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OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 8: Mechanical forcing 2 (rotation/Coriolis)

Outline

- ▶ rotation of Earth, **Coriolis “force”** (recall from OCES 2001)
 - rotation axis
 - consequences for flow
 - **Rossby number** + **geostrophic balance**
- ▶ **thermal wind balance**
 - hydrostatic (vertical) + geostrophic (horizontal) balance
 - SSH anomaly example revisited (see Lec. 6 + 7)

Key terms: Coriolis “force”, Rossby number, geostrophic balance/flow, thermal wind (balance)

Recap: equations of motion

Denoting $\mathbf{u} = (u, v)$ and $\mathbf{u}_3 = (u, v, w)$, to numerous approximations (!!!) (see OCES 3203) ocean dynamics is governed by

$$\rho_0 \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} \right) = -\nabla p + F_u + D_u \quad (1)$$

$$\frac{\partial p}{\partial z} = -\rho g \quad (2)$$

$$\nabla \cdot \mathbf{u}_3 = 0 \quad (3)$$

$$\left(\frac{\partial T}{\partial t} + \mathbf{u}_3 \cdot \nabla T \right) = F_T + D_T \quad (4)$$

$$\left(\frac{\partial S}{\partial t} + \mathbf{u}_3 \cdot \nabla S \right) = F_S + D_S \quad (5)$$

$$\rho = \rho(T, S, p) \quad (6)$$

Respectively, (1) momentum equation, (2) hydrostatic balance, (3) incompressibility, (4) temperature equation, (5) salinity equation, and (6) equation of state (EOS)

Recap: hydrostatic pressure

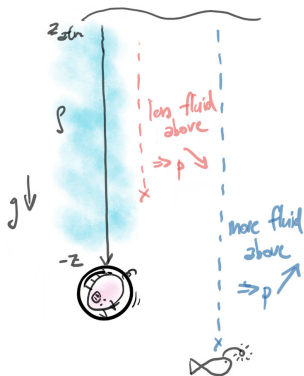


Figure: Schematic of hydrostatic pressure

- **hydrostatic approximation:**
pressure **approximately equal** to weight above when static
→ **weight** is $F = mg$ so for force balance,

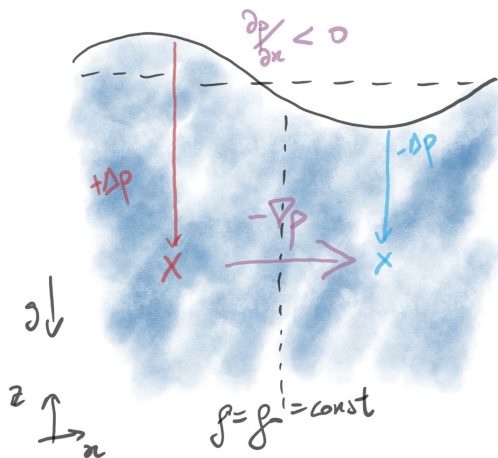
$$F = mg = g \int_{-z}^{z_{atm}} \rho \, dz = p ,$$

with $g \approx 9.81 \, \text{m s}^{-2}$

→ if $\rho = \text{const}$ then $p = \rho g z + p_{atm}$

$$\frac{\partial p}{\partial z} = -\rho g$$

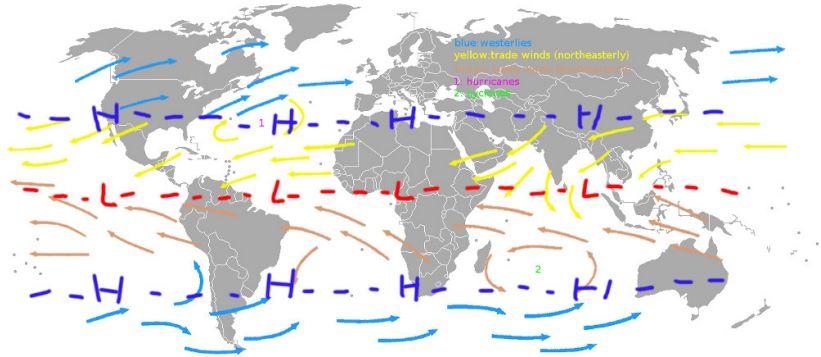
Recap: pressure gradients and flows



- ▶ assuming hydrostatic balance, water moves from $+\Delta p$ to $-\Delta p$ because there is a **net force** (negative pressure gradient $-\nabla p$)
→ important for **geostrophic flows**

Figure: Horizontal effect because of hydrostatic pressure.

