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OCES 2003 :
Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 10: Mechanical forcing 4
(diffusion and friction)

Outline

- energy + momentum
  → recap of concept
  → sources and sinks

- concept of diffusion and friction
  → inherently small-scale
  → largely driven by dynamics!
  → Reynolds/Ekman number (cf. Rossby number)
  → Ekman boundary layers revisited

Key terms: friction, diffusion, viscosity/diffusivity, boundary layers, Reynolds/Ekman number

Recap: equations of motion

Denoting $\boldsymbol{u} = (u, v)$ and $\boldsymbol{u}_3 = (u, v, w)$, to numerous approximations (!!!) (see OCES 3203) ocean dynamics is governed by

$$\rho_0 \left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} \right) = -\nabla p + \boldsymbol{F_u} + \boldsymbol{D_u} \tag{1}$$

$$\frac{\partial p}{\partial z} = -\rho g \tag{2}$$

$$\nabla \cdot \boldsymbol{u}_3 = 0 \tag{3}$$

$$\left(\frac{\partial T}{\partial t} + \boldsymbol{u}_3 \cdot \nabla T \right) = F_T + D_T \tag{4}$$

$$\left(\frac{\partial S}{\partial t} + \boldsymbol{u}_3 \cdot \nabla S \right) = F_S + D_S \tag{5}$$

$$\rho = \rho(T, S, p) \tag{6}$$

Respectively, (1) momentum equation, (2) hydrostatic balance, (3) incompressibility, (4) temperature equation, (5) salinity equation, and (6) equation of state (EOS)

Recap: wind forcing

Figure: Schematic of Ekman layer (boundary denoted by orange).

- wind a source of momentum for the ocean
- but influence has vertical limit

- direct influence only over the Ekman (boundary) layer
- difference in wind/current speed $\Rightarrow$ transfer of momentum ocean (usually into ocean and hence **source**; why?)

  $\rightarrow$ molecular diffusive rate $\Rightarrow$ very slow! (see next Lec.)

  $\rightarrow$ instabilities $\Rightarrow$ much faster (because on dynamical time-scales; see Lec. 17)

Wind forcing the chief source of momentum into ocean, but where is the sink? (vertical transfer in Lec 13)

Recap: wind forcing

- Ekman transport (mass flux) perpendicular to wind vector
- Ekman suction/pumping (i.e. up/downwelling) related to wind stress curl

$$w_e \sim \frac{1}{f} \boldsymbol{e}_z \cdot (\nabla \times \boldsymbol{\tau})$$

$\rightarrow$ related to fluid di/convergence $\nabla \cdot \boldsymbol{u}$

$\rightarrow$ analogous relations with vertical component of fluid vorticity $\omega = \boldsymbol{e}_z \cdot (\nabla \times \boldsymbol{u})$

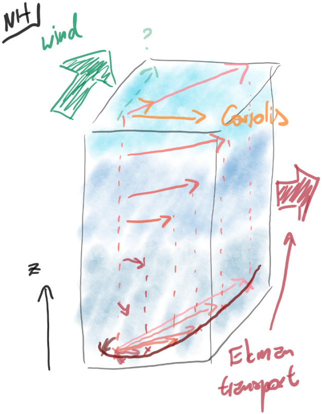

Figure: Schematic of Ekman spiral.

What sets Ekman layer depth?

Recap (?): momentum and energy

Recall that (Lec. 4)

- (linear) momentum $\boldsymbol{p} = m\boldsymbol{v}$ (units = ???)

  $\rightarrow$ here it will be $\boldsymbol{p} = \rho \boldsymbol{u}$
- Newton's law refers to **momentum** <u>not</u> **velocity**

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  $\rightarrow$ note this is a scalar (why?)

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dynamics $\leftrightarrow$ mass/momentum/energy transfer + conservation

Diffusion

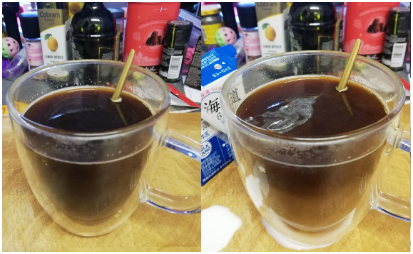


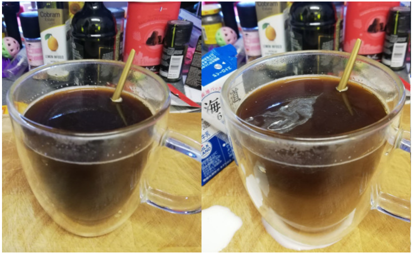
Figure: Making a mess of coffee + milk...

