Imperial College London

AMCG

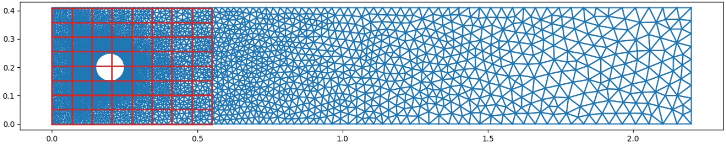
Reduced-Order Modelling

Subdomain approach

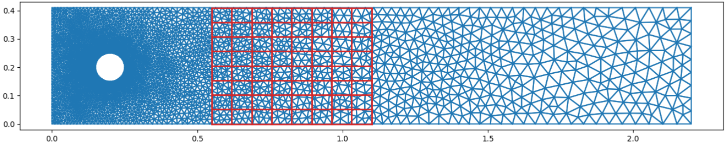
Subdomain approach to model long pipes with NIROM:

- split the domain into a number of subdomains
- interpolate solution onto a grid in each subdomain
- compress the solutions in each subdomain
- train a GAN with information from each subdomain and its two neighbouring subdomains
- online stage will involve iterating over subdomains

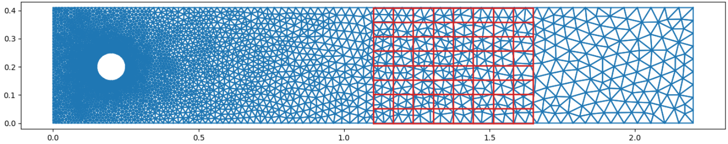
Subdomain approach
Interpolate solution onto a grid in each subdomain



Subdomain approach
Interpolate solution onto a grid in each subdomain

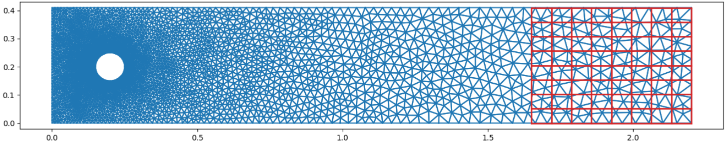


Subdomain approach
Interpolate solution onto a grid in each subdomain

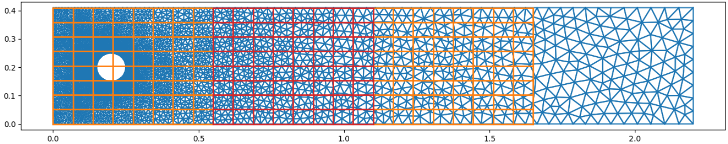


Subdomain approach

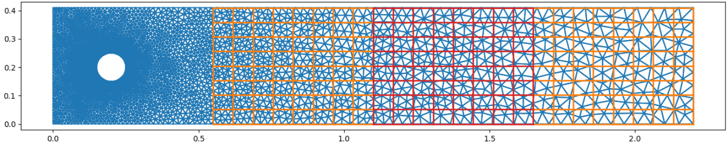
Interpolate solution onto a grid in each subdomain



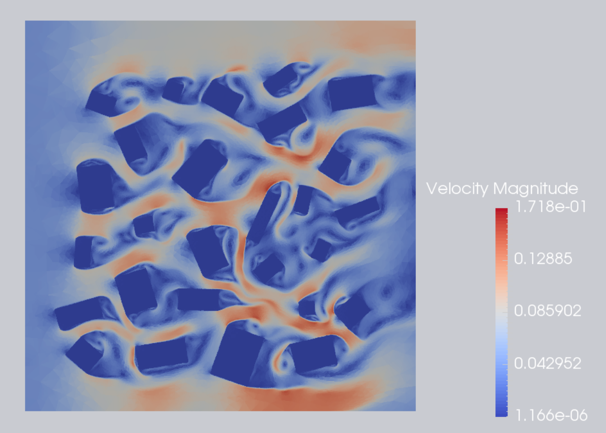
Subdomain approach
Iterate over subdomains

