OCES 2003 Assignment 1, Spring 2021

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Set on: Tue 16th Feb; due: Tue 23th Feb

Model solutions and mark scheme

Problems

1. - Gulf Stream, east coast of N. America, west part of Northern Hemisphere Pacific
   - Brazil Current, east coast of S. America, west part of Southern Hemisphere Pacific
   - Aghulas Current, east coast of Africa, west part of Indian Ocean
   - Kuroshio, east coast of Asia, west part of Northern Hemisphere Pacific
   - East Australian Current, east coast of Australia, west part of Southern Hemisphere Pacific

   Might have missed some. Other answers acceptable if the current is on the west and sufficiently deep (more than 500 m) *(1 mark for getting any four, 0.5 mark for missing some locations or getting only three, 0 marks for anything less)*

2. This question concerns the *Leeuwin current*.
   (a) The Leeuwin current is on the west coast of Australia *(1 mark for everything, 0 otherwise)*
   (b) The Leeuwin current is poleward, taking equatorial warm water to the poles *(1 mark for both points, 0.5 for one, 0 for neither)*
   (c) With 1 knot = 1.852 km per hour, we have

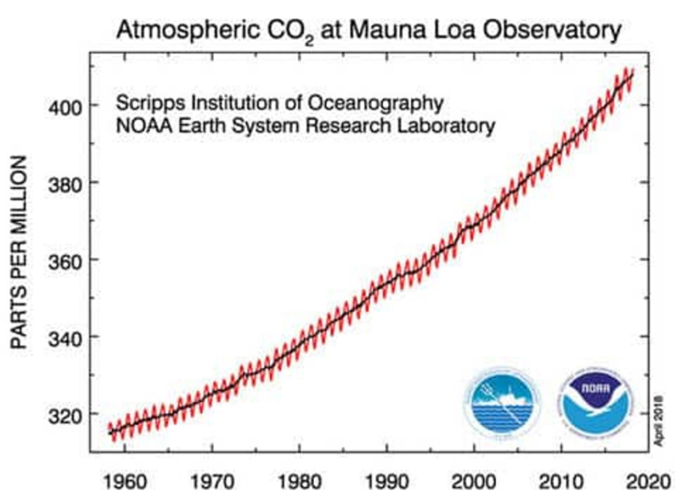
$$\mathrm{km} = 10^3\ \mathrm{m}, \qquad \mathrm{hr}^{-1} = (60 \times 60\ \mathrm{s})^{-1} = (1/3600)\ \mathrm{s}^{-1},$$

   so

$$1.852\ \mathrm{km\ hr}^{-1} = 1.852 \times 10^3/3600\ \mathrm{m\ s}^{-1} = 0.51444....\ \mathrm{m\ s}^{-1} = 0.51\ \mathrm{m\ s}^{-1}$$

   (answer and working for 2 marks, minus 0.5 marks for missing units, minus 0.5 for not giving it to 2dp, largely correct working but wrong answers give no more than 1 mark)

3. The following graph showing CO_2 concentration against time and is known as the *Keeling curve*, after Charles Keeling (not to be confused with his son Ralph Keeling):



$\partial[CO_2]/\partial t > 0$ i.e. positive because $[CO_2]$ is increasing with time. *(0.5 marks for sign of derivative but no justification)*

4. The second derivative $\partial^2[CO_2]/\partial t^2$ is the rate of change of the rate of change, and since the curve curves upwards, this is positive. This means the rate of increase itself is increasing, i.e. the increase in $[CO_2]$ is larger with time. *(0.5 marks for the sign, 0.5 marks for the justification, 1 mark for the physical interpretation)*

5. Convince yourself that

   (a) $-\boldsymbol{e}_x \times \boldsymbol{e}_z = \boldsymbol{e}_y$

   (b) $\boldsymbol{e}_z \times \boldsymbol{e}_y = -\boldsymbol{e}_x$

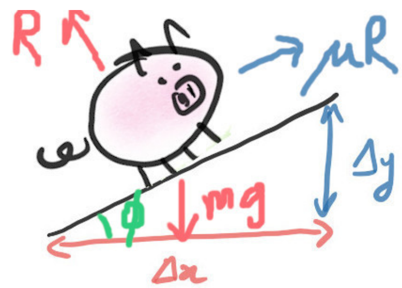
   (c) $\boldsymbol{e}_x \cdot (\boldsymbol{e}_y \times \boldsymbol{e}_x) = 0$ (because the curl generates a vector perpendicular to both inputs, and the dot product of perpendicular vectors is zero)

