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The repository principally contains the compiled products rather than the source for size reasons.

- ▶ Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 18: Dynamics 4 (tides)

Outlook of the next few lectures

Dynamics important

- ▶ **waves** (this Lec. + 16, 18) and **instabilities** (Lec. 17)

→ because waves are easier to talk about without maths...

Highlight gross features (i.e. those that can be drawn...)

- ▶ how to describe waves (Lec. 15)
- ▶ types of waves (Lec. 16)
 - consequence + leading to instabilities
- ▶ instabilities (Lec. 17)
 - **parcel**-type (mechanistic) arguments for instability
- ▶ tides (particularly as **internal gravity waves**) (Lec. 18)

Outline

► tides

→ as deformation of sea surface arising from gravitational attraction **only** (rotation **leads** to evolution but does not **partake** in force balance)

→ observations and features

→ frequency + period

→ forcing

→ propagation

► internal tides (basically internal gravity waves)

→ dynamics and transfer

→ statistics (Garrett–Munk spectrum)

Key terms: high/low/spring/neap tides, tidal forcing, internal tides, wavenumber and length-scales Garrett–Munk spectrum

Recap: shear instabilities and diapycnal mixing (Lec. 17)



Figure: Schematic of mixing by (irreversible) wave breaking, with contours reconnecting leading to e.g. diapycnal mixing.

- ▶ growing disturbance, **instability**
→ mixing of material **across** isopycnals after reconnection, i.e. **diapycnal mixing**
- ▶ shear instabilities can be interpreted as a **constructive interference** of waves
→ key role of **vorticity** and induced velocity

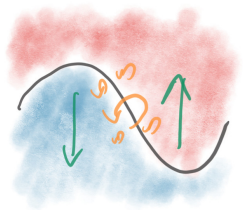


Figure: Velocity shear from waves can lead to mixing.

Recap: internal gravity waves (Lec. 16)

Recall that since $N \gg f$ on Earth, so **internal inertia-gravity waves** satisfy

$$\omega \approx \pm \sqrt{f^2 + \frac{N^2 k_x^2}{k_z^2}} \quad (\text{for } |k_z| \gg |k_x|)$$

- internal **gravity waves** **influenced by rotation**
 - main restoring force is **gravity**, with influences from Coriolis effect
 - remember $|f| \leq |\omega| \leq |N|$

Recap: internal gravity waves (Lec. 16)

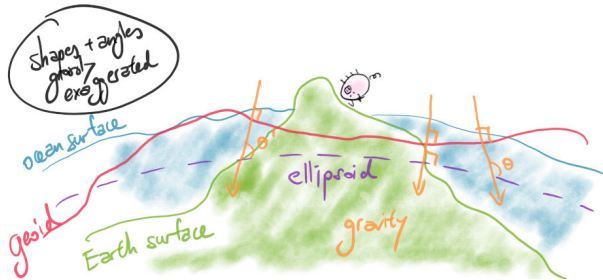
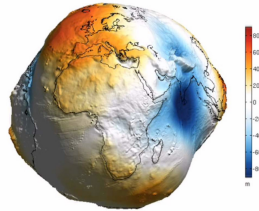
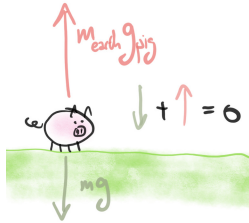
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internal tides = internal gravity waves with tidal forcing

Recap: gravity and geoid (Lec. 7)



Tides: observations



Figure: Tides at (top) St. Michael Mount in Cornwall (near Penzance), and Holy Island causeway (Lindisfarne) in Northumberland, UK. Images by Michael Marten (<http://www.michaelmarten.com/>), image taken from (twistedsifter.com).

Tides: observations



Figure: High (or flood) and low (or ebb) tide at Tobermory, Isle of Mull, Scotland, using the pastel pink and red house as references. Modified images from www.thechaoticscot.com (left) and from myself (right).

