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  `https://github.com/julianmak/academic-notes`

  The repository principally contains the compiled products rather than the source for size reasons.
- Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCES 2003 :
Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 11: Gyres 1 (overview + Sverdrup balance)

Outline

- (wind driven?) gyres and features

  → subtropical/subpolar gyres

  → Western Boundary Currents (WBCs) (e.g. Gulf stream, Kuroshio)
- β-plane + Sverdrup balance

  → simple model of wind-driven gyre (on β-plane)

  → wind balancing gradient of Coriolis
- depth-independent model with no topography: Sverdrup balance

  → interior dynamics

Key terms: subtropical/subpolar gyres, $f = f_0 + \beta y$, Sverdrup balance

Aim of these two lectures

combine material so far for a theory of wind-driven gyres

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plan of attack:

1. recall material on gyres (Lec 2)
2. construct model (wind, rotation, friction; Lec. 4, 7-10)
   $\rightarrow$ β-plane
   $\rightarrow$ assumption for density (but see next Lec.) (Lec. 5 + 6)
3. analyse and deduce via balance arguments
   $\rightarrow$ Sverdrup balance (wind + rotation; Lec. 8 + 9)
   $\rightarrow$ mass conservation (Lec. 4?)
   $\rightarrow$ vorticity balance (wind + friction; Lec. 9 + 10)

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"All models are wrong, but some are useful"

– attributed to George Box

Recap: gyres

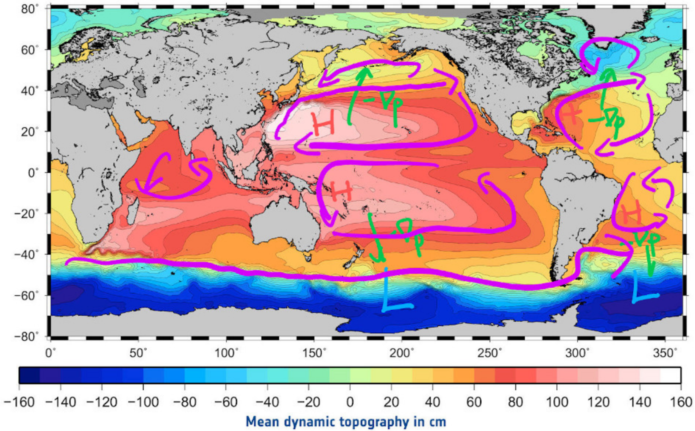


Figure: Time-mean global SSH (also called mean dynamic topography, with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al* (2011), J. Geophys. Res: Oceans.

- contours of SSH + geostrophic balance ⇒ flow, important part of MOC

Recap: gyres + WBCs

- subtropical and subpolar gyes (former shown here)

  → anti-cyclonic and cyclonic respectively (in both hemispheres) (see Lec 8 + 9)
- Western Boundary Current as a part of system

  → Gulf stream here, Kuroshio in Pacific

  → transports tropical + warm water towards high latitudes

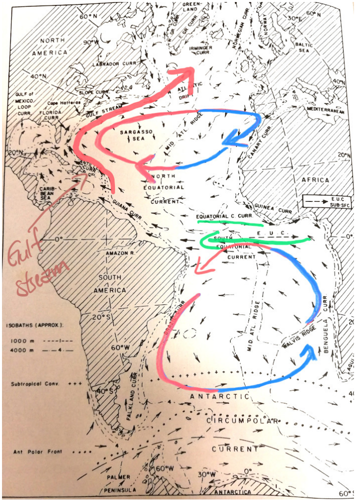


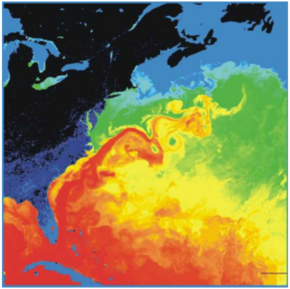
Figure: Features in the Atlantic Ocean. Modified Figure 7.9 from Pickard & Emery (1990), 5th edn.

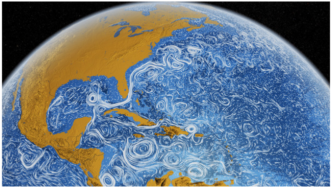
Recap: gyres + WBCs

Q. why Western and not Eastern? (see next Lec.)

Q. processes leading to eddies? (see Lec. 17)

Q. fluctuations + role in climate? (see Lec. 17 + OCES 4001)





Gulf stream in temperature (left) and surface current speed, from NASA

Recall: Coriolis effect and parameter (Lec. 8)

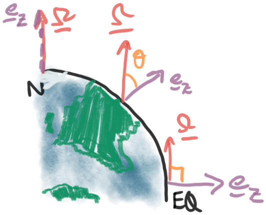


Figure: Mis-alignment of $\boldsymbol{\Omega}$ and $\boldsymbol{e}_z$ used locally for depth.

