Imperial College London

Eigenmode Analysis of the Bloch Point

Arnav Avad *[1] Martin Lang [2] Hans Fangohr [2 3 4] Marijan Beg [1 2]

[1]Imperial College London, United Kingdom [2]University of Southampton, United Kingdom [3]Max Planck Institute for the Structure and Dynamics of Matter, Germany [4]Center for Free-Electron Laser Science, Germany *arnav.avad@outlook.com

Master's of Science Project

Introduction

- No **standard open-source eigenmode analysis** tool exists containing all methods for convenient eigenmode analysis.

Note: **Finmag** [1] exists as a **finite element** eigenvalue method and **Magnum.np** [2] was released as a **finite difference** eigenvalue method in summer, 2023.

- The importance of the project is derived from the need to identify micromagnetic states by using **ferromagnetic resonance absorption** [3]. Hence, when shooting electromagnetic waves of matching frequencies to the eigenmodes, a clear absorption can be observed to identify the state. This experimental method is inexpensive to operate with highly available instruments, creating a computational tool for analysis will shorten the research process.

Hence, the aims were to:

- Create an eigenmode analysis Python package for the **ringdown methods** and the **semi-analytical eigenvalue method**.
- Make it compatible/integrable with **Ubermag** [4].
- Use the tool for Bloch Point [5] eigenmode analysis.

Initially demonstrated by Beg et al. [5], a **Bloch point** is a **discontinuity** formed by adjacent magnetization vectors.

Method

Ringdown Method [6]

Spatially Averaged Method: $P_{\text{sa}}(f) = \sum_{k=x,y,z} \left| \sum_{j=1}^{n} \langle \Delta m_k(t_j) \rangle e^{-i2\pi f t_j} \right|^2$

Spatially Resolved Method: $P_{\text{sr}}(f) = \sum_{k=x,y,z} \frac{1}{N} \sum_{i=1}^{N} \left| \sum_{j=1}^{n} m_k(\mathbf{r}_j, t_j) e^{-i2\pi f t_j} \right|^2$

Finite Difference Eigenvalue Method [7, 8]

Using an undamped LLG equation and a perturbed relaxed magnetisation state, the eigenvalue method can be derived.

$$\frac{\partial}{\partial t}(\mathbf{m}_0 + \mathbf{v}(t)) = -\gamma_0^*(\mathbf{m}_0 + \mathbf{v}(t)) \times \mathbf{H}_{\text{eff}}(\mathbf{m}_0 + \mathbf{v}(t))$$

$$\begin{aligned} \frac{\partial}{\partial t}\mathbf{v}(t) &= -\gamma_0^* \left(-\mathbf{m}_0 \times h_0 \mathbf{v}(t) + \mathbf{m}_0 \times (\mathbf{H}'_{\text{eff}}(\mathbf{m}_0) \cdot \mathbf{v}(t)) \right) \\ &= \mathbf{A} \cdot \mathbf{v}(t) \end{aligned}$$

Using an ansatz of $\mathbf{v}(t) = \text{Re}(\mathbf{v}_0 e^{i\omega t})$:

$$\begin{aligned} \mathbf{A} \cdot \mathbf{v}_0 &= i\omega \mathbf{v}_0 \\ &= 2i\pi f \mathbf{v}_0 \end{aligned}$$

To increase **memory efficiency**, **dimensionality reduction** can be used within the eigenvalue problem.

New basis:

$$\begin{aligned} \mathbf{e}_{1,n} &= -\mathbf{m}_{0,n} \times (\mathbf{e}_z \times \mathbf{m}_{0,n}) \\ \mathbf{e}_{2,n} &= \mathbf{e}_z \times \mathbf{m}_{0,n} \\ \mathbf{e}_{3,n} &= \mathbf{m}_{0,n} \end{aligned}$$

Basis transformation Matrix:

$$\mathbf{R}_n = \begin{pmatrix} \mathbf{e}_{1,n} \cdot \mathbf{e}_x & \mathbf{e}_{2,n} \cdot \mathbf{e}_x & \mathbf{e}_{3,n} \cdot \mathbf{e}_x \\ \mathbf{e}_{1,n} \cdot \mathbf{e}_y & \mathbf{e}_{2,n} \cdot \mathbf{e}_y & \mathbf{e}_{3,n} \cdot \mathbf{e}_y \\ \mathbf{e}_{1,n} \cdot \mathbf{e}_z & \mathbf{e}_{2,n} \cdot \mathbf{e}_z & \mathbf{e}_{3,n} \cdot \mathbf{e}_z \end{pmatrix}$$

Magnetisation dynamics are shown using the equation $\mathbf{m}(t) = \mathbf{m}_0 + \mathbf{v}_0 e^{i\omega t}$

Bloch Point Eigenmode Analysis — Ringdown Method

Method to simulate the Bloch point for eigenmode analysis is shown below, similar to method used for skyrmions in the paper, 'Dynamics of skyrmionic states in confined helimagnetic nanostructures' [9].

