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# OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 15: Dynamics 1 (intro to waves)

# Outline

- Recap: circulation and dependence on small-scale dynamics

- waves: fundamental concepts
  → periodicity, crest/trough/node
  → wavelength + period
  → frequency + wavenumber
  → restoring force + dispersion relation
  → propagation, phase/group velocity

**Key terms**: waves, wavenumber, frequency, period, dispersion relation, phase/group velocity

# Reacp: MOC

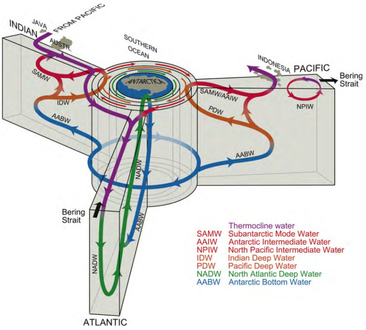


**Figure:** Schematic of the 3d MOC with watermass distributions. From Talley *et al.* (2011), *Descriptive Physical Oceanography*; see more in their Fig. 14.11. Format after Arnold Gordon (1991).

- MOC important for climate, carbon storage, ecology, etc.

  $\rightarrow$ e.g. warming of Western Europe by AMOC

  $\rightarrow$ e.g. carbon storage by deep water formation

- mostly along-isopycnal flow

- isolated places for watermass transformation + deep/abyssal water formation (deep convection)

# Recap: what goes down must come up

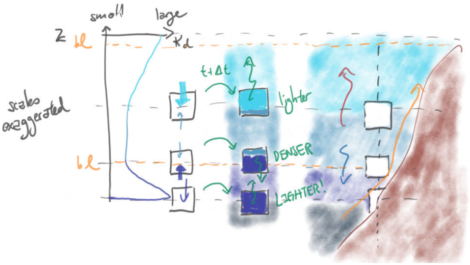


**Figure:** Schematic of the diffusive upwelling.

- diapycnal mixing contribute upwelling, strongest in boundary layers

$\rightarrow$ ~~broad diffusive~~ boundary intensified upwelling

**what causes the boundary intensification of $\kappa_d$?**

# Recap: what goes down must come up

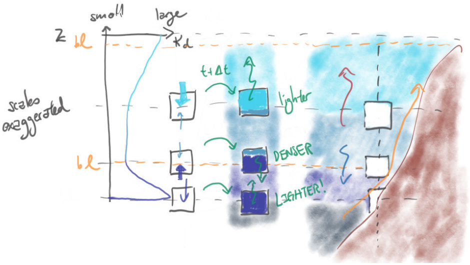


**Figure:** Schematic of the diffusive upwelling.

- diapycnal mixing contribute upwelling, strongest in boundary layers

  $\rightarrow$ ~~broad diffusive~~ boundary intensified upwelling

**what causes the boundary intensification of $\kappa_d$?** dynamics!

- at the surface, lots of things... (convection, waves, Langmuir turbulence etc.)
- at the bottom, probably tidal conversion (Lec. 18) $\rightarrow$ internal gravity waves (Lec. 16) $\rightarrow$ shear instabilities (Lec. 17)

# Recap: form stress and SO overturning

Role also of baroclinic instability (Lec. 13, see also Lec. 17), important for

- vertical momentum transfer by interfacial form stress
- scale transfer of energy
  → mesoscale eddies, conduit between large-scales and submesoscales
- along-isopycnal mixing and also MOC

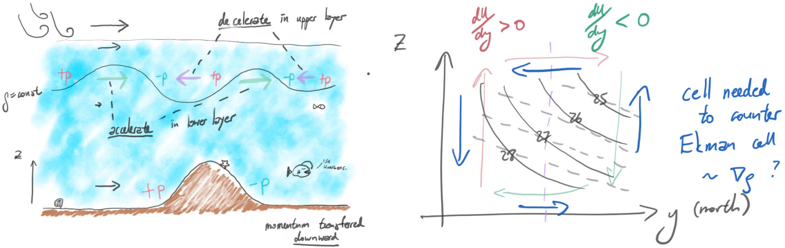


**Figure:** Schematic of form stress and eddy induced overturning cell in Southern Ocean (see Lec. 14)

# Outlook of the next few lectures

Dynamics important, next four lectures on

- waves (this Lec. + 16, 18) and instabilities (Lec. 17)

  $\rightarrow$ because waves are easier to talk about without maths...

