OCES 2003 Assignment 3, Spring 2022

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Set on: Thur 21st Apr; due: Thur 28th Apr

Blurb

- Assignments have a maximum mark out of 20, although you will see that there are 22 marks available to get in total, i.e. if you get 22/20 you still only get credit for 20/20
  - 16-17 is roughly around the A- boundary
  - anything below 8 is probably a fail
- Please show working in calculation
  - no working + wrong answer = no credit whatsoever
  - some working + wrong answer = partial credit
  - generically, give things to 2 decimal place and provide the appropriate units (marks are allocated for these), unless otherwise specified
- No answers except the 'hard' ones should need more than a paragraph / half a page, and excess answers that are not to the point will be penalised
- Type up the assignment or send a photo of your written up work in (the former is preferred), and the only request I have is no Microsoft Word documents (you can type up things with Word but export it as a pdf if you do)
  - write in full sentences where appropriate
  - particularly poor and/or scrappy presentation will have a mark that can be taken off
- There will be a rigid mark scheme, and model solutions will be available in due course
  - the TAs only mark the stuff, you should come to the instructor for arguing marks, and note the re-marking can result in marks going up or down

!!! By handing something in, you agree to the usual Academic Honour code and Integrity declarations. For more, see `http://qa.ust.hk/aos/academic_integrity.html`. Cases for plagiarism (whether intended or not, it is the "act" that matters) gets a penalty ranging from

- zero on the question concerned
- a fixed penalty starting from around 1/3 of the total marks
- zero for the whole assignment/midterm/final

The following counts as plagiarism (and is a non-exhaustive list):

- copying word for word *any* (i.e. one or more) sentence without quote marks regardless of whether it is cited or not, e.g. *Yer a Jedi, Harry* (Gandalf of House Stark)
  - use quote marks if need be, e.g. "*Yer a Jedi, Harry*" (Gandalf of House Stark), although don't do it too often, because then one could argue you are not passing any of your thoughts through
  - any more than around three usages in text is probably excessive
- copying without citation or wrong citation, e.g. "*Yer a Jedi, Harry*", or "*Yer a Jedi, Harry*" (Jon Snow of Tatooine)
- changing a few words but sentence largely the same, e.g. *You, Harry, sir, are a Jedi* (Mithrandir of Winterfell)

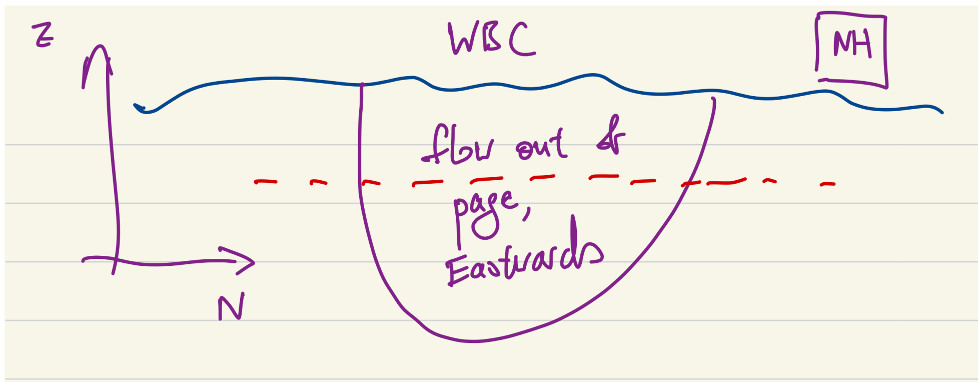
- Turnitin will pick out most of the aforementioned things
- Cases can be contested but will lead to an official review, where the penalty may go up and/or down, and could result in an Academic Misconduct case being filed (see `https://acadreg.ust.hk/generalreg.html#b`)

Problems

1. (a) Explain in your own words (and using not more than half a page) the thermal wind balance (or thermal wind shear relation), and under what conditions thermal wind balance holds. [3 marks]

   (b) If the Earth's rotation slowed down but the density profile remains the same, what can you say about the change in the zonal and meridional flow's vertical shear? Justify your answer. [2 marks]

   (c) Consider the schematic below for the flow profile of a Northern Hemisphere Western Boundary Current:



   State the signs of the Coriolis parameter f and the zonal velocity u in the Western Boundary Current. Assuming thermal wind balance holds, sketch the *temperature* profile you might expect going North on the red dashed line, and justify your answer. For simplicity you can assume the flow inside the sketched Western Boundary Current region has $\partial u_g/\partial z$ = contant > 0, but the flow is zero outside of it. [3 marks]

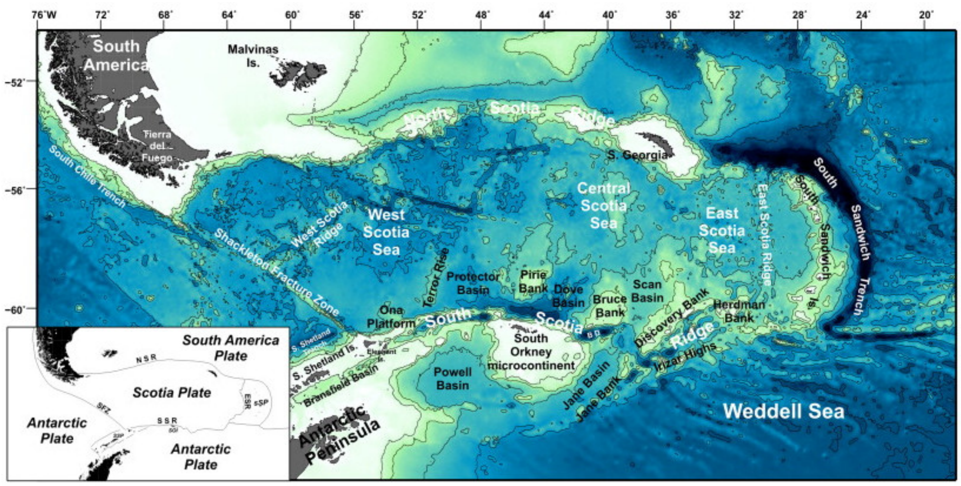
2. Recall that potential vorticity can be defined as