- tidal signal most obvious in coastal sea level
 - but generally in SSH η (recall Lec. 7)
 - η/H largest near coasts (because of $H \ll 1$)

Tides: observations

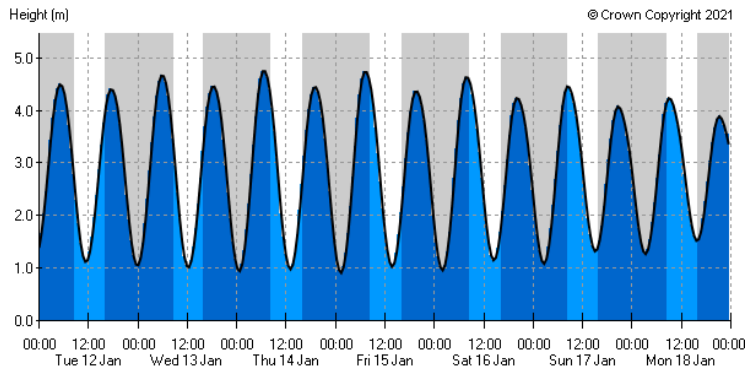


Figure: Predicted (!) tidal charts at Tobermory, Isle of Mull, Scotland between over a period of seven days. Taken from <http://www.ukho.gov.uk>.

- high and low tide at around 12 hr periods
→ longer term variability (cf. wave packets, Lec. 15)

Tides: observations

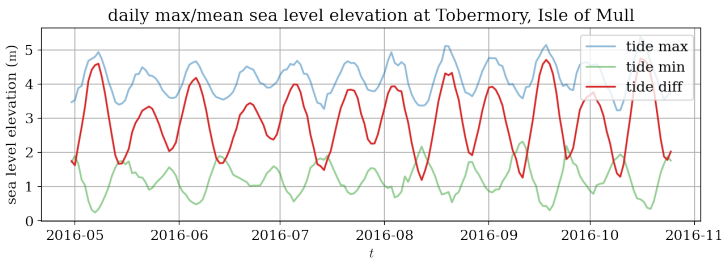


Figure: Daily maximum and minimum sea surface elevation (blue and green) and their difference (red) over a six month period. Data from BODC, see `tobermory_tides.ipynb`.

- ▶ longer term variability
- ▶ oscillation of around **two weeks** between max and min difference

→ **spring** and **neap tides**

(spring not to do with the season, more like the boing-boing mechanical spring)

Tides: observations

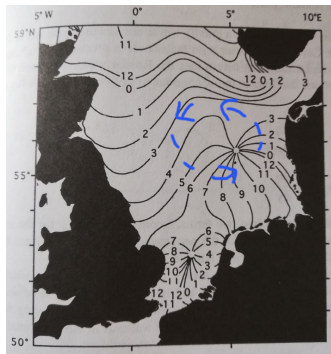


Figure: Phase lines (denoting peak of SSH, in hours in GMT) of the semi-diurnal tidal signal. Notice the anti-clockwise propagation (with boundary to the right). Diagram modified from Knauss (1997), Fig. 10.5.

- ▶ tides over the whole globe
→ wave-like structure
- ▶ propagation characterised by **phase lines** (cf. c_p in Lec. 15)
→ **anti-clockwise** propagation in this NH example (cf. **cyclonic**)
→ note signal propagates with the boundary to the **right** of it
→ **Coriolis effect** again?

<https://www.youtube.com/watch?v=u6ZNnGnc-cQ>

Tides: forcing

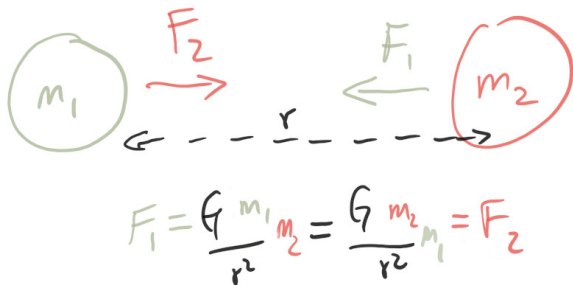


Figure: Schematic of gravitational attraction for two masses, from Lec. 7.