- Relax initial magnetisation state using Symmetric Exchange, Demagnetisation and DMI of **opposite chiralities** terms for top and bottom discs.
- Add **sinc Zeeman field** with 0.25ns shift and 100GHz frequency to excite eigenmodes below this frequency. $\text{H}_{\text{max}} = 40\mathbf{e}_{plane}\text{kAm}^{-1}$
- Simulate for 0.5ns to excite all eigenmodes below 100GHz.
- Remove sinc Zeeman field.
- Simulate for 2ns to enable the system to relax for noise reduction.
- Simulate for 25ns to sample the oscillations 5000 times.

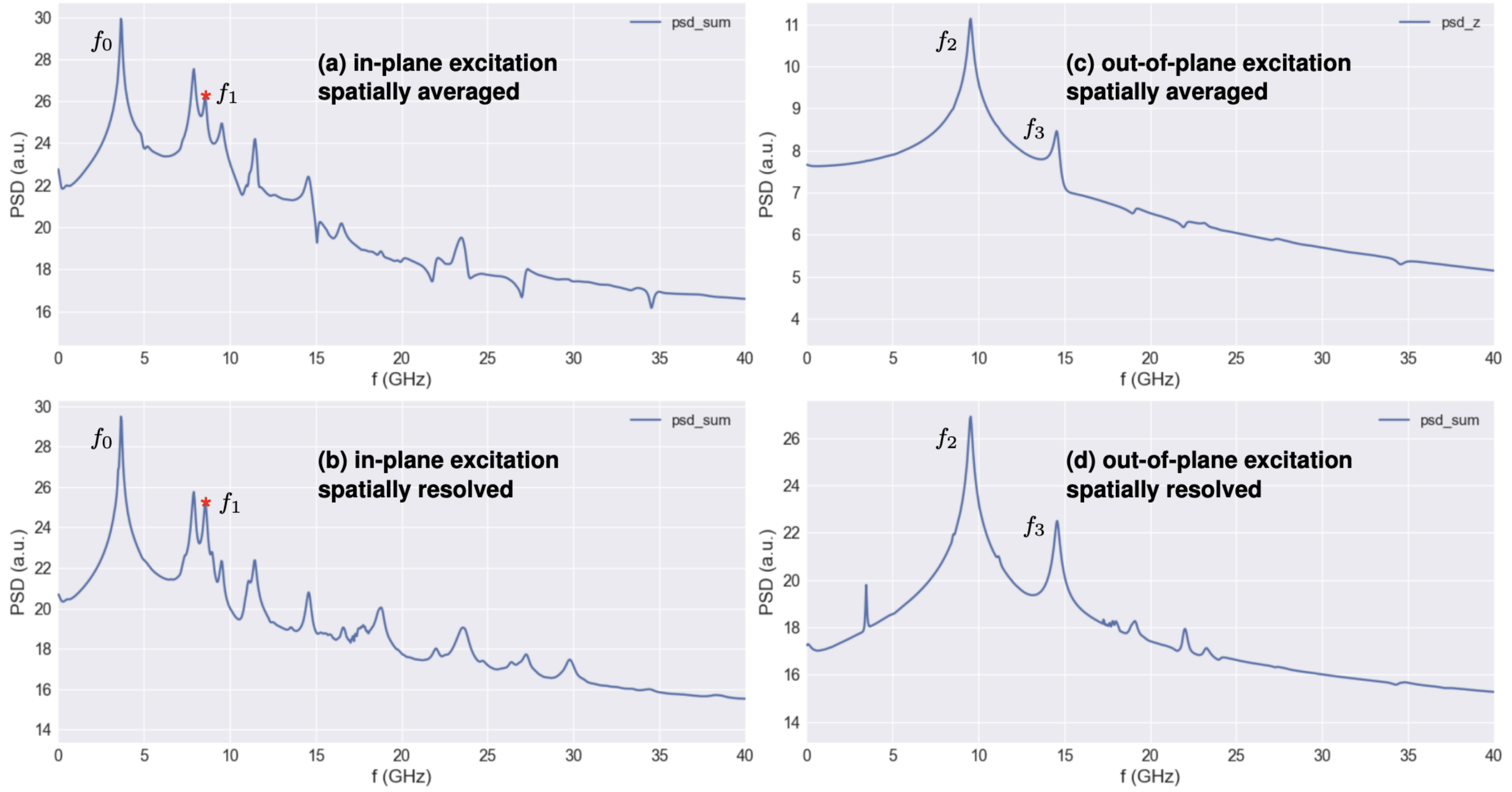