Beyond advection (more later), momentum, energy and other tracers (e.g. temperature, salinity, oxygen) can transfer by diffusion

- if no stirring then, milk will gradually **spread**

  $\rightarrow$ but very **slowly**!

Diffusion



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- tendency to spread = diffusion

  $\rightarrow$ diffusion aims to erase **gradients** of "stuff"

  $\rightarrow$ a **redistribution**, i.e. total amount of "stuff" **conserved**

Diffusion

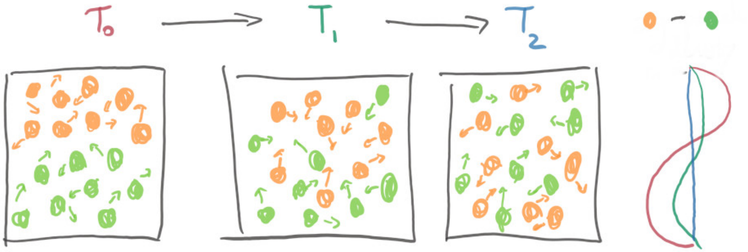

Figure: Schematic of microscopic motion leading to macroscopic diffusion. Here it is actual particle types, but can also imagine particles carrying "stuff", bumping into each other and transferring "stuff", and eventually the distribution of "stuff" evens out (but the total is conserved).

- at small-scale (!), "random" motion of particles (Brownian motion)
- collision ⇒ transfers ⇒ spreading

  → physics (!) ⇒ **gradient erasing** transfers more likely than **gradient increasing ones** (e.g. entropy)
- molecular diffusion, inherently **small-scale**

Diffusion

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- to a good approximation (!!!) momentum "spreading" can be treated as a diffusion
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- **molecular** diffusivities/viscosities (units of $\mathrm{m}^2\ \mathrm{s}^{-1}$) are substance dependent and **known**

Diffusion

material	κ_T	κ_S	ν
seawater	10^{-7}	10^{-9}	10^{-6}
air	10^{-5}	—	10^{-5}
honey	10^{-6}	—	10^{-2}
lava	10^{-7} (!)	—	depends, 10^{0}?
steel	10^{1}	—	big (!?)

Table: Table of kinematic molecular diffusivity/viscosity values at some control conditions. All numerical entries have units of $\mathrm{m}^2\ \mathrm{s}^{-1}$.

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- κ has units of $\mathrm{m}^2\ \mathrm{s}^{-1}$, define a diffusion time as

$$t_m = L^2/\kappa$$

$\rightarrow$ time t_m to travel distance L is transport is only by molecular diffusion with diffusivity κ

$\rightarrow$ for sea water, if $L = 100$ m (surface to thermocline say), temperature diffusion time is about **3000 yrs**! (homework)

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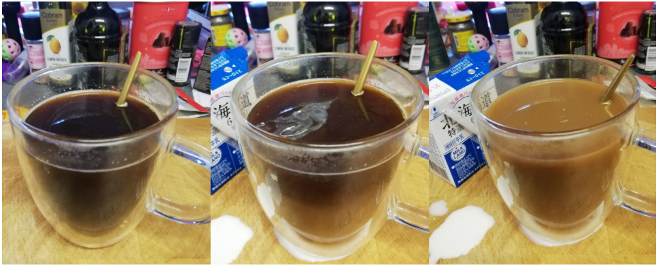

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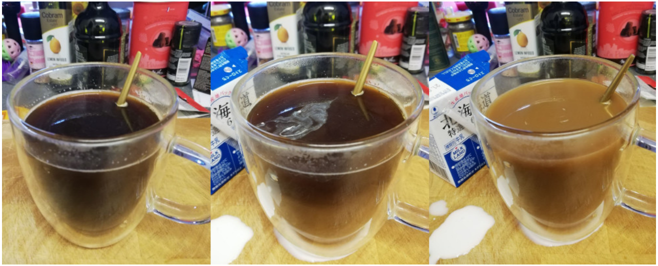

Figure: Making a mess of coffee + milk...

