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- ▶ Associated Python code (as Jupyter notebooks mostly) will be held on the same repository. The source data however might be big, so I am going to be naughty and possibly just refer you to where you might get the data if that is the case (e.g. JRA-55 data). I know I should make properly reproducible binders etc., but I didn't...
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OCES 2003 : Descriptive Physical Oceanography

(a.k.a. physical oceanography by drawing pictures)

Lecture 16: Dynamics 2 (waves and dynamic mechanisms)

Outlook of the next few lectures

Dynamics important, next few lectures on

- ▶ **waves** (this Lec. + 16, 18) and **instabilities** (Lec. 17)

→ because waves are easier to talk about without maths...

Highlight gross features (i.e. those that can be drawn...)

- ▶ how to describe waves (Lec. 15)
- ▶ **types of waves** (Lec. 16)
 - consequence + leading to instabilities
- ▶ instabilities (Lec. 17)
 - **parcel**-type (mechanistic) arguments for instability
- ▶ tides (particularly as **internal gravity waves**) (Lec. 18)

Outline

- ▶ gravity waves
 - gravity/buoyancy as restoring mechanism (\sqrt{gH})
- ▶ inertial waves
 - Coriolis as restoring mechanism (f)
 - e.g. Rossby waves, Kelvin waves
- ▶ inertial-gravity + internal waves (\sqrt{gH} or N , and f)
 - extra depth dimension to deal with
 - Brunt–Väisälä or buoyancy frequency N
- ▶ propagation mechanism (Rossby wave example)
 - kinematic argument with **vorticity**

Key terms: buoyancy frequency, gravity waves, inertial waves, Rossby waves, Kelvin waves, vorticity inversion

Recap: what goes down must come up

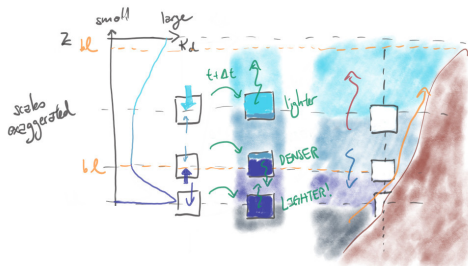


Figure: Schematic of the diffusive upwelling.

- diapycnal mixing contribute upwelling, strongest in boundary layers
→ broad diffusive boundary intensified upwelling

what causes the boundary intensification of κ_d ? **dynamics!**

- at the surface, lots of things... (convection, waves, Langmuir turbulence etc.)
- at the bottom, probably tidal conversion (Lec. 18) → internal gravity waves (Lec. 16) → shear instabilities (Lec. 17)

Recap: waves and dispersion relation

- ▶ **waves** are ubiquitous physical features
→ depends on physics
- ▶ wave described by the **dispersion relation** $\omega = \mathcal{F}(k)$
→ physics dictate the form of \mathcal{F}

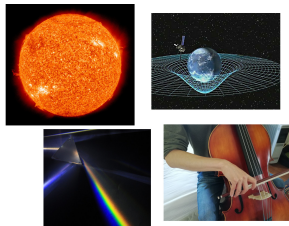


Figure: Examples of systems supporting waves. All figures from Wikipedia except the cello one.

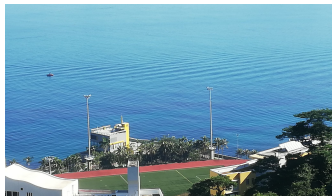


Figure: Gravity waves with signal at the sea surface (as darker and lighter bands). Taken at HKUST.

- ▶ (linear) waves can **interfere** with each other
→ **constructive** or **destructive**
→ interference can lead to **steepening** and **breaking** (“becoming”
nonlinear)

Recap: wave propagation

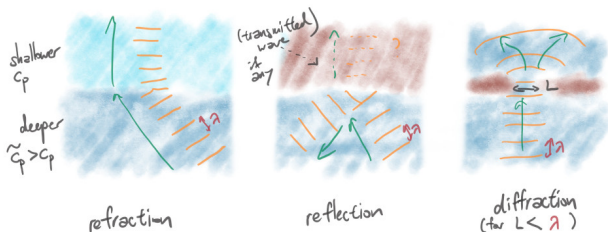


Figure: Schematic of **refraction**, **reflection** (and **transmission**), and **diffraction**. See Lec. 15.