- for a spherical Earth we take rotation axis to be z-axis, i.e. $\boldsymbol{\Omega} = \Omega \boldsymbol{e}_z$ (this a vector), but locally, z is depth...
- introduce the latitudinally varying Coriolis parameter

$$f = 2\Omega \sin(\text{latitude})$$

- want to further simplify this, spherical (i.e. (lon, lat, depth)) to Cartesian geometry (i.e. (x, y, z)) (cf. Lec 8, when rationalising Coriolis effect)

β-plane

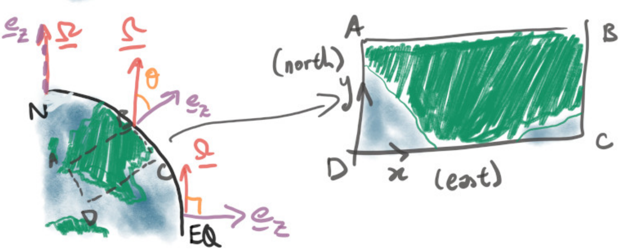


Figure: β-plane schematic. (lon, lat) $\rightarrow (x, y)$ with $f = f_0 + \beta y$ on the plane.

- $x \leftrightarrow$ lon, $y \leftrightarrow$ lat, $z \leftrightarrow$ depth
- $f = 2\Omega \sin(\text{lat}) \leftrightarrow \boxed{f = f_0 + \beta y}$

  $\rightarrow f_0$ (units: s^{-1}, same as f) the uniform rotation frequency

  $\rightarrow \beta = \partial f / \partial y$ (units: exercise)

  $\rightarrow$ **not** haline contraction (context should be clear)

will use β-plane extensively

Model preliminaries

For simplification, going to assume:

- NH β-plane ($f = f_0 + \beta y > 0$), domain is square
- subtropical wind profile (for subtropical gyre)

  $\rightarrow$ only for simplificty, will extend

  $\rightarrow$ note: negative wind stress curl
- assume depth-independece (either $\rho = \rho_0$ or vertically integrate)

  $\rightarrow$ sometimes barotropic (I don't like this term for technical reasons)
- lateral frictional boundary layers

  $\rightarrow$ main **sink** of stuff is going to be over boundary layers

Single (subtropical) gyre: schematic

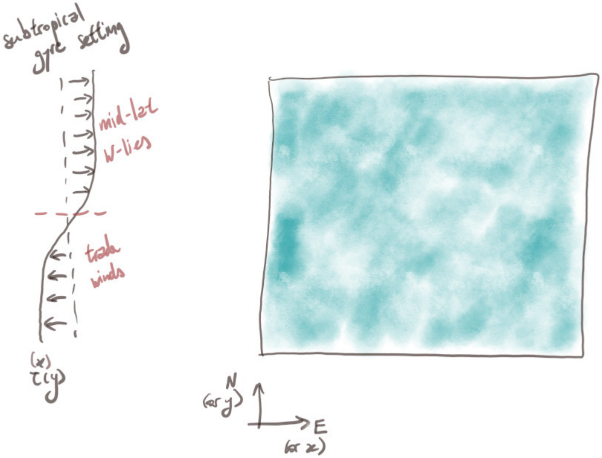


Figure: Schematic of wind-drive model (NH, assume subtropical wind profile, β-plane, square, homogeneous in density)

Equations

Original equations something like (in vertical vorticity ω):

$$\frac{\partial \omega}{\partial t} + \underbrace{\boldsymbol{u} \cdot \nabla \omega}_{\text{inertia}} + \underbrace{\beta v}_{\text{Coriolis}} = \underbrace{-r\omega}_{\text{drag}} + \underbrace{F_\tau(x, y)}_{\text{wind}}$$

- 2d equations in (x, y) (assumed no vertical variation)
- no pressure gradients (took a $\nabla \times$ of momentum equation)
- Coriolis effect appears through βv (as $f = f_0 + \beta y$)
  $\rightarrow$ it is **meridional** velocity v that shows up here
- $F_\tau = \boldsymbol{e}_z \cdot \nabla \times \boldsymbol{\tau}$ is wind stress curl
  $\rightarrow F_\tau < 0$ for wind profile in previous slide
- friction parameterised as linear drag
  $\rightarrow$ important near boundaries

