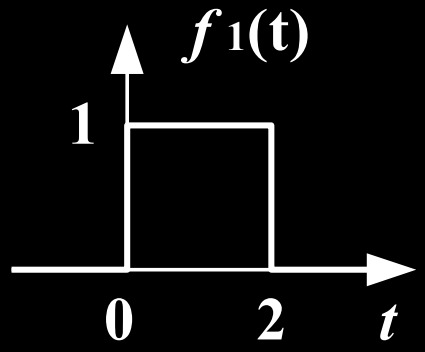
卷积性质例1

例1：$f_1(t)$ 如图，$f_2(t) = e^{-t}\varepsilon(t)$，求 $f_1(t) * f_2(t)$



解：$f_1(t) * f_2(t) = f_1'(t) * f_2^{(-1)}(t)$

$f_1'(t) = \delta(t) - \delta(t-2)$

$$f_2^{(-1)}(t) = \int_{-\infty}^{t} e^{-\tau}\,\varepsilon(\tau)\,d\tau = \left[\int_{0}^{t} e^{-\tau}\,d\tau\right]\varepsilon(t) = -e^{-\tau}\Big|_0^t \cdot \varepsilon(t) = (1-e^{-t})\varepsilon(t)$$

$f_1(t) * f_2(t) = (1- e^{-t})\varepsilon(t) - [1- e^{-(t-2)}]\varepsilon(t-2)$

注意：当 $f_1(t)=1$，$f_2(t) = e^{-t}\varepsilon(t)$，套用 $f_1(t) * f_2(t) = f_1'(t) * f_2^{(-1)}(t) = 0 * f_2^{(-1)}(t) = 0$ 显然是错误的。