OCES 2003 Assignment 2, Spring 2023

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Set on: Wed 8th Mar; due: Wed 15th Mar

Blurb

- Assignments have a maximum mark out of 20, although you will see that there are 22 marks available to get in total, i.e. if you get 22/20 you still only get credit for 20/20
  - 16-17 is roughly around the A- boundary
  - anything below 8 is probably a fail
- Please show working in calculation
  - no working + wrong answer = no credit whatsoever
  - some working + wrong answer = partial credit
  - generically, give things to 2 decimal place and provide the appropriate units (marks are allocated for these), unless otherwise specified
- No answers except the 'hard' ones should need more than a paragraph / half a page, and excess answers that are not to the point will be penalised
- Type up the assignment or send a photo of your written up work in (the former is preferred), and the only request I have is no Microsoft Word documents (you can type up things with Word but export it as a pdf if you do)
  - write in full sentences where appropriate
  - particularly poor and/or scrappy presentation will have a mark that can be taken off
- There will be a rigid mark scheme, and model solutions will be available in due course
  - the TAs only mark the stuff, you should come to the instructor for arguing marks, and note the re-marking can result in marks going up or down

!!! By handing something in, you agree to the usual Academic Honour code and Integrity declarations. For more, see `http://qa.ust.hk/aos/academic_integrity.html`. Cases for plagiarism (whether intended or not, it is the "act" that matters) gets a penalty ranging from

  - zero on the question concerned
  - a fixed penalty starting from around 1/3 of the total marks
  - zero for the whole assignment/midterm/final

  The following counts as plagiarism (and is a non-exhaustive list):

  - copying word for word *any* (i.e. one or more) sentence without quote marks regardless of whether it is cited or not, e.g. *Yer a Jedi, Harry* (Gandalf of House Stark)
    * use quote marks if need be, e.g. "*Yer a Jedi, Harry*" (Gandalf of House Stark), although don't do it too often, because then one could argue you are not passing any of your thoughts through
    * any more than around three usages in text is probably excessive
  - copying without citation or wrong citation, e.g. "*Yer a Jedi, Harry*", or "*Yer a Jedi, Harry*" (Jon Snow of Tatooine)
  - changing a few words but sentence largely the same, e.g. *You, Harry, sir, are a Jedi* (Mithrandir of Winterfell)

- Turnitin will pick out most of the aforementioned things
- Cases can be contested but will lead to an official review, where the penalty may go up and/or down, and could result in an Academic Misconduct case being filed (see `https://acadreg.ust.hk/generalreg.html#b`)

Problems

1. Estimate the Rossby number for the following:

   (a) ACC circulation,

   (b) gyre circulation,

   (c) Western Boundary Current dynamics,

   (d) rivers,

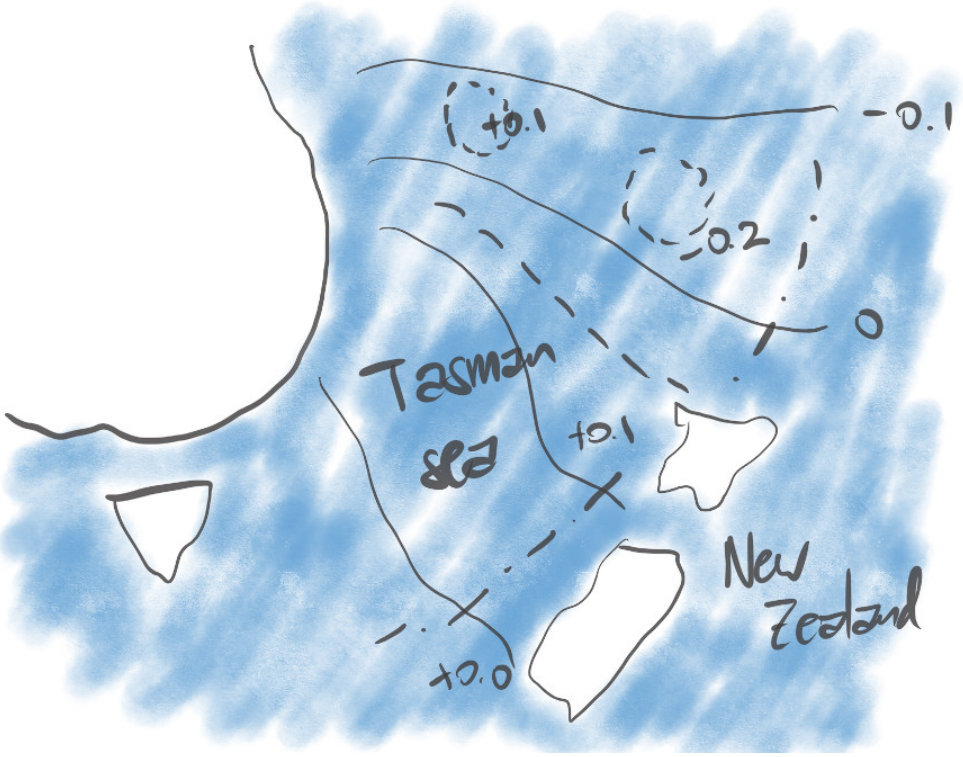
   (e) toilet flow.

