Boring but important disclaimers:

- If you are not getting this from the GitHub repository or the associated Canvas page (e.g. CourseHero, Chegg etc.), you are probably getting the substandard version of these slides Don't pay money for those, because you can get the most updated version for free at

  `https://github.com/julianmak/academic-notes`

  The repository principally contains the compiled products rather than the source for size reasons.
- Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
- I do not claim the compiled products and/or code are completely mistake free (e.g. I know I don't write Pythonic code). Use the material however you like, but use it at your own risk.
- As said on the repository, I have tried to honestly use content that is self made, open source or explicitly open for fair use, and citations should be there. If however you are the copyright holder and you want the material taken down, please flag up the issue accordingly and I will happily try and swap out the relevant material.

OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 11: Gyres 1 (overview + Sverdrup balance)

Outline

- (wind driven?) gyres and features

  → subtropical/subpolar gyres

  → Western Boundary Currents (WBCs) (e.g. Gulf stream, Kuroshio)

- β-plane + Sverdrup balance

  → simple model of wind-driven gyre (on β-plane)

  → wind balancing gradient of Coriolis

- depth-independent model with no topography: Sverdrup balance

  → interior dynamics

Key terms: subtropical/subpolar gyres, $f = f_0 + \beta y$, Sverdrup balance

Aim of these two lectures

combine material so far for a theory of wind-driven gyres

Aim of these two lectures

combine material so far for a theory of wind-driven gyres

plan of attack:

1. recall material on gyres (Lec 2)
2. construct model (wind, rotation, friction; Lec. 4, 7-10)
   $\rightarrow$ β-plane
   $\rightarrow$ assumption for density (but see next Lec.) (Lec. 5 + 6)
3. analyse and deduce via balance arguments
   $\rightarrow$ Sverdrup balance (wind + rotation; Lec. 8 + 9)
   $\rightarrow$ mass conservation (Lec. 4?)
   $\rightarrow$ vorticity balance (wind + friction; Lec. 9 + 10)

Aim of these two lectures

combine material so far for a theory of wind-driven gyres

plan of attack:

1. recall material on gyres (Lec 2)
2. construct model (wind, rotation, friction; Lec. 4, 7-10)

   → β-plane

   → assumption for density (but see next Lec.) (Lec. 5 + 6)
3. analyse and deduce via balance arguments

   → Sverdrup balance (wind + rotation; Lec. 8 + 9)

   → mass conservation (Lec. 4?)

   → vorticity balance (wind + friction; Lec. 9 + 10)

"All models are wrong, but some are useful"

– attributed to George Box

Recap: gyres

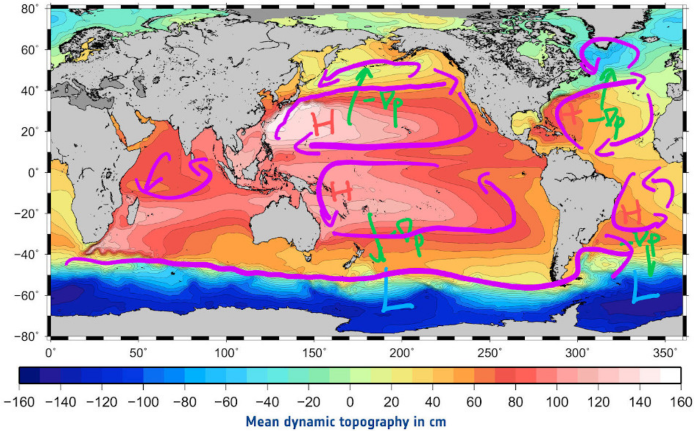


Figure: Time-mean global SSH (also called mean dynamic topography, with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al* (2011), J. Geophys. Res: Oceans.

