OCES 2003 Assignment 4, Spring 2021

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Set on: Thur 29th$^{\text{th}}$ Apr; due: Thur 6$^{\text{th}}$ May

Blurb

- Assignments have a maximum mark out of 20, although you will see that there are 22 marks available to get in total, i.e. if you get 22/20 you still only get credit for 20/20
  - 16-17 is roughly around the A- boundary
  - anything below 8 is probably a fail
- Please show working in calculation
  - no working + wrong answer = no credit whatsoever
  - some working + wrong answer = partial credit
  - generically, give things to 2 decimal place and provide the appropriate units (marks are allocated for these), unless otherwise specified
- No answers except the 'hard' ones should need more than a paragraph / half a page, and excess answers that are not to the point will be penalised
- Type up the assignment or send a photo of your written up work in (the former is preferred), and the only request I have is no Microsoft Word documents (you can type up things with Word but export it as a pdf if you do)
  - write in full sentences where appropriate
  - particularly poor and/or scrappy presentation will have a mark that can be taken off
- There will be a rigid mark scheme, and model solutions will be available in due course
  - the TAs only mark the stuff, you should come to the instructor for arguing marks, and note the re-marking can result in marks going up or down

!!! By handing something in, you agree to the usual Academic Honour code and Integrity declarations. For more, see `http://qa.ust.hk/aos/academic_integrity.html`. Cases for plagiarism (whether intended or not, it is the "act" that matters) gets a penalty ranging from

- zero on the question concerned
- a fixed penalty starting from around 1/3 of the total marks
- zero for the whole assignment/midterm/final

The following counts as plagiarism (and is a non-exhaustive list):

- copying word for word *any* (i.e. one or more) sentence without quote marks regardless of whether it is cited or not, e.g. *Yer a Jedi, Harry* (Gandalf of House Stark)
  * use quote marks if need be, e.g. "*Yer a Jedi, Harry*" (Gandalf of House Stark), although don't do it too often, because then one could argue you are not passing any of your thoughts through
  * any more than around three usages in text is probably excessive
- copying without citation or wrong citation, e.g. "*Yer a Jedi, Harry*", or "*Yer a Jedi, Harry*" (Jon Snow of Tatooine)
- changing a few words but sentence largely the same, e.g. *You, Harry, sir, are a Jedi* (Mithrandir of Winterfell)

- Turnitin will pick out most of the aforementioned things
- Cases can be contested but will lead to an official review, where the penalty may go up and/or down, and could result in an Academic Misconduct case being filed (see `https://acadreg.ust.hk/generalreg.html#b`)

Problems

1. We are going to have a look at a Doppler shift question based on GRACE, with some fairly heavy handed assumptions for illustration purposes. The question looks quite wordy and complicated, but the calculations are actually fairly simple. The principles are illustrated for GRACE, but works for other applications (e.g. ADCPs, police speed cameras, ultrasound to detect heart disease...)

   For objects moving at non-relativistic speeds, i.e. substantially slower than speed of light, Doppler shifts of waves follow the formula
$$f = \left(\frac{c \pm v_r}{c \pm v_s}\right) f_0 \approx \left(1 + \frac{\Delta v}{c}\right) f_0,$$
   where

   - f is the Doppler-shifted frequency,
   - f_0 is the original frequency,
   - c is the wave speed,
   - Δv is the velocity differences between the observer and source, positive if source and receiver are moving towards each other.

   The latter approximation comes from assuming that $|v_{r,s}| \ll |c|$, i.e., the relative speeds between source and observer are much slower than the wave speed. We are going to be using the latter approximation because we are going to consider electromagnetic waves (which travel at the speed of light $c = 3 \times 10^8$ m s^{-1}) and non-relativistic examples ($|\Delta v| \ll |c|$).

   (a) The GRACE satellites (both Tom and Jerry) essentially work with the K-band ranging frequencies of 24 GHz and 32 GHz (1 GHz = 10^9 Hz). If $\Delta v = -0.1$ m s^{-1}, work out $\Delta f = f - f_0$ for both bands. Give your answers in units of Hz to the nearest Hz. [2 marks]

   NOTE: depending on your calculator, you may want to multiply Δv by f_0 first before dividing by c, because if you divide first then multiply you might get zero if your calculator does not store enough decimal places.

   (b) Does Δf calculated above represent a blue shift or a red shift in this above case? Justify your answer. [1 mark]

   (c) Suppose this Δv occurred for Jerry (the one being chased) over a period of 1 second. Work out the acceleration assuming this acceleration is uniform in time. [1 mark]

   (d) Recalling that we have Newton's second law $F = ma$, and assuming the only forces acting on Jerry is gravity, work out the signed (positive or negative) change in δg, i.e. the anomaly in the gravitational acceleration. [1 mark]

   NOTE: You now essentially have a link between a measured (Doppler shifted) frequency time series with the gravitational acceleration, from which you can get the anomalies in the gravitational field if you wanted. In practice you get four time series (two bands each from Tom and Jerry), from which you can cross calibrate.

