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<https://github.com/julianmak/academic-notes>

The repository principally contains the compiled products rather than the source for size reasons.

- ▶ Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 7: Mechanical forcing 1 (pressure and gravity)

Outline

- ▶ recall forcing on ocean
 - thermodynamic (T and $S \Rightarrow \rho$ and buoyancy)
 - mechanical (wind, gravity, pressure, rotation etc.)
- ▶ gravity + pressure (alluded to last Lec.)
 - geoid (see also Lec. 18)
 - sea surface height (SSH)
 - weight
 - hydrostatic pressure
 - some consequences for flow

Key terms: geoid, SSH, hydrostatic pressure

Recap: forces

- Newton's second law: objects are in **steady state** (at **rest** or **steady speed**) unless there is a **net force**

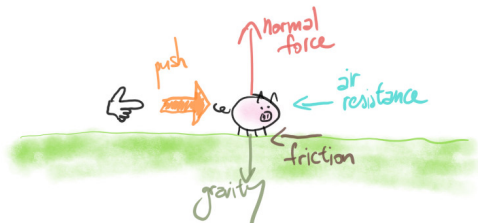


Figure: Forces acting on a (physicist joke: uniform point-mass, spherical) pig (not in a vacuum because we have air resistance + abuse of animal rights).

- thermodynamic forcing: affects T and S
- mechanical forcing: affects momentum

thermodynamic variables affects momentum via **pressure**

Recap: buoyancy forces

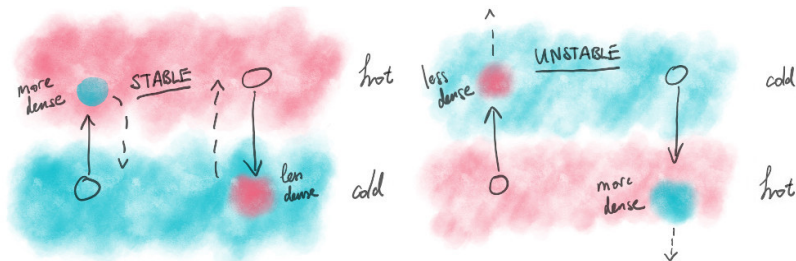


Figure: (Un)stable temperature configurations.

- **buoyancy** of fluid ultimately depends on **density**
 - lighter density water, more 'floaty'
 - heavier things (less buoyant water) = more **weight**, imbalance and sinks

Recap: in-situ vs. potential/neutral density

- ▶ $\rho = \rho(T, S, p)$ via the EOS, but want to neglect p contribution to ρ because **non-dynamic** (from a **work done** point of view)
- ▶ **in-situ density** ρ says you basically have no up-down motion in the deep
→ but we know we have a bit!
→ contributions from p included here
- ▶ **potential densities** referenced to different levels says you can
→ **some** p contribution removed
- ▶ **you want the p resulting from ρ but without p in the ρ** (otherwise, a circular argument?)

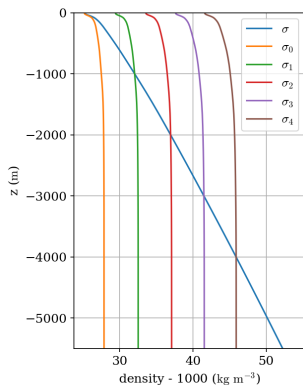


Figure: Vertical profiles of in-situ and potential density (referenced to various depths) at the same location as in the previous graph. See `plot_eos.ipynb`

Recap: equations of motion

Denoting $\mathbf{u} = (u, v)$ and $\mathbf{u}_3 = (u, v, w)$, to numerous approximations (!!!) (see OCES 3203) ocean dynamics is governed by

$$\rho_0 \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} \right) = -\nabla \mathbf{p} + \mathbf{F}_u + \mathbf{D}_u \quad (1)$$

$$\frac{\partial p}{\partial z} = -\rho g \quad (2)$$

$$\nabla \cdot \mathbf{u}_3 = 0 \quad (3)$$

$$\left(\frac{\partial T}{\partial t} + \mathbf{u}_3 \cdot \nabla T \right) = F_T + D_T \quad (4)$$

$$\left(\frac{\partial S}{\partial t} + \mathbf{u}_3 \cdot \nabla S \right) = F_S + D_S \quad (5)$$

$$\rho = \rho(T, S, p) \quad (6)$$

Respectively, (1) momentum equation, (2) hydrostatic balance, (3) incompressibility, (4) temperature equation, (5) salinity equation, and (6) equation of state (EOS)