Figure 1. The **power spectral densities (PSDs)** resulting from the spatially averaged and spatially resolved ringdown methods using an in-plane and out-of-plane excitation in the form of a sinc function. Note: the PSD of only the z component is used in figure c due to the negative values of the x and y-component PSDs.

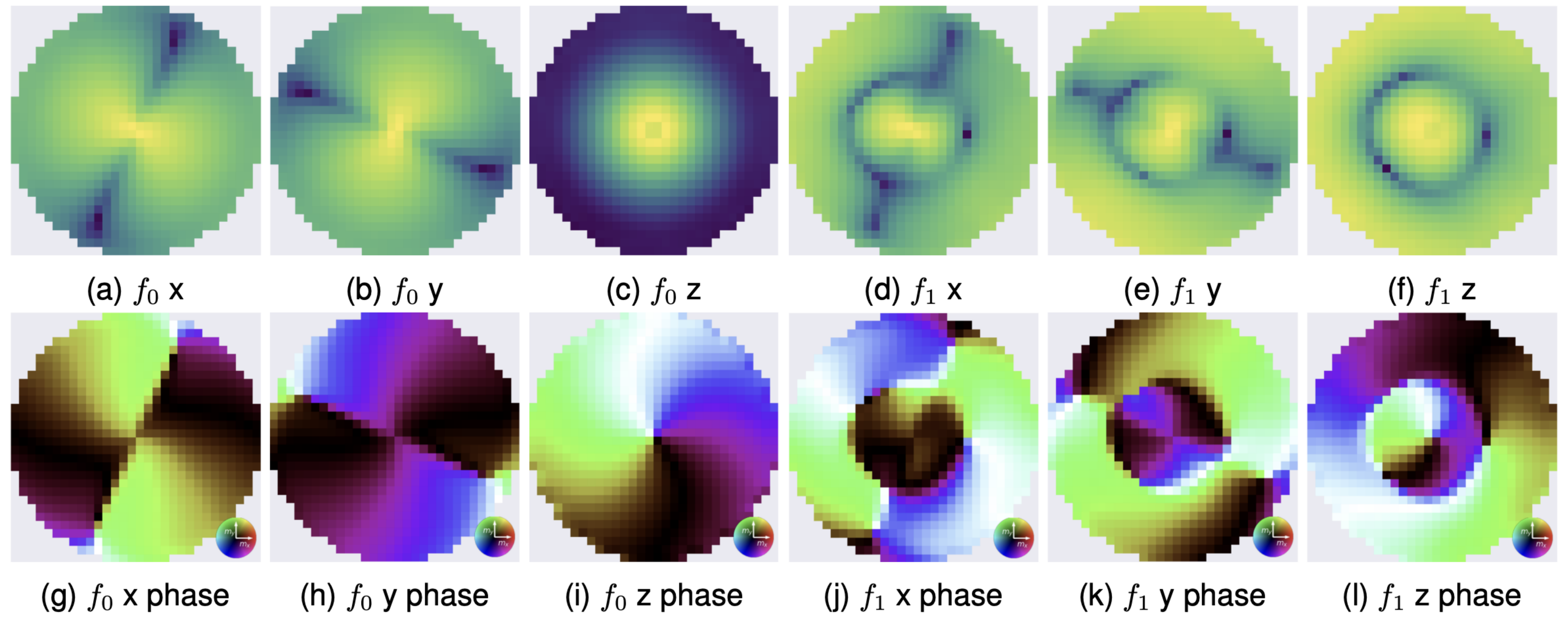

Figure 2. The **magnitude** and **phase** of the **in-plane excitation** are shown according to the spatially resolved method. Note that $f_0 = 3.68\text{GHz}$ and $f_1 = 8.60\text{GHz}$