- contours of SSH + geostrophic balance ⇒ flow, important part of MOC

Recap: gyres + WBCs

- subtropical and subpolar gyres (former shown here)

  → anti-cyclonic and cyclonic respectively (in both hemispheres) (see Lec 8 + 9)

- Western Boundary Current as a part of system

  → Gulf stream here, Kuroshio in Pacific

  → transports tropical + warm water towards high latitudes

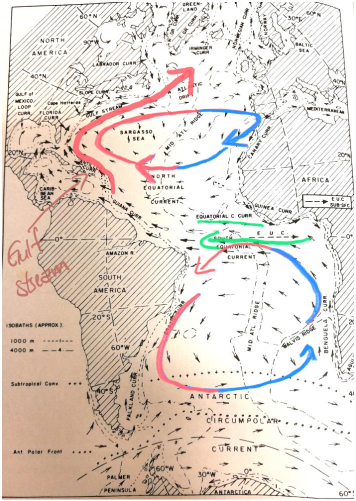


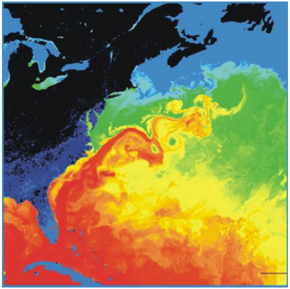
Figure: Features in the Atlantic Ocean. Modified Figure 7.9 from Pickard & Emery (1990), 5th edn.

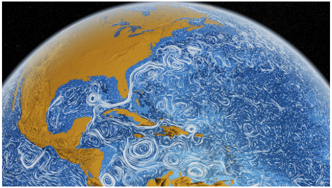
Recap: gyres + WBCs

Q. why Western and not Eastern? (see next Lec.)

Q. processes leading to eddies? (see Lec. 17)

Q. fluctuations + role in climate? (see Lec. 17 + OCES 4001)





Gulf stream in temperature (left) and surface current speed, from NASA

Recall: Coriolis effect and parameter (Lec. 8)

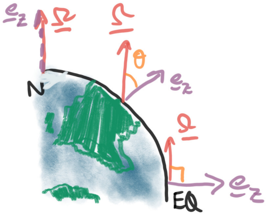


Figure: Mis-alignment of $\boldsymbol{\Omega}$ and $\boldsymbol{e}_z$ used locally for depth.

- for a spherical Earth we take rotation axis to be z-axis, i.e. $\boldsymbol{\Omega} = \Omega \boldsymbol{e}_z$ (this a vector), but locally, z is depth...
- introduce the latitudinally varying Coriolis parameter

$$f = 2\Omega \sin(\text{latitude})$$

- want to further simplify this, spherical (i.e. (lon, lat, depth)) to Cartesian geometry (i.e. (x, y, z)) (cf. Lec 8, when rationalising Coriolis effect)

β-plane

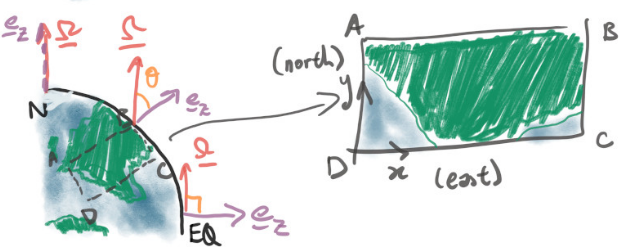


Figure: β-plane schematic. (lon, lat) $\rightarrow$ (x, y) with $f = f_0 + \beta y$ on the plane.

- $x \leftrightarrow$ lon, $y \leftrightarrow$ lat, $z \leftrightarrow$ depth
- $f = 2\Omega \sin(\text{lat}) \leftrightarrow f = f_0 + \beta y$

  $\rightarrow f_0$ (units: s^{-1}, same as f) the uniform rotation frequency

  $\rightarrow \beta = \partial f / \partial y$ (units: exercise)

  $\rightarrow$ **not** haline contraction (context should be clear)

will use β-plane extensively

Model preliminaries

For simplification, going to assume:

- NH β-plane ($f = f_0 + \beta y > 0$), domain is square
- subtropical wind profile (for subtropical gyre)

