OCES 2003 Assignment 3, Spring 2021

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Set on: Tue 13$^{\text{th}}$ Apr; due: Tue 20$^{\text{th}}$ Apr

Blurb

- Assignments have a maximum mark out of 20, although you will see that there are 22 marks available to get in total, i.e. if you get 22/20 you still only get credit for 20/20
  - 16-17 is roughly around the A- boundary
  - anything below 8 is probably a fail
- Please show working in calculation
  - no working + wrong answer = no credit whatsoever
  - some working + wrong answer = partial credit
  - generically, give things to 2 decimal place and provide the appropriate units (marks are allocated for these), unless otherwise specified
- No answers except the 'hard' ones should need more than a paragraph / half a page, and excess answers that are not to the point will be penalised
- Type up the assignment or send a photo of your written up work in (the former is preferred), and the only request I have is no Microsoft Word documents (you can type up things with Word but export it as a pdf if you do)
  - write in full sentences where appropriate
  - particularly poor and/or scrappy presentation will have a mark that can be taken off
- There will be a rigid mark scheme, and model solutions will be available in due course
  - the TAs only mark the stuff, you should come to the instructor for arguing marks, and note the re-marking can result in marks going up or down

!!! By handing something in, you agree to the usual Academic Honour code and Integrity declarations. For more, see `http://qa.ust.hk/aos/academic_integrity.html`. Cases for plagiarism (whether intended or not, it is the "act" that matters) gets a penalty ranging from

- zero on the question concerned
- a fixed penalty starting from around 1/3 of the total marks
- zero for the whole assignment/midterm/final

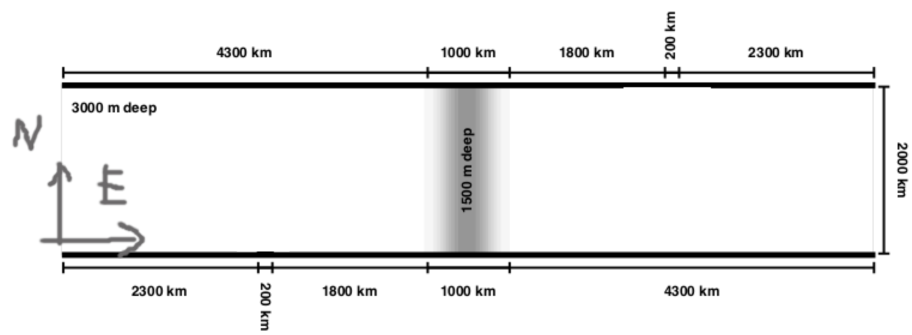
The following counts as plagiarism (and is a non-exhaustive list):

- copying word for word *any* (i.e. one or more) sentence without quote marks regardless of whether it is cited or not, e.g. *Yer a Jedi, Harry* (Gandalf of House Stark)
  - use quote marks if need be, e.g. "*Yer a Jedi, Harry*" (Gandalf of House Stark), although don't do it too often, because then one could argue you are not passing any of your thoughts through
  - any more than around three usages in text is probably excessive
- copying without citation or wrong citation, e.g. "*Yer a Jedi, Harry*", or "*Yer a Jedi, Harry*" (Jon Snow of Tatooine)
- changing a few words but sentence largely the same, e.g. *You, Harry, sir, are a Jedi* (Mithrandir of Winterfell)

- Turnitin will pick out most of the aforementioned things
- Cases can be contested but will lead to an official review, where the penalty may go up and/or down, and could result in an Academic Misconduct case being filed (see `https://acadreg.ust.hk/generalreg.html#b`)

Problems

1. Since geostrophic flow has a tendency to conserve potential vorticity contours that are well approximated by f/H, sketch on (you don't need to quantify) some f/H = constant contours (and thus essentially the path of the geostrophic flow) on the following diagram of the bathymetry on a β plane (diagram courtesy of Dave Munday) if we are in the Southern Hemisphere:



Justify your answers. How would your sketch differ if this was in the Northern Hemisphere instead?

[3 marks]

2. Sound waves in fluids are slightly different to the waves we have talked about in the Lectures since they are *compression*-type or *longitudinal* waves (the ones in class are *transverse* waves); you can think of sounds waves in fluids as pressure waves. The propagation speeds are given by the Newton–Laplace formula

$$c_{\text{sound}} = \sqrt{\frac{K_s}{\rho}},$$

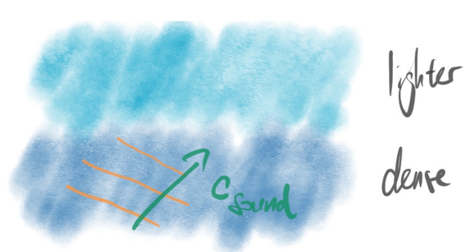
where K_s is a measurement of stiffness of medium, and ρ is the density (not going to make a distinction which density here, but we are really looking at *isentropic* processes).