Geostrophic flows: atmosphere



Winds do not go direct from high to low p ? (more on wind patterns next Lec.)

Geostrophic flows: atmosphere

日期/Date: 14.10.2020 香港時間/HK Time: 14:00 香港天文台 Hong Kong Observatory

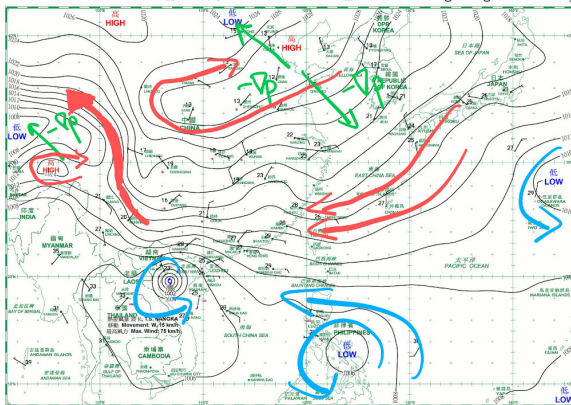


Figure: Atmospheric weather chart with isobars (in units of hPa = 100 Pa = 1 mbar) and wind directions. From HKO.

- note that flow doesn't go in the direction of $-\nabla p!$
→ **along** rather than **across** isobars (Coriolis effect, see next Lec.)

Geostrophic flows: ocean

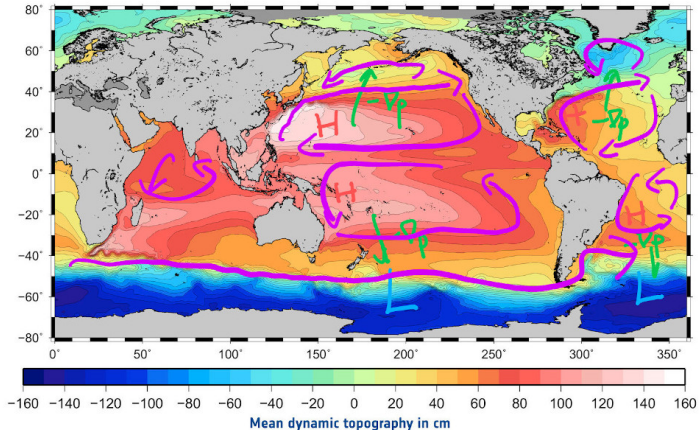


Figure: Time-mean global SSH (also called **mean dynamic topography**, with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al* (2011), J. Geophys. Res: Oceans.

- contours of SSH related to isobars via **hydrostatic balance**
 - flow is **along** rather than **across** isobars (**Coriolis effect**, see next Lec.)

Coriolis effect

The Earth rotates around the **rotation axis**

- ▶ the **geographical North** (as opposed to magnetic north)
- ▶ **rate** of rotation is the **angular frequency** Ω (units: s^{-1}), with

$$\Omega = \frac{2\pi}{T}$$

→ T is the **period** (see again in Lec. 15 - 18), time needed to do one rotation (2π radians or 360°)

→ for Earth (units!)

$$\Omega = \frac{2\pi}{3600 \times 24} \approx 7.29 \times 10^{-5} \text{ s}^{-1}$$

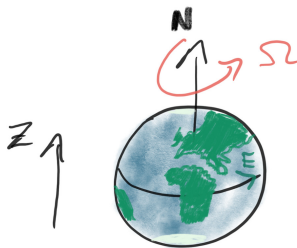


Figure: Rotation axis and angular frequency Ω .

