

时域卷积定理的证明

$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau$$

$$F[f_1(t) * f_2(t)] = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau \right] e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} f_1(\tau) \left[\int_{-\infty}^{\infty} f_2(t - \tau) e^{-j\omega t} dt \right] d\tau$$

$$= \int_{-\infty}^{\infty} f_1(\tau) \underbrace{F_2(j\omega) e^{-j\omega \tau}}_{f(t-t_0) \longleftrightarrow e^{-j\omega t_0} F(j\omega)} d\tau$$

$$f_1(t) * f_2(t) \longleftrightarrow F_1(j\omega) F_2(j\omega)$$