- **phase speed** (in a direction) and **group velocity** as (note $\omega = \mathcal{F}(k)$)

$$c_{p,x} = \frac{\omega}{k}, \quad c_{g,x} = \frac{\partial \omega}{\partial k}$$

→ individual wave vs. **wavepacket** behaviour

→ contribute to wave phenomenon (e.g. refraction from $c_p = c_p(x)$)

Wave steepening and breaking



Figure: Schematic of mixing by (irreversible) wave breaking, with contours reconnecting leading to e.g. diapycnal mixing.

- ▶ growing waves by **instability**
 - convective and/or shear (see Lec. 17)
 - mixing of material **across** isopycnals after reconnection, leading to **diapycnal mixing**
- ▶ feedback onto MOC (see Lec. 14)

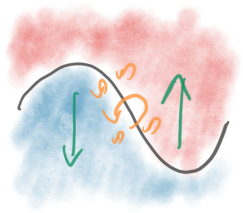


Figure: Velocity shear from waves can lead to mixing.

Gravity waves

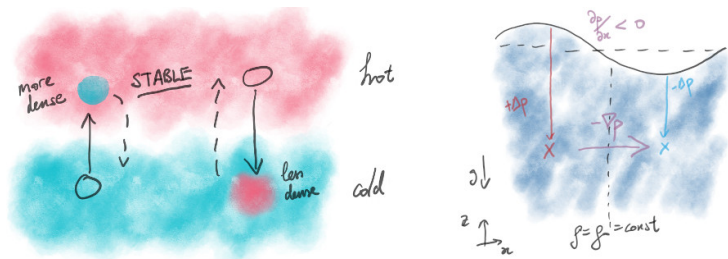


Figure: Gravity as restoring force. Pictures adapted from ones used in Lec. 7.

- ▶ deviation from resting **isopycnal** experiences restoring force from gravity (buoyancy)
 - left case: internal isopycnal (as an **isotherm**)
 - right case: sea surface is the isopycnal
- ▶ weak damping, restoring force, overshoots \Rightarrow oscillatory motion (up and down in this case)

Gravity waves

For simplicity, consider a **homogeneous** (i.e. $\rho = \text{const}$) layer of fluid (cf. Lec. 11 + 12, right case in previous Figure) until we get to **internal waves**

Gravity waves

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In this instance, dispersion relation for **gravity waves** is given by (without derivation)

$$\omega^2 = gk \tanh(kH) \quad \Rightarrow \quad \omega = \pm \sqrt{gk \tanh(kH)}$$

- ▶ H is water depth, and \tanh = hyperbolic tangent, goes from -1 to 1
 - note **symmetry** in both directions (the \pm sign)
 - ($\omega \leq 0$ cases are just **shifts** in the **phase**)

Gravity waves

$$\omega = \pm \sqrt{gk \tanh(kH)}$$

- ▶ for **deep** water waves ($kH \gg 1$) and **shallow** water waves ($kH \ll 1$),

$$\omega_{\text{deep}} = \pm \sqrt{gk}, \quad \omega_{\text{shallow}} = \pm k \sqrt{gH}$$

→ $kH \gg 1$ so $\tanh(kH) \rightarrow 1$

→ $kH \ll 1$ with $\tanh(kH) \approx kH + O((kH)^3)$ (do a Taylor expansion)

- ▶ deep water waves are **depth-independent** and **dispersive**
- ▶ shallow water waves are slower in shallow waters ($c_p \sim \sqrt{H}$) and **non-dispersive** ($c_p = c_g$)

