depends** on the signal. Known as

Signal Dependant Additive Noise.

This model assumes the imaging system is space invariant and linear.

Still rather diff cult to deal with, since the noise depends on the signal.

Low Contrast Approximation

If we assume the image is *Low Contrast* so

$$\langle f\rangle(i,j) \approx \text{const} = \mu$$

we have that

$$p(n(i,j)) = \frac{1}{(2\pi\mu)^{1/2}} \exp\left(\frac{-n(i,j)^2}{2\mu}\right)$$

which is **independent** of the structure of the image.

Additive noise, with PDF being **Zero Mean Gaussian** with variance μ, the "mean" of the expected image value,

$$\mu = \langle(\langle f\rangle(i,j))\rangle$$

This is the typical assumption for image noise models with are

Signal Independant Additive Gaussian

This assumes imaging system is:

- Linear and Space Invariant
- High brightness (many photons/electrons)
- Low contrast

for this noise model to be valid.

Validity of Additive Noise

In taking *additive* signal independent model, we assume *High brightness* & *Low contrast*.

Assumption valid in many imaging systems,

1. Video images (Many thousands on photons/electrons)
2. Infra-red (Usually very low contrast)
3. Electron microscope (Usually very low contrast)
4. CT and MRI Medical imaging (low contrast)

Not valid in,

1. γ-camera (small count number)
2. Astronomical images (small count number **and/or** very high contrast).
3. Image intensifi r systems (small count number)
4. High magnif cation electron microscope (small count number)

If the additive signal independant noise model is not valid, processing is much more diffic lt.

Most processing used additive signal independant noise model, even if it is not *really* valid.

Properties of Additive Noise

Take the additive noise model as:

$$f(i,j) = s(i,j) + n(i,j)$$

where $s(i,j)$ is “signal” and $n(i,j)$ is “noise”.

The PDF of $n(i,j)$ is zero mean Gaussian so:

$$\begin{aligned} \langle n(i,j) \rangle &= 0 \\ \left\langle |n(i,j)|^2 \right\rangle &= \sigma_n^2 \end{aligned}$$

so that

$$\langle s(i,j) \rangle = \langle f(i,j) \rangle = \mu$$

Noise is independent of “signal” $s(i,j)$, so un-correlated, mathematically that:

$$\langle s(i,j) n(i,j) \rangle = 0$$

so the variance of $f(i,j)$ is:

$$\sigma_f^2 = \left\langle | f(i,j) - \langle f(i,j) \rangle |^2 \right\rangle$$

By substitution, and above properties, this can be expanded to give

$$\sigma_f^2 = \sigma_s^2 + \sigma_n^2$$

where σ_s^2 is the Variance of the signal.

So the addition of noise alters the Variance but not the Mean.

We have assumed that each image point is independant, so the noise is not correlated "with-itself", mathematially this means:

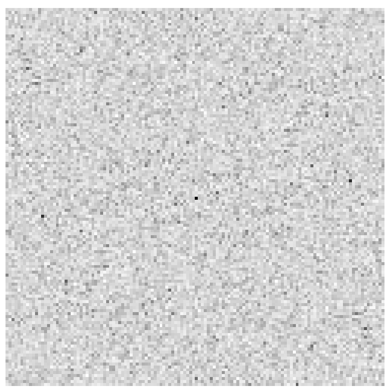
$$n(i,j) \otimes n(i,j) = \delta_{i,j}\langle |n(i,j)|^2 \rangle = \delta_{i,j}\sigma_n^2$$

so the Auto-correlation is a δ-Function.

From the (auto)-correlation theorm, the Fourier Transform of the *Auto-correlation* is the *Power Spectrum*, so:

$$|N(k,l)|^2 = \text{Constant}$$

Known as *White Noise* with equal power at all spatial frequencies.



Power Spectrum of single "realisation" not actually constant, but equal power over each region of Fourier space.

Parseval's Theorm

We have that the power in real space and Fourier space is the same, so that

$$\langle |N(k,l)|^2 \rangle = \langle |n(i,j)|^2 \rangle = \sigma_n^2$$

Processing of Noisy Images

Unlike “Fixed Pattern” and “Striping” this noise in an intrinsic part of the imageing process and **cannot** be “subtracted” from the image.