Highlight gross features (i.e. those that can be drawn...)

- **how to describe waves** (Lec. 15)
- types of waves (Lec. 16)

  $\rightarrow$ consequence + leading to instabilities
- instabilities (Lec. 17)

  $\rightarrow$ parcel-type (mechanistic) arguments for instability
- tides (particularly as internal gravity waves) (Lec. 18)

# Examples of waves

**Figure:** Gravity waves with signal at the sea surface (as darker and lighter bands). Taken at HKUST.

Surface gravity waves[1]

- restoring force is buoyancy

  $\rightarrow$ treat air-sea as one fluid with a giant jump in density

[1] According to Richard Feynman, while water waves are "*...easily seen by everyone... are the worse possible example [of waves], because... they have all the complications that waves can have* (From Feynman, Leighton & Sands, 1971, *Lectures of Physics*).

# Examples of waves

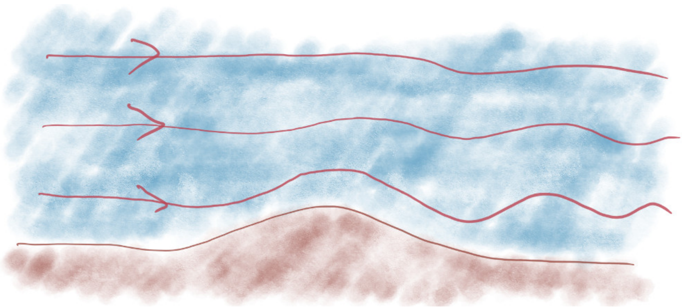

**Figure:** Flow over topography (e.g. tidal motion) leading to wave generation.

Tides and/or internal gravity waves forced by tidal motion

- restoring force is still buoyancy

  → wave breaking contributing to mixing

# Examples of waves

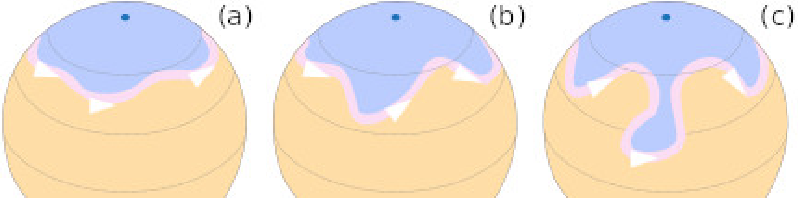


**Figure:** Example of jet stream meanders as Rossby waves. Figure from Wikipedia.

Rossby waves (cf. same Rossby as Rossby number)

- restoring force is Coriolis (or background gradient in vorticity)

  $\rightarrow$ planetary-scale waves, generic in rotating systems (e.g. rotating tanks, Jupiter, galactic disks)

# Examples of waves

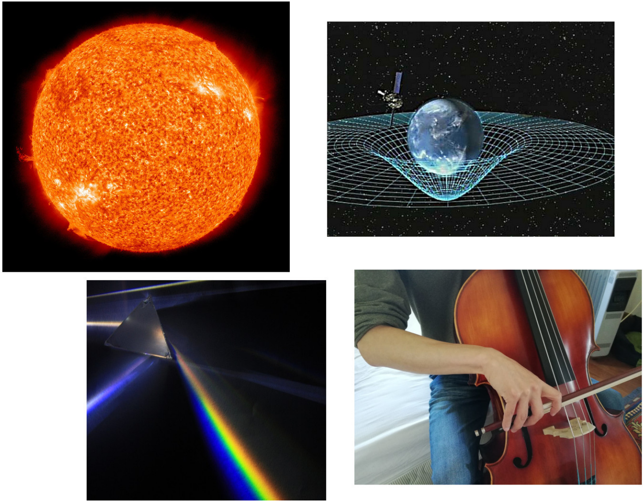

**Figure:** Example of some other waves in systems that support waves: Alfv́en waves (magnetic field + Lorentz force), gravitational waves (spacetime + gravity), electromagnetic waves (but also wave-partical duality), sound waves (mechanical forcing + any medium). All figures from Wikipedia except the cello one.