- driving and restoring force being gravity, what objects drive the tides then?

Esoteric pop culture interlude

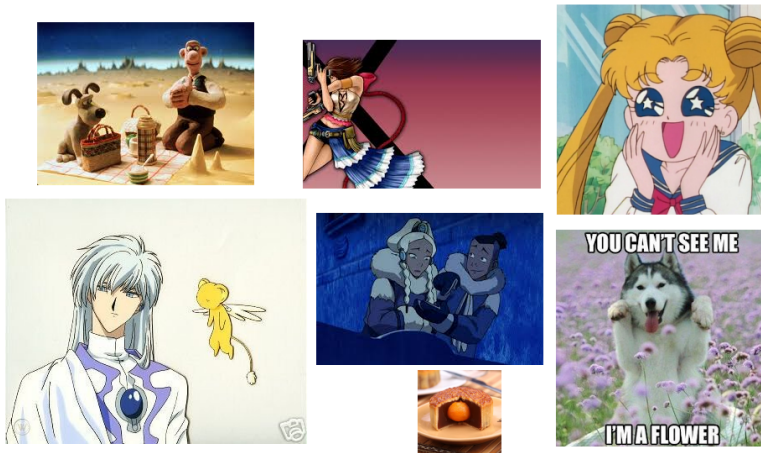


Figure: What is the thing that unites all of these? Bonus: can you name all of these? (Copyright probably with Aardman studios, Square Enix, Naoko Takeuchi + Toei animation, Clamp, Nickelodeon, and two not sures).

Tides: forcing

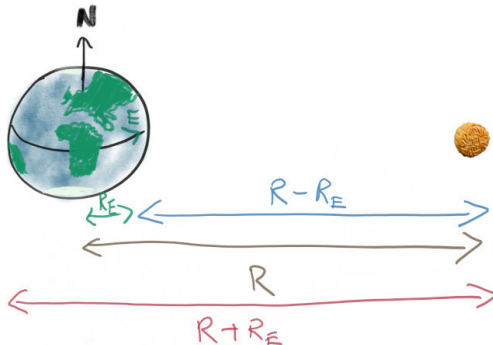


Figure: Schematic of tidal forcing by an astronomical body. Assume instantaneous response (“equilibrium theory”). No rotation is assumed here.

► define some distances

Tides: forcing

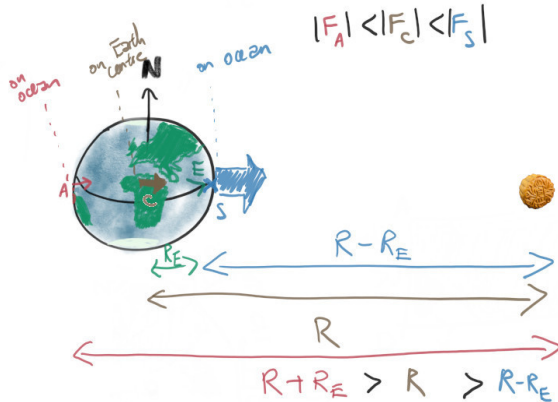


Figure: Schematic of tidal forcing by an astronomical body. Assume instantaneous response (“equilibrium theory”). No rotation is assumed here.

- differences in gravitational force because distances

Tides: forcing

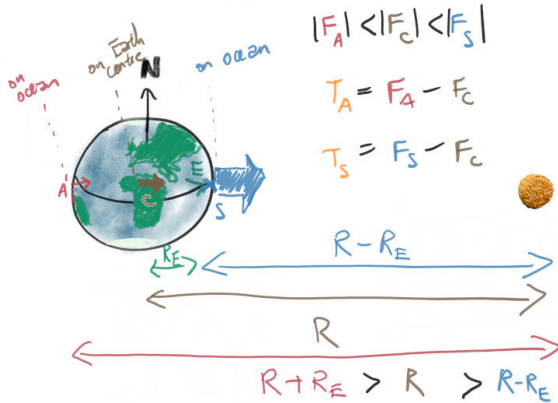


Figure: Schematic of tidal forcing by an astronomical body. Assume instantaneous response (“equilibrium theory”). No rotation is assumed here.