  $\rightarrow$ only for simplicity, will extend

  $\rightarrow$ note: negative wind stress curl
- assume depth-independence (either $\rho = \rho_0$ or vertically integrate)

  $\rightarrow$ sometimes barotropic (I don't like this term for technical reasons)
- lateral frictional boundary layers

  $\rightarrow$ main **sink** of stuff is going to be over boundary layers

Single (subtropical) gyre: schematic

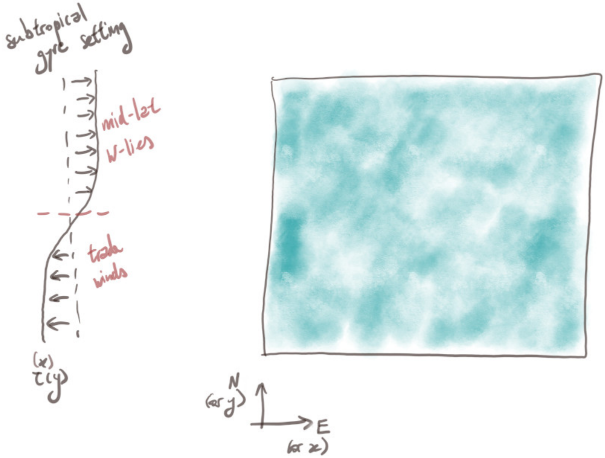


Figure: Schematic of wind-drive model (NH, assume subtropical wind profile, β-plane, square, homogeneous in density)

Equations

Original equations something like (in vertical vorticity ω):

$$\frac{\partial \omega}{\partial t} + \underbrace{\boldsymbol{u} \cdot \nabla \omega}_{\text{inertia}} + \underbrace{\beta v}_{\text{Coriolis}} = \underbrace{-r\omega}_{\text{drag}} + \underbrace{F_\tau(x, y)}_{\text{wind}}$$

- 2d equations in (x, y) (assumed no vertical variation)
- no pressure gradients (took a $\nabla \times$ of momentum equation)
- Coriolis effect appears through βv (as $f = f_0 + \beta y$)

  $\rightarrow$ it is **meridional** velocity v that shows up here
- $F_\tau = \boldsymbol{e}_z \cdot \nabla \times \boldsymbol{\tau}$ is wind stress curl

  $\rightarrow$ $F_\tau < 0$ for wind profile in previous slide
- friction parameterised as linear drag

  $\rightarrow$ important near boundaries

Sverdrup balance

Throw away time derivative and inertia term (cf. Ro $\ll$ 1) gives Stommel's model (Stommel, 1948, *Trans. Amer. Geophys. Union*)

$$\underbrace{r\omega}_{\text{drag}} + \underbrace{\beta v}_{\text{rotation}} = \underbrace{F_\tau(x,y)}_{\text{wind}}$$

Sverdrup balance

Throw away time derivative and inertia term (cf. Ro $\ll$ 1) gives Stommel's model (Stommel, 1948, *Trans. Amer. Geophys. Union*)

$$\underbrace{r\omega}_{\text{drag}} + \underbrace{\beta v}_{\text{rotation}} = \underbrace{F_\tau(x,y)}_{\text{wind}}$$

In **interior** friction is unimportant, so we have Sverdrup balance

$$\beta v \sim F_\tau(x,y)$$

- Coriolis balancing wind stress curl

  $\rightarrow$ geostrophy is Coriolis balancing **pressure gradient** (but no pressure in vorticity equation)

$\beta > 0$ so v related to wind stress curl in interior

Note: The standard derivation involves looking at Ekman up/downwellings associated with the wind, then imply the Sverdrup interior