(e) In reality we have uncertainties in the measurements so we want to try and quantify those. Suppose for argument sake you can measure on the 32 GHz band accurate to 1 Hz (that's an uncertainty of 1 part in about 10 billion from the frequency point of view by the way). Assuming we have perfect knowledge of c and f_0, work out the associated magnitude of uncertainties in Δv, giving your answer in standard units accurate to four decimal places. [1 mark]

(f) Making the same assumptions as the above parts, assuming we can measure time and mass perfectly, and taking $\Delta t = 1$ second, work out the uncertainty in δg associated with an accuracy of 1 Hz on the 32 GHz band, giving your answer in standard units accurate to four decimal places. [1 mark]

NOTE: The GRACE sensors are in fact substantially more accurate than the example given above. The much higher degree of accuracy is required because all the assumptions thus far is not entirely true (particularly the one where the only force acting on the satellites is gravity), leading to bits of error everywhere, though the principles would be essentially as illustrated above. In reality of course one would code the calculations up on a computer and let the computer do the dirty work, but the point is that to get the computer to do the dirty work you need to be able to tell it what dirty work to do in the first place...

(g) Why might we use the microwave band instead of a lower frequency band (e.g. infra-red)? Give an argument based on Doppler shifts. [1 mark]

(h) (Slightly less obvious) From the previous part of the question it might suggest we want to use a higher frequency band, but why might we not want do that from a practical point of view? Give an argument and cite source(s) as necessary. [1 mark]

(i) Can GRACE contribute to tides on Earth, and does it matter? Justify your answer. [1 mark]

2. The acceleration due to tidal generating forces acting on Earth can be shown to be given by

$$a_{\text{tide}} \approx 2Gm\frac{r_{\text{Earth}}}{r^3},$$

where the terms have their usual meaning, and r is the distance between Earth and some other object.

(a) This part aims to justify the statement "*the tidal acceleration arising from the moon is roughly twice as strong as the one from the sun*". Using numbers taken from[1]

`https://en.wikipedia.org/wiki/Tidal_force`

but *only accurate to one decimal place* in the form of 1.2×10^3, do the calculation for a_{tide} for both the Sun and the moon and give your answer in standard units, give it accurate to 1 decimal place in the form of 1.2×10^3. [2 mark]

(b) Suppose Miffy was inspired by Diego Maradona and makes use of her hand (paw?) of God, and decided to move the moon so that the Sun and the moon now lead to the same tidal acceleration on Earth (for cat reasons). Calculate this new moon-Earth distance in units of meters, and give it accurate to 1 decimal place in the form 1.2×10^3. [1 mark]

(c) Why do we not bother with including Jupiter for calculating tides on Earth (we absolutely need it when working out planetary orbits in the Solar system)? Look up the mass and distance of Jupiter and, by calculation, argue why we tend to drop Jupiter. [1 mark]

[1] If you copy the link manually you may want to make sure the there is an underscore between 'Tidal' and 'force'.

(d) (Harder; after Lamb 1932) Suppose Miffy decided to restore the orbit of the Moon to its original distance from the Earth, but in her excitement smashed the moon in two equal halves so that the two new moons have equal mass $m_{\text{moon}}/2$, and kicked one part to the other side, so that the orbit of the new moons are in the same orbital plane but separated in phase by 180° (or π radians). What happens to the resulting tidal amplitudes and frequencies on Earth? Justify your answer, making the assumption that the distance between Earth and the individual new moons are exactly the same as the distance between Earth and the original moon.

(Hint: draw a picture, and assume equilibrium tides) [2 mark]

3. There is a phenomenon called *brinicles*, a.k.a. the underwater fingers of death (look it up on YouTube say), that is known to occur around ice forming regions, and known to end up freezing creatures like starfish and crabs on the sea floor (making the assumption here that the distance from the base of the ice is not that far away from the sea floor). Using what you have learn from the lectures on equation of state, static instability, and possibly additional sources on the internet, <u>draw</u> (not *write*) a set of schematics to explain how these brincicles work.

   Most of the marks are for the *drawings*. You can use words to reinforce the drawings, but limit those to short-ish bullet points.

   (Hint: you will probably need to look up the concept of *brine rejection* if you don't know what that is.)

   [6 marks]

!? (for interest mainly) NOAA has a (fairly old?) description of what leads to tides, given at (NASA has a similar one)[2]

`https://oceanservice.noaa.gov/education/tutorial_tides/tides03_gravity.html`

Find as many scientific points to criticise about this article as possible. I can immediately see four problematic things here (in an attempt to make the explanation "accessible" the explanation actually becomes confusing and/or wrong).

[2] Again, if you are copying the link manually, make sure the underscores are there.