# OCES 2003 Assignment 4, Spring 2022

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Set on: Thur 5th$^{\text{th}}$ May; due: Thur 12$^{\text{th}}$ May

## Blurb

- Assignments have a maximum mark out of 20, although you will see that there are 22 marks available to get in total, i.e. if you get 22/20 you still only get credit for 20/20
  - 16-17 is roughly around the A- boundary
  - anything below 8 is probably a fail
- Please show working in calculation
  - no working + wrong answer = no credit whatsoever
  - some working + wrong answer = partial credit
  - generically, give things to 2 decimal place and provide the appropriate units (marks are allocated for these), unless otherwise specified
- No answers except the 'hard' ones should need more than a paragraph / half a page, and excess answers that are not to the point will be penalised
- Type up the assignment or send a photo of your written up work in (the former is preferred), and the only request I have is no Microsoft Word documents (you can type up things with Word but export it as a pdf if you do)
  - write in full sentences where appropriate
  - particularly poor and/or scrappy presentation will have a mark that can be taken off
- There will be a rigid mark scheme, and model solutions will be available in due course
  - the TAs only mark the stuff, you should come to the instructor for arguing marks, and note the re-marking can result in marks going up or down

- !!! By handing something in, you agree to the usual Academic Honour code and Integrity declarations. For more, see `http://qa.ust.hk/aos/academic_integrity.html`. Cases for plagiarism (whether intended or not, it is the "act" that matters) gets a penalty ranging from

  - zero on the question concerned
  - a fixed penalty starting from around 1/3 of the total marks
  - zero for the whole assignment/midterm/final

  The following counts as plagiarism (and is a non-exhaustive list):

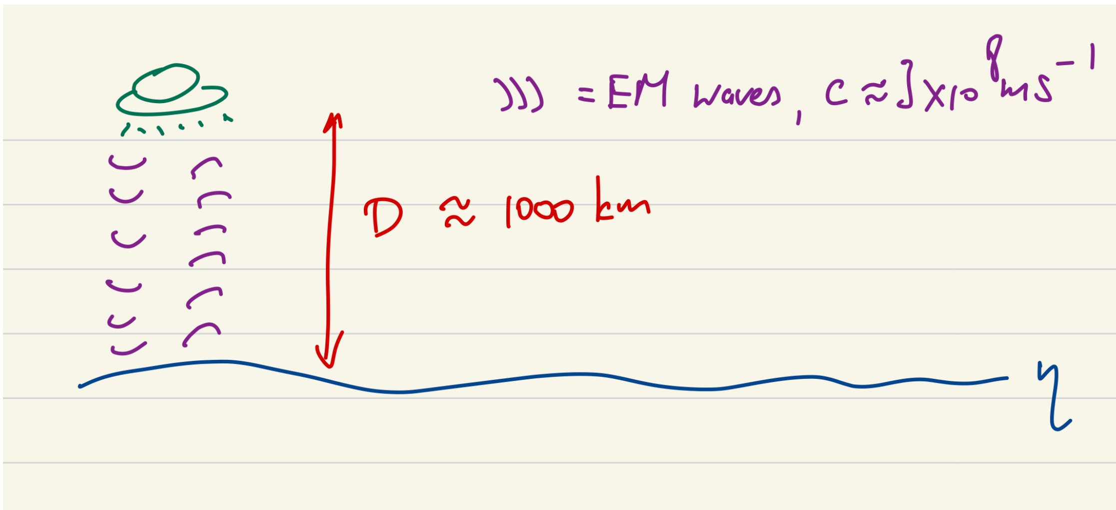
  - copying word for word *any* (i.e. one or more) sentence without quote marks regardless of whether it is cited or not, e.g. *Yer a Jedi, Harry* (Gandalf of House Stark)
    - use quote marks if need be, e.g. "*Yer a Jedi, Harry*" (Gandalf of House Stark), although don't do it too often, because then one could argue you are not passing any of your thoughts through
    - any more than around three usages in text is probably excessive
  - copying without citation or wrong citation, e.g. "*Yer a Jedi, Harry*", or "*Yer a Jedi, Harry*" (Jon Snow of Tatooine)
  - changing a few words but sentence largely the same, e.g. *You, Harry, sir, are a Jedi* (Mithrandir of Winterfell)

- Turnitin will pick out most of the aforementioned things
- Cases can be contested but will lead to an official review, where the penalty may go up and/or down, and could result in an Academic Misconduct case being filed (see `https://acadreg.ust.hk/generalreg.html#b`)

## Problems

1. For this problem we are going to go through a few rough but quantitative details relating to satellite altimetry.

   (a) Consider the diagram below:

   Suppose the satellite (or UFO whatever) is emitting electro-magnetic waves (which travel at the speed of light $c$). Explain and derive the expression

   $$D = \frac{c\Delta t}{2},$$

   where $D$ is the distance between the satellite and the sea surface $\eta$, and $\Delta t$ is the elapsed time between when the satellite sends out the signal and when it receives the rebound signal.