- **tide generating force** = force on ocean - force on center

Tides: forcing

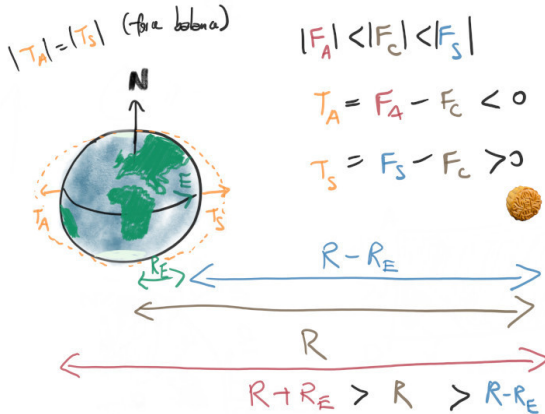


Figure: Schematic of tidal forcing by an astronomical body. Assume instantaneous response ("equilibrium theory"). No rotation is assumed here.

- force balance, equal and opposite, pile water \Rightarrow two bulges

Tides: forcing

- ▶ Earth rotates with period 24 hours, one bulge when moon overhead and one when on far side
→ i.e. **semi-diurnal** signal
- ▶ above apply equally to the Sun too
→ note while $M_{\text{Sun}} \gg M_{\text{moon}}$, $R_{\text{Sun-Earth}} \gg R_{\text{moon-Earth}}$, so since tidal acceleration is roughly

$$a_{\text{tide}} \sim \frac{M}{r^3},$$

magnitude of Solar influence turns out to only be about half as strong (homework: show this explicitly)

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magnitude of Solar influence turns out to only be about half as strong (homework: show this explicitly)

- !!! above is for **equilibrium** theory, **dynamic** theory absolutely cares about rotation because then we need to care about wave propagation

Tides: forcing

symbol	period (in solar hrs)	rel. amp (to M_2)	name
M_2	12.42	1	principal lunar (semi-diurnal)
K_1	23.93	0.58	luni-solar (diurnal)
S_2	12.00	0.47	principal solar (semi-diurnal)
O_1	25.82	0.42	principal lunar (diurnal)
N_2	12.66	0.19	larger lunar elliptic (semi-diurnal)
\vdots	\vdots	\vdots	\vdots
Mf	327.85 (≈ 14 days)	0.09	lunar fortnightly
Mm	661.30 (≈ 28 days)	0.05	lunar monthly
SSa	4382.86	0.04	solar semi-annual

Table: Some sample tidal forcings sorted by relative amplitude to the M_2 tide (which is the largest forcing for Earth). Subset of Table 6.2 given in Wunsch (2015). The last few entries are weak and long term but they are there.

- ▶ M_2 and K_1 the dominant ones
→ usually do include these two in **numerical models**
- ▶ notice the periods are close to multiples of 12 hours

Tides: springs and neaps



Figure: Schematic of spring and neap tides.

- **spring tide**: max difference in highs and lows

Tides: springs and neaps

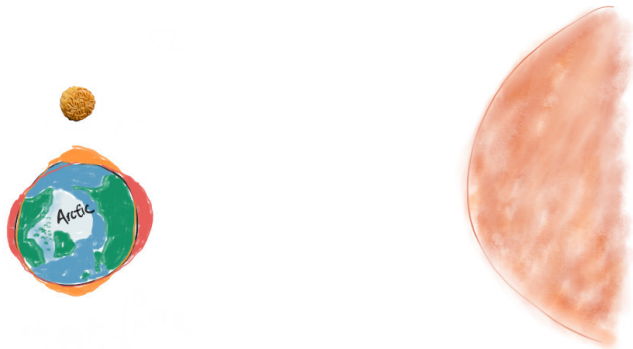


Figure: Schematic of spring and neap tides.

- moon(cake) and Sun in quadrature

Tides: springs and neaps



Figure: Schematic of spring and neap tides.

