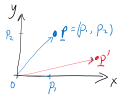
COMP MATH - L3



$\underline{p} = \underbrace{p_1 \underline{i} + p_2 \underline{j}}$

$$\underline{p}' = A\underline{p} = A(p_1 \underline{i} + p_2 \underline{j})$$
$$= A p_1 \underline{i} + A p_2 \underline{j}$$
$$= p_1 A \underline{i} + p_2 A \underline{j}$$

$A\underline{i} = A\begin{pmatrix}1\\0\end{pmatrix} = \underline{a}_{:1}$ i.e. the first col.
$A\underline{j} = \underline{a}_{:2}$ i.e. the 2nd col.

$$\underline{p}' = A\underline{p} = p_1 \underline{a}_{:1} + p_2 \underline{a}_{:2}$$

Shear: example $A = \begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}$

$$A\underline{v} = \lambda \underline{v}$$

$\underline{v} = \underline{0}$ is a soln to this but I'm not interested in that soln

$$\Leftrightarrow A\underline{v} - \lambda\underline{v} = \underline{0}$$
$$\Leftrightarrow (A - \lambda I)\underline{v} = \underline{0}$$

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e.g. $A = \begin{pmatrix}3 & 1\\1 & 3\end{pmatrix}$

I want $\det(A - \lambda I) = 0$

$$0 = \left|\begin{pmatrix}3 & 1\\1 & 3\end{pmatrix} - \lambda\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\right|$$
$$= \begin{vmatrix}3-\lambda & 1\\1 & 3-\lambda\end{vmatrix}$$
$$= (3-\lambda)^2 - 1$$
$$= 9 - 6\lambda + \lambda^2 - 1$$
$$= \lambda^2 - 6\lambda + 8$$
$$= (\lambda - 4)(\lambda - 2)$$

$\Rightarrow \lambda = 4$ or $\lambda = 2$

$\lambda_1 = 4$: $(A - \lambda_1 I)\underline{v} = \underline{0}$

$$\Rightarrow \begin{pmatrix}-1 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}v_1\\v_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$$\Rightarrow \begin{cases}-v_1 + v_2 = 0\\ v_1 - v_2 = 0\end{cases}$$

e.g. $\underline{v} = \begin{pmatrix}1\\1\end{pmatrix}$ is a soln to this

$\underline{v} = \begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$ is the normalised soln

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$$\alpha_1 \underline{v}_1 + \alpha_2 \underline{v}_2 + \alpha_3 \underline{v}_3 = \underline{0}$$

this has or does not a soln $\underline{\alpha} \neq \underline{0}$

$$\Leftrightarrow \underline{v}_2 = \frac{-1}{\alpha_2}(\alpha_1 \underline{v}_1 + \alpha_3 \underline{v}_3)$$

$$A = P\Lambda P^{-1}$$
$$\Leftrightarrow P^{-1}AP = P^{-1}P\Lambda P^{-1}P$$
$$= I\Lambda I = \Lambda$$

$P\Lambda P^{-1}$ cf. $A_b A A_b^{-1}$

$A = \begin{pmatrix}3 & 1\\1 & 3\end{pmatrix}$ $\lambda_1 = 4$ $\underline{v}_1 = \begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$

$\lambda_2 = 2$ $\underline{v}_2 = \begin{pmatrix}-1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$

$$A = \begin{pmatrix}1/\sqrt{2} & -1/\sqrt{2}\\1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}4 & 0\\0 & 2\end{pmatrix}\begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2}\\-1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}$$

So now $A\underline{p} \Leftrightarrow P\Lambda P^{-1}\underline{p}$

$$\frac{dx}{dt} = f(t, x)$$

linear ODE in x

$$\frac{d\underline{x}}{dt} = A\underline{x}$$

$$\boxed{\text{Scalar} \quad \frac{dx_1}{dt} = \alpha x_1 \quad \Rightarrow x_1(t) = C e^{\alpha t}}$$

Vector case $\frac{d\underline{x}}{dt} = A\underline{x}$

$$\Rightarrow \underline{x} = \exp(tA)\underline{c}$$

Above: $A = P\Lambda P^{-1}$

or $A = Q\Lambda Q^T$

Now $A = U\Sigma V^T$

Not the same matrix but are square

Not square