

## 0-和0+初始值举例2

**例2：**描述某系统的微分方程为

$$y''(t) + 3y'(t) + 2y(t) = 2f'(t) + f(t)$$

已知 $y(0_-)=2$ ,  $y'(0_-)=0$ ,  $f(t)=\delta'(t)$ , 求 $y(0_+)$ 和 $y'(0_+)$ 。

**解：**将输入 $f(t)=\delta'(t)$ 代入上述微分方程得

$$y''(t) + 3y'(t) + 2y(t) = 2\delta''(t) + \delta'(t) \quad (1)$$

**利用系数匹配法分析：**

令 $y''(t)=a\delta''(t)+b\delta'(t)+C\delta(t)+r_1(t)$ ,  $r_1(t)$ 中不含冲激

$$y'(t)=a\delta'(t)+b\delta(t)+r_2(t), \quad r_2(t)=C\varepsilon(t)+r_1^{(-1)}(t)$$

$$y(t)=a\delta(t)+r_3(t), \quad r_3(t)=b\varepsilon(t)+r_2^{(-1)}(t)$$

将上述关系代入式 (1), 并整理得

$$\begin{aligned}
 &a\delta''(t) + b\delta'(t) + c\delta(t) + r_1(t) \\
 &\quad + 3a\delta'(t) + 3b\delta(t) + 3r_2(t) \\
 &\quad + 2a\delta(t) + 2r_3(t) = 2\delta''(t) + \delta'(t)
 \end{aligned}$$

比较等式两边冲激项系数，有

$$a=2$$

$$b+3a=1$$

$$c+3b+2a=0$$

解得：  $a=2$ ,  $b=-5$ ,  $c=11$ , 故

$$y''(t) = 2\delta''(t) - 5\delta'(t) + 11\delta(t) + r_1(t),$$

$$y'(t) = 2\delta'(t) - 5\delta(t) + r_2(t),$$

$$y(t) = 2\delta(t) + r_3(t),$$

$$y''(t)=2\delta''(t)-5\delta'(t)+11\delta(t)+r_1(t),$$

$$y'(t)=2\delta'(t)-5\delta(t)+r_2(t),$$

$$y(t)=2\delta(t)+r_3(t),$$

对 $y''(t)$ 从 $0_-$ 到 $0_+$ 积分得

$$y'(0_+)-y'(0_-)=11, \quad y'(0_+)=y'(0_-)+11=11$$

对 $y'(t)$ 从 $0_-$ 到 $0_+$ 积分得

$$y(0_+)-y(0_-)=-5, \quad y(0_+)=y(0_-)-5=2-5=-3$$