- **neap tide**: min difference in highs and lows

Tides: springs and neaps

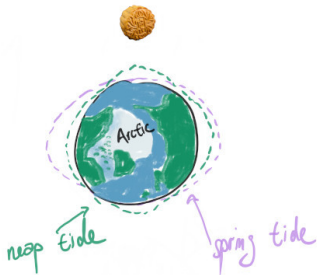


Figure: Schematic of spring and neap tides.

- occurs every two weeks or so (in line with lunar rotation)

Tides: propagation + Kelvin waves

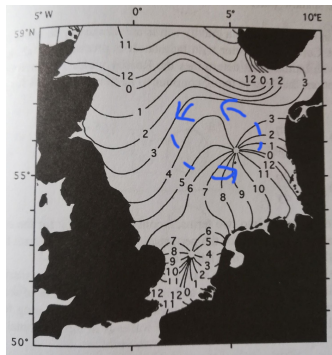


Figure: Phase lines (denoting peak of SSH, in hours in GMT) of the **semi-diurnal** tidal signal. Notice the anti-clockwise propagation (with boundary to the right). Diagram modified from Knauss (1997), Fig. 10.5.

- ▶ anti-clockwise signal in NH from SSH from **Kelvin waves** (see Lec 16)
 - Kelvin waves propagate **cyclonically**
 - disturbance from tide generating force
- ▶ fast propagation (because gravity wave speed)
- ▶ non-dispersive
- ▶ decays over **Rossby deformation radius**
- ▶ cares about rotation
 - **dynamic theory of tides**

Internal tides: generation

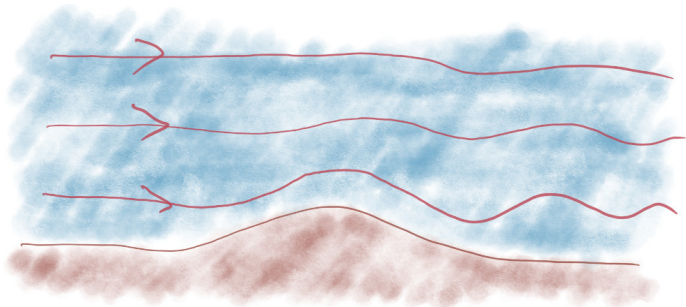


Figure: Flow over topography (e.g. tidal motion) leading to wave generation.

- ▶ flow over topography can lead to wave generation
→ **internal gravity waves** (Lec. 16)
- ▶ repeated “sloshing” by tide generating force
→ period of “sloshing” (e.g. M_2 is around 12 hrs)

Internal tides: decreasing in scale

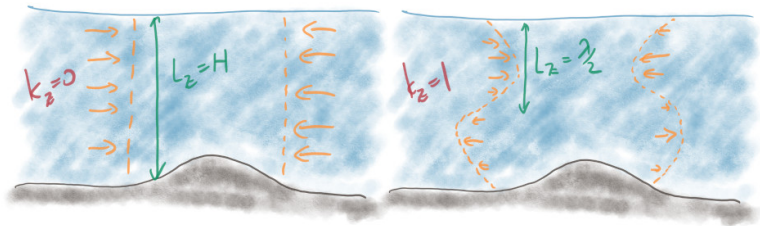


Figure: Motion associated to theoretical $k_z = 0$ and $k_z = 1$ (with boundary as ocean surface). Note associated with k_z is a length-scale.

- ▶ tidal forcing is to first approximation uniform over depth of ocean
→ think $k_z = 0$
(commonly barotropic but I don't like "barotropic" used like this...)
- ▶ generating waves of increasing vertical variation (barotropic conversion, but, again...)
→ e.g. $k_z = 1$, increase in frequency, **decrease** in vertical wavelength or vertical scale

Internal tides: breaking

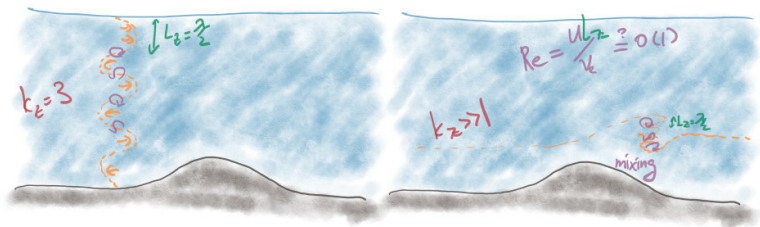


Figure: Motion associated to theoretical $k_z = 3$ and a more realistic case with nominal $k_z \gg 1$, via the observation that k_z can be linked to a length-scale.