Single (subtropical) gyre: Sverdrup interior

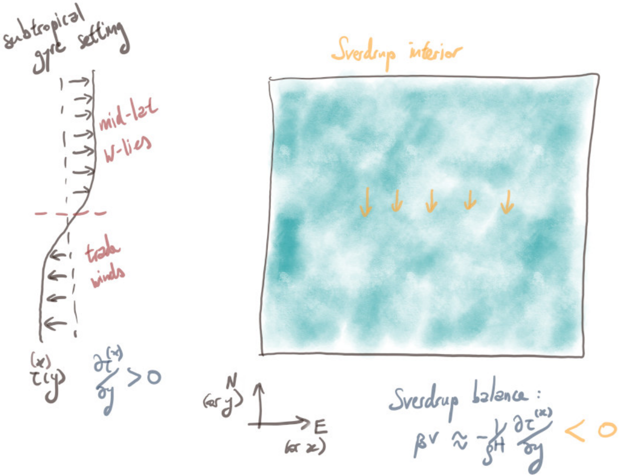


Figure: Schematic of wind-drive model (NH, assume subtropical wind profile, β-plane, square, homogeneous in density)

Single (subtropical) gyre: mass conservation

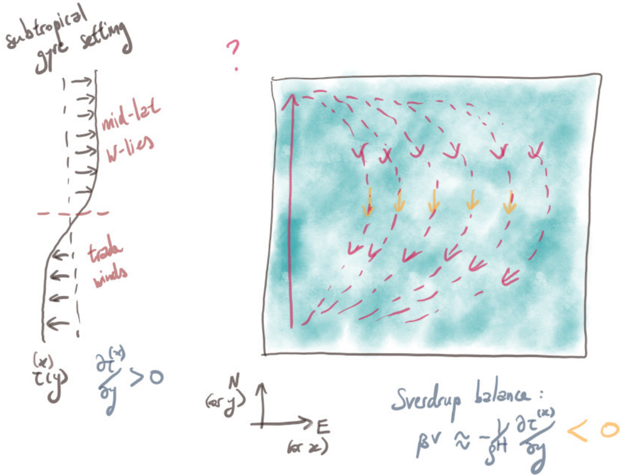


Figure: Schematic of wind-drive model (NH, assume subtropical wind profile, β-plane, square, homogeneous in density)

Single (subtropical) gyre: mass conservation

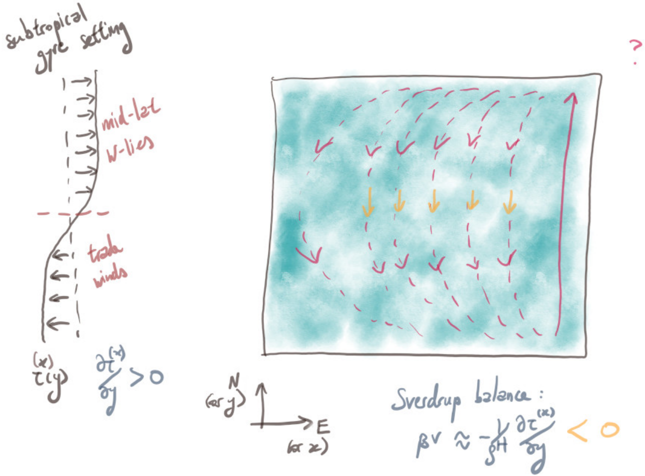


Figure: Schematic of wind-drive model (NH, assume subtropical wind profile, β-plane, square, homogeneous in density)

Single (subtropical) gyre: mass conservation

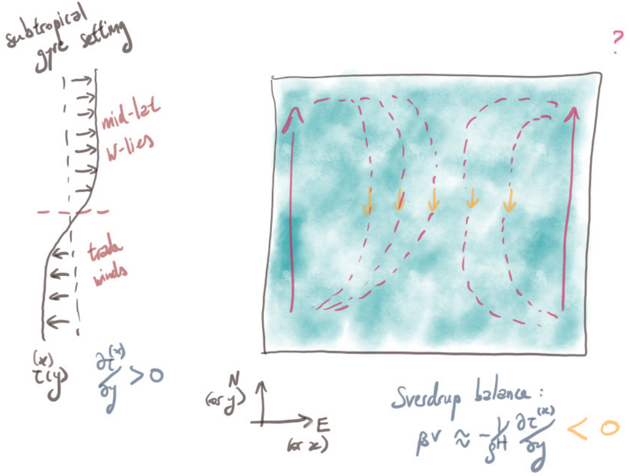


Figure: Schematic of wind-drive model (NH, assume subtropical wind profile, β-plane, square, homogeneous in density)