Gravity

- **attraction** between bodies of different **masses**

→ note it is a purely attractive force (cf. magnetism)

$$F = G \frac{m_1 m_2}{r^2}, \quad G = 6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

→ G the **gravitational constant**

(exercise: check the LHS and RHS units agree)

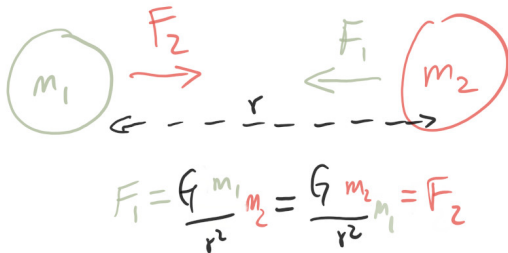


Figure: Schematic of gravitational attraction for two masses. If $m_1 \gg m_2$ (e.g. Earth and a pig) then forces on each body are equal, but its effect on the pig is much larger than it is for the Earth (recall $F = ma$).

Gravity and weight

- Let's take Earth as an example:

$$F = G \frac{m_{\text{earth}}}{r_{\text{earth}}^2} m,$$

taking (units!)

$$G = 6 \times 10^{-11}$$

$$m_{\text{earth}} = 6 \times 10^{24}$$

$$r_{\text{earth}} = 6400 \text{ km} \approx 6 \times 10^6 \text{ m}$$

$$F = 6 \times 10^{-11} \frac{6 \times 10^{24}}{(6 \times 10^6)^2} m = \frac{6^2}{6^2} \times 10^{-11+24-12} m = 10m \equiv mg$$

- the **gravitational acceleration on Earth** (recall $F = ma$) is
 $g \approx 10 \text{ m s}^{-2}$ (exercise: don't drop the decimal places like I did above and repeat the calculation)

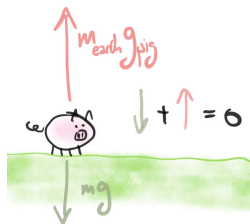


Figure: Gravity as applied on Earth. Note that g_{pig} is tiny (exercise: make an estimate of g_{pig}).

Mass vs. weight

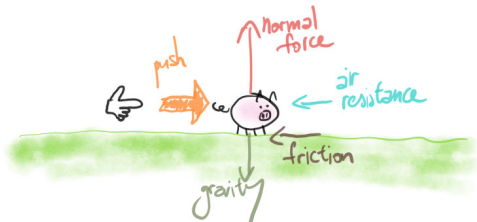


Figure: Forces acting on a (physicist joke: uniform point-mass, spherical) pig (not in a vacuum because we have air resistance + abuse of animal rights).

- the pig above with **mass** m has **weight** mg
 - **mass** is how much 'stuff' a body has
 - **weight** is a **force** and dependent on value of g
 - e.g. pig has same mass on moon but **weighs less** there because $g_{\text{moon}} \approx (1/6)g$ (exercise: why is g_{moon} smaller?)

Geoid

- picture for spherical bodies with uniform mass (then r is distance between centre of gravity), but Earth is not quite spherical...

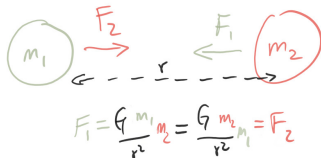


Figure: Schematic of gravitational attraction for two masses.



Figure: Cartoon of spherical vs. ellipsoid earth (it's inflated slightly at the Equator from Earth's spinning). Modified picture from NASA.

Geoid

- ...nor is the mass uniformly distributed!
→ where there is more mass there is more gravitational attraction
- **geoid** is the surface that the ocean surface would trace out if we only had **gravity** and **rotation**

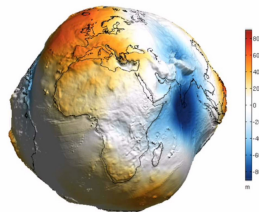


Figure: The “lumpy potato” Earth, variations in the geoid height magnified by several orders of magnitude to highlight difference. From Earth Gravitational Model 2008.