In Fourier space we have that

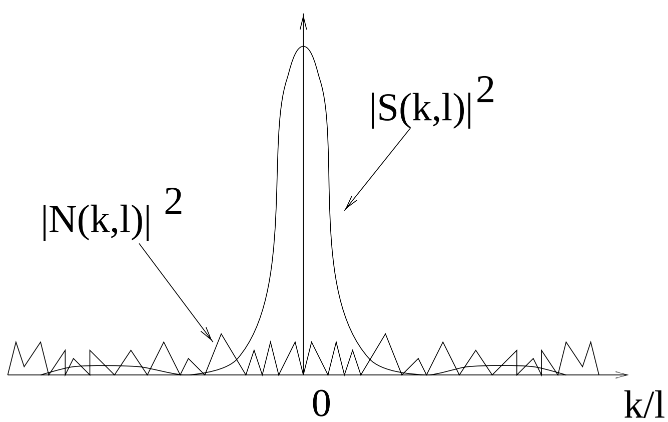
$$F(k,l) = S(k,l) + N(k,l)$$

we know that

$$\begin{aligned} S(k,l) &= \text{Sharply peaked about low spatial frequencies} \\ N(k,l) &= \text{Constant at all spatial frequencies} \end{aligned}$$

so that when

$$\begin{array}{lcll} |S(k,l)| & \gg & |N(k,l)| & \text{Little effect} \\ |S(k,l)| & \approx & |N(k,l)| & \text{Signal corrupted} \end{array}$$

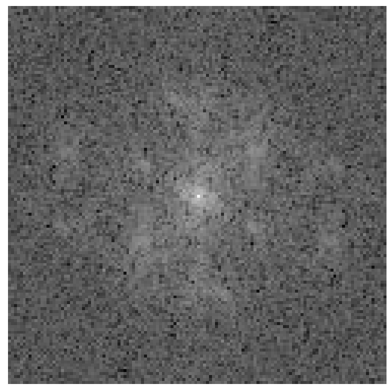


Noise has greatest effect at high spatial frequencies where $S(k,l)$ is small. So high spatial frequencies corrupted by the noise.

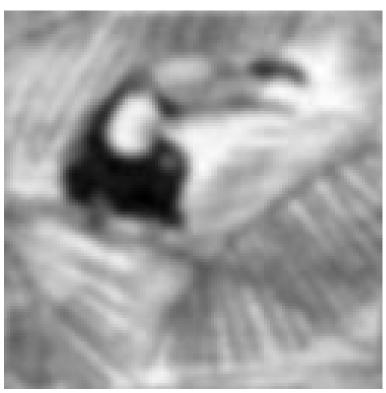
Reduce effect of “noise” by Low-Pass filtering

- Fourier low pass
- Real Space averaging
- Median filter

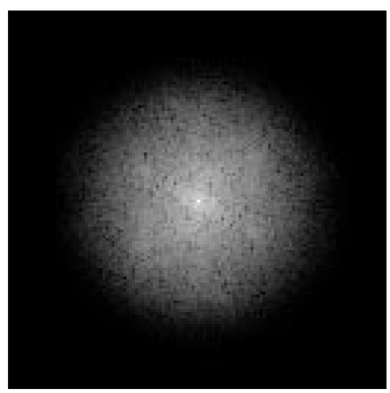
Noisy Image



Fourier Transform



Low-pass filtered



Fourier Transform

Deatils in next two lectures.

Signal to Noise Ratio

The use of SNR (Signal to Noise Ratio) is a confusing topic, since

- There is a range (about 10) def nitions.
- Can a single number really classify "how good an image is"
- In practice image quality is very strongly image dependant.

For signal independent additive noise,

$$f(i,j) = s(i,j) + n(i,j)$$

Signal & Noise variances are:

$$\begin{aligned}\sigma_s^2 &= \langle |s(i,j) - \langle s(i,j)\rangle|^2\rangle \\ \sigma_n^2 &= \langle |n(i,j)|^2\rangle\end{aligned}$$

Defi e SNR by

$$\mathrm{SNR} = \frac{\sigma_s}{\sigma_n}$$

Noting that the signal and the noise are uncorrelated, we have

$$\sigma_f^2 = \sigma_s^2 + \sigma_n^2$$

so we can write the SNR as:

$$\mathrm{SNR} = \sqrt{\frac{\sigma_f^2}{\sigma_n^2} - 1}$$

Calculation of SNR

To calculate SNR, need 2 of σ_s, σ_f, or σ_n.