Sverdrup balance

Throw away time derivative and inertia term (cf. Ro $\ll$ 1) gives

Stommel's model (Stommel, 1948, *Trans. Amer. Geophys. Union*)

$$\underbrace{r\omega}_{\text{drag}} + \underbrace{\beta v}_{\text{rotation}} = \underbrace{F_\tau(x,y)}_{\text{wind}}$$

Sverdrup balance

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$$\underbrace{r\omega}_{\text{drag}} + \underbrace{\beta v}_{\text{rotation}} = \underbrace{F_\tau(x,y)}_{\text{wind}}$$

In **interior** friction is unimportant, so we have Sverdrup balance

$$\beta v \sim F_\tau(x,y)$$

- Coriolis balancing wind stress curl

  $\rightarrow$ geostrophy is Coriolis balancing **pressure gradient** (but no pressure in vorticity equation)

$\beta > 0$ so v related to wind stress curl in interior

Note: The standard derivation involves looking at Ekman up/downwellings associated with the wind, then imply the Sverdrup interior

Single (subtropical) gyre: Sverdrup interior

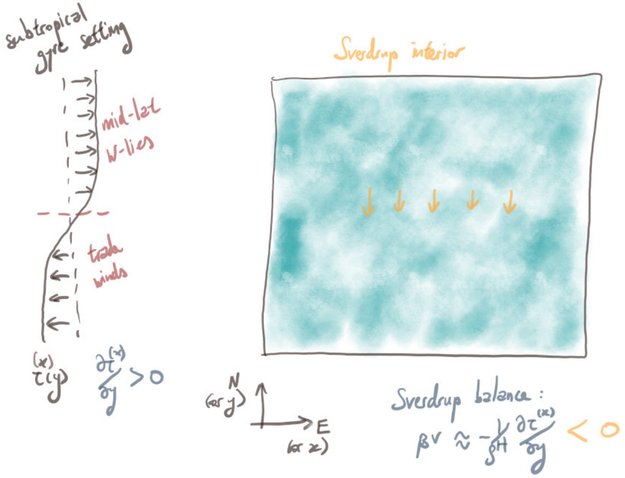
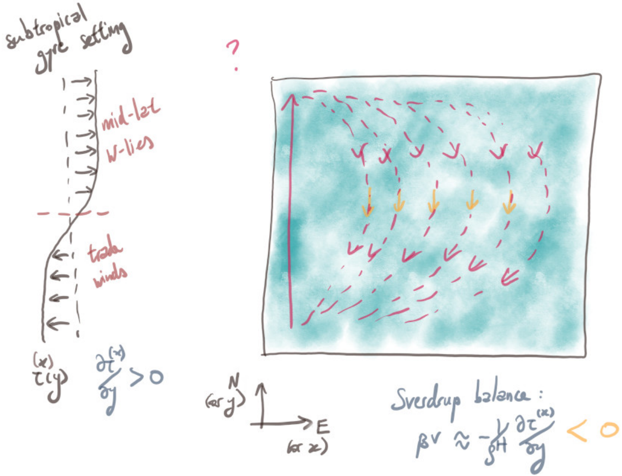


Figure: Schematic of wind-drive model (NH, assume subtropical wind profile, β-plane, square, homogeneous in density)

Single (subtropical) gyre: mass conservation



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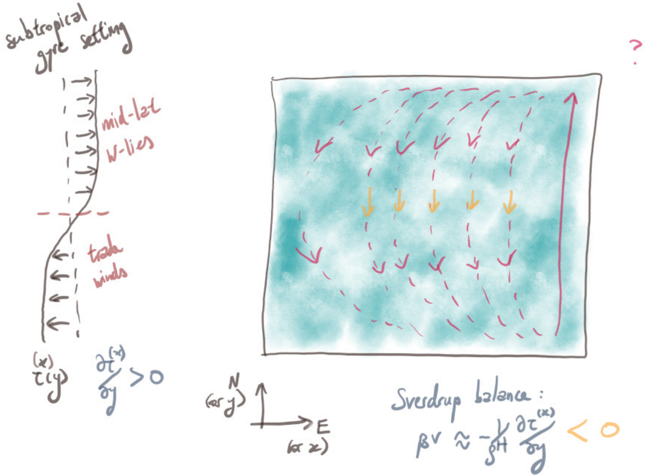

