Imperial College London

Department of Earth Science Engineering

MSc in Applied Computational Science and Engineering

Independent Research Project

Project Plan

Conservative Interpolation Between Unstructured Meshes that Preserves Heterogeneity and Structure for Modeling Three-Dimensional Bone

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1 Introduction

1.1 Preface

Mesh interpolation is a vital topic of discussion in many fields such as nuclear reactor dynamics, fluid dynamics, and weather predictions. In such problems, accuracy is an integral element but so is speed. Thus, in trying to obtain accurate results in a timely manner, one runs into challenges of discretization, storage, and solving the linear system. In this project as well, when simulating the fracture of a bone, we will naturally run into the same problems. Thus, it is essential to minimize the error during the simulation and attempt to decrease the time it takes to produce it.

1.2 Literature Review

1.2.a Introduction to the Finite Element Method and Implementation with MATLAB by Gang Li

The total degrees of freedom for a finite element mesh is equal to the total number of nodes multiplied by the degrees of freedom of each node, which obviously is a number that grows quite rapidly as we increase the number of nodes in our grid. 3D models, of course, have more nodes and more degrees of freedom. Assuming that we will have to solve a linear system involving this mesh, in the best-case scenario, if we have access to state-of-the-art computers, the fastest we can get that done is in N.log2(N) time, where N is the degrees of freedom mentioned earlier. Moreover, this solution is entitled to have an error caused mainly by discretization and integration (in addition to numerical rounding, but this is nothing we can alleviate by refining the mesh). This error can be made smaller if our elements are smaller, which will undoubtedly lead to a higher computational cost.

There are several types of computational meshes that one could choose from depending on the problem at hand. A mesh can thus be uniform which means the elements are of the same volume and shape, structured which means it has a uniform geometry template overlapped on a region of irregular geometry, or unstructured which means the volumes and shapes follow an irregular pattern.

One way to construct an unstructured grid is to use the Delaunay Triangulation method. This essentially results in a mesh composed of triangular-shaped nodes (or simplices.)

1.2.b Conservative Interpolation Between Unstructured Meshes via Supermesh Construction

When interpolating new solution fields from previous ones, standard mesh adaptivity methods for unstructured meshes do not necessarily conserve a system’s essential physical quantities (density, pressure, volume fractions, etc.) This most definitely leads to an error that propagates with the timesteps and consequently grows. Thus, the conservative interpolation method proposed aims to conserve these quantities by using a 2D supermesh (a union of the old and new meshes) for the case where elements are linear. The supermesh assists in using projection operators as mesh-to-mesh interpolars. As a result, the preservation of the conservation properties will improve long-term accuracy.

There exist several mesh interpolation methods. Firstly, there is the Arbitrary Lagrangian-Eulerian Method that takes Lagrangian timesteps and rezones the mesh periodically and then interpolates it. Secondly, there are global re-mapping methods that assume no link between the base and target meshes, which requires calculating the volume integral of the intersection of the two meshes. The proposed method that uses the supermeshing technique is different from the aforementioned ones since it is minimally diffusive and performs bounded interpolation between the unstructured meshes.

2 Description of Problem and Objectives

I am interested in interpolating between 3D unstructured meshes that represent bones that have undergone a fracture. When a bone is subjected to a hit (or anything else that might cause it to break), a fracture will gradually grow with time, which evidently changes the mesh that represents this bone, as shown in Figure 1. In examining this propagation, we would be working on a very small scale of nanometers (Sabet et al., 2016). Consequently, we end up having very large unstructured meshes stored in memory. And in order to model how the fracture will grow at each timestep, we need to solve a linear system of equations using this grid. Given that a small time step is taken to reduce the discretization error, this ends up being a costly task in terms of execution time (Li et al., 2020). In order to mitigate this issue, we can instead use a mesh interpolation method that maps the values of a previous mesh to the subsequent one, while also being conservative. The method chosen for this project is the supermeshing technique (Farrell et al., 2009). In this method, we begin by constructing a supermesh of both grids. There is no out-of-the-box C++ supermesh generator that can do this for us, so this is something I will have to implement in code as a next step. We also need to create two mappings: one from the old to the supermesh and one from the new to the supermesh. Afterward, we use the supermesh to find the integral value for each element of the new mesh and sum all of the values. Finally, we compute the nodal values using a Galerkin Projection.

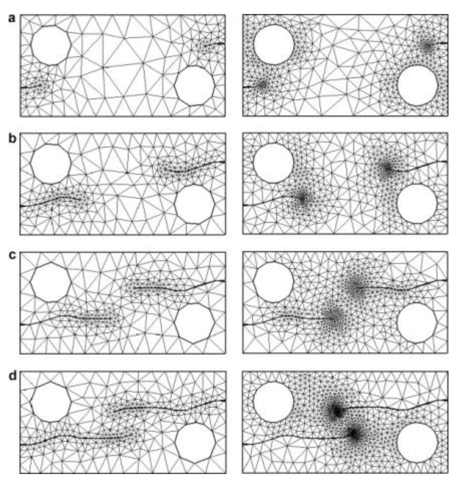

Fig 1 Modeling of crack propagation via an automatic adaptive mesh refinement based on modified superconvergent patch recovery technique, A.R.Khoei, H.Azadi, H.Moslemi

3 Conclusion

3.1 Progress

So far, I have been invested in reading about various mesh adaptivity techniques and algorithms. I have read chapters 1-6 in the “Introduction to the Finite Element Method" book to ensure I have a sturdy mathematical and theoretical background in modeling using meshes and the Finite Element Method. Moreover, I have read and understood the paper on the conservative interpolation of meshes using a supermesh that discusses 2D models. I have also been researching 3D supermeshing techniques. One last thing I have done is download the GMSH library and run the tutorials that come with the library’s SDK. The tutorials include creating structured and unstructured meshes using points, surfaces, and curves.

3.2 Future Goals

This following list is a Gantt Chart showing what I will be working on for the following 8 weeks:

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8
- create a 3D supermesh using GMSH - create mappings for supermesh	In Progress							
- devise an algorithm to apply the conservative interpolation			Not Started					
- work on larger complex meshes that represent bones					Not Started			
- benchmark my code’s preformance - begin writing up the final report						Not Started		
- finalize the final report and code								Not Started

4 References

Farrell, P.E et al. “Conservative Interpolation Between Unstructured Meshes via Supermesh Construction.” Computer methods in applied mechanics and engineering 198.33 (2009): 2632–2642. Web.

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Sabet, Fereshteh A., et al. “Modelling of Bone Fracture and Strength at Different Length Scales: A Review.” *Interface Focus*, vol. 6, no. 1, 2016, p. 20150055., doi:10.1098/rsfs.2015.0055.