# Features of waves

Some observations:

- waves have some oscillation/periodicity
  $\rightarrow$ want a measure of **period**
- waves to propagate
  $\rightarrow$ **speed/velocity** associated with waves
- waves need a medium to travel through
  $\rightarrow$ subtlety with Electro-Magnetic waves (not touched on here)
- characterised by a restoring force (follows from medium)
- waves can increase in amplitude and steepen
  $\rightarrow$ wave breaking and mixing
- can disperse, refract, interfere etc. (used in Lec. 17, 20)

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**physics $\Rightarrow$ dispersion relation, identifies the type of waves**

# Features of waves

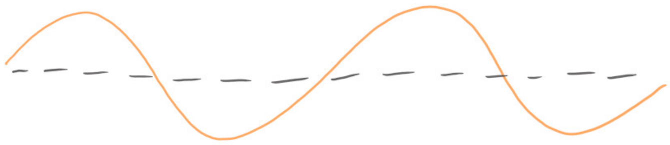
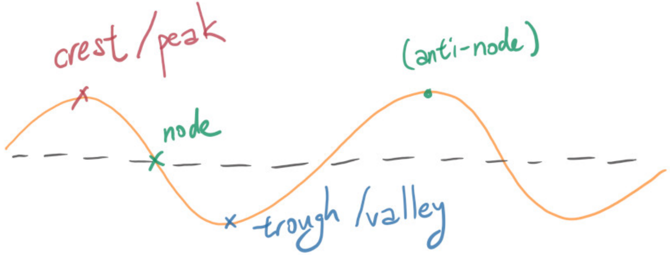


**Figure:** Schematic of wave features. Box length $L = 2\pi$ for simplicity.

- displacement $\eta$ described by (could also be sine)

$$\eta \sim \cos(x)$$

# Features of waves

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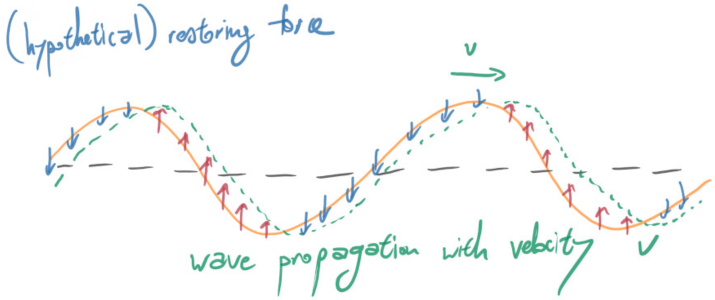


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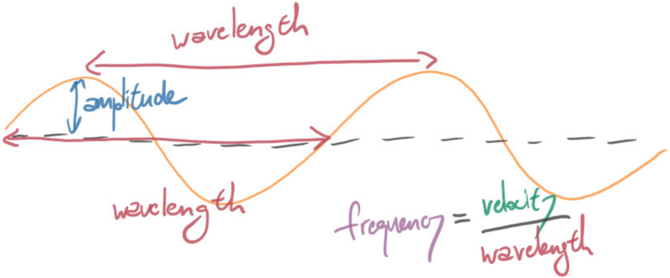


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$$\eta \sim A\cos(x - vt), \qquad \gamma = v/\lambda$$

# Features of waves

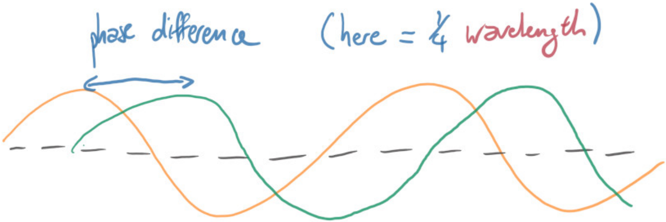


**Figure:** Schematic of wave features. Box length $L = 2\pi$ for simplicity.

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$$\eta \sim A\cos(x), \qquad \eta \sim A\cos(x - \lambda/4)$$