- or, geoid is the surface where gravity is everywhere **perpendicular** to it (I like this one more...)
- wind and tidal action move sea surface around
- important concept in dynamics + **sea level science**

Geoid

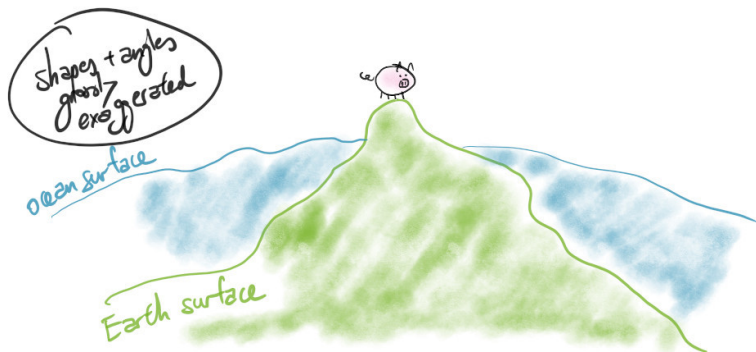


Figure: Schematic of the ellipsoid and geoid.

Geoid

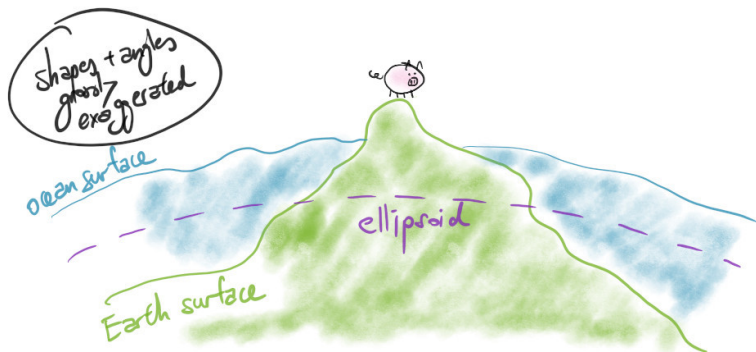


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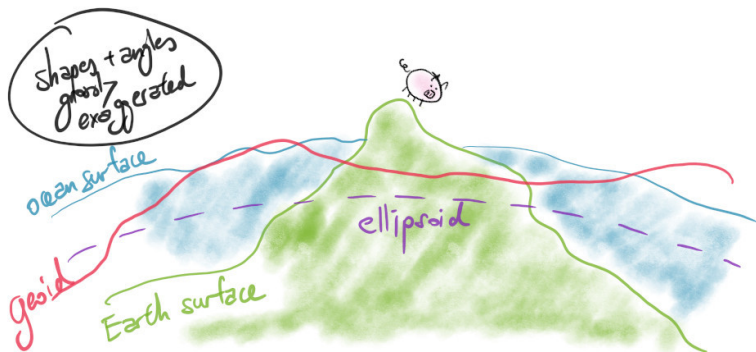


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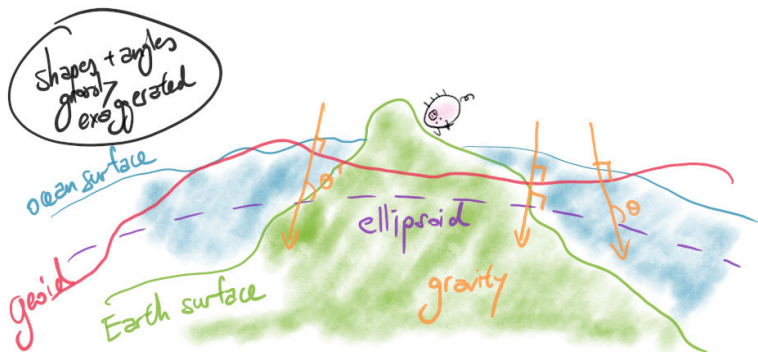


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Gravity + weight

- ▶ differences greatly exaggerated above, in reality gravity variations are very small
 - that's why it was very difficult to get the geoid!
 - really needed **satellites** (see Lec. 20)

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- ▶ for most intents and purposes we can take g to be a constant
 - remember the ocean is quite thin ($H/L \ll 1$)
- ▶ it does matter when we are talking about things like **sea level** (see OCES 4001)
 - sea level change but **relative to what?**
 - e.g. ellipsoid? ground? geoid?

