OCES 2003 Assignment 1, Spring 2022

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Set on: Tue 15th Mar; due: Tue 22rd Mar

Blurb

- Assignments have a maximum mark out of 20, although you will see that there are 22 marks available to get in total, i.e. if you get 22/20 you still only get credit for 20/20
 - 16-17 is roughly around the A- boundary
 - anything below 8 is probably a fail
- Please show working in calculation
 - no working + wrong answer = no credit whatsoever
 - some working + wrong answer = partial credit
 - generically, give things to 2 decimal place and provide the appropriate units (marks are allocated for these), unless otherwise specified
- No answers except the 'hard' ones should need more than a paragraph / half a page, and excess answers that are not to the point will be penalised
- Type up the assignment or send a photo of your written up work in (the former is preferred), and the only request I have is no Microsoft Word documents (you can type up things with Word but export it as a pdf if you do)
 - write in full sentences where appropriate
 - particularly poor and/or scrappy presentation will have a mark that can be taken off
- There will be a rigid mark scheme, and model solutions will be available in due course
 - the TAs only mark the stuff, you should come to the instructor for arguing marks, and note the re-marking can result in marks going up or down

- !!! By handing something in, you agree to the usual Academic Honour code and Integrity declarations. For more, see http://qa.ust.hk/aos/academic_integrity.html. Cases for plagiarism (whether intended or not, it is the "act" that matters) gets a penalty ranging from
 - zero on the question concerned
 - a fixed penalty starting from around 1/3 of the total marks
 - zero for the whole assignment/midterm/final

The following counts as plagiarism (and is a non-exhaustive list):

- copying word for word *any* (i.e. one or more) sentence without quote marks regardless of whether it is cited or not, e.g. *Yer a Jedi, Harry* (Gandalf of House Stark)
 - * use quote marks if need be, e.g. "Yer a Jedi, Harry" (Gandalf of House Stark), although don't do it too often, because then one could argue you are not passing any of your thoughts through
 - * any more than around three usages in text is probably excessive
- copying without citation or wrong citation, e.g. "Yer a Jedi, Harry", or "Yer a Jedi, Harry" (Jon Snow of Tatooine)
- changing a few words but sentence largely the same, e.g. *You, Harry, sir, are a Jedi* (Mithrandir of Winterfell)
- Turnitin will pick out most of the aforementioned things
- Cases can be contested but will lead to an official review, where the penalty may go up and/or down, and could result in an Academic Misconduct case being filed (see https://acadreg.ust.hk/generalreg.html#b)

Problems

1. Consider the linear equation of state given by

$$\rho = \rho_0 [1 - \alpha (T - T_0) + \beta (S - S_0)].$$

Take the references as $\rho_0 = 1000 \text{ kg m}^{-3}$, $T_0 = 25 \text{ °C}$, and $S_0 = 36 \text{ g kg}^{-1}$. Calculate the density associated with the below scenarios, and give your answer as *density anomalies*, i.e. $\rho - \rho_0$, accurate to two decimal places in the sense of say *ab.cd* with appropriate units:

(a)
$$\alpha = 2 \times 10^{-4} \, {}^{\circ}\text{C}^{-1}$$
, $\beta = 2 \times 10^{-4} \, \mathrm{g}^{-1} \, \mathrm{kg}$, $T = 24 \, {}^{\circ}\text{C}$, $S = 34 \, \mathrm{g \, kg}^{-1}$? [2 marks]

(b)
$$\alpha = 2 \times 10^{-4} \text{ K}^{-1}$$
, $\beta = 2 \times 10^{-4} \text{ psu}^{-1}$, $T = 26 \,^{\circ}\text{C}$, $S = 38 \,\text{g kg}^{-1}$? [2 marks]

- (c) Calculate the *magnitude* (i.e. I don't care about the signs) of the buoyancy $b = g(\Delta \rho/\rho_0)$ between case (a) and (b) above, taking $g = 10 \text{ m s}^{-2}$, giving your answer as a fraction or in exponential form accurate to two decimal places, i.e. $a.bc \times 10^d$, and with the appropriate units. [2 marks]
- (d) Suppose I have a blob of water of density ρ_0 and volume 1 m³ and it is at rest. If it were to be subject to the above answer for buoyancy (not going to clarify its nature or the units since this is part of the last sub-question) by its surroundings, work out the absolute minimum force (or, to be more precise, the lower bound of the force) required to move the blob of water in the *along*-isopycnal direction, assuming all other forces experienced by the blob of water at rest are negligible compared to the associated buoyancy force. Give your answer in appropriate units to two decimal places.

[2 marks]

half a page blank for formatting reasons, Q2, 3 and bonus question to come

2. The question concerns geostrophic balance, various curls/vorticities and Ekman upwelling. Below is an extremely detailed map that I spent a very large amount of nano-seconds making:

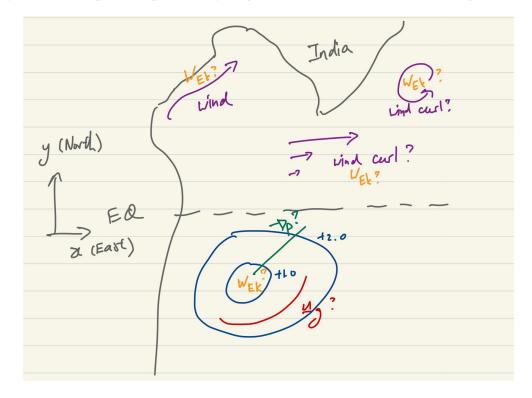


Figure 1: A high quality map.

For the below questions, you could make a copy of the map and draw on the answers, but I still want a one line explanation to your answer somewhere. Use the convention that anti-clockwise flow is positive vorticity, and that positive $w_{\rm Ek}$ means upwelling.

- (a) Taking the blue lines to be contours of SSH in whatever units you like, assuming hydrostatic balance, draw on the direction of the negative pressure gradient (i.e. add the right arrowhead to the green line). [1 mark]
- (b) Assuming geostrophic balance, give the direction of the associated geostrophic flow for the aforementioned SSH anomaly (i.e. add the right arrowhead to the red line). [1 mark]
- (c) Suppose the contours of SSH in blue denote an eddy in geostrophic balance, state whether this eddy has positive, negative or zero vorticity. Also state whether this is a cyclonic eddy, anti-cyclonic eddy, or neither. [2 marks]
- (d) Does the wind curl directly south of India has positive, negative or no curl? [1 mark]
- (e) State whether the wind curl East of India is cyclonic, anti-cyclonic, or neither. [1 mark]
- (f) There are four w_{Ek} drawn on the map. For each of these, state whether there is an implied geostrophic Ekman upwelling, downwelling or neither. [4 marks]

3. One of the dynamical processes that can lead to substantial downward motion of seawater is the process of cabbeling, relying on the nonlinear equation of state. Look up and describe what that is in your own words, possibly with the aid of a diagram. Furthermore, pictorially or otherwise, explain (*a*) why you cannot get cabbeling in a linear equation of state, and (*b*) construct a (not necessarily physical) example where you have a nonlinear equation of state but you do not get cabbeling that helps with deep water formation.

[4 marks]

!? (No marks bonus question) A slight complication when we talk about the Coriolis effect is that we often invoke what is known as the *traditional approximation*. Look up what that is, explain in your own words how that differs to the full Coriolis effect, and where it might matter, and what it could affect. Cite any sources you use.