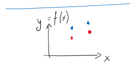
COMPUTATIONAL MATHEMATICS LI

input $\longrightarrow \boxed{\text{Model}} \longrightarrow$ output



Temperature

$$\underset{\substack{\uparrow \\ \text{dependent var.}}}{T} \equiv T(\underbrace{x, y, z, t}_{\text{independent var}})$$

$$x_{n+1} = r x_n (1 - x_n)$$

$$x_n \in [0, 1]$$

$x_n \ll 1$

$$\Rightarrow x_{n+1} = r x_n - r x_n^2$$

$$\approx r x_n$$

$$x_{n+1} = x_n + r x_n (1 - x_n)$$

Fixed points

When is $x_{n+1} = x_n$

$$\Leftrightarrow \cancel{x_n} + r x_n (1 - x_n) = \cancel{x_n}$$

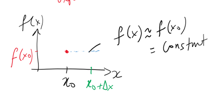
$$\Rightarrow r x_n (1 - x_n) = 0.$$

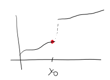
[assume $r > 0$] $\Rightarrow x_n = 0$ or $x_n = 1$.

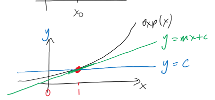
In 1D.

in $\longrightarrow \boxed{f(x)} \longrightarrow$ out.

expensive







Want to find a function

$$y \equiv y(t)$$

dependent var.

t is the only indpt. var.

forms of an ODE

e.g. $\dfrac{dy}{dt} = y' = \exp(t)$

or $\dfrac{dy}{dt} = y$

or $\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} = t + y^3$

General form

$$F(t, y, y', y'') = 0$$

1st order problem can be written

$$\boxed{y' = f(t, y)}$$

Given. Find $y(t)$

Taylor series provides an approx

$$\boxed{y_{n+1} = y_n + \Delta t \, f(t_n, y_n)}$$

Forward Euler

Explicit Euler.

$$\frac{dy}{dt} = y \Rightarrow \int \frac{dy}{y} = \int dt$$

$$\Rightarrow \log(y) = t + c$$

$$\Rightarrow y = e^{t + c_1}$$

$$= e^t e^{c_1}$$

$$= C_2 e^t$$

Constant of integration

I.C. $\quad 1 = y(0) = C_2 e^0$

$$= C_2$$

$$\therefore y(t) = e^t$$