(a) What standard units should K_s have, if ρ has units of mass per unit volume, and c_{sound} has units of velocity? Show your working (just stating the answer gets no credit). [1 mark]

(b) c_{sound} is about four times larger in water than in air, but is that primarily because water is denser or because water is more 'stiff'? [1 mark]

(c) If water as a medium can be considered essentially non-dispersive, is c_{sound} a phase or a group velocity? Justify your answer. [1 mark]

(d) Draw on and justify with the Newton–Laplace formula what the phase lines (the orange lines) would look like if we have a wave propagating into a region with a different density characteristic (but for simplicity assume same stiffness) as illustrated below:



What is this phenomenon called? [2 marks]

(e) Supposing sound waves travel at 5220 km hr^{-1} in sea water, find the wavelength of a sound wave that is pitched at the modern standard concert pitch A (you have to look up what this means), giving your answers in units of meters accurate to 2 decimal places. (Your answer should be about four times larger than the equivalent in *air*.) [2 marks]

3. Recall that the thermal wind balance from Lec 13 can be written as component form as

$$\frac{\partial u_g}{\partial z} = \frac{g}{\rho_0 f}\frac{\partial \rho}{\partial y}, \qquad \frac{\partial v_g}{\partial z} = -\frac{g}{\rho_0 f}\frac{\partial \rho}{\partial x}$$

Question is about atmosphere but it's the same dynamical principle for the ocean.

In the atmosphere and in the troposphere there are *jet streams* that are very fast eastward flowing air streams that can reach up to speeds of 100 km h^{-1}) or so. There are usually two of these in both hemispheres, the *subtropical jet* and the *polar jet*, where the polar jet is the one at a higher latitude and surrounding the polar regions. Given these jet streams obey small Rossby number dynamics and satisfy hydrostatic balance reasonably well, explain why these jet streams arise, why they are *eastward* jets in both hemispheres, and why the polar jets is generally faster than the subtropical jets.

[5 marks]

4. (Somewhat open ended) Speculate what happens to the ocean stratification, the overturning circulation and the physical (not chemical and/or biological) aspects of the global climate if the Drake passage was *closed* and/or *blocked off* (recall we had this discussion briefly around Lec 2 + 3). Justify your answers accordingly, using concepts that you have learnt so far, and quote sources you use as appropriate.

[4 marks]

5. (Probably quite hard) In the lectures we demonstrated how we can rationalise Rossby wave propagation by vorticity anomalies. It turns out we can rationalise gravity wave propagation also within the vorticity picture. Try and see if you can go reproduce some the broad and pictorial arguments for gravity wave propagation given in Harnik *et al.* (2008) and/or Rabinovich *et al.* (2011) for the propagation of gravity waves accordingly (don't go through the maths unless you are feeling particularly adventurous). Assume a stable stratification with $N^2 > 0$.

The former paper is probably more accessible than the latter. If need be, one of my post-doc papers might also be helpful (while it is for magnetic or Alfvén waves the arguments are conceptually identical).

- Harnik, N., Heifetz, E. Umurhan, O. M., and Lott, F. (2008)
"A buoyancy-vorticity wave interaction approach to stratified shear flow"
Journal of Atmospheric Sciences, **65**, 2615–2630, doi: `10.1175/2007JAS2610.1`
- Rabinovich, A., Umurhan, O. M., Harnik, N., Lott, F, and Heifetz, E. (2011)
"Vorticity inversion and action-at-a-distance instability in stably stratified shear flow"
Journal of Fluid Mechanics, **670**, 301–325, doi: `10.1017/S002211201000529X`
- Heifetz, E., Mak, J., Nycander, J., and Umurhan, O. M. (2015)
"Interacting vorticity waves as an instability mechanism for magnetohydrodynamic shear instabilities"
Journal of Fluid Mechanics, **767**, 199–225, doi: `10.1017/jfm.2015.47`

[3 marks]

!? (Bonus question, no marks + for interest only) As Q.5, but do it for an unstable stratification with $N^2 < 0$, which provides an alternative vorticity view of the Rayleigh–Taylor instability (see Lec 17).

Some notes for Q. 5 + bonus: The main difference between the Rossby waves and gravity waves is that while Rossby waves were assumed to conserve vorticity, so that vorticity anomalies are immediately associated with the displacement, for gravity waves vorticity conservation does not hold, but we can rely on the *buoyancy* as an intermediary to give rise to vorticity anomalies (roughly: displacement leads to buoyancy anomalies, buoyancy anomalies induce a baroclinic torque, which leads to vorticity anomalies). The resulting argument leads to some differences that you actually are aware of: Rossby waves only have one branch ($\omega = -\beta k_x/(k_x^2 + k_y^2)$) and the propagation is westwards, while gravity waves have two branches ($\omega \sim \pm\sqrt{N^2/k}$ essentially) and can propagate in either direction.