Gravity waves

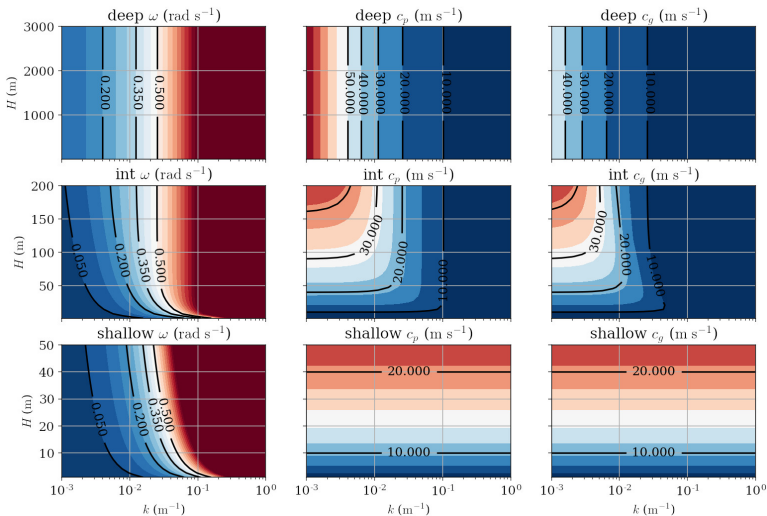


Figure: Water wave ω , c_p and c_g over (k, H) space, with k shown on a log axis. k chosen so wavelengths are roughly between 50 m to 5 km (recall $k = 2\pi/\lambda$). Also note the transitions from shallow to intermediate to deep are really to do with $kH \sim H/\lambda$. See `waves.ipynb`.

Inertial waves

Inertial waves has the Coriolis “force” act as the restoring force

- ▶ generic for rotating systems
→ planetary interiors, stars, galactic disks
- ▶ mostly arise in context of **internal waves**
→ at surface buoyancy effects can dominate
- ▶ limited in frequency by **inertial frequency** f_0 (cf. Coriolis parameter)
→ revisit later when talking about **inertia-gravity waves**

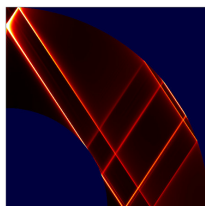
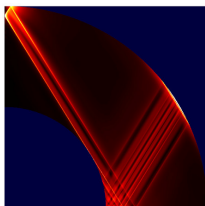


Figure: Inertial wave attractors in a homogeneous planetary interior at different tidal forcings. From Gordon Ogilvie (2009, *Mon. Not. Royal. Astro. Soc.*).

Inertial-Gravity waves

In reality **Coriolis** and **buoyancy** effects both contribute

- ▶ large-scale and/or slow \Rightarrow Coriolis important (because $Ro \ll 1$), classify as inertial waves
 \rightarrow e.g. Rossby waves
- ▶ small-scale and/or fast \Rightarrow Coriolis unimportant
 \rightarrow e.g. internal gravity waves
- ▶ somewhere in between? **Poincaré** or **inertia-gravity** waves

$$\omega = \pm \sqrt{f_0^2 + gH(k_x^2 + k_y^2)}$$

\rightarrow for $gH(k_x^2 + k_y^2) \gg f_0$, recover gravity waves

\rightarrow for $gH(k_x^2 + k_y^2) \ll f_0$, recover inertial waves

Rossby deformation radius

$$\omega = \pm \sqrt{f_0^2 + gH(k_x^2 + k_y^2)}$$

- boundary between gravity and inertial regimes is roughly

$$L_d = \frac{\sqrt{gH}}{f_0}$$

- the **Rossby deformation radius** (for shallow water system)
 - roughly also the boundary where **geostrophic approximation** should hold (see Lec. 8 + 13)
 - estimates in a few slides

Internal waves

- introduce a useful quantity

$$N = \sqrt{-\frac{g}{\rho_0} \frac{\partial \rho}{\partial z}}$$

- Brunt–Väisälä or buoyancy frequency (units: s^{-1})

→ N^2 normally used

→ note $\partial \rho / \partial z < 0$ for **stable** stratification, i.e.

$$N^2 > 0$$

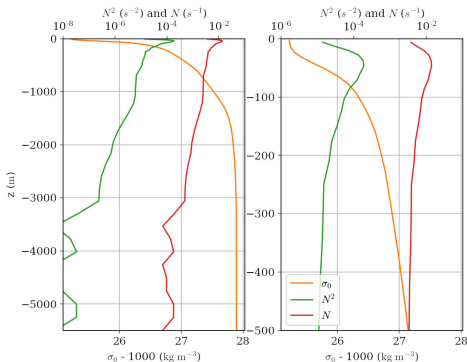


Figure: σ_0 (see Lec. 6) and the associated N^2 and N . $N^2 \ll 1$ means weakly stratified (weak density gradients), whilst $N^2 < 0$ shows unstable stratification (none in this case, but see Lec. 17). See `plot_eos.ipynb`.