Coriolis effect: co-ordinates

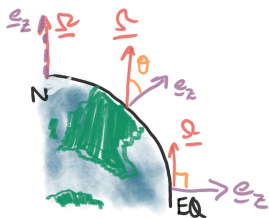


Figure: Mis-alignment of Ω and e_z used locally for depth.

- ▶ for a spherical Earth we take rotation axis to be z-axis, i.e. $\Omega = \Omega e_z$ (this a vector), but locally, z is depth...
- ▶ introduce the latitudinally varying **Coriolis parameter**

$$f = 2\Omega \sin(\text{latitude})$$

- ▶ to take into account of mis-alignments between Ω and the local e_z for depth
 - Coriolis = $-2\Omega \times u$ (global case, z is North)
 - Coriolis $-f e_z \times u$ (local case, z is depth) (mostly going to use this one)

Frame of reference

(sort of) a demonstration of a **frame of reference**

- ▶ one could work in global picture (with Ω and z being North) or local picture (with f and z being depth)
→ change in point-of-view, **co-ordinate system**

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→ e.g. tracking change in meaning of z vs. remembering f varies in latitude

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- ▶ underlying physics should still be the **same**, although description might **look** different (e.g. Ω vs. $f\mathbf{e}_z$)
- ▶ **freedom in choice of frame!**
→ e.g. frame rotating with the planet, others...
→ fine as long as we keep **consistency**

Coriolis effect

You can think of it as a “hack”

- ▶ Newton's laws are formulated for **inertial frames**
→ non-accelerating frames

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can't use Newton's laws then?

- ▶ “Hack”: put in a **fictitious forces** to compensate the fact we are not in an inertial frame
→ can then treat non-inertial frames **as if it were inertial**
→ proceed as normal with that caveat

Coriolis effect

Earth's **spin** has a major influence on large-scale winds

- ▶ extra “force” from the spinning: **Coriolis** “force” $2\boldsymbol{\Omega} \times \boldsymbol{u}$

(Gaspard-Gustave de Coriolis, 1792-1843, French mathematician)

→ apparent deflection \Leftrightarrow a net force on it

- ▶ apparent “force” because not being in **inertial frame**
 - can think of it arising only because of perspective...
 - note Coriolis “force” does **no work** (see assignment)

- ▶ try it yourself! (seriously this really helped me...)

→ on a piece of paper, try drawing a straight line while rotating the paper underneath

→ something similar but on e.g. a basketball

Coriolis effect

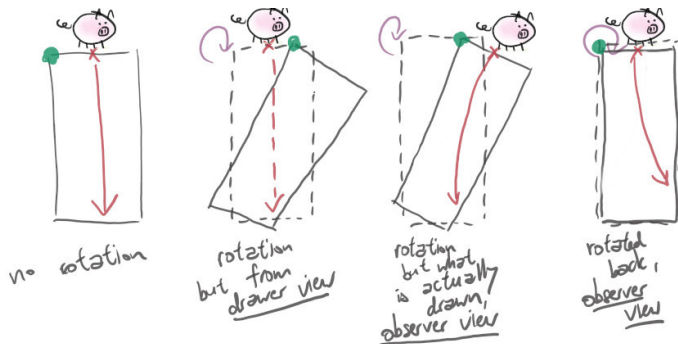


Figure: Schematic of apparent deflection from Coriolis "force". From the drawer perspective, the drawer is doing a straight line and sees a straight line. By from observer's perspective, there is a deflection. The action is the same, but to describe it from the observer's point of view, we need to additionally describe this apparently deflection arising from the system's rotation.

- on a piece of paper, try drawing a straight line while rotating the paper underneath

Coriolis effect

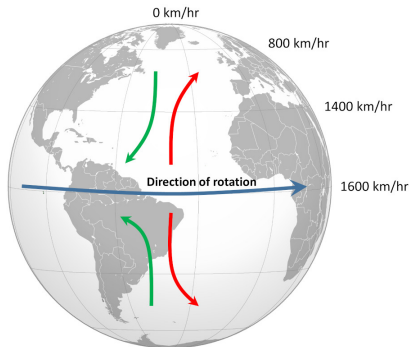


Figure: Schematic of apparent deflection from Coriolis “force”, from Vallis (2011).