SSH to be instantaneous height relative to ellipsoid

Pressure (recall last Lec.)

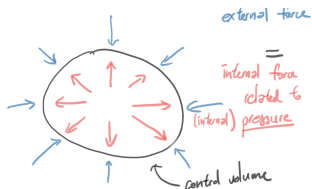


Figure: Fluid volume in force balance.

Consider a body (e.g. balloon) of fixed volume

- fixed volume \Rightarrow steady
- steady \Rightarrow in **force balance**, no net force

- **pressure** = force per area,

$$p = F/A, \quad \text{units: } \text{N m}^{-2} \equiv \text{Pa}$$

1 bar = 10^5 Pa (Pascals) (see e.g. Wikipedia for others)

→ cf. millibars (mbar) in atmosphere

→ lines of constant pressure = **isobar**

Pressure: atmospheric example

日期/Date: 14.10.2020 香港時間/HK Time: 14:00 香港天文台 Hong Kong Observatory

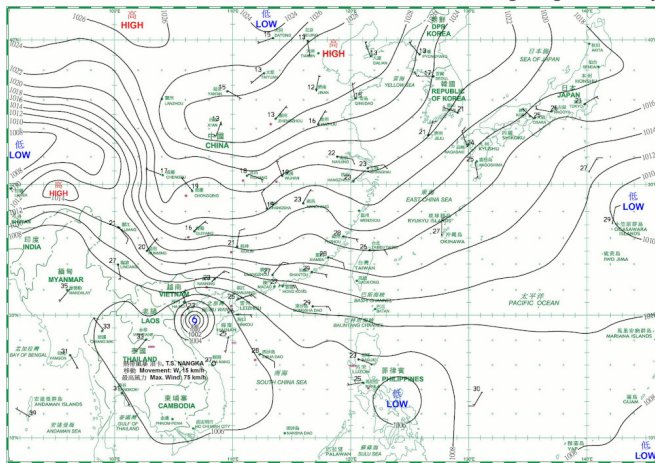


Figure: Atmospheric weather chart with isobars (in units of hPa = 100 Pa = 1 mbar) and wind directions. From HKO.

Hydrostatic balance (recall last Lec.)

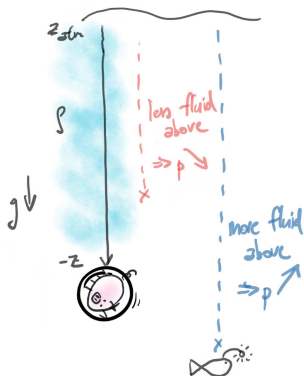


Figure: Schematic of hydrostatic pressure

- **hydrostatic approximation:**
pressure **approximately equal** to weight above when static
→ **weight** is $F = mg$ so for force balance,

$$F = mg = g \int_{-z}^{z_{atm}} \rho(z') dz' = p,$$

with $g \approx 9.81 \text{ m s}^{-2}$

→ if $\rho = \text{const}$ then $p = \rho g z + p_{atm}$

Horizontal effect?

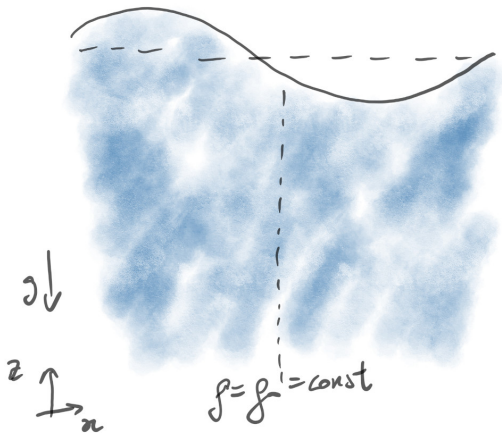


Figure: Horizontal effect because of hydrostatic pressure.

Horizontal effect?