# Features of waves

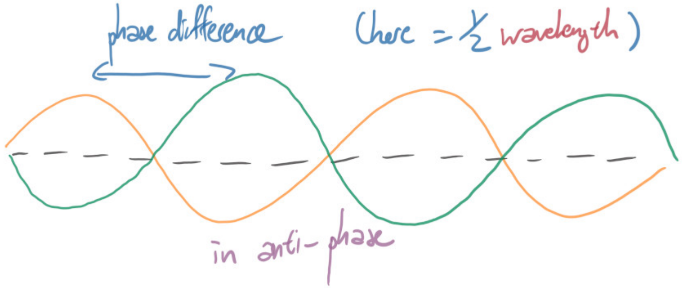
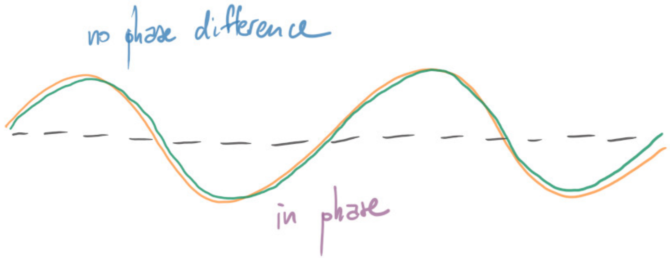



**Figure:** Schematic of wave features. Box length $L = 2\pi$ for simplicity.

- displacement $\eta$ described by (could also be sine)

$$\eta \sim A\cos(x), \qquad \eta \sim A\cos(x - \lambda/2) \sim -A\sin(x)$$

# Features of waves

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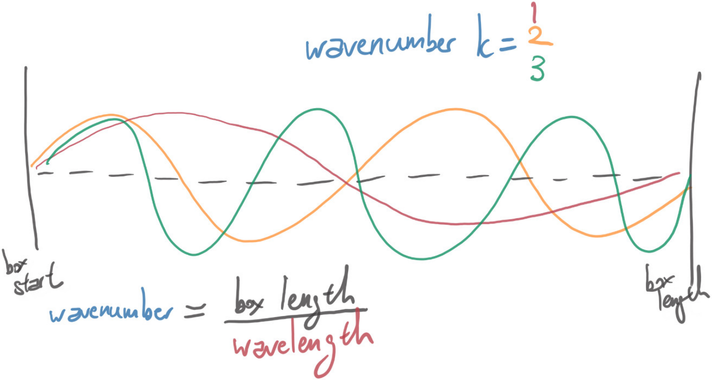


**Figure:** Schematic of wave features. Box length $L = 2\pi$ for simplicity.

- displacement $\eta$ described by (could also be sine)

$$\eta \sim A\cos(2x), \qquad \eta \sim A\cos(1x), \qquad \eta \sim A\cos(3x)$$

# Features of waves

$$\gamma = \frac{v}{\lambda}, \qquad k = \frac{2\pi}{\lambda}$$

- $\gamma$ the frequency (units: $s^{-1} = \text{Hz}$)

  $\rightarrow$ how quickly the wave oscillates
- $v = c_p$ the **phase** velocity

  $\rightarrow$ how fast the wave itself moves around
- $\lambda$ the wavelength

  $\rightarrow$ how long the wave is
- $k$ the wavenumber

  $\rightarrow$ intuitively how many waves can you fit in a box (so $k \sim \lambda^{-1}$)

  $\rightarrow$ does not necessarily have to be an integer

# Features of waves: dispersion relation

Usually describe waves in terms of wavenumber $k$ and the **angular frequency** $\omega = 2\pi\gamma$, i.e.

$$\eta = A\cos(kx - \omega t)$$

- generally, for $\boldsymbol{x} = (x, y, z)$, we would have

$$\eta = A\cos(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)$$

$\rightarrow$ $\boldsymbol{k}$ is the **wavevector**

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Aside: If you know your **complex numbers**, the above is neatly encapsualted as

$$\eta = \mathrm{Real}\left[A\mathrm{e}^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t - \theta_0)}\right],$$

where e is Euler's number, $\mathrm{i} = \sqrt{-1}$, and $\theta_0$ denotes a phase shift if any (could be sucked into the amplitude). For calculating the dispersion relation this form is substantially nicer to deal with (don't have to keep track of sines and cosines when taking derivatives).