- ▶ apparent deflection **to the right** in NH
→ looking down from North Pole, rotating **anti-clockwise**
- ▶ apparent deflection **to the left** in SH
→ looking down from South Pole, now rotating **clockwise**

Rossby number

Note there is a competition between **fluid velocity** (intended path) and **rotation**

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- ▶ matter of **time-scales**

- if fluid moves quickly relative to system spinning (**inertial period**), then Coriolis influence small

- if fluid moves relatively slowly, more time for Coriolis effect to act

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► measured by the **Rossby number**: for $f = 2\Omega \sin(\text{latitude})$,
(Carl-Gustav Rossby, 1898-1957, Swedish meteorologist)

$$\text{Ro} = \frac{U/L}{f} = \frac{1/T}{f} = \frac{\text{advection}}{\text{rotation}}$$

→ note f decreases to zero at EQ (mis-alignment of rotation axis, recall $-2\mathbf{\Omega} \times \mathbf{u}$)

Rossby number

$$\text{Ro} = \frac{U/L}{f} = \frac{1/T}{f} = \frac{\text{advection}}{\text{rotation}}$$

- For large-scale motion in Earth's atmosphere in mid-lats (say 50°N) and $\Omega = 2\pi/\text{day}$,

$$\text{Ro} = \frac{10 \text{ m s}^{-1}/1000 \text{ km}}{2 \times 2\pi \text{ day}^{-1} \times \sin(50^\circ)} \approx \frac{10^1 \times 10^{-6}}{10^{-4}} = 0.1,$$

so rotational effects dominant

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→ very fast flows but also very large length scales

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- ▶ $\text{Ro} \sim O(0.01)$ for Jupiter <https://www.youtube.com/watch?v=exkeE6aIlpc>
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- ▶ flows in the interior of the Sun, $\text{Ro} = O(1)$
→ Sun not spinning too fast, rotationally influenced

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- ? what about in a toilet bowl? (see assignment)

Geostrophic flow

After **non-dimensionalisation** (see OCES 3301 or ask me), momentum equation is (other numbers appear for forcing + dissipation, assume small; see Lec 9 + 10)

$$\text{Ro}\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) + \rho 2\boldsymbol{\Omega} \times \mathbf{u} = -\nabla p + \dots$$

If $\text{Ro} \ll 1$, dominant force balance is:

$$2\boldsymbol{\Omega} \times \mathbf{u}_g = -\frac{1}{\rho} \nabla p$$

► geostrophic balance

→ given p and $\boldsymbol{\Omega}$, \mathbf{u}_g (the **geostrophic flow**) has to be **something** so resulting forces balance

what is the implied velocity \mathbf{u} then?

Geostrophic flow: rationalisation

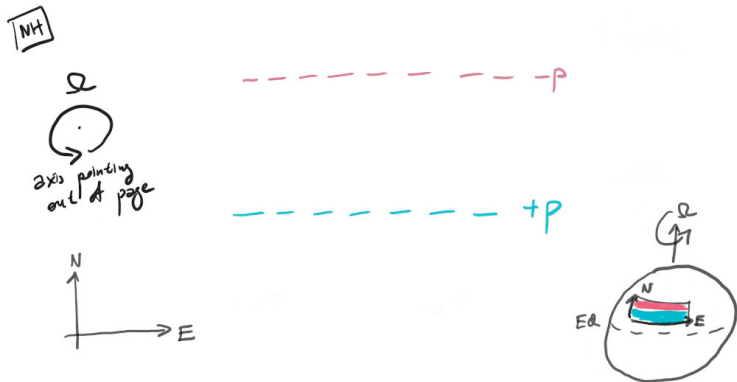


Figure: Geostrophic balance and resulting geostrophic flow u_g in Northern Hemisphere. Note u_g is **along** (rather than **across**) isobars.