$$q = \frac{\zeta + f}{H} = \frac{\boldsymbol{e}_z \cdot \nabla \times \boldsymbol{u} + (f_0 + \beta y)}{H} \sim \frac{U/L + f_0}{H},$$

where $\zeta = \boldsymbol{e}_z \cdot \nabla \times \boldsymbol{u}$ is the vertical component of the flow vorticity, while U and L are typical velocity and length scales for the flow, and the other symbols have their usual meanings.

   (a) Provide some numerical estimates of U/L and values of f_0 for geostrophic flows; provide references as appropriate (hint: try assignment 1). Argue why for geostrophic flows we can generally take $q \approx f/H$. [2 marks]

   (b) Below diagram shows the bathymetry for the Drake passage (taken from Lecture 13 and Civile *et al.*, 2012), where grey colours are land, and light blue colours are shallower regions:



Argue from potential vorticity conservation why the Antarctic Circumpolar Current (which is eastward flowing) is expected to deflect *northwards* as it flows through the Drake passage.

It is possible to get full marks for this by drawing an appropriately labelled diagram. [3 marks]

(c) Taking $f_0 = -10^{-4}$ s^{-1} (since we are in the Southern Hemisphere), $\beta = 2 \times 10^{-11}$ m^{-1} s^{-1}, and assuming $q \approx (f_0 + \beta y)/H$, how much northward deflection would we expect from potential vorticity conservation if the Antarctic Circumpolar Current is flowing from the west in a region with water depth $H = 4000$ m, into a region in the east with water depth $H = 2000$ m? Give you answers in to the nearest *degree* making the assumption that 1 degree = 100 km. For simplicity you can set $y = 0$ as the Antarctic Circular Current's starting latitude. [3 marks]

(Of course this would just the suggested value you get from PV conservation. In reality other things are at play, but it does give you a not implausible value that is on the large side compared to observations.)

3. An important example of non-dispersive waves in the ocean is the (barotropic) Kelvin wave (a boundary trapped inertial-gravity wave), which has dispersion relation

$$\omega = k\sqrt{gH}$$

and a spatial structure

$$\eta \sim \mathrm{e}^{-y/L_d}, \qquad L_d = \frac{\sqrt{gH}}{|f_0|},$$

where x is the direction the wave is propagating in, k is the x-wavenumber, y is the distance from the coast (or the wave guide at the equator, it doesn't really matter), and L_d is the Rossby deformation radius.

(a) State what it means for waves to be non-dispersive, and draw a picture illustrating the difference between the phase and group velocities for non-dispersive waves propagating in x. [2 marks]

(b) Given the above information, compute the phase speed for the equatorial (barotropic) Kelvin wave phase speed using $g = 10$ m s^{-2} and $H = 4$ km, giving your answer to the nearest m s^{-1}. From that work out the *crossing time* for the equatorial Kelvin to cross the Pacific, assuming the Pacific is about 20,000 km wide, giving your answer to the nearest *hour*. [2 marks]

(These barotropic waves are *fast*! This is what we might see for tides, but for things like El-Nino it is the *first baroclinic Kelvin wave* that we want, which is quite a bit slower and the crossing time is closer to *months*; see OCES 4001 maybe.)

(c) For simplicity lets take $y = 0$ to be at the boundary, and $|f_0| = 10^{-4}\ \mathrm{s}^{-1}$. For the coastal Kelvin wave with $H = 400$ m, workout the length after which the amplitude of the wave drops off by a factor of 10, giving your answer to the nearest *degree* and making the assumption that 1 degree = 100 km. (Hint: What you want is "$\ln = \log_e$" rather than "$\log = \log_{10}$"; recall that $\ln \mathrm{e}^x = x$.) [2 marks]

(Again the computed value will be quite a bit larger than what you might see quoted on the internet, because usually we are interested in the *first baroclinic Kelvin wave*, which has a smaller *equivalent depth* $\tilde{H} \ll H$; see OCES 3203 maybe.)

!? (Bonus question, no marks + for interest only)

- Plot out the Kelvin wave amplitude

$$\eta \sim \mathrm{e}^{-y/L_d} \cos(kx - \omega t), \qquad L_d = \frac{\sqrt{gH}}{|f_0|},$$

for some choices of L_d and k in a graphing utility of your choice, and investigate what happens if you take $f + \beta y$ instead of f_0 in the definition of L_d. (The book of Knauss has an example of what it should look like as a function of t and space.)

- Show explicitly that the group velocity of internal waves are orthogonal (perpendicular) to the phase velocity. Be careful however that the thing you actually want to show is that

$$\frac{\boldsymbol{k}}{|\boldsymbol{k}|} \cdot \boldsymbol{c}_g = 0,$$

since the phases propagate in the direction of the normalised wave-vector[1].

- Animate some internal waves in 2d. See `waves.ipynb` provided in the GitHub repository for some code that might help you along.

[1]The vector formed from components of the phase velocity is not the velocity that phases propagate at...see the book of Vallis for a discussion of this.