$f(t, y(t)), \quad \frac{dy}{dt} = y' = \dot{y}$
$= f(t, y)$

$\Rightarrow y' = y$ and $y' = c$

L1 (Taylor series):
$y(t + \delta t) = y(t) + \delta t\, y'(t) + \frac{\delta t^2}{2} y''(t) + O(\delta t^3)$

For small δt ...

$y(t + \delta t) \approx y(t) + \delta t\, y'(t)$

Forward Euler $\qquad f(t, y)$

L2. discretise $\Rightarrow$ ...

$y_{n+1} = y_n + \delta t\, f(t_n, y_n)$

Model problem 1:
$\frac{dy}{dt} = k(y_* - y)$

$y(t) = y_* + (y_0 - y_*) e^{-kt}$

F.E. derivation #2:
$\frac{dy}{dt}\Big|_{t_n} = f(t, y)$

$\Rightarrow \boxed{y_{n+1} = y_n + \delta t\, f(t_n, y_n)}$

$\frac{dy}{dt} = f(t, y) = Ay$

$\frac{dy}{dt} = f(t, y)$

Stability — consider the model problem

$\boxed{\frac{dy}{dt} = \lambda y}$

$y(t) = C e^{\lambda t}$

$\boxed{y(t) = y_0 e^{\lambda t}}$

For the model problem: $\frac{dy}{dt} = \lambda y$

$\frac{dy}{dt} = \lambda y$

$\frac{dy}{dt} = \lambda y$

$y_{n+1} = y_n + \delta t\, \lambda y_n$
$\Rightarrow y_{n+1} = (1 + \delta t\, \lambda) y_n$

$y_{n+1} = (1 + \lambda \delta t)^n y_0$

F.E. "absolute" problem
$\lambda : -\lambda \in \mathbb{R}$

$\Rightarrow 0 < \delta t < \frac{2}{\lambda}$

Multistep (e.g. BDF)

Implicit methods