OCES 2003 Assignment 2, Spring 2023

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Model solutions and mark scheme

Problems

1. (a) For ACC, $U = 0.01$ m s^{-1}, $L = 10000$ km $= 10^7$ m (want the extent in the along-stream direction), so $\mathrm{Ro} = 10^{-2}/(10^7 \times 10^{-4}) = 10^{-3}$, and geostrophic balance holds.

   (b) For gyre flow away from the WBC, $U = 0.01$ m s^{-1}, $L = 1000$ km $= 10^6$ m, so $\mathrm{Ro} = 10^{-2}/(10^6 \times 10^{-4}) = 10^{-2}$, and geostrophic balance should hold (could take L larger).

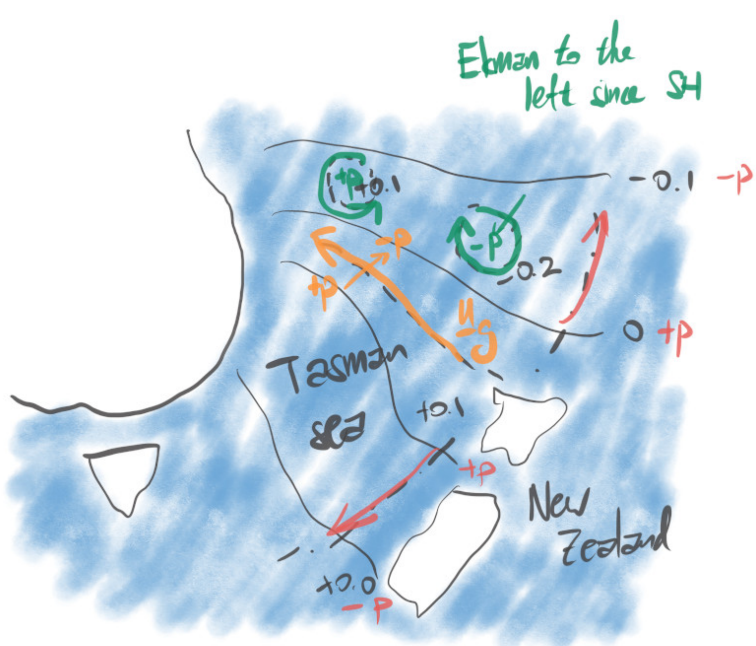
   (c) For the Gulf Stream, $U = 0.1$ m s^{-1}, $L = 1000$ km $= 10^6$ m (want the extent in the along-stream direction), so $\mathrm{Ro} = 10^{-1}/(10^6 \times 10^{-4}) = 10^{-1}$. If $U = 1$ m s^{-1} then $\mathrm{Ro} = 1$. Geostrophic balance should hold. (Could take L smaller, then we are getting close to the $\mathrm{Ro} = 1$ regime.)

   (d) For a river, $U = 1$ m s^{-1}, and we are probably talking $L = 10$ km $= 10^4$ m (probably less), so $\mathrm{Ro} = 10^0/(10^4 \times 10^{-4}) = 10$, so we are rotationally influenced maybe.

   (e) For a big toilet, $U = 1$ m s^{-1} and $L = 1$ m, $\mathrm{Ro} = 10^0/(10^0 \times 10^{-4}) = 10^4$, so rotational effects are negligible (and so the claim about how water flushes the other way in different hemispheres has very little dynamics basis; the geometry of the toilet bowl probably has a stronger influence).

   (1 mark for each sensible estimate, differences of about an order of magnitude is fine. Take 0.5 marks off if choices of scale are not sensible, take another 0.5 mark off if whether geostrophic balance holds is inconsistent with the given estimate of the Rossby number.)

2. This is recycled from the midterm of 20/21, so I am going to be lazy and not make a new drawing. The model answer for the midterm 20/21 was:



But FOR THIS ASSIGNMENT, there was an explicit mention that we are still in the Southern Hemisphere but **the Earth is rotating the other way round**. The arrows associated with $-\nabla p$ are not affected, but the arrows associated with the **flow** should be FLIPPED in orientation.

