# OCES 2003 Assignment 4, Spring 2022

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Set on: Thur 5th$^{\text{th}}$ May; due: Thur 12$^{\text{th}}$ May

## Model solutions and mark scheme

## Problems

1. (a) Speed is distance over time, and since the round trip (down and then up) is $2D$, we have

$$c = \frac{2D}{\Delta t} \qquad \Rightarrow \qquad D = \frac{c\Delta t}{2}.$$

*(0.5 marks for the speed is distance over time, 0.5 marks for the answer.)*

(b) Substituting some numbers I make it

$$\Delta t = \frac{2\times 10^6}{3\times 10^8} = \frac{2}{3}\times 10^{-2}\ \text{s} = 6.66\ldots.\times 10^{-3}\ \text{s} \approx 7\ \text{ms}.$$

*(0.5 marks for the answer and 0.5 marks for the degree of accuracy.)*

(c) Geostrophic balance gives the relation between geostrophic flow and horizontal gradient of pressure. Hydrostatic balance suitably integrated in the vertical gives the relation between pressure and sea level height, and combining the two gives the stated relation possibly up to some factors.
*(1 mark for geostrophic balance and 1 mark for hydrostatic balance with an integration. Can be a bit lax about the integration bit.)*

(d) Notice that I gave an estimate for $N^2$ while the $L_d$ wants $N$, so plugging some numbers in gives

$$L_d = \frac{\sqrt{10^{-5}}\times 4000}{10^{-4}} = 126.49\ \text{km} \approx 100\ \text{km}.$$

*(0.5 marks for the calculation and 0.5 marks for the degree of accuracy.)*

(e) We are looking to take horizontal derivatives relevant to geostrophic motions, and given $L_d$ is a typical length scale of geostrophic motion $L_d$ would be a reasonable choice for $L$.
*(0.5 marks for something about geostrophic motion, and 0.5 marks for $L_d$ being relevant to geostrophic motion.)*

(f) So denoting the uncertainty with little deltas $\delta$, we have

$$\delta u_g \sim \frac{g}{f}\nabla\delta\eta \sim \frac{g}{fL_d}\delta\eta = \frac{10}{10^{-4}10^5}\delta\eta = \delta\eta.$$

So if $\delta u_g \sim 10^{-2}$, then $\eta \sim 10^{-2} = 0.01$ m = 1 cm.
*(1 mark for taking $\nabla\delta\eta \sim 10^{-5}\delta\eta$, 1 mark for the calculation, and 1 mark for the degree of accuracy. Be generous with this part of the question.)*

(g) The accuracy would be $\delta\eta/D = 10^{-2}/10^6 = 10^{-8}$, so 1 part in 100 million.
*(0.5 marks for dividing previous answer by $D$, and 0.5 marks for the calculation.)*

2. (a) This part is actually the bonus question last year. Notable differences include the webpage's explanation invoking rotation and inertial effects, when in the way I went through in the lectures, equilibrium tides only care about the differential in the gravitational field, and no rotation is required (or we could just be working in an inertial frame, and then there would be no inertial forces).

   *(Give 1 mark for something about rotation, inertial effects, centrifugal forces etc.)*

   (b) I can either do $r_{\mathrm{Earth}} + \delta r$ or $r + \delta r$, but noting that $\delta r = 4000$ m is much smaller than $r_{\mathrm{Earth}} \sim 10^6$ m or $r \sim 10^8$ m from Earth to moon, the change in $a$ is going to be fairly negligible.

   If you compute $a$ you get something which is about $10^{-6}$ m s$^{-2}$ for both cases, and the differences between the two cases is on the order of $10^{-10}$ m s$^{-2}$, i.e. around 0.01%, so it's really small.

   *(Give 2 marks if using an algebraic and scaling argument as I did above, or 1 mark for the computation of the two values of a and 1 mark for noting their differences are small from a raw value or percentage point of view.)*

   (c) For $g$ we are dealing with $1/r_{\mathrm{Earth}}^2$ while for $a$ we are dealing with $1/r^3$. Given $r > r_{\mathrm{Earth}}$ and the exponents involve, $g$ should be much bigger than $a$.