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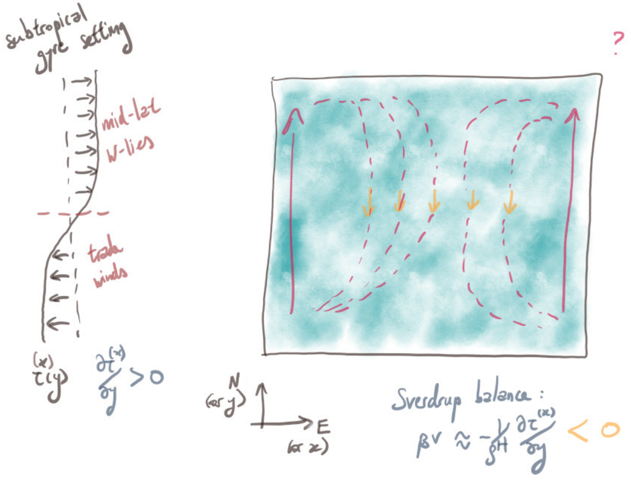

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Double (subtropical + subpolar) gyre

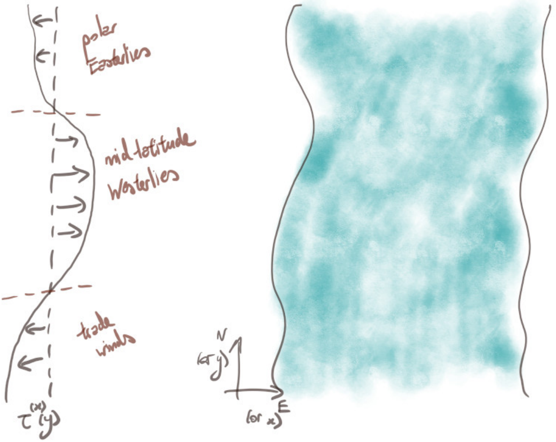

Figure: Schematic of wind-drive model (NH, β-plane, homogeneous in density)

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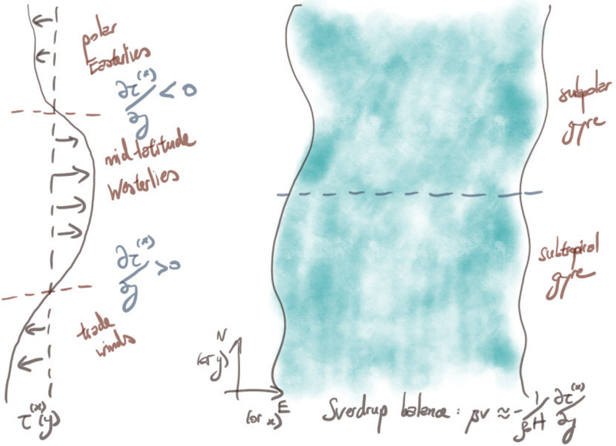

Figure: Schematic of wind-drive model (NH, β-plane, homogeneous in density)

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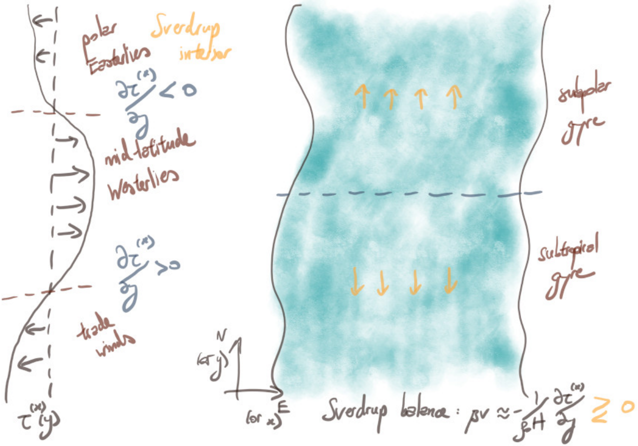


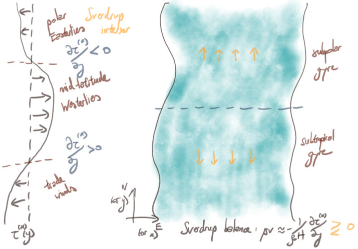
Figure: Schematic of wind-drive model (NH, β-plane, homogeneous in density)

Summary

- Sverdrup balance:

$$\beta v \sim F_\tau(x, y)$$

$\rightarrow$ Sverdrup interior,
meridional flow related to wind stress curl



- mass balance + continuity implies essentially two possibilities

- intuition: flow should be western intensified

  $\rightarrow$ energetic argument, flow should go in direction of wind

next lecture: support intuition by vorticity balance argument