COMP. MATH L2

$A\underline{x} = \underline{b}$

underline = bold font = vector

$$\begin{pmatrix} a_{11} & a_{12} & \cdots \\ a_{21} & a_{22} & \\ \vdots & & \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$\underline{a}_{:1}$ $\underline{a}_{:2}$

$= [\underline{a}_{:1}, \underline{a}_{:2}, \underline{a}_{:3}]$

A real $m \times n$ matrix A can be written as

$A \in \mathbb{R}^{m\times n}$

real numbers

$A = [a_{ij}]_{m\times n}$

" $\forall$ " = for all

" $\exists$ " = there exists.

$A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$ $A^T = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$

$A = A^T \Rightarrow a_{ii} = 0 \ \forall i$

$(ABC)^T = (A(BC))^T$

$= \quad)^T$

$= ?$

$((AB)C$

$A \in \mathbb{R}^{m\times n}$ $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & & \vdots & \vdots \\ a_{31} & & \vdots & \vdots \\ \vdots & & & \\ a_{m1} & & a_{m2} & a_{mn} \end{pmatrix}$

$\underline{x} \in \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$, $\underline{b} = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$

$\underline{x} \in \mathbb{R}^n \xrightarrow{A\underline{x}} \underline{b} \in \mathbb{R}^m$

$$\boxed{\underline{b}_i = \sum_{j=1}^{n} a_{ij} x_j \quad \forall i = 1, \dots, m}$$

$A\underline{x} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\underline{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} a_{11} + a_{12} \\ a_{21} \end{pmatrix}$

interpret

$= \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix} + \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix} = \underline{a}_{:1} + \underline{a}_{:2}$

linear

$$\boxed{A\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \dots = \alpha \underline{a}_{:1} + \beta \underline{a}_{:2}}$$

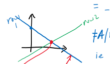
$A\underline{x} = \underline{b} \dots \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$

$2x + 3y = 7$

$\Leftrightarrow 3y = 7 - 2x$

$\Leftrightarrow y = \frac{7}{3} - \frac{2}{3}x$

$= -\frac{2}{3}x \quad \frac{7}{3}$



$\cancel{A}/ mx + c$

ie a line

slope $-\frac{2}{3}$, $\frac{7}{3}$

every (x,y) pair on this line satisfies the first row of the linear system

unique (x,y) that satisfies the whole linear system.

$\det \begin{pmatrix} \alpha & & \\ 1 & \beta & \\ 2 & 3 & \gamma \end{pmatrix}$

$A\underline{x}$

$\Rightarrow A^{-1}A\underline{x} = {}^{-1}\underline{b}$

$\Rightarrow I\underline{x} = A^{-1}\underline{b}$

$\Rightarrow \underline{x} = A^{-1}\underline{b}$

$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$\det(A) =$

$= 1\times 4 - 2\times 3$

$= -2$

$\left| \alpha \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = \begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix} = \alpha^3$

$= \alpha^3 . 1.$

$= \alpha^3 |A|$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\det \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & 0 & 0 \end{pmatrix} = 0$

$\overset{0}{0} = \alpha\beta\gamma$

$A\underline{x} = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$

↳ ∞'ly many solns.

$\begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = x\begin{pmatrix} 2 \\ 1 \end{pmatrix} + y\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

$= \underline{b}$

If $\exists\, \underline{x} \neq \underline{0}$ s.t. $A\underline{x} = \underline{0}$

$x_1 \underline{a}_{:1} + x_2 \underline{a}_{:2} + \dots = 0$

$\therefore \underline{a}_{:1} = \frac{1}{x_1}(-x_2 \underline{a}_{:2} + \dots)$

i.e we can write one of the cols as the weighted sum of the others

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$A\underline{x} = \underline{b}$

& $\exists\, \underline{y} \neq \underline{0}$ s.t. $A\underline{y} = \underline{0}$

then $A(\underline{x} + \underline{y}) = A\underline{x} + A\underline{y}$

$= \underline{b} + \underline{0}$

$= \underline{b}.$

ie. non-uniqueness of solns

Sparse A e.g. $\begin{pmatrix} -2 & 1 & 0 & 0 & 0 & \cdots \\ 1 & -2 & 1 & & & 0 \\ 0 & 1 & -2 & 1 & & \\ \vdots & 0 & & \ddots & & \end{pmatrix}$

inv of a sparse matrix in general is dense.