Double (subtropical + subpolar) gyre

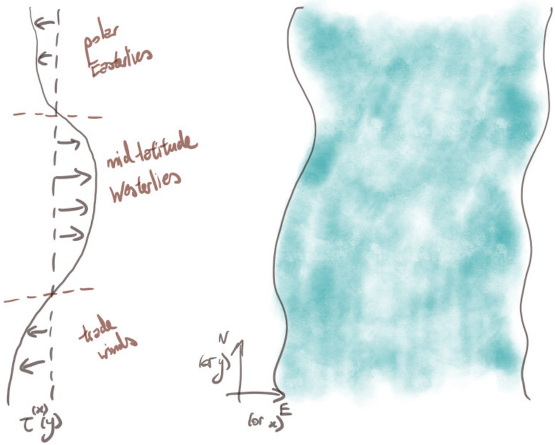


Figure: Schematic of wind-drive model (NH, β-plane, homogeneous in density)

Double (subtropical + subpolar) gyre

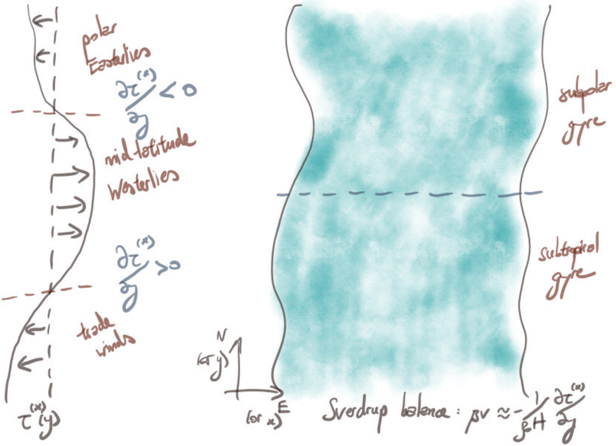


Figure: Schematic of wind-drive model (NH, β-plane, homogeneous in density)

Double (subtropical + subpolar) gyre

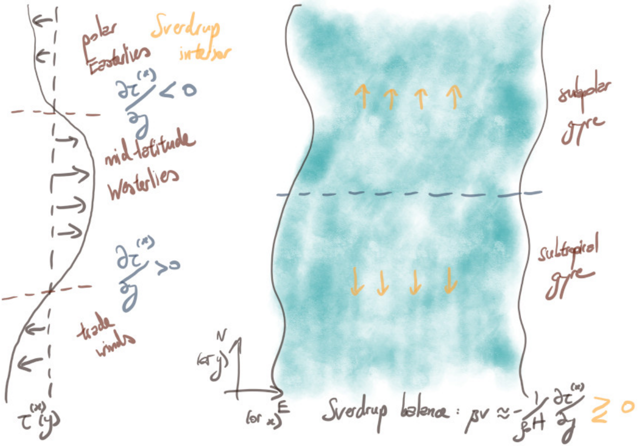


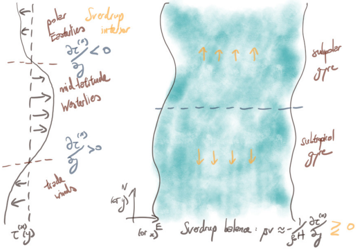
Figure: Schematic of wind-drive model (NH, β-plane, homogeneous in density)

Summary

- Sverdrup balance:

$$\beta v \sim F_\tau(x, y)$$

  $\rightarrow$ Sverdrup interior, meridional flow related to wind stress curl



- mass balance + continuity implies essentially two possibilities
- intuition: flow should be western intensified

  $\rightarrow$ energetic argument, flow should go in direction of wind

next lecture: support intuition by vorticity balance argument