   (d) $(\boldsymbol{e}_x \times \boldsymbol{e}_y) \times \boldsymbol{e}_z = \boldsymbol{0}$ (you get $\boldsymbol{e}_z \times \boldsymbol{e}_z$ which is the zero vector, mathematically because of the anti-symmetric property, or physically because you can't uniquely generate a vector that is perpendicular to a single input in 3d space, so by default you set it to zero otherwise the operation is not well-defined)

   (half a mark each as long as working is sensible)

6. A uniform flow has no curl or div (because the derivative of any constant is zero). *(1 mark or nothing)*

7. Our neighbourhood friendly pig on an inclined slope:



   (a) "*the pig weighs 100 kg*" is confusing a force (a vector) and a mass (a scalar). "The pig has mass 100 kg" or "the pig weighs 1000 Newtons taking $g = 10$ m s^{-2}" would be better. *(0.5 mark for each for identification and providing a fix)*

   (b) Weight and normal force as above in red. The pig should slide/roll down because there is a net force down the slope. *(0.5 marks for mg being vertically down, 0.5 marks for drawing an R, 0.5 marks for drawing R to be perpendicular to the ground, and 0.5 marks for justifying there is motion because of net forces)*

   (c) As above in blue as μR. Friction acts against motion (which is down the slope) so it has to be pointing up the slope. *(0.5 marks for drawing it up the slope, and 0.5 marks for justifying it has to act against motion)*

   (d) Frictional force is a vector because it has a direction (frictional force *magnitude* would be a scalar). *(0.5 marks for vector and 0.5 marks for justifying)*

8. This slightly more open-ended question is inspired by *the Messinian salinity crisis*. If you block of the Strait of Gibraltar for whatever reason, then this restricts the flow between the Atlantic and Med sea. Now, we also know from the lectures part of the reason the salinity signature is so strong in the Med Sea is because of the large EmP *(0.5 marks)*, and it turns out actually EmP trumps also the river runoff *(0.5 marks)*, so the Med Sea will lose mass as well as gain salinity over time *(0.5 marks)*. The evaporated water is dumped somewhere else, suggesting global sea level should rise *(1 mark only if the answer was justified by some of the preceding arguments)*.

   (0.5 marks for each of the points below, or any other sensible-ish points not in the list below if they are backed up by references, up to a maximum of 2.5 marks) Over long enough time-scales, the Med Sea will probably evaporate off, leaving it in a hypersaline state possibly not unlike the Dead Sea. The global sea level may rise up to 10m or so as a result of the Med Sea drying out. This has in fact happened before in the past, called the *the Messinian salinity crisis*. The Med Sea came out of this kind of state when the Strait opened and Atlantic water flooded the Med Sea. Something similar may happen again in the 'near' future as the African plate moves into Europe, though 'near' is in reference to geological time-scales. The lack of water may have contributed to migration of life from Africa into Europe.

   (1 mark for using and citing sources; I used Wikipedia for all of the above stuff.)

!? (No marks bonus question for those more mathematically inclined.) The angle ϕ is drawn above. Convince yourself that the force balance *perpendicular* to the ground is

$$mg\cos\phi = R,$$

and these do balance otherwise the pig is going to sink into the ground. For the bit parallel to the ground, friction needs to be larger than the force component making the pig slide down, so

$$\mu R = \mu mg\cos\phi \geq mg\sin\phi.$$

Since $mg \neq 0$ and we are going to assume $\phi \in [0, 90^\circ)$ so that $\cos\phi > 0$ (otherwise you have to flip the inequality when you divide both sides by $\cos\phi$), and we get $\mu \geq \tan\phi$ (note this is independent of the object's weight). If $\mu \leq 1$, the greatest angle of elevation that allows for no motion is when $1 = \tan\phi$, i.e. $\pi/4$ or 45°.