simplistic view (!): $\sqrt{gH} \rightarrow N$

Internal waves

Generally then, **internal inertia-gravity waves** described by

$$\omega = \pm \sqrt{\frac{f_0^2 k_z^2 + N^2 k_x^2}{k_x^2 + k_z^2}} \approx \pm \sqrt{f_0^2 + \frac{N^2 k_x^2}{k_z^2}} \quad (\text{for } |k_z| \gg |k_x|)$$

- atmosphere and ocean has $N/f_0 = O(10^1 \text{ to } 10^2)$
 - so really we have **gravity waves influenced by rotation**
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- ▶ note that $|f_0| \leq |\omega| \leq |N|$
 - since $0 \leq k_{x,z}^2 / (k_x^2 + k_z^2) \leq 1$
 - frequency is **much lower** than gravity waves

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internal tides to be seen as internal waves (Lec. 18)

Internal waves

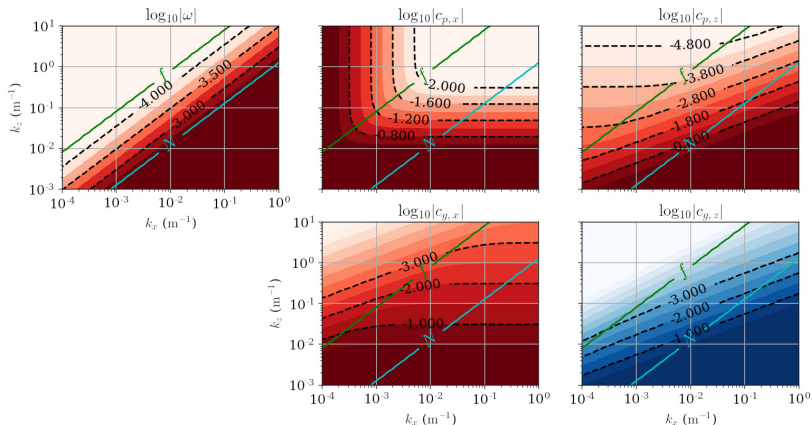


Figure: Inertial-gravity waves (with the $k_z \gg k_x$ approximation) ω , $c_{p,x}$, $c_{p,y}$, $c_{g,x}$ and $c_{p,y}$ as a log-log plot in (k_x, k_z) space, with $f = 5 \times 10^{-5}$ and $N = 3 \times 10^{-3}$ (oceanic relevant values). The contours denote the exponent x of $|10^x|$ and the colour shading denotes the sign (more blue = more negative *actual* values rather than exponents, more red = more positive *actual* values rather than exponents); since k_x and k_z is chosen to be positive, everything except $c_{g,z}$ is positive. Contours of f and N plotted with an offset plotted to show the boundary beyond which everything is either gravity waves or inertial oscillations. See `waves.ipynb`.

Internal waves

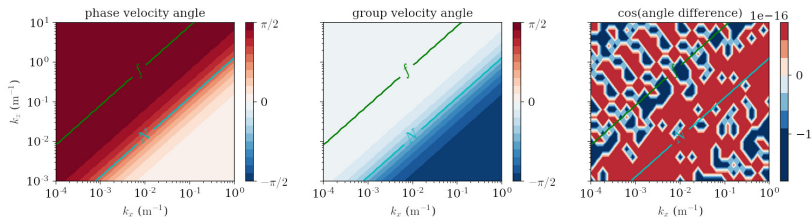


Figure: Inertial-gravity waves (with the $k_z \gg k_x$ approximation) phase velocity c_p angles and group velocity c_g angles (in radians, relative to the horizontal, and note $\pi/2 = 90^\circ$). The final panel shows $c_p \cdot c_g = |c_p||c_g| \cos \theta$ (which is zero up to rounding errors). Contours of f and N plotted with an offset plotted as in the previous diagram. See `waves.ipynb`.