# Features of waves: dispersion relation

Note that

$$\omega = 2\pi\gamma = 2\pi\frac{v}{\lambda} = vk$$

- the physics tells you how $v = v(k)$
- the dispersion relation is given by

$$\boxed{\omega = \mathcal{F}(k; \ldots)}$$

for some function $\mathcal{F}$

$\rightarrow$ the dispersion identifies the types of wave (see Lec. 16), e.g.

$$\omega = \sqrt{gk}, \qquad \omega = -\frac{\beta}{k}, \qquad \omega = B_0 k, \qquad \omega = \frac{\hbar k^2}{2m},$$

# Superposition

(Linear) waves can be superimposed, leading to interference

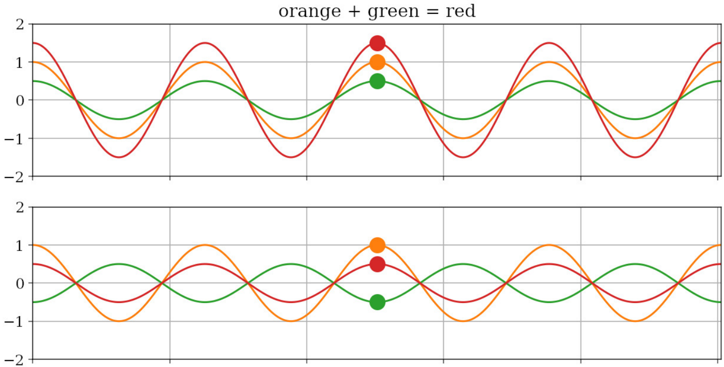


**Figure:** Interference of waves with Red = Orange + Green. For waves in phase (constructive inteference) and waves in anti-phase (destructive inteference).

Q. but what about waves not quite in phase or anti-phase?

# Superposition

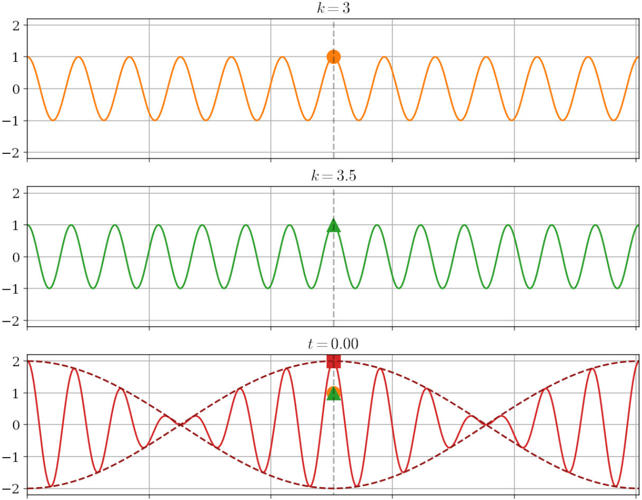


**Figure:** Superposition of two waves slightly out of phase, again with Red = Orange + Green. The crests have a marker marked on to track its progress later.

# Wave propagation

Recall that $v = v(k)$ is the wave velocity. More precisely,

- the phase speed in a direction ($= v$) is defined as

$$c_{p,x} = \frac{\omega}{k}$$

$\rightarrow$ how the wave by itself travels

# Wave propagation

Recall that $v = v(k)$ is the wave velocity. More precisely,

- the phase speed in a direction ($= v$) is defined as

$$c_{p,x} = \frac{\omega}{k}$$

  $\rightarrow$ how the wave by itself travels

- the group velocity $c_g$ is defined as

$$c_{g,x} = \frac{\partial \omega}{\partial k}$$

- in higher space dimensions,

$$c_{p,x} = \frac{\omega}{k_x}, \qquad c_{p,y} = \frac{\omega}{k_y}, \qquad c_{p,z} = \frac{\omega}{k_z}, \qquad \boldsymbol{c}_g = \nabla_{\boldsymbol{k}} \omega$$

  $\rightarrow$ NOTE! Phase propagates in the direction of $\boldsymbol{k}$

  (subtlety: $c_{p,x}$ is not a component of the velocity that phases propagate at; see Ch.5 App of Vallis (2006))