- in NH top-down view, rotation is **anti**-clockwise

Geostrophic flow: rationalisation

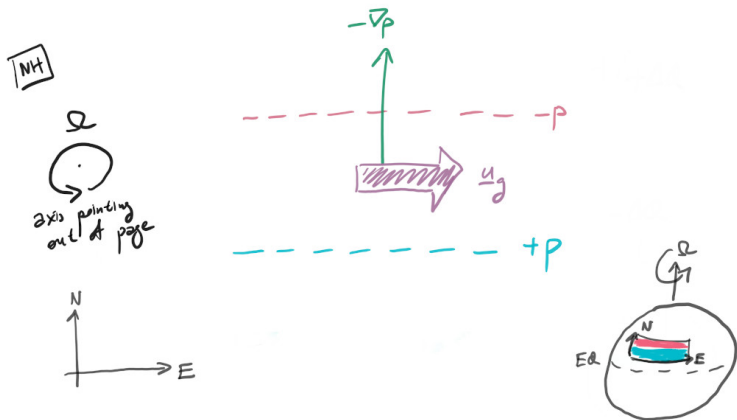


Figure: Geostrophic balance and resulting geostrophic flow u_g in Northern Hemisphere. Note u_g is **along** (rather than **across**) isobars.

- in NH, deflection to the **right** of $-\nabla p$

Geostrophic flow: rationalisation

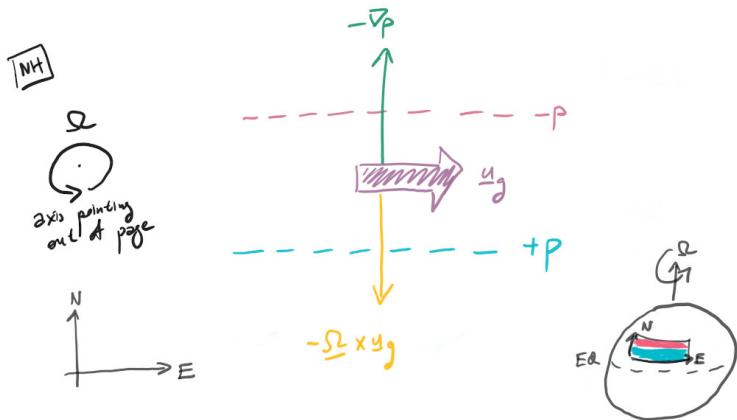
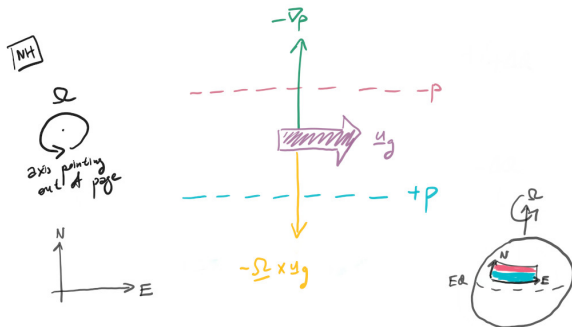


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- for **force balance**, Coriolis has to be opposite of $-\nabla p$

Geostrophic flow: rationalisation



- ▶ or, really, **because** of force balance, u_g has to be to the right of $-\nabla p$ in NH
- ▶ $\Omega \sim +e_z$ (Earth NH), $-\nabla p \sim e_y$, so

$$-2\Omega \times u = -\frac{1}{\rho} \nabla p \quad \Rightarrow \quad -e_z \times u_g \sim e_y \quad \text{or} \quad u_g \times e_z \sim e_y,$$

so $u \sim +e_x$ only possibility, i.e. to the E (right of N is E)

Geostrophic flow: rationalisation

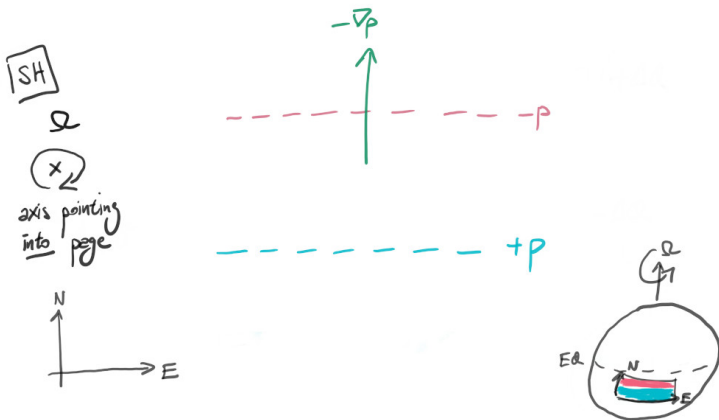


Figure: Geostrophic balance and resulting geostrophic flow u_g in Southern Hemisphere. Note u_g is **along** (rather than **across**) isobars.