- note that, for inertial-gravity waves (left as a bonus exercise),

$$c_p \cdot c_g = 0$$

→ i.e. phase and group velocities are **perpendicular** to each other (see Lec. 4)

Deformation radius

- ▶ boundary given again by the **Rossby deformation radius** (for the continuously stratified case)

$$L_d = \frac{NH}{f}$$

- $L_{d,\text{atmos}} = O(1000 \text{ km})$, scale of **cyclones** and **anti-cyclones**, i.e. weather systems form (synoptic structures)
- $L_{d,\text{ocean}} = O(50 \text{ km})$, scale of **ocean eddies**
- ▶ latitude (through f) and H dependent
 - smaller L_d for **high** latitudes and **shallow** regions
 - consequence for **geostrophic approximation?** (e.g. shelves and coasts)

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 - smaller L_d for **high** latitudes and **shallow** regions
 - consequence for **geostrophic approximation?** (e.g. shelves and coasts)
- ▶ internal L_d defined analogously (normally smaller than above)

Kelvin waves (more on this in Lec. 18)

A type of boundary wave

- ▶ need f and a **boundary**
 - could be land (**coastal Kelvin waves**) (see Lec. 18)
 - could be a **wave guide** (e.g. equator where f changes sign, **equatorial Kelvin waves**) (see OCES 4001, El-Niño, QBO etc.)
- ▶ needs f but propagates at the **gravity** wave speed, with

$$\omega = k\sqrt{gH}$$

- **non-dispersive**
- fairly fast (gravity wave speed)

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- ▶ **NOTE the lack of \pm !**

Kelvin waves (more on this in Lec. 18)

- ▶ boundary introduces **asymmetry** in this case: general solution like

$$\eta \sim e^{\pm f_0 y / \sqrt{gH}} \cos(kx - \omega t)$$

→ take $y \leq 0$ to be **boundary**, if $f_0 > 0$ (NH), need minus sign, and vice-versa

→ wave propagates **cyclonically** (same sign as f)

- ▶ taking $f_0 > 0$ (NH),

$$\eta \sim e^{-y/L_d} \cos(kx - \omega t),$$

so decay over the $L_d = \sqrt{gH}/f_0$

Rossby waves (more on this later)

A (particularly important) type of **inertial** wave

- ▶ requires a **gradient** in background vorticity
→ $\partial f / \partial y = \beta$ (planetary case)
→ background flow $-\partial U / \partial y \sim \nabla \times \mathbf{u}$ (see later and Lec. 17)
- ▶ dispersion relation given by (on β -plane)

$$\omega = -\frac{\beta k_x}{k_x^2 + k_y^2}$$

→ note that Rossby waves propagate to the **west** (more generally, **retrograde** or against the mean flow) **since**

$$c_{p,x} = \frac{\omega}{k_x} = -\frac{\beta}{k_x^2 + k_y^2} < 0,$$

and **long** waves ($k_x \ll 1$) are **fast(er)**

Rossby waves (more on this later)

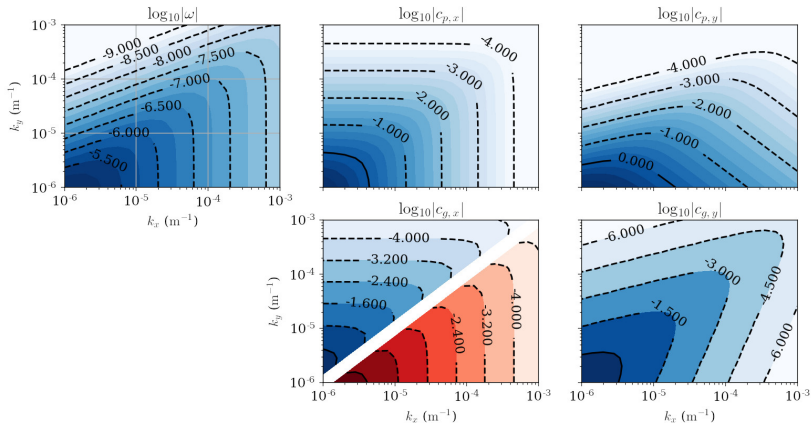


Figure: Rossby waves ω , $c_{p,x}$, $c_{p,y}$, $c_{g,x}$ and $c_{p,y}$ as a log-log plot in (k_x, k_y) space, with magnitude also as logs. The contours denote the exponent x of $|10^x|$ and the colour shading denotes the sign (more blue = more negative *actual* values, more red = more positive *actual* values); since k_x and k_y is chosen to be positive, everything except $c_{g,x}$ is negative. Choice of k_x and k_y correspond to wavelengths roughly between 6 km to 6000 km (Rossby waves are usually seen as planetary-scale waves). See `waves.ipynb`.