   [1 mark]

   *(The point here is that the right hand side contains stuff you know or can measure, and the left hand side is the stuff you want.)*

   (b) Given $c = 3 \times 10^8$ m s$^{-1}$ and the orbiting height of the satellite is around 1000 km above sea level, give an estimate of $\Delta t$. Give your answer to the nearest millisecond ($10^{-3}$ s). [1 mark]

   *(Temporal variation in those elapsed times is going to give you the variation in sea/land surface if you know the geographical location of the satellite; that is a separate problem we are not going to deal with here.)*

   (c) Describe in words how we might get a relation between the sea surface height $\eta$ and the geostrophic velocity $\boldsymbol{u}_g$, as something like (you don't need to derive this; yes I have dropped a factor of $\rho$)

   $$\boldsymbol{e}_z \times \boldsymbol{u}_g \sim \frac{g}{f}\nabla\eta \tag{1}$$

   [2 marks]

*(So now you can in principle measure $\eta$ as a varying function of geographic location, and taking gradients and multiplying some numbers you can infer a geostrophic velocity out of that. In practice you might care about the fact that $g$ and $f$ are also functions of space, and you probably want to do some data tidying to reduce possibilities of bad sampling.)*

(d) Estimate the Rossby deformation radius $L_d = NH/f$ taking (fairly typical values of) $H = 4000$ m, $N^2 = 10^{-5}$ s$^{-1}$ and $f = 10^{-4}$ s$^{-1}$. Give your answer to the nearest 100 km. [1 mark]

(e) Given $\nabla\eta \sim \eta/L$ where $L$ is some horizontal characteristic length scale, argue why we might take $L = L_d$ as computed in the previous part. [1 mark]

(f) Using the relation in Eq.(1) and part (*e*) of this question, show that in order to have an uncertainly/error in the geostrophic velocity of no more than $10^{-2}$ m s$^{-1}$, we need to be able to measure the sea level $\eta$ accurate to about 1 cm. [3 marks]

(g) Show additionally this implies that the satellite has an accuracy of about 1 part in a 100 million (1 part in $10^8$). (Hint: how far away is the satellite?) [1 mark]

*(Details do matter, but this is in principle how satellite altimetry works, and can be automated on a computer. What always impresses me is that 1 part in 100 million is like someone shooting a bullet from the top of Mount Everest to a tree at the bottom where a mosquito is sitting, and hitting the mosquito every time.)*

2. This question is about tides.

(a) The following link has a (rather out of date and in my opinion wrong) description of how tides manifest:

`https://oceanservice.noaa.gov/education/tutorial_tides/tides03_gravity.html`

State two notable differences in their description of why there are two bulges to the equilibrium tide theory that was introduced in lecture 18. [1 marks]

(b) It can be derived that the acceleration due to tidal forces is given by (e.g. see Assignment 4 last year)