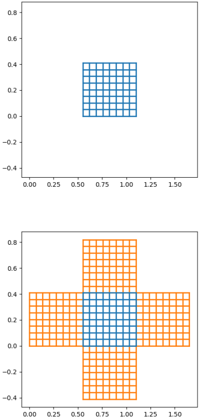


Subdomain approach
Iterate over subdomains



DD-GAN for flow past buildings
Sampling approach





Project tasks

Starting point POD-GAN and DD-POD

Jon, project 1:

1. split GAN code into training and prediction parts using old FPC data
2. use new FPC data and obtain improved results (POD-GAN)
3. modify GAN to work (train and predict) for more than one subdomain
4. apply the above framework to slug flow data

Zef, project 2:

1. help Jon with tasks of organising/re-writing code
2. develop AEs in a domain decomposition framework (SVD-AE, 2D classical CAE and adversarial AE) and compare with DD-POD for FPC
3. train GANs and compare DD-AE-GAN with DD-POD-GAN for FPC
4. apply the above framework to slug flow data

Project tasks

Tianyi, project 3:

1. take the POD coefficients for FPC and train a MLP in two ways
   (a) predict the next time level given the current time level $\boldsymbol{\alpha}^{n+1} = f(\boldsymbol{\alpha}^n)$
   (b) predict the time derivative given the current time level $\dot{\boldsymbol{\alpha}}^n = f(\boldsymbol{\alpha}^n)$
2. compare both POD-MLPs with POD-GAN
3. train GANs that predict the derivative and compare with POD-GAN

$$\mathcal{G}(\boldsymbol{z}^n) = \left\{ \begin{array}{c} \dot{\boldsymbol{\alpha}}^n \\ \boldsymbol{\alpha}^n \end{array} \right\}$$

   at this stage, we have POD-MLP, POD-MLP-deriv, POD-GAN, POD-GAN-deriv

4. apply these methods of time prediction to DD-POD-GAN for flow past a cylinder
5. apply these methods of time prediction to DD-POD-GAN for slug flow

Project tasks

Stella, project 4:

1. take the POD coefficients for FPC and train a MLP without and with a physics-informed term in the loss function
   (a) MSE:

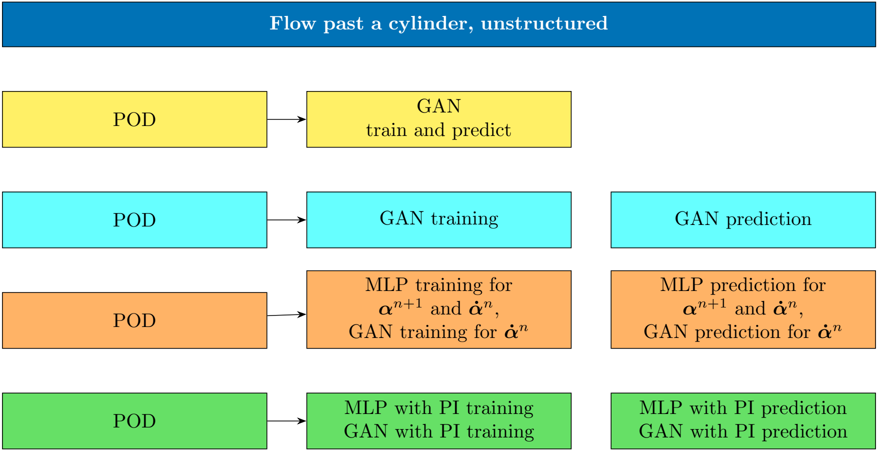
$$\mathcal{L}_1 = \frac{1}{N^{ex}} \sum_{n=1}^{N^{ex}} (\tilde{\boldsymbol{\alpha}}^n - \boldsymbol{\alpha}^n) \cdot (\tilde{\boldsymbol{\alpha}}^n - \boldsymbol{\alpha}^n)$$

   (b) PI MSE:

$$\mathcal{L}_1 + \frac{1}{N^c} \sum_{j=1}^{N^c} \left| \mathcal{F}(\boldsymbol{u}; \boldsymbol{x}_j^c) \right| \text{ where } \mathcal{F}(\boldsymbol{u}; \boldsymbol{x}) = \boldsymbol{\nabla} \cdot \boldsymbol{u} \Big|_{\boldsymbol{x}} = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \Bigg|_{\boldsymbol{x}}$$