Propagation mechanism: Rossby waves

Rossby waves propagate **west-ward** (or, more generally, **retrograde**)

$$c_{p,x} = \frac{\omega}{k_x} = -\frac{\beta}{k_x^2 + k_y^2} < 0$$

why?

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why?

Key bits to the pictorial/parcel (cf. Lec 5 for temperature) argument:

- ▶ the initial wave **conserves** and carries **vorticity** (spini-ness, recall Lec. 4, 11, 12) into the external environments
→ these are now vorticity **anomalies**
- ▶ vorticity anomalies induces a velocity/flow (because spini-ness)
- ▶ induced flow seen to self-advect the wave and move it to the **West** (**retrograde** in the general case)

Propagation mechanism: Rossby wave example

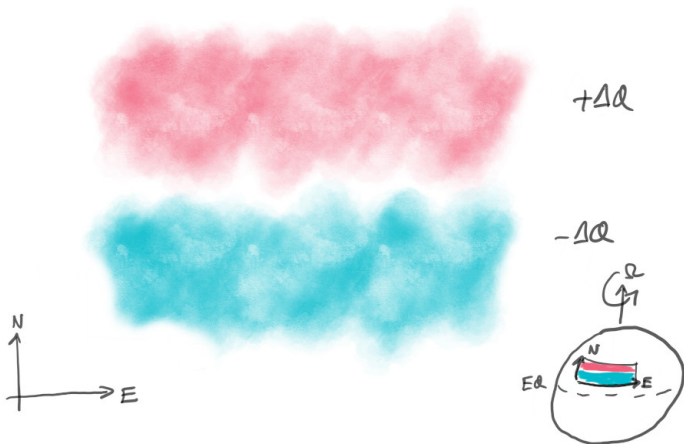


Figure: Rossby wave propagation schematic.

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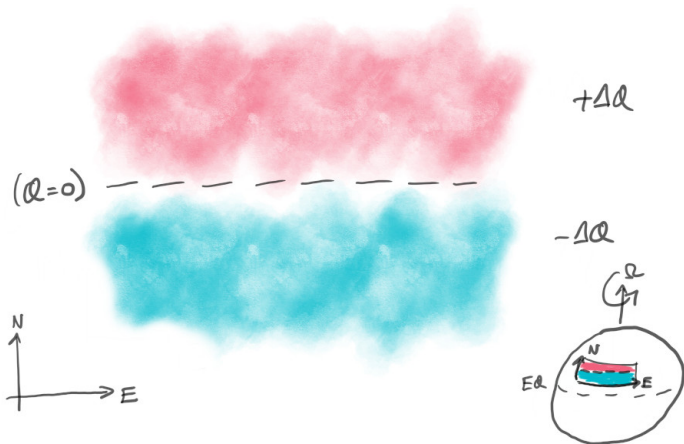


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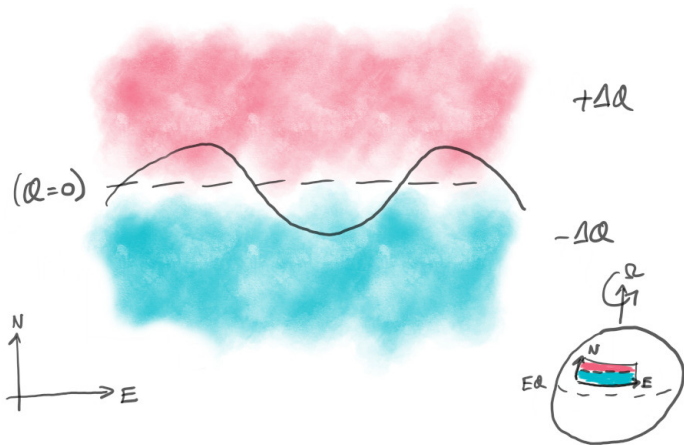