- ▶ instabilities (Lec. 17) lead to more small-scales
→ e.g. k_z keeps increasing, so vertical scale keeps decreasing...
- ▶ eventually a dynamical Re (Lec. 10) starts becoming important
→ reconnecting isopycnals, **diapycnal mixing** (Lec. 10, 14, 16, 17)

Internal tides: statistics

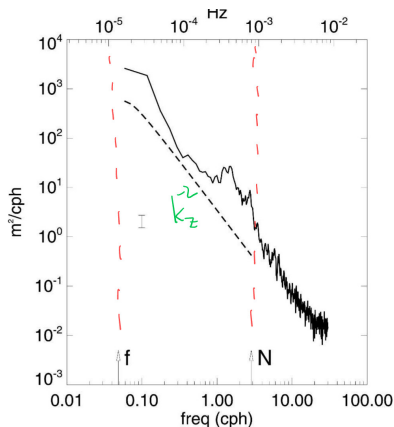


Figure: Observation of ocean displacement as a power spectrum, with the Garrett-Munk wavenumber (so frequency) spectrum dependence put in. Note also the frequency boundaries marked on since internal gravity waves should satisfy $f < \omega < N$. From Stevens *et al.* (2005), *J. Phys. Oceanogr.*, modified from their Fig. 3.

- general characteristics of internal waves?
→ **Garrett-Munk spectrum**

(Garrett & Munk, 1972, *Geophys. Fluid. Dyn.*)

$$E \sim \omega^{-2} \sim k_z^{-2} \quad (\omega \gg f)$$

- universal behaviour?
- initially fit to observations, but can derive from first principles?

→ e.g. **wave turbulence formalism**

(way beyond here; see works by Victor L'vov or book by Sergey Nazarenko)

Summary

- ▶ **tides** as deformation arising solely from gravitational attraction (Lec. 4 + 7)
 - **tide generating force** as a differential force
- ▶ signal particularly in SSH (Lec. 7), but also can generate **internal waves** (Lec. 16)
 - low mode forcing to high mode motion (decreasing length scale) (Lec. 15)
 - **Garrett–Munk spectrum**, $E \sim k_z^{-2}$
 - wave breaking (Lec. 17) lead to **diapycnal mixing** (Lec. 10), feedback onto **MOC** (Lec. 14)

Summary

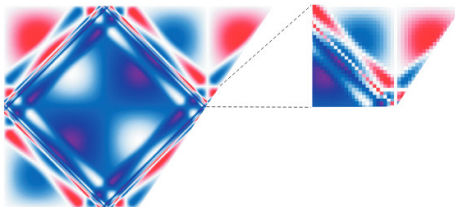


Figure: Internal gravity wave attractors in a uniformly stratified channel. Image from Maas (2005), *Int. J. Bifurcat. Chaos*, reworking of Maas *et al.* (1997), *Nature*.

- ▶ what do waves do when they break?
- ▶ where do wave break?
- ▶ how do waves break?
- ▶ how to model the above?
- ▶ implications for overall system?

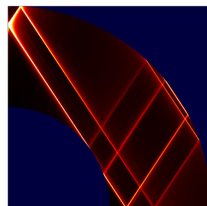
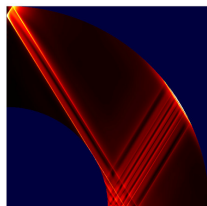


Figure: Inertial wave attractors in a homogeneous planetary interior at different tidal forcings. From Gordon Ogilvie (2009, *Mon. Not. Royal. Astro. Soc.*).