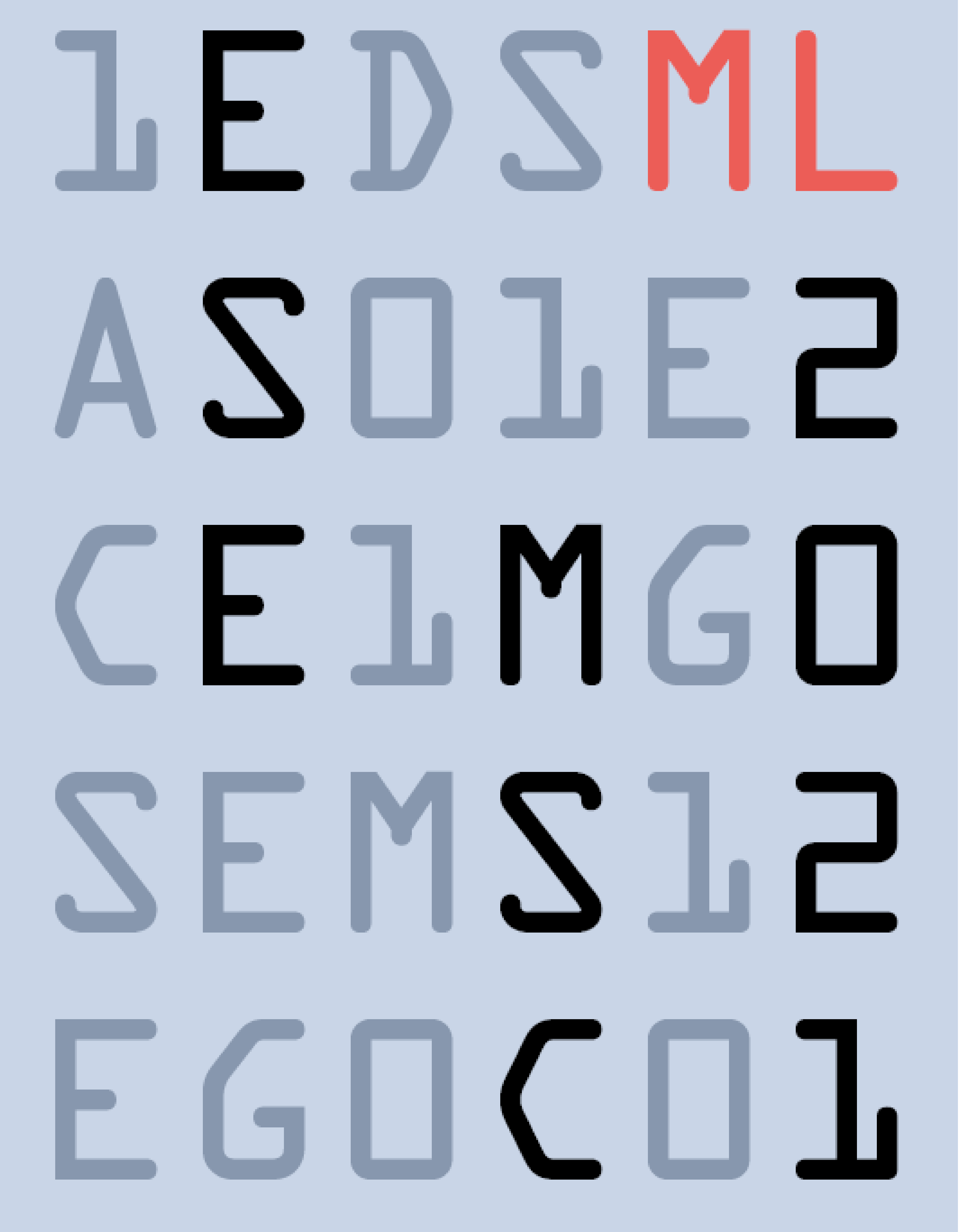


Recurrent Neural Networks and Long Short-Term Memory

Lluis Guasch & Carlos Cueto

GTAs online:
Alex
George

RNNs and LSTMs

1. Recurrent Neural Networks (RNNs)
2. Long Short-Term Memory (LSTM) networks
3. Text representation

1. **Recurrent Neural Networks (RNNs)**
2. Long Short-Term Memory (LSTM) networks
3. Text representation

Sequential data

Examples of sequential data:

- Text considered as a sequence of words or characters
- Continuous parameter which is a function of time (e.g. stock price)
- Sequence of images in a video-clip
- Sequence of labels in a genome sequence
- Vertical sequences of lithologies in the subsurface
- … plus many others

