OCES 2003 Assignment 3, Spring 2021

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Set on: Thur 21st Apr; due: Thur 28th Apr

Model solutions and mark scheme

Problems

1. (a) Combination of hydrostatic balance (vertical pressure gradient balanced by weight) and geostrophic balance (horizontal pressure gradient balanced by Coriolis effect), leading to a relation between the vertical flow shear and horizontal gradients in the density, obtained from eliminating the pressure.

   Applies when both of the balances hold, so would work for large aspect ratio and low Rossby number flows, which is satisfied on Earth for low Rossby number flows. What this says is that geostrophic flow tends to go hand-in-hand with horizontal gradients in the density.

   (1 mark for hydrostatic balance and geostrophic balance, 1 mark for vertical flow shear and horizontal density gradients (0.5 marks off if "shear" is left out), 1 mark for large-scale flows. Give some credit if any of the above are missing but equations are written out, or an example is given without explanation, or diagrams are drawn but explanation not explicit.)

   (b) Recalling that

   $$f\frac{\partial u_g}{\partial z} = \frac{g}{\rho_0}\frac{\partial \rho}{\partial y}, \qquad f\frac{\partial v_g}{\partial z} = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial x},$$

   so if f decreases the flow shear is actually going to increase in magnitude if the density profile doesn't change.

   (1 mark for f decreasing and 1 for flow shear increasing.)

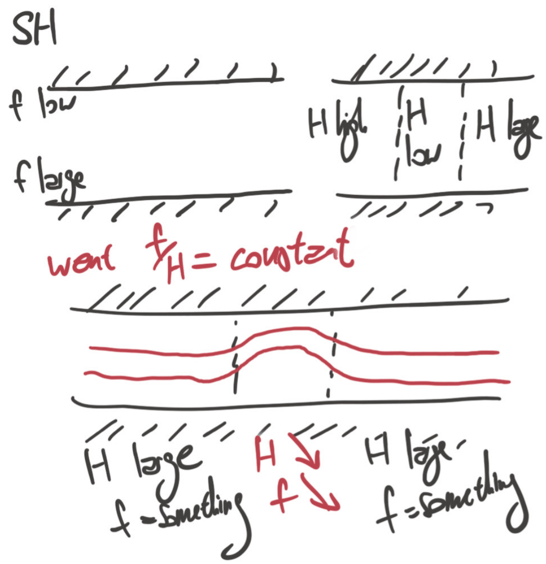
   (c) Both f and u are positive by convention. Since the question tells you you can assume $\partial u_g/\partial z =$ contant > 0 and the flow outside the WBC region is zero, $\partial\rho/\partial y$ should be positive in the WBC, so anything that is constant like outside of the boundary with a positive gradient going North will do it. However, the question asks for *temperature*, so the temperature should be large south of the WBC, decreasing in the WBC, and smaller in magnitude north of the WBC.

   (1 mark for positive f and u, 1 mark for some argument about the sign of $\partial\rho/\partial y$ and profile, and 1 mark for giving the right answer for the temperature. If the density part is skipped and the temperature part is correct, give full credit. Give partial credit if argument is vaguely sensible.)

2. (a) Assignment gives $U \sim 0.01\ \mathrm{m\ s^{-1}}$ and $L \sim 2\times10^6$ m, and I normally take $f \sim 10^{-4}$, so I make $U/L \sim 10^{-8} \ll 10^{-4} \sim |f_0|$. The first term is tiny by comparison so we can probably drop that term (this is generally a reasonable approximation for low Rossby number flows).

   (1 mark for some computation for U/L and some guess/quote of $|f_0|$ (references required if calculation is not explicit), and 1 mark for saying the first term is tiny compared to the second term.)

   (b) Pictorial argument as below (and is the same one given in assignment 3 last year):



(1 mark for noting f is larger towards the poles, 1 mark for H is low over the Drake passage, 1 mark for saying that if f/H is to be constant then a flow going into a smaller region of H needs to decrease f, in this case by going north. Give similar marks if diagram says basically the same thing.)

(c) If f/H is to be constant then

$$\frac{f_0 + \beta y_{\mathrm{before}}}{H_{\mathrm{before}}} = \frac{f_0 + \beta y_{\mathrm{after}}}{H_{\mathrm{after}}}.$$

By assumption I am going to take $y_{\mathrm{before}} = 0$, so rearranging and plugging in the values of H in I get

$$\frac{f_0}{2} - f_0 = \beta y_{\mathrm{after}},$$

and throwing the f_0 and β values in I make it 2.5×10^6 m, or 25 degrees with 1 degree = 100 km = 10^5 m.

(1 mark for setting up an equation using f/H, 1 mark for working towards the answer, 1 mark for the final answer.)

3. (a) This one should be a freebie, since you can just take the answer from my notes. A non-dispersive wave has the phase and group velocities being the same. The phase speed is when a single wave propagates, and the group speed is for the propagation of the envelope (and for non-dispersive waves these are the same.)

(1 mark for written definition and 1 mark for drawing the wave and the envelop propagating at the phase and group velocities. 0.5 marks if the latter part is given without drawing a picture. Give marks if the word wave packet is used instead of envelop; it should really be the latter, but I was sloppy in the notes...)

(b) I make the phase speed to be $c_p = \sqrt{40000} = 200$ m s^{-1}, and since $U = L/T$ the crossing time would be $T = L/U = 2 \times 10^7/(2 \times 10^2) = 10^5$ s, which is a 27.777... hours, or 28 hours rounded up.

(1 mark for working out the velocity, 1 mark for giving answers in days. Give 0.5 marks if answer not at the specified degree of accuracy.)

(c) This one requires playing around with the log function a bit. Taking $\log_e = \ln$, for the amplitude to drop by a factor of 10, what we have is

$$\frac{1}{10} \sim \mathrm{e}^{-y/L_d},$$

which upon taking the log of both sides gives

$$\log \frac{1}{10} \sim -\frac{y}{L_d} \quad \Rightarrow \quad y \sim L_d \log_e 10.$$

Here $L_d = 10^4 \times \sqrt{4000}$, so putting everything into a calculator I get 1456282 m, which I make to be 14.5 degrees with the above conversion, so would be 15 degrees to the specified degree of accuracy. (If you used the the 'wrong' log, most common being $\log_{10}$, then the answer comes out as 6 degrees.) *(1 mark for algebraic manipulations and 1 mark for giving the answer. Give 0.5 marks for answer not to the right degree of accuracy, or to 6 degrees is using the wrong log, but no marks if the choice of log and the degree of accuracy are both incorrect.)*