- stirring leads to larger-than-molecular transport, spreading and erasing gradients (!!! eventually anyway...)
  $\rightarrow$ quantify as an effective diffusivity $\kappa_e \gg \kappa_{\mathrm{molecular}}$

But we don't know κ_e, because it is dynamics dependent!

Diffusion

- resulting dynamic time-scales $t_d = \kappa_e \ll t_m$
  $\rightarrow$ e.g., for $\kappa_{e,z} = O(10^{-2}\ \mathrm{m}^2\ \mathrm{s}^{-1})$, then for example above t_d is about **10 days** ($\ll t_m = 3000$ yrs!)

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  $\rightarrow \kappa_{e,z} = 10^{-4} - 10^{-5}\ \mathrm{m^2\ s^{-1}}$

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Understanding dynamics fundamental in getting ocean transport/circulation right! (see Lec. 15 – 20)

Friction

Friction = resistance to **relative** motion

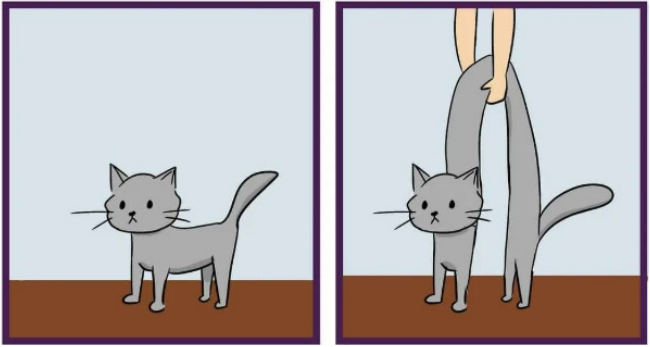

Figure: Cat physics. Picture from Meowingtons.

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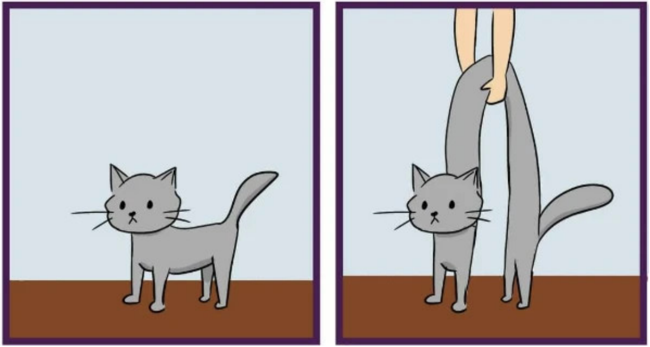

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- resistance to **relative** motion $\Rightarrow$ needs **motion**
  $\rightarrow$ function of velocity $\boldsymbol{u}$ (κ can sometimes be function of $\boldsymbol{u}$ too...)
  $\rightarrow$ e.g. linear drag $-r\boldsymbol{u}$ (moving a pig along the floor)
  $\rightarrow$ e.g. quadratic drag $-C_d|\boldsymbol{u}|\boldsymbol{u}$ (air resistance after a certain point)

Friction and diffusion

- internal stresses of object resisting relative motion

  $\rightarrow$ contributes to fluid viscosity (cats are viscous then?)

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Figure: Momentum diffusion and friction. Friction arises because there is relative motion, so it might look like there is momentum loss if we are looking only at the ocean as it's own system. But really what is happening is that the ocean is transferring momentum into the land, and from the ocean + earth system point of view there is no momentum lost.

Diffusion and friction

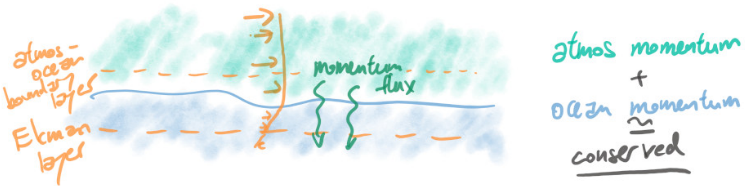

Figure: Same as above but for atmosphere and land. Atmosphere acts as a **source** of momentum for ocean most of the time (equivalently, the ocean acts as a **sink** of momentum for the atmosphere as well as a **drag** for the atmospheric winds).