(a) (red lines) isobars are related to SSH so $-\nabla p$ points towards the edge of the drawing

(b) (green lines) geostrophic transport is to the RIGHT since we are in the southern hemisphere but Earth is rotating the other way round (so double negatives), and since the +0.1 eddy is a high-pressure, while the -0.2 is a low pressure since we are in the region where the background SSH is between 0 and 0.1, so the associated flow is CLOCKWISE and ANTI-CLOCKWISE respectively.

(c) (orange lines) within the +0.1 and 0 contour, $-\nabla p$ is pointing north, so since the geostrophic flow is to the RIGHT, the geostrophic flow is pointing OUT OF Australia.

(2 marks each for each part, but only if the justification is correct.)

3. (a) Equator is at 0° so $f = 0$ there.
(1 mark)

(b) A purely meridional velocity is pointing in the same direction as $\mathbf{\Omega}$, so $2\mathbf{\Omega} \times \mathbf{u}$ is zero by definition. The vector that is both perpendicular to the zonal velocity and meridionally pointing $\mathbf{\Omega}$ is radial (in or out of the Earth), so has no horizontal component. There is thus no horizontal Coriolis effect on a horizontal velocity at the equator.

(1 mark each for the three explanations. Accept mathematical demonstration if provided. Give some sympathy marks / partial credit if definition of cross product is mentioned.)

(c) Simply because w is in generally much smaller than u and v throughout the globe, and since $2\mathbf{\Omega} \times \mathbf{u}$ scales like the velocity, the Coriolis effect arising from a vertical component is going to be tiny.

(1 mark for w small relative to u, v, and 1 mark for Coriolis effect scaling like the velocity. Give some sympathy marks if reasoning incomplete but citations used; the citation needs to support the reasoning.)

(d) If $u > 0$ the velocity is pointing East, then $2\mathbf{\Omega} \times \mathbf{u}$ by the right-hand-screw convention is pointing *into the Earth*. In the radial / locally vertical direction we have gravity and buoyancy forces, so that's going to swamp this contribution of the Coriolis effect if the fluid is stratified.

(1 mark for Coriolis effect pointing into Earth, one for buoyancy dominating. Accept mathematical explanations and/or gravity dominating. Give some sympathy marks if reasoning incomplete but citations used; the citation needs to support the reasoning.)

(e) From the previous two parts, the traditional approximation might not be great when w is significant, and in unstratified cases where buoyancy might not dominate. It might also be a bit dubious if we are not in the shallow fluid regime, as hydrostatic balance is then not a great approximation (formally the traditional approximation and the shallow approximation are separate, but they are invoked together for consistency reasons). These can occur for example when associated with convective events, or strong vertically propagating waves. Such cases are slightly less common in the ocean, but occur in atmosphere, and in astrophysical examples (e.g. some planets and stars are known to have deep convection zones, which is generally unstratified or only very weakly stratified).

The paper by Gerkema *et al.* (2008) in Review of Geophysics is pretty comprehensive about the traditional approximation. More recent works include those of

- Colin de Verdière (2012), Journal of Physical Oceanography
- Tort *et al.* (2016), Journal of Fluid Mechanics
- Yano (2017), Journal of Fluid Mechanics

- Zeitlin (2018), Physics of Fluids.

(1 mark for w large, 1 mark for weak or unstratified mediums, 1 mark for slowly rotating bodies. Give some sympathy marks if reasoning incomplete but citations used; the citation needs to support the reasoning.)

!? My incomplete understanding is that θ is not very conservative as it is being moved around by the flow, and is itself not a very good measure of 'heat', which causes problems when we want to talk about the energy cycles in the Earth system. Θ on the other hand is an internally consistent thermodynamic variable (related to *potential enthalpy*), which is much more conservative, and is a much more appropriate variable to measure 'heat', which is needed for working out things like *ocean heat uptake* for example. Θ is the variable being advocated to be used as the international standard (TEOS-10).