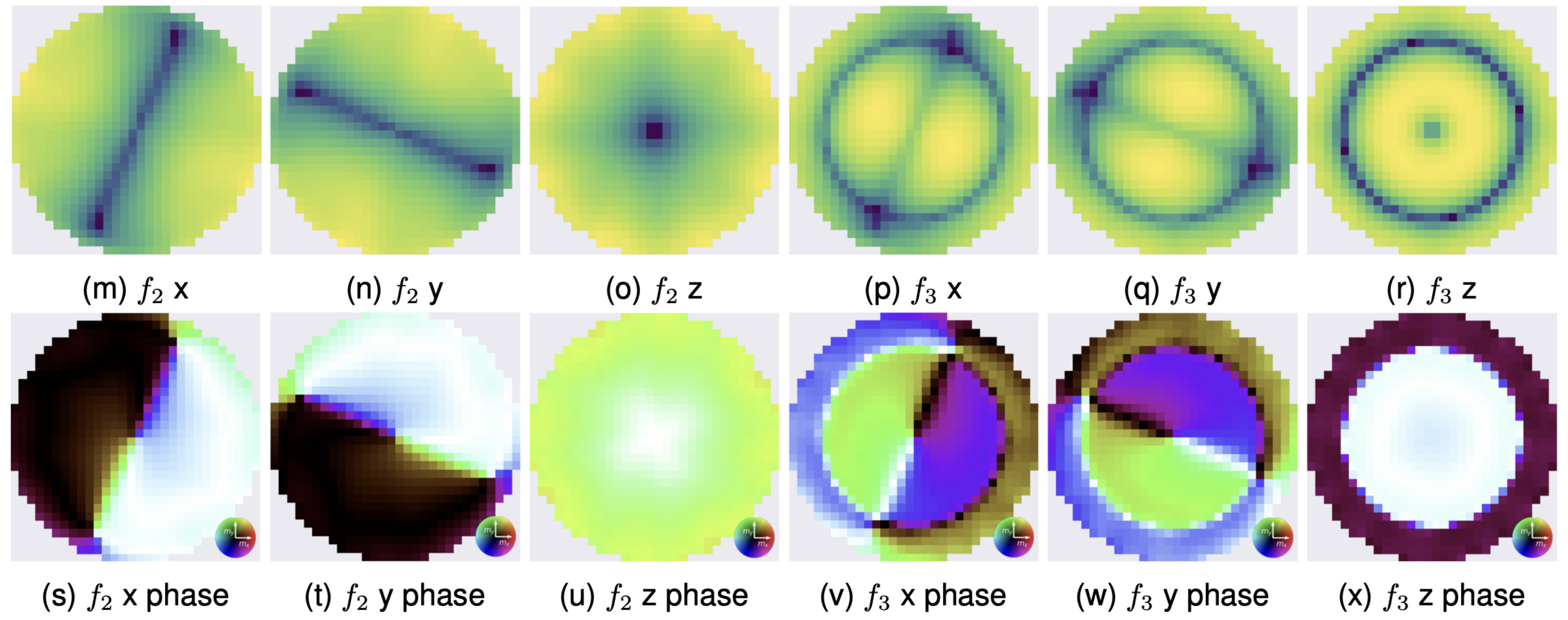

Figure 3. The magnitude and phase of the **out-of-plane excitation** are shown according to the spatially resolved method. Note that $f_2 = 9.56\text{GHz}$ and $f_3 = 14.60\text{GHz}$.

Note: The finite difference eigenmode analysis was not completed on the Bloch point as per d'Aquino et al [7] on the effect of the **symmetry** of the **linear operator** which calculates the cross product with $\mathbf{m}_0$. For example, with symmetry comes all real eigenvalues but since the DMI coefficient is not constant throughout the material, this statement does no longer hold.

Conclusion

- The spatially averaged and spatially resolved ringdown methods were implemented. Furthermore, the semi-analytical finite difference eigenvalue method was implemented. This also includes methods for **imaging** the **magnitude**, **phase** and **dynamics** of the eigenmodes. The entire package was built to Ubermag compatibility.
- Both eigenmode analysis methods were Validated using the **FMR Standard Problem** [10].
- Bloch Point analysis using the ringdown method was done to show the power spectral densities of in-plane and out-of-plane excitations, hence, their eigenfrequencies. And, eigenmode images of how each component varies in terms of the phase and magnitude were depicted.
- This tool can be used to aid in experiments — Bloch Point creation and the respective **FMR absorption data** can be analysed.

References

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