# Example: 1d Rossby waves (animation)

# Wave propagation

- group velocity describes

  $\rightarrow$ how a collection of waves travel as a **group** or wavepacket

  $\rightarrow$ velocity that "stuff" propagates at

- a type of wave is non-dispersive if

$$c_g = c_p$$

  $\rightarrow$ e.g. $\omega = B_0 k$ and $\omega = k\sqrt{gH}$ are non-dispersive

  $\rightarrow$ if non-dispersive, wavepacket and phase travel together

- example just now is dispersive

  $\rightarrow$ 1d Rossby waves, $\omega = -\beta/k$ (exercise: show $c_g = -c_p$ for this case)

# Wave propagation

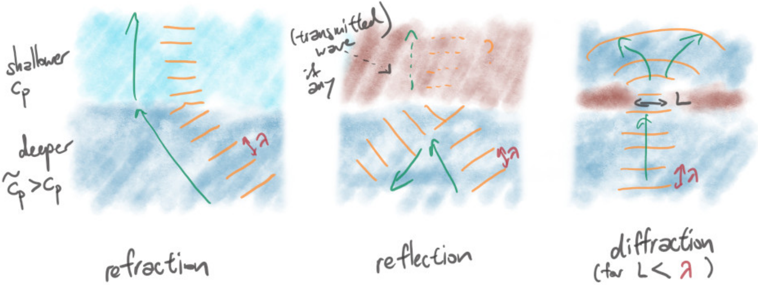


**Figure:** Schematic of refraction, reflection (and transmission), and diffraction, nominally using **monochromatic** (i.e. one choice of $k$) surface gravity wave as an example. The orange lines are phase lines (e.g. think wave crests).

## refraction, reflection, and diffraction (used in Lec. 17, 20)

- resulting interference of waves can lead to wave steepening and wave breaking

# Wave propagation

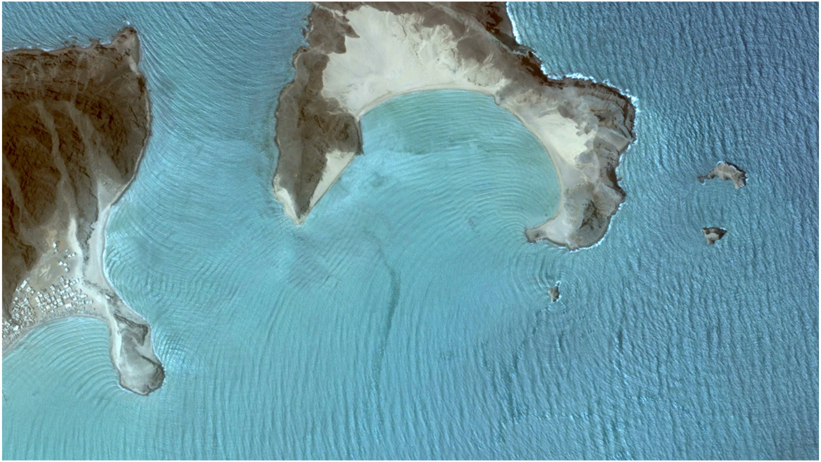

**Figure:** Picture of (presumably non-monochromatic) waves over the Arabian sea. Image taken from https://www.earthglance.com/post/133835790223/wave-diffraction-on-the-arabian-sea.

# Summary

**smaller-scale dynamics affects large-scale circulation**

- waves are ubiquitous physical features

  $\rightarrow$ depends on physics
- wave described by the dispersion relation $\omega = \mathcal{F}(k)$

  $\rightarrow$ physics of the system dictates what $\mathcal{F}(k)$ is

  $\rightarrow$ usually use angular frequency $\omega$ and wavenumber $k$

  (absorbs factors of $2\pi$ floating around)

  $\rightarrow$ $k \sim \lambda^{-1}$ sometimes used to characterise scale of motion

  (more on this in Lec. 18)
- difference in $c_p$ and $c_g$

  $\rightarrow$ individual (former) and collective (latter) velocity

**wave breaking contributes to diapycnal mixing** (see Lec. 16 + 17)