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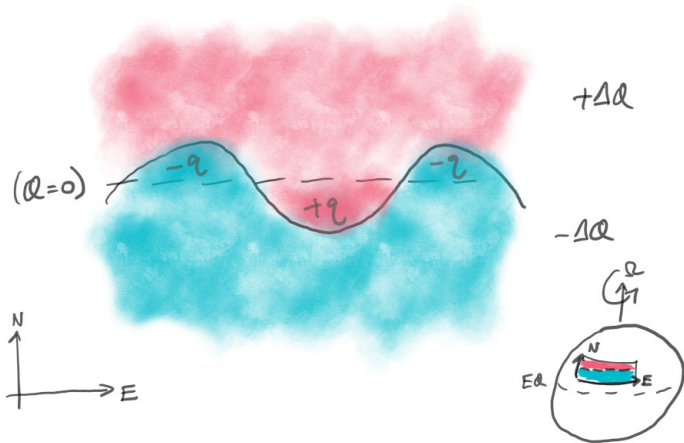


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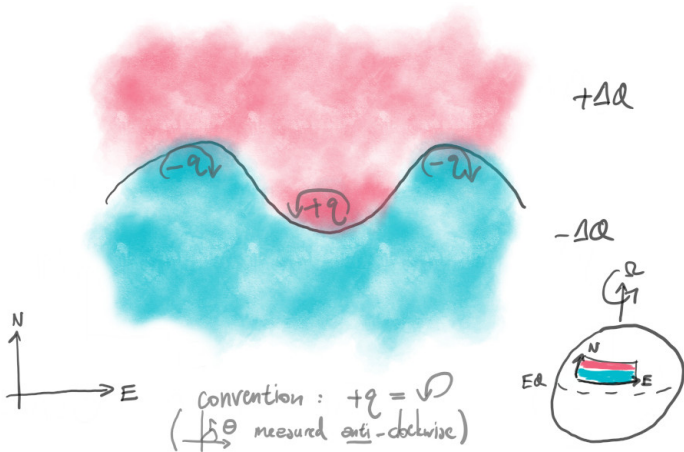


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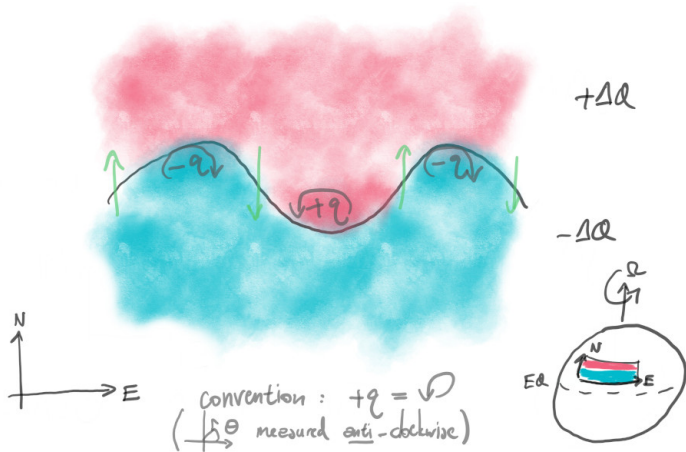


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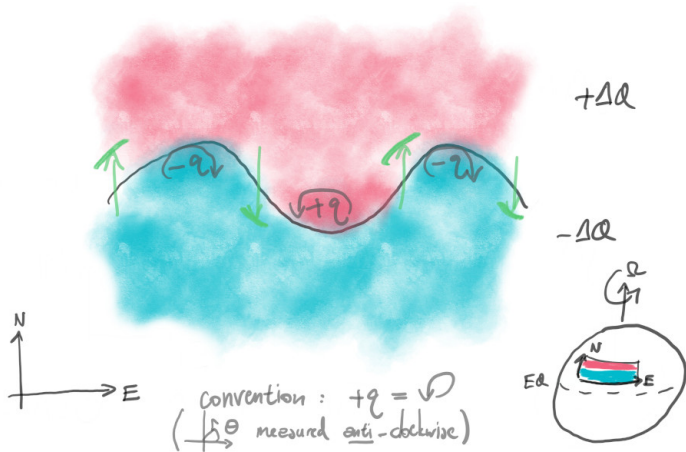


