

# 尺度变换证明

证明:

$$F[f(at)] = \int_{-\infty}^{\infty} f(at) e^{-j\omega t} dt$$

对于  $a > 0$  ,

$$F[f(at)] = \int_{-\infty}^{\infty} f(\tau) e^{-j\omega \frac{\tau}{a}} \frac{1}{a} d\tau = \frac{1}{a} F\left(j\frac{\omega}{a}\right)$$

对于  $a < 0$  ,

$$\begin{aligned} F[f(at)] &= \int_{\infty}^{-\infty} f(\tau) e^{-j\omega \frac{\tau}{a}} \frac{1}{a} d\tau = -\frac{1}{a} \int_{-\infty}^{\infty} f(\tau) e^{-j\omega \frac{\tau}{a}} d\tau \\ &= -\frac{1}{a} F\left(j\frac{\omega}{a}\right) \end{aligned}$$

因此,  $f(at) \longleftrightarrow \frac{1}{|a|} F\left(j\frac{\omega}{a}\right)$