OCES 2003 Assignment 1, Spring 2021

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Set on: Tue 16$^{\text{th}}$ Feb; due: Tue 23$^{\text{th}}$ Feb

Model solutions and mark scheme

Problems

1. (a) To get a volume flux (in units of $\mathrm{m}^3\ \mathrm{s}^{-1}$) from flow and the given area you simply multiply the two, but noting that you need to convert units:

$$(1\ \mathrm{m\ s}^{-1})(2\ \mathrm{cm}^2) = (1\ \mathrm{m\ s}^{-1})(2\ (10^{-2}\ \mathrm{m})^2) = 2\times 10^{-4}\mathrm{m}^3\ \mathrm{s}^{-1} = 2.00\times 10^{-10}\ \mathrm{Sv}.$$

(0.5 marks for getting answer and convert units properly, 0.5 marks for giving answer in the form requested.)

(b) Here we need a conversion of the velocity, but the cross section is simply $50\times 2 = 100\ \mathrm{m}^2$, so

$$7\ \mathrm{miles\ hr}^{-1}100\ \mathrm{m}^2 = \frac{7\cdot 1600\ \mathrm{m}}{3600\ \mathrm{s}}100\ \mathrm{m}^2 = 3.11\times 10^{-4}\ \mathrm{Sv}.$$

(1 mark for getting answer and convert units properly, 0.5 marks for giving answer in the right accuracy, 0.5 marks for giving answer in the form requested.)

(c) Since transport is speed multiplied by cross section, and cross section here is $2000\times 10^3\cdot 1000\ \mathrm{m}^2$, so velocity is

$$\frac{130\times 10^6\ \mathrm{m}^3\ \mathrm{s}^{-1}}{2000\times 10^3\cdot 4000\ \mathrm{m}^2} = \frac{130}{2\cdot 4000}\ \mathrm{m\ s}^{-1} = \frac{13}{800}\ \mathrm{m\ s}^{-1} = 0.016\ \mathrm{m\ s}^{-1}.$$

(0.5 mark for getting answer and 0.5 marks for giving answer in the form requested.)

(d) The same manipulation as above gives a speed of $0.300\ \mathrm{m\ s}^{-1}$. The flow speeds of the ACC and the Gulf Stream are in about the right ball park (see Talley *et al.*, 2011 for example). The ACC, while slower, is so much bigger, so the transport is substantially larger than that of the Gulf Stream.

(0.5 mark for getting answer and 0.5 marks for giving answer in the form requested. 1 mark for looking up the velocities and providing an appropriate reference.)

2. The graph shown here has time going *backwards*, and this is sometimes what you do see in palaeoclimate or paleo-oceanography literature. I have been specific about t to be going *forward*, so in this instance the region indicated has positive $\partial\eta/\partial t$, i.e. sea level is going up as we go towards the present day. Positive $\partial^2\eta/\partial^2 t$ in this region means the sea level increase is accelerating.

(1 mark for identifying t is going the "wrong" way, 1 mark for $\partial\eta/\partial t > 0$, 1 mark for sea level increase is accelerating. Give 1.5 mark for $\partial\eta/\partial t < 0$ AND arguing $\partial^2\eta/\partial t^2 > 0$ means sea level decrease *is* decelerating, *but only for this combination of answer.)*

3. The answer I cooked up to all be "Up".

   (0.5 marks for each answer, but give credit only if some justification has been given.)

4. (a) According to Wikipedia (`https://en.wikipedia.org/wiki/List_of_rivers_by_discharge`) I make it:

river	average discharge ($m^3\ s^{-1}$)
Amazon	209000
Congo	41200
Ganges	38129
Orinoco	37740
Yangtze	30146

   The discharge rate in Sverdrups is just those numbers multiplied by 10^{-6}.

   (1 mark for listing the rivers, 1 mark for giving the answer in Sverdrups. Not too bothered in this case about the accuracy as such.)

   (b) The density of interest here is 1000 kg m^{-3}. Summing up the above numbers and multiplying by 1 yr = $3600 \cdot 24 \cdot 365$ s, I make the answer to be 1.12×10^{16} kg.

