

## 指数形式傅氏级数推导

$$f(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos(n\Omega t + \varphi_n)$$

$$= \frac{A_0}{2} + \sum_{n=1}^{\infty} \frac{A_n}{2} \left[ e^{j(n\Omega t + \varphi_n)} + e^{-j(n\Omega t + \varphi_n)} \right]$$

$$= \frac{A_0}{2} + \frac{1}{2} \sum_{n=1}^{\infty} A_n e^{j\varphi_n} e^{jn\Omega t} + \frac{1}{2} \sum_{n=1}^{\infty} A_n e^{-j\varphi_n} e^{-jn\Omega t}$$

上式中第三项的n用-n代换, $A_{-n}=A_n$ , $\varphi_{-n}=-\varphi_n$ , 则上式写为

$$\frac{A_0}{2} + \frac{1}{2} \sum_{n=1}^{\infty} A_n e^{j\varphi_n} e^{jn\Omega t} + \frac{1}{2} \sum_{n=-1}^{-\infty} A_n e^{j\varphi_n} e^{jn\Omega t}$$

因为
$$A_0 = A_0 e^{j\varphi_0} e^{j0\Omega t}$$
,  $\varphi_0 = 0$ 

所以
$$f(t) = \frac{1}{2} \sum_{n=-\infty}^{\infty} A_n e^{j\varphi_n} e^{jn\Omega t} \qquad a_n = A_n \cos \varphi_n$$

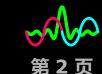
令复数 
$$\frac{1}{2}A_n e^{j\varphi_n} = |F_n| e^{j\varphi_n} = F_n$$
 
$$a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos(n\Omega t) dt$$

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$$F_{n} = \frac{1}{2} A_{n} e^{j\varphi_{n}} = \frac{1}{2} (A_{n} \cos \varphi_{n} + j A_{n} \sin \varphi_{n}) = \frac{1}{2} (a_{n} - j b_{n})$$

$$= \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos(n\Omega t) dt - j \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin(n\Omega t) dt$$

$$= \frac{1}{T} \int_{\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-j n\Omega t} dt$$





$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\Omega t}$$

$$F_n = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\Omega t} dt$$

表明:任意周期信号f(t)可分解为许多不同频率的虚指数信号之和。 $F_0 = A_0/2$ 为直流分量。