- in SH top-down view, rotation is **clockwise**

Geostrophic flow: rationalisation

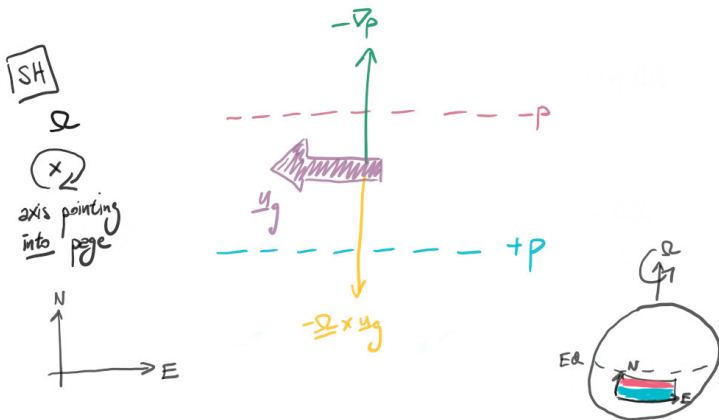


Figure: Geostrophic balance and resulting geostrophic flow u_g in Southern Hemisphere. Note u_g is **along** (rather than **across**) isobars.

- u_g to the **left** of $-\nabla p$ (same arguments as but $\Omega \rightarrow -\Omega$)

Geostrophic flow

Suppose hydrostatic as well as geostrophic balance:

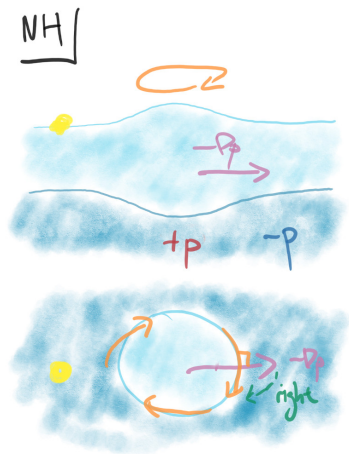


Figure: Schematic for an anti-cyclonic (warm core) eddy.

- ▶ bulge ($+\Delta h$) so $+\Delta p$ in the centre
 $\rightarrow -\nabla p$ points **away** from region
- ▶ **geostrophic current** u_g to the **right** of $-\nabla p$ (since NH)
 \Rightarrow **clockwise** around bulge
- ▶ **opposite** sense to planet rotation f (in NH),
anti-cyclonic eddy (in NH)
 \rightarrow other direction in SH (since $f < 0$)

Geostrophic flow

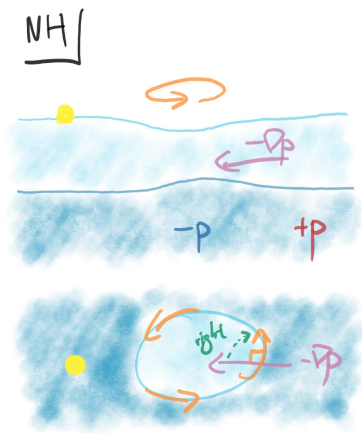


Figure: Schematic for a cyclonic (cold core) eddy.