2. compare the results of POD-PI-MLP with POD-MLP
3. use the physics-informed loss in the GAN and compare POD-PI-GAN with POD-GAN
4. apply these methods within the DD framework for FPC
5. apply these methods within the DD framework for slug flow

Plan


Flow past a cylinder, unstructured
POD
GAN
train and predict
POD
GAN training
GAN prediction
POD
MLP training for
$\boldsymbol{\alpha}^{n+1}$ and $\dot{\boldsymbol{\alpha}}^n$,
GAN training for $\dot{\boldsymbol{\alpha}}^n$
MLP prediction for
$\boldsymbol{\alpha}^{n+1}$ and $\dot{\boldsymbol{\alpha}}^n$,
GAN prediction for $\dot{\boldsymbol{\alpha}}^n$
POD
MLP with PI training
GAN with PI training
MLP with PI prediction
GAN with PI prediction

Plan

Flow past a cylinder, DD, structured

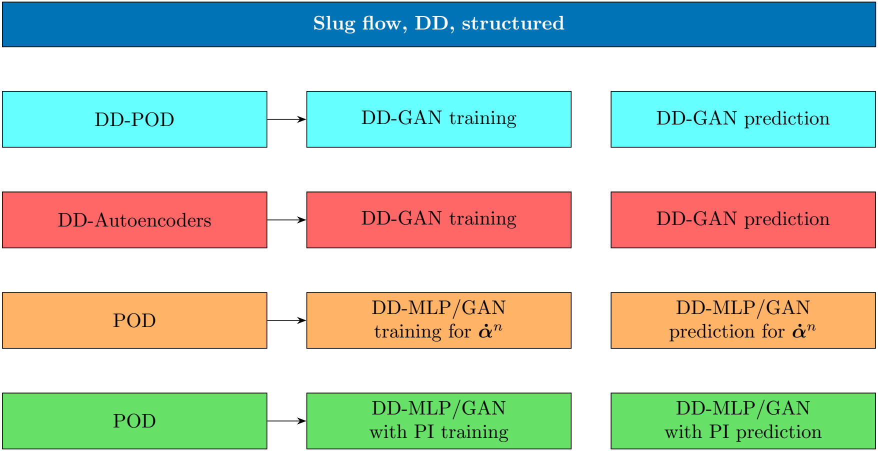
DD-POD → DD-GAN training | DD-GAN prediction

DD-Autoencoders → DD-GAN training | DD-GAN prediction

POD → DD-MLP/GAN training for $\boldsymbol{\alpha}^{n+1}$ and/or $\dot{\boldsymbol{\alpha}}^{n}$ | DD-MLP/GAN prediction for $\boldsymbol{\alpha}^{n+1}$ and/or $\dot{\boldsymbol{\alpha}}^{n}$

POD → DD-MLP/GAN with PI training | DD-MLP/GAN with PI prediction

Plan


Slug flow, DD, structured
DD-POD
DD-GAN training
DD-GAN prediction
DD-Autoencoders
DD-GAN training
DD-GAN prediction
POD
DD-MLP/GAN training for $\dot{\boldsymbol{\alpha}}^n$
DD-MLP/GAN prediction for $\dot{\boldsymbol{\alpha}}^n$
POD
DD-MLP/GAN with PI training
DD-MLP/GAN with PI prediction

DD-GAN for flow past buildings

Hanna and Xiangqi, projects 5 & 6:

- apply the (best) of the above methods to flow past buildings
- sampling replaces domain decomposition

As a start:

- Hanna and Xiangqi to think about sampling the vtu files to form the snapshots / data
- Hanna to apply (POD) and 2D classical CAE to learn the locations of the buildings
- Xiangqi to apply (POD) and adversarial AE to learn the locations of the buildings
- Hanna to apply MLP/GAN to predict the flow patterns
- Xiangqi to apply MLP/GAN (training for the derivative or increment) to predict the flow patterns