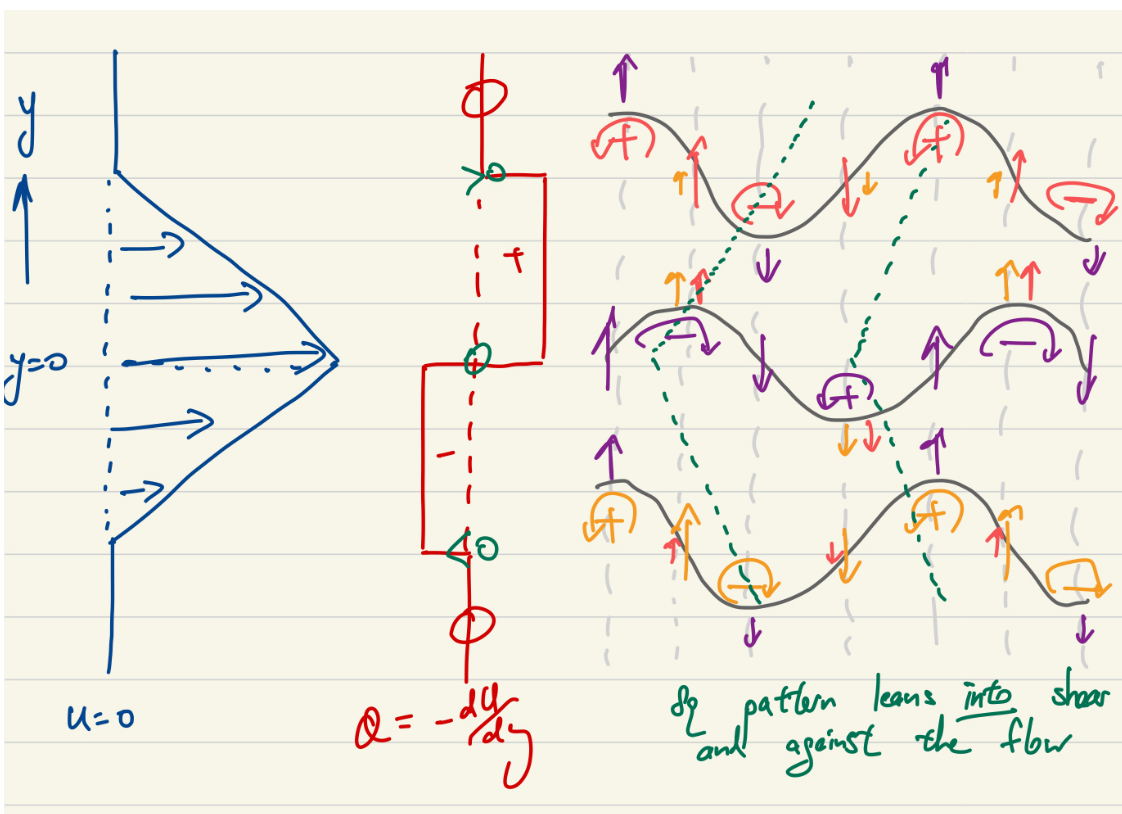
   Following from previous part, $g/a \sim 10/10^{-6} = 10^7$ difference, so $g \gg a$.

   *(Give 1 mark for either the algebraic or numerical explanation.)*

   (d) The tidal effect is from water convergence arising from the non-radial part of the forces, rather than from direct pulling by the radial part of the forces.

   *(1 mark for something about water convergence, be generous with remaining mark regarding argument relating to radial and non-radial forces, diagrams, whatever. Accept horizontal/vertical instead of non-radial/radial.)*

3. See the rather busy diagram below:



- The zero regions of the flow have zero background vorticity. The lower flank with increasing velocity has *negative* background vorticity, and the upper flank has decreasing velocity so *positive* background vorticity.

  *(0.5 mark each for each of the positive or negative background vorticity.)*

- Starting with the middle wave (the purple stuff), we assume the wave carries essentially zero vorticity, so if it is protruding up then it is a negative anomaly (since it is zero going into a region with positive anomaly), and if it is protruding down it is a positive anomaly by the same argument. This wave propagates to the left and against the flow.

  *(0.5 marks for the vorticity anomalies, 0.5 marks for the argument, 0.5 marks for going left.)*

  For the upper wave (the pink stuff), it is something mildly positive going into a region of zero or something more positive, while for the lower wave (the orange stuff), it is some mildly negative going into a region of something more negative or something zero. Both cases give a positive anomaly when protruding up and negative anomaly when protruding down. Both waves go to the right.

  *(Mark both waves at the same time: 0.5 marks for the vorticity anomalies, 0.5 marks for the argument, 0.5 marks for going right. Give 0.25 marks accordingly if a part is only right for one of the waves.)*

- The outer waves should *lag* the middle wave if we are looking at the displacement, or *lead* the middle wave if we are looking at the vorticity anomalies (the green dashed line case) by a quarter of a wavelength. The outer wave and the middle wave interact such that they amplify the displacement (see the coloured arrows). The outer waves are in phase with each other and serve to speed up each other.

  *(1 mark for the configuration, and 1 mark for demonstrating explicitly that the three waves are all amplifying. Give 0.5 marks accordingly if say only one of the pairs are amplifying.)*