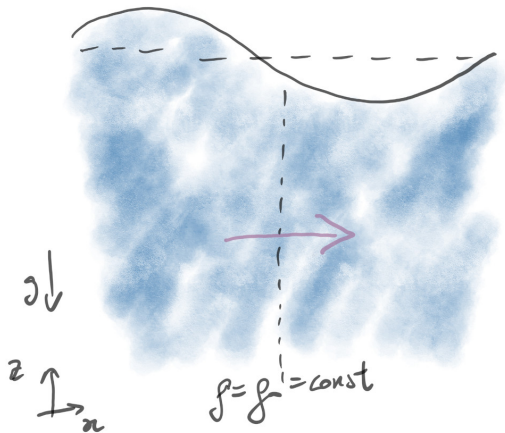


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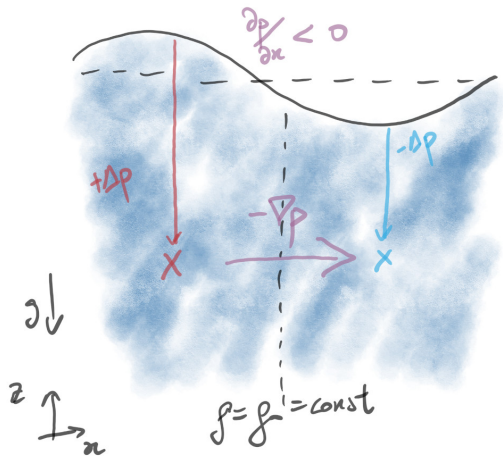
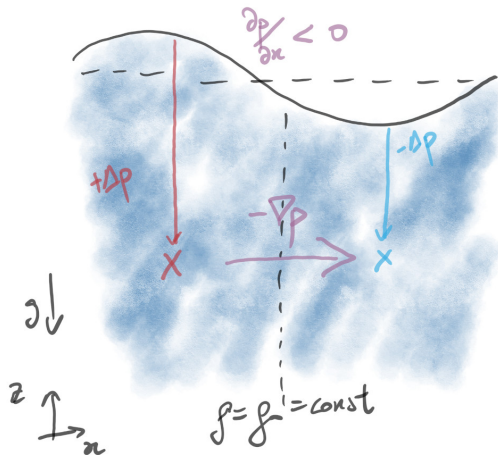


Figure: Horizontal effect because of hydrostatic pressure.

Horizontal effect?



- assuming hydrostatic balance, water moves from $+\Delta p$ to $-\Delta p$ because there is a **net force** (negative pressure gradient $-\nabla p$)
→ important later for **geostrophic flows** (see next Lec.)

Figure: Horizontal effect because of hydrostatic pressure.

Atmospheric example revisited

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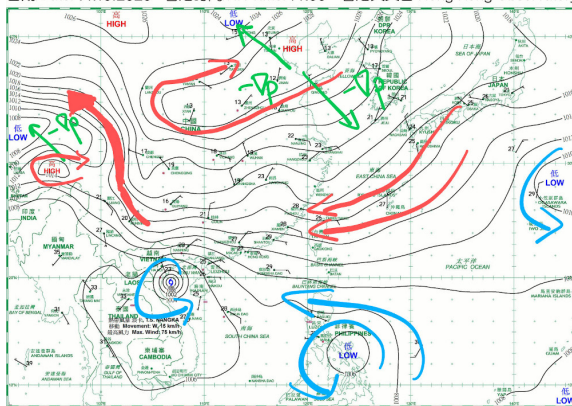


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- note that flow doesn't go in the direction of $-\nabla p!$
→ **along** rather than **across** isobars (Coriolis effect, see next Lec.)

Oceanic example

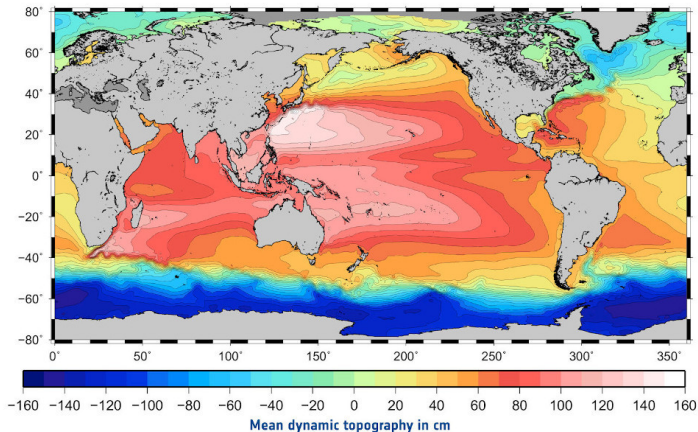


Figure: Time-mean global SSH (also called **mean dynamic topography**), with time-mean currents drawn on (notice the orientation around high/low SSH regions). Modified from Rio *et al.* (2011), J. Geophys. Res: Oceans.