$$a = \frac{2Gmr_{\mathrm{Earth}}}{r^3},$$

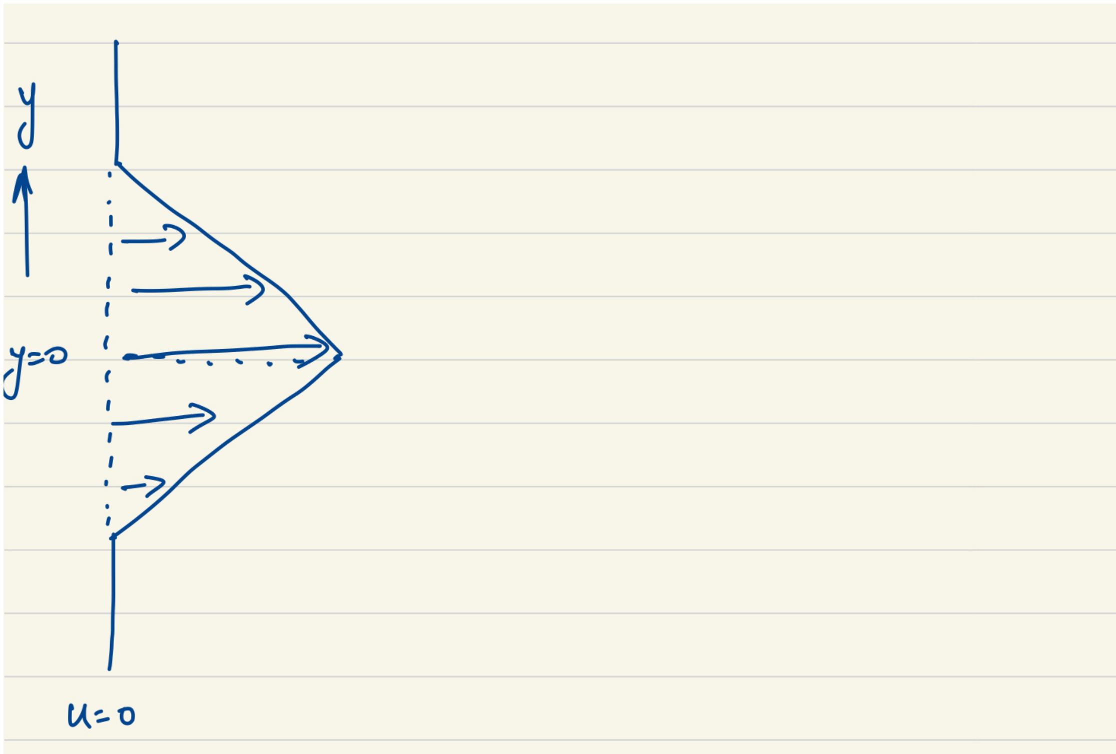
where you should look up the previous assignment 4 for the meaning of the terms (it's on the GitHub repository). Show by algebraic reasoning or computational arguments that the magnitude of the tidal acceleration $a$ due to the moon on Earth varies by so little over the ocean depth that we can basically assume it is constant (cf. $g$ varying over the ocean depth). Use $H = 4000$ m as above if you really want to compute some numbers. [2 marks]

(c) Show by algebraic reasoning or computational arguments that $a$ is tiny compared to $g$ (hint: it's about a $10^7$ difference in the raw value if you want to go down the computational route). [1 mark]

(d) (Probably requires a bit of reading) Following on from previous part, we note that we as humans under the influence of $g$ itself struggle to be jump more than about a meter or two off the ground. Given $a$ is so much smaller than $g$, explain briefly how we are actually getting such tidal signal in the sea surface that is around a meter in magnitude. (Regarding "brief", it is entirely possible to get full marks with a single sentence answer of around 20 words.)

The written course notes provide a hint of this. Otherwise, this (`https://iopscience.iop.org/article/10.1088/0031-9120/50/2/159`) or this (`https://www.lockhaven.edu/~dsimanek/scenario/tides.htm`) will probably help. [2 marks]

3. Consider the horizontal shear flow (not subject to rotation, stratification, viscosity etc.) below:




Draw a unstable scenario using interacting vorticity/Rossby waves that is relevant for this flow profile, such that all three vorticity waves in this system are growing. For this you might want to split the working in a few steps, in the manner of the diagrams in lecture 17. While my model solution gives all of this within a single diagram, I would suggest you split the question into the following parts and do them in order:

- State/draw on the sign of the vorticity associated with the profile in each linear segment and where it is zero, noting that vorticity is given by $-\mathrm{d}U/\mathrm{d}y$.

  [1 mark]

- Assume the waves are on the sharp edges where the vorticity jumps are, these edge waves conserve vorticity, and carries with it a vorticity value that is exactly half of the neighbouring vorticity strips. Draw on the associated vorticity anomalies associated with the wave forms, and from those derive the direction that each of the three waves are propagating in. (Hint: the middle should be different to the outer two, but show this explicitly.)

  [3 marks]

- Figure out the phase differences you need to make sure all three vorticity waves are growing at the same time, and demonstrate all three waves are really amplifying in the configuration you have drawn on. You might need to try this a few times; it took me a three tries and I would suggest you fix the middle wave and shift the outer ones accordingly. Drawing on some grid lines really helped.

  [2 marks]

(Hint: The answer is actually implicitly in Tamarin *et al.* 2015 available at `https://doi.org/10.1175/JPO-D-15-0139.1`, who tackles this triangular jet profile, but it is not immediately obvious where the answer is unless you actually know what is going on anyway, hence why I am not that bothered giving you this hint.)

!? (Bonus question, no marks + for interest only)

Relating to Q3, it is known there are two unstable modes (an even and an odd one about $y = 0$) for profiles that are even about $y = 0$, and the answer I had in mind above is probably the unstable configuration associated with the even mode. See if you can draw out what the odd mode might look like.

(A function $f$ is even about $y = 0$ if $f(-y) = f(y)$, and odd about $y = 0$ if $f(-y) = -f(y)$.)