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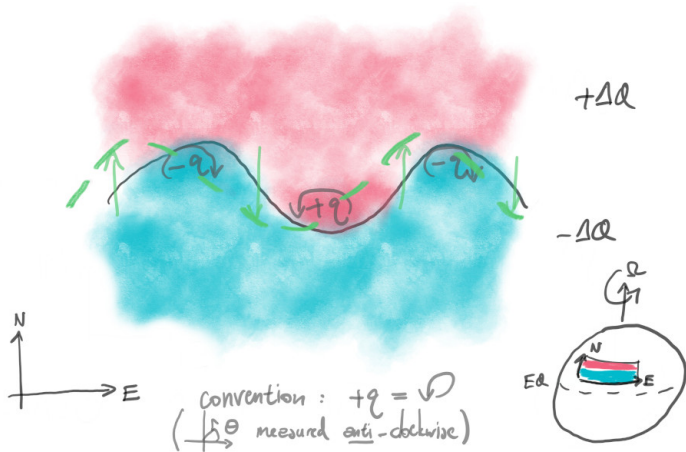


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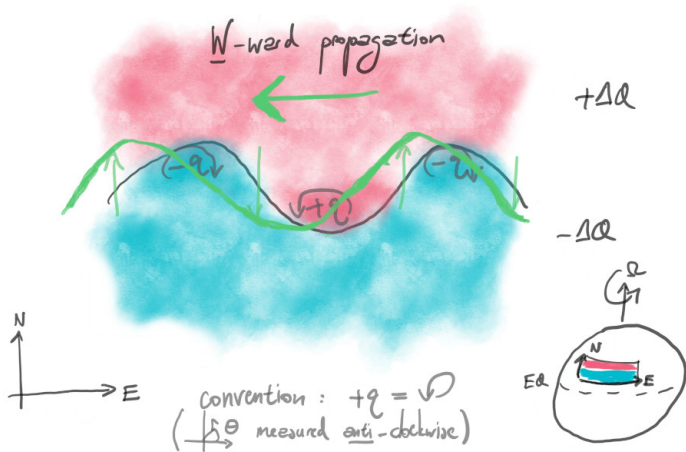


Figure: Rossby wave propagation schematic.

Summary

- gravity waves
(gravity/buoyancy)
- inertial waves (Coriolis)
- inertial-gravity waves (general)
→ internal waves have
 $|f| \leq |\omega| \leq |N|$

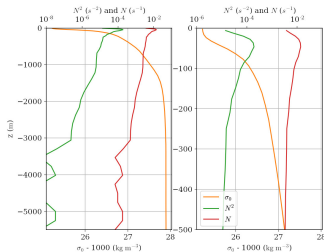


Figure: σ_0 (see Lec. 6) and the associated N^2 and N . See `plot_eos.ipynb`.

- Brunt–Väisälä or buoyancy frequency N

$$N = \sqrt{-\frac{g}{\rho_0} \frac{\partial \rho}{\partial z}}$$

→ measure of stratification strength (see also Lec. 17)

Summary

- ▶ parcel argument for **west-ward Rossby wave** propagation
 - conservation of **vorticity**
 - vorticity anomalies induces flow
 - self-advecting

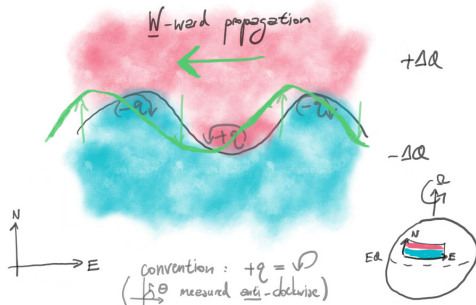


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- ▶ generalisations exist (e.g. internal gravity waves in Harnik *et al.*, 2008, *J. Atmos. Sci.*)

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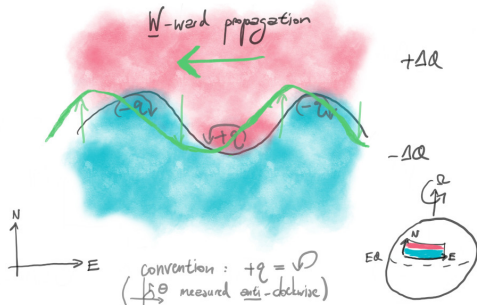


Figure: Rossby wave propagation schematic.

- ▶ generalisations exist (e.g. internal gravity waves in Harnik *et al.*, 2008, *J. Atmos. Sci.*)
- ▶ two such waves interacting? (see Lec. 17)
 - potential for **instabilities**