- similar picture but between fluid of different density too

  $\rightarrow$ more details about interfacial + topographic form stress later (see Lec 13)

Diffusion and friction

Some key points:

- diffusion reduces gradients + variance of "stuff', but conserves "stuff"

  → e.g. viscosity reduces K and $\nabla \boldsymbol{p}$ (bit sloppy here with notation), but conserves total $\boldsymbol{p}$
- effective diffusion = molecular + dynamics, dynamics usually the most important contribution across the length-scales (see Reynolds number later)
- friction removes velocity differences (and therefore K)

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Fundamentally dependent on dynamics!

Boundary layers

Figure: Schematic of Ekman layer (boundary denoted by orange).

- momentum input by atmosphere via frictional stress

  $\rightarrow$ details actually complicated and current research topic

  (e.g. search for bulk formulas, mixed layer dynamics etc.)

Q. what sets the boundary layer extent?

Q. where/when is diffusion (and thus friction) important?

Non-dimensional numbers

Recall $\mathrm{Ro} = (U/L)/f$ is a measure of relative importance of advection and rotation

- $\mathrm{Ro} \ll 1$ means rotation important

  $\rightarrow$ could use it as "how small does L need to be for rotation to not matter" (cf. diffusion t_m above)

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- for general diffusion the Péclet number is defined as

$$\boxed{\mathrm{Pe} = \frac{UL}{\kappa}} = \frac{\text{advective transport}}{\text{diffusive transport}}$$

Reynolds + Ekman number

- when dealing with **viscosity** it is the Reynolds number

$$\boxed{\mathrm{Re} = \frac{UL}{\nu}}$$

$\rightarrow$ if $\mathrm{Re} \ll 1$ (e.g. L and/or U small, ν large) then **viscous** effects are important

- for rotating fluid systems the Ekman number

$$\boxed{\mathrm{Ek} = \frac{\nu}{fL^2}} = \frac{(U/L)/f}{UL/\nu} = \frac{\mathrm{Ro}}{\mathrm{Re}}$$

is usually the important one

$\rightarrow$ might see other conventions and analogues (e.g. plasma physics?)

Boundary layers

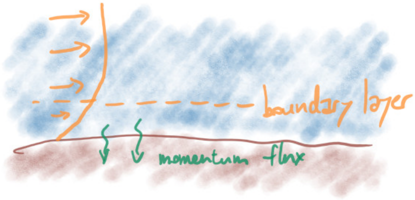

Figure: Schematic of a Prandtl boundary layer.

- boundary layer is where diffusive effects important, define it as e.g. $\mathrm{Re} \approx 1$, i.e.

$$\mathrm{Re} = \frac{UL}{\nu} \qquad \Rightarrow \qquad L_{bl} \sim \frac{\nu}{U}$$

$\rightarrow$ complication since U depends on L
$\rightarrow$ one standard solution has $L_{bl} \sim 1/\sqrt{\mathrm{Re}}$, i.e. boundary layer width **decreases** with increasing Reynolds number

Boundary layers

- similarly, the Ekman layer depth could be when $\mathrm{Ek} \approx 1$, i.e.

$$\mathrm{Ek} = \frac{\nu}{fL^2} \qquad \Rightarrow \qquad L_{ek} \sim \sqrt{\frac{\nu}{f}}$$

  $\rightarrow$ since (!) $f \sim 10^{-4}\ \mathrm{s}^{-1}$, if we take $\nu = \nu_{e,z} = 10^{-2}\ \mathrm{m}^2\ \mathrm{s}^{-1}$, we have

$$L_{ek} \sim \sqrt{\frac{10^{-2}}{10^{-4}}} = \sqrt{10^2} = 10\ \mathrm{m}$$

- if we use the molecular ν the Ekman layer depth as well as Prandtl boundary layer width will be tiny (homework)

  $\rightarrow$ $\mathrm{Re} \gg 1$ for geophysical flows

  $\rightarrow$ $\mathrm{Ek} \ll 1$ because ν and f small

Summary

- diffusion is **spreading** (**erasing** gradients)

  → **redistributing** things (variance decreasing but conserved total)

  → depends on magnitude of gradients and diffusivity κ

  → molecular vs. effective (latter dynamics dependent)

- friction resists **relative** motion

  → motion dependent

  → **sink/source** of momentum/energy of ocean (to land/atmosphere) (see Lec. 13)

- inter-system transfers through boundary layers

  → where Re and/or Ek $= O(1)$

  → these are usually **thin** (important consequences!)

another reason why dynamics is important!