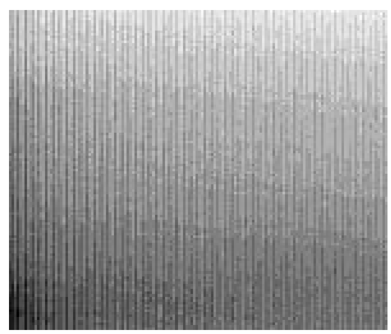
Single Image
From single image can **only** fi d σ_f.

If "region" of image with **NO** signal can estimate σ_n from that region.

This method works for an image that contains a large region of "water" or "sky" where these is not signal.

Example:

Whole Image



Piece of Sky

Sky region shows typical CCD array f xed pattern noise.

Calculated vales are $\sigma_f^2 = 5287$ and $\sigma_n^2 = 1.85$, so that

$$\mathrm{SNR} \approx 53.4$$

Multiple Images

Assume that we have **two** realisation of same scene, so:

$$\begin{aligned} f(i,j) &= s(i,j) + n(i,j) \\ g(i,j) &= s(i,j) + m(i,j) \end{aligned}$$

which is equivalent to two image of the same scene take at different times.

The noise in each realsiation have the same PDF, so that

$$\langle n(i,j)\rangle = \langle m(i,j)\rangle = 0$$
$$\langle |n(i,j)|^2\rangle = \langle |m(i,j)|^2\rangle = \sigma_n^2$$

The two noise realisations were measured at *different* times, so they are uncorrelated, so:

$$\langle n(i,j)m(i,j)\rangle = 0$$

In both cases the noise is uncorrelated with the signal, so that

$$\langle s(i,j)n(i,j)\rangle = \langle s(i,j)m(i,j)\rangle = 0$$

If we then define the *Normalised* Correlation between $f(i,j)$ and $g(i,j)$ as:

$$r = \frac{\langle (fg - \langle f\rangle\langle g\rangle)\rangle}{[\langle |f-\langle f\rangle|^2\rangle\langle |g-\langle g\rangle|^2\rangle]^{1/2}}$$

Then be expanding, collecting terms, and cancelling all zero terms, *it-can-be-shown* that

$$r = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_n^2}$$

to we can form the SNR from

$$\mathrm{SNR} = \sqrt{\frac{r}{1-r}}$$

So allowing direct calculation of the SNR independant of the type of the signal.

If there is more than two realiastion available a better estimate for SNR can be found by forming the normalised correlation between pairs of images and averaging.

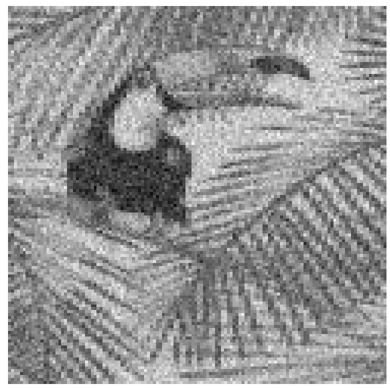
In practice this measure of SNR “looks about right” for most images. For examples:

SNR	$>$	20	Little visible noise
SNR	$\approx$	10	Some noise visibile
SNR	$\approx$	4	Noise clearly visible
SNR	$\approx$	2	Image severly degraded
SNR	$\approx$	1	Is there an image?

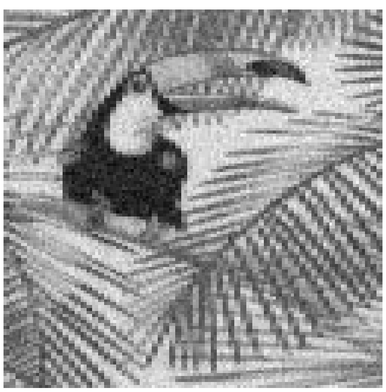
Digital Simulation

Digitally simulated “noisy” images:

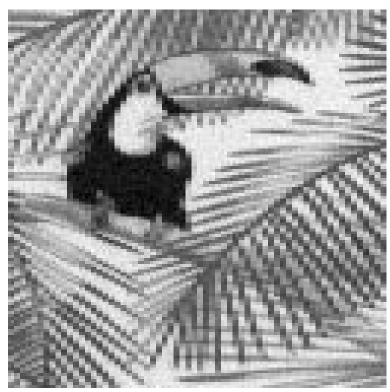
SNR = 1



SNR = 2



SNR = 4



SNR = 8

Images formed by addition of Gaussian random noise.