- ▶ depression ($-\Delta h$) so $-\Delta p$ in the centre
→ $-\nabla p$ points **into** region
- ▶ **geostrophic current** u_g to the **right** of $-\nabla p$ (since NH)
⇒ **anti-clockwise** around depression
- ▶ **same** sense as rotation of planet f (in NH), **cyclonic eddy** (in NH)
→ other direction in SH (since $f < 0$)

Geostrophic flow

Above was for NH, for SH:

- ▶ pressure and $-\nabla p$ still related to hydrostatic balance
- ▶ deflection is to the **left** (cf. opposite rotation sense)
- ▶ u_g is now
 - anti-clockwise around bulge
 - clockwise around depression
- ▶ BUT!
 - bulges are still **anti-cyclonic**
 - depressions are still **cyclonic**

Geostrophic flow

Above was for NH, for SH:

- ▶ pressure and $-\nabla p$ still related to hydrostatic balance
- ▶ deflection is to the **left** (cf. opposite rotation sense)
- ▶ u_g is now
 - anti-clockwise around bulge
 - clockwise around depression
- ▶ BUT!
 - bulges are still **anti-cyclonic**
 - depressions are still **cyclonic**
- ▶ cf. atmosphere, low pressures \leftrightarrow depressions \leftrightarrow cyclonic
 - in atmosphere low pressures are **convergence zones**, related to unsettled weather (and vice-versa in high pressures) (relation to **Ekman up/downwelling** next Lec.)

Geostrophic flow from SSH

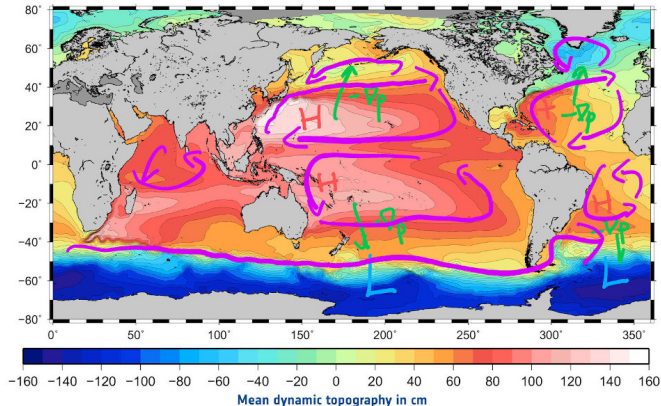


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 - flow is **along** rather than **across** isobars (**Coriolis effect**, see next Lec.)

Geostrophic flow from SSH

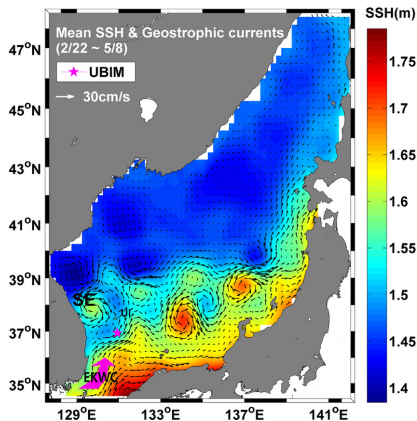


Figure: SSH and inferred currents from AVISO satellite altimeter data. Figure 4 of Son *et al.* (2014), *Biogeosciences*.

- ▶ observed SSH from **AVISO** (see Lec. 20) + inferred geostrophic velocities near Japan
→ anti-cyclonic around positive SSH anomalies (anti-clockwise in NH)
- ▶ note current is strongest where gradients are large (the arrow lengths)

Summary

geostrophic flow goes to the right of pressure gradient in NH

- ▶ flip this in SH (because rotation is “reversed”)
 - again, **Coriolis effect is frame dependent**, i.e. depend on your point of view, a **pseudo-force**
 - Coriolis force does **no work** (cf. Lec 5 + 6; see assignment)
- ▶ **anti-cyclonic** eddies \sim bulges in SSH (in SH and NH)
 - rotation same sense as planet, $\nabla \times \mathbf{u} \sim f$
 - clockwise in NH (because $f > 0$), reverse in SH
- ▶ **cyclonic** eddies \sim depressions in SSH (in SH and NH)
 - rotation opposite sense as planet, $\nabla \times \mathbf{u} \sim -f$
 - anti-clockwise in NH (because $f > 0$), reverse in SH

Summary

Turns out the deflection aspect is also (largely) true for wind forced flows (see next Lec.):

Ekman transport is to the right of the wind in NH (flip in SH)

→ wind drives flow at surface in direction of wind...

→ but flow needs to turn 90° (or $\pm\pi/2$) at depth?

► Ekman spirals