   (0.5 marks for stating the relevant density value, 1 mark for the answer, 0.5 marks for the degree of accuracy and giving the answer in the specified form. Give the full 1 mark for the answer as long as the answer is around 10^{16} kg.)

5. Just from considering units of c_p you would expect to have

$$Q = c_p M \Delta T,$$

   where Q is the energy content/input, M is the mass, and ΔT is the change in temperature.

   (a) From the above and converting the units so as to have 1 ml = 10^{-6} m^3, with $\Delta T = 100 - 20 = 80$ K, $M = (1000 \text{ kg m}^{-3})(300 \times 10^{-6} \text{ m}^3) = 0.3$ kg, we have

$$Q = 0.3 \text{ kg} \cdot 4000 \text{ J kg}^{-1} \text{ K}^{-1} \cdot 80 \text{ K} = 96000 \text{ J} = 96 \text{ kJ}.$$

   (0.5 mark for unit conversion, 1 mark for the calculation, 0.5 marks for the degree of accuracy and units.)

   (b) The time needed to 96000 J given an input of 700 J s^{-1} is $96000/700 \approx 137$ s ≈ 2 mins.

   (0.5 mark for the calculation, 0.5 marks for giving the to the degree of accuracy and unit requested.)

   (c) So here it is potentially useful to sort out the algebra first before dealing with the numbers. To get the energy input from the average power P (W m^{-2}), we need the time period over which power is being applied Δt, and the area A, giving

$$Q = PA\Delta t.$$

   To get the mass from the density, we need the volume, and we are given area A and depth Δz, so

$$M = \rho A \Delta z.$$

Substituting accordingly and noting that the question is asking for ΔT, we notice that the factors of area A cancel out, and

$$\Delta T = \frac{P\Delta t}{\rho \Delta z c_p}.$$

Then with $P = 340$ W m^{-2}, $\Delta t = 3600 \cdot 24 = 86400$ s, $\rho = 1000$ kg m^{-3}, $\Delta z = 1$ m, and $c_p = 4000$J kg^{-1} K^{-1}, we have $\Delta T = 7.344 \approx 7$ K.

(0.5 marks for calculation and argument turning power into energy, 0.5 marks for calculation and argument turning density in a mass, and 1 mark for putting it all together.)

(d) Reasons I can think of are:

- $P = 340$ W m^{-2} is solar radiation at the top of the atmosphere, and clouds could block of some this;
- Radiation that get through could be reflected from the ocean before it gets absorbed;
- The assumption here is all the energy goes into the first meter of the ocean, when radiation penetration is probably deeper than that;
- Not all radiation absorbed gets turned into heat;
- Sea water has a non-zero temperature and it would emit some of that radiation back out, thus losing heat;
- Fluid motion (atmosphere or ocean) could take heat away;
- Conduction I suppose could too, but that's probably weak over the time-scale of a day;
- Transfer into biological matter?

(1 mark each up or something which is plausible, up to a maximum of 2 marks.)

!? (No marks bonus question.) So the number I got for input power via stirring is 10-millionth (from "*What if?*" by Randall Monroe, creator of XKCD), which I am interpreting as 10^{-7} W. Assuming perfect efficiency and no loss whatsoever, to put in the 96000 J you need 9.6×10^9 s, which I make it to be over 30000 years (if I interpret the power input as 10^{-5} W then I would get a cup of boiling tea at lightning time of only 300 years).

If you use a blender for whatever reason you might think you are getting a larger power in since the blender requires a larger power input, but you have to remember also that most of that is lost as heat while the machine is running, and only a small portion of it actually goes into the stirring. Unless you somehow get all that machine heat loss into the cup water too then it's probably still going to take a while to heat up that water. And we haven't even talked about heat loss[1] and things like latent heat...

Note even going to try to do this for the ocean. It is generally thought that marine stirring of the ocean is somewhat negligible, although there are estimates saying that while it is small, it is potentially comparable to the wind forcing and is not that negligible. The jury is out, because we don't really have a good estimate of power input by the whole set of marine organisms.

[1] The Stefan–Boltzmann law tells you energy emission should go like T^4, i.e. emission of energy is larger the hotter a body gets.