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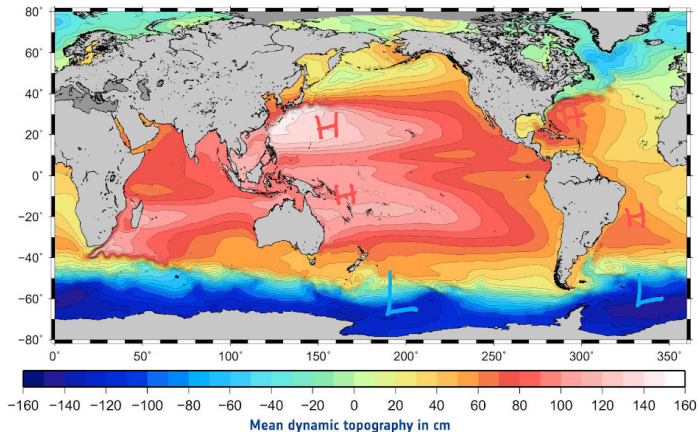


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- contours of SSH related to isobars via **hydrostatic balance**

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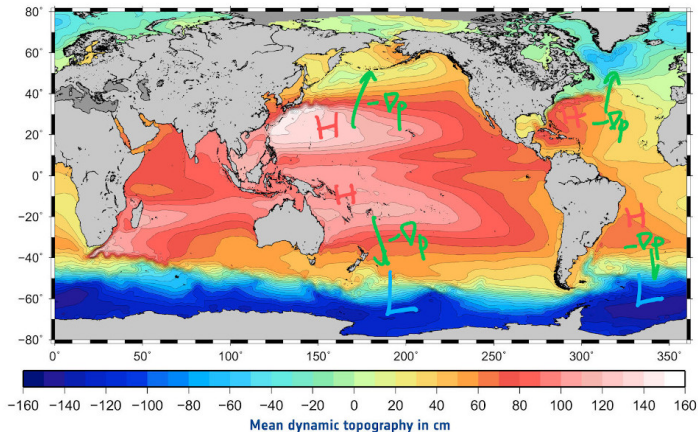


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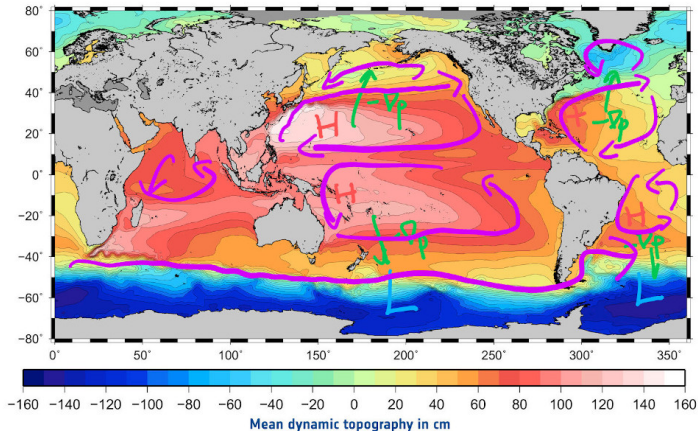


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- contours of SSH related to isobars via **hydrostatic balance**
→ flow is **along** rather than **across** isobars (Coriolis effect, see next Lec.)

Summary

- ▶ gravity + **geoid**
 - astronomical forcing on ocean
(see Lec. 18)
 - geoid important for e.g. **sea level change** (see Lec. 18 + OCES 4001)
- ▶ **hydrostatic pressure**
 - pressure proportional to weight of fluid above
- ▶ **buoyancy** (thermodynamic stuff) affects **pressure**...
- ▶ ...leading to pressure **gradients** (mechanical force) driving a **flow**...
 - ...but **rotation** can influence resulting flow! (see next Lec.)

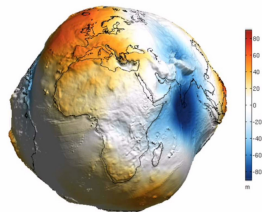


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