   For each of these, look up some relevant numbers for the U and L, but take $f = 10^{-4}$ s^{-1} (which would be f at around 30° N). I just need an order of magnitude, i.e. values of U, L and the eventual answer in the form 10^a with appropriate units for some integer a will do. In each of these cases state whether geostrophic balance is applicable. Cite sources as appropriate.

   [5 marks]

   (Hint: Most of the relevant numbers are provided in either this year or in previous years' assignments.)

2. Consider the following sketch where the numbers refer to SSH about the time mean (units of meters if you like), and the contours are isolines of SSH:



   Assume that we are in the Ro $\ll 1$ regime, we are still in the Southern Hemisphere, **but Earth is rotating the other way round**. Draw on the arrows (or draw your own copy if you want more space to draw things on) associated with the following:

(a) $-\nabla p$ associated with the two dot-dashed lines ($- \cdot - \cdot -$),

(b) rotation sense associated with the two eddies drawn on near the top of the diagram as the dashed lines ($- - -$),

(c) the sense of geostrophic flow along the middle dashed line ($- - -$) between the 0 and +0.1 contour, assuming only those two contours contribute to the geostrophic flow in question.

Justify your answer, and note that answers without a justification gets zero credit (so guessing by drawing arrows on by itself gets no marks). Note that none of the actual things drawn on necessarily correspond to what is happening in the region (particularly since we make the assumption here that the Earth is spinning the other way around), so even if you look up what actually happens in the region it won't help you.

[6 marks]

3. The question concerns the Coriolis effect and what is known as the *traditional approximation*, which in a nut shell leads to the Coriolis effect only in the *horizontal* momentum equation (hence the $2\boldsymbol{\Omega} \times \mathbf{u}$ with a horizontal velocity; some other terms are also dropped for reasons to possibly be explored here).

(a) What is the value of $f = 2\Omega \sin(\text{latitude})$ at the equator?

[1 mark]

(b) Starting from the full Coriolis effect given by $2\boldsymbol{\Omega} \times \mathbf{u}$, where $\boldsymbol{\Omega}$ is the rotation axis pointing north, argue pictorially or otherwise why, at the equator,

i. the Coriolis effect on a purely meridional velocity $\mathbf{u} = (0, v, 0)$ is zero,

ii. there is no horizontal Coriolis effect on a purely zonal velocity $\mathbf{u} = (u, 0, 0)$.

What does this say about the horizontal Coriolis effect on an arbitrary two-dimensional horizontal velocity $\mathbf{u} = (u, v, 0)$ with no vertical component at the equator?

[3 marks]

(c) The above answer for the horizontal Coriolis effect is not true when there is a locally vertical (or radial) component to the velocity, i.e. $w \neq 0$ in $\mathbf{u} = (u, v, w)$, but for most intents and purposes we would probably ignore it anyway. Why is that a reasonable thing to do? Give some numbers to back up your claim if you want, and cite any sources you do use.

[2 marks]

(d) Notice I have also been very explicit in talking about the *horizontal* Coriolis effect in the above, and a velocity with a zonal component $u \neq 0$ will provide a *vertical* Coriolis effect at the equator. If $u > 0$ at the equator, work out pictorially or otherwise whether $2\boldsymbol{\Omega} \times \mathbf{u}$ points into the Earth or out of the Earth, but also argue why we would probably drop that anyway. Give some numbers to back up your claim if you want, and cite any sources you do use.

[2 marks]

(Dropping terms involving w as well as dropping the radial / locally vertical component of the Coriolis effect is mostly the *traditional approximation*, although strictly speaking the latter part is more to do with the small aspect ratio approximation.)

(e) (Extra reading might help) The above argument only looks at the equator but the traditional approximation would be applied over the whole globe. We make the approximation because it holds

over most dynamical scenarios on Earth, but look up and/or argue under what circumstances (geographical, dynamical, or otherwise) when such an approximation might not be so applicable. (Hint: Try a Google search, or failing that, contents in the Vallis book, or Gerkema *et al.* (2008) in Reviews of Geophysics.)

[3 marks]

!? (No marks bonus question) It turns out *potential* temperature θ is actually not a great thermodynamic variable to use. Look up *conservative* temperature (denoted Θ, capital theta) and write maybe a page or two (with appropriate referencing) on what is so bad about θ, and why Θ is a more sensible thing to use. The references on the Wikipedia page is a good place to start; the TEOS-10 webpage has some tutorial slides on the left hand bottom corner, which might be more accessible than the original paper of McDougall (2003).