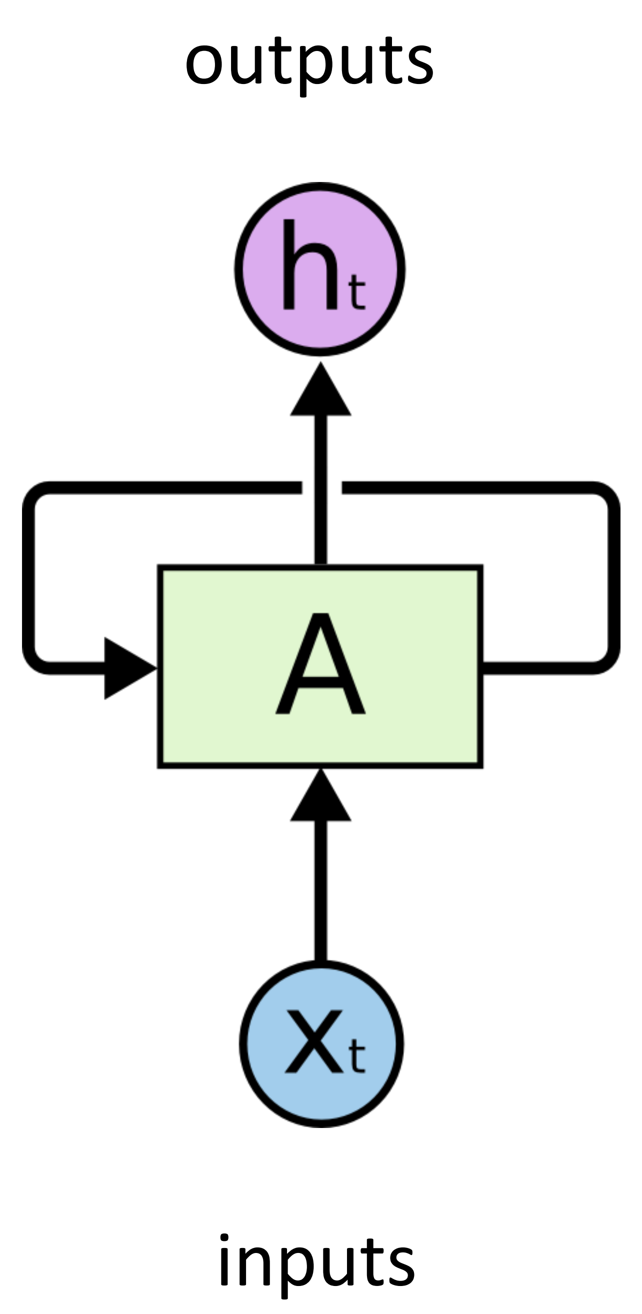
Sequential data

Applications of RNNs to sequential data:

- Text generation “in the style of”
- Provide ”sentiment analysis” on a piece of text
- Automatic “speech-to-text” as in MS Teams
- Automatic foreign language translation
- Prediction of stock price on the basis of historical data
- Label sequence of images in a video-clip

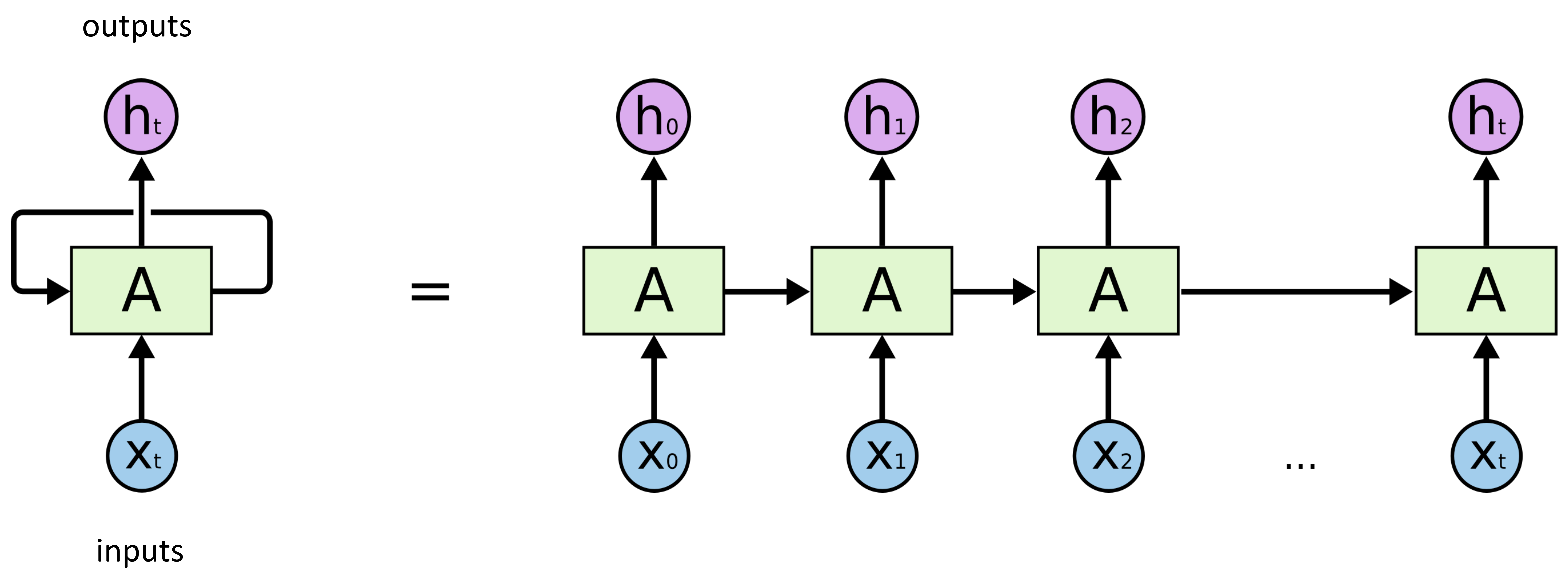
RNNs

Basic structure of RNNs:



RNNs

Basic structure of RNNs:



RNNs

Simple (unrealistic) example:

Example:
Based on a (usually long) text used for training, we wish to generate “similar” text on a character by character basis.

Given a training string: `Hello world!`

We identify the dictionary associated with the input (ignoring capitalisation) contains 9 characters: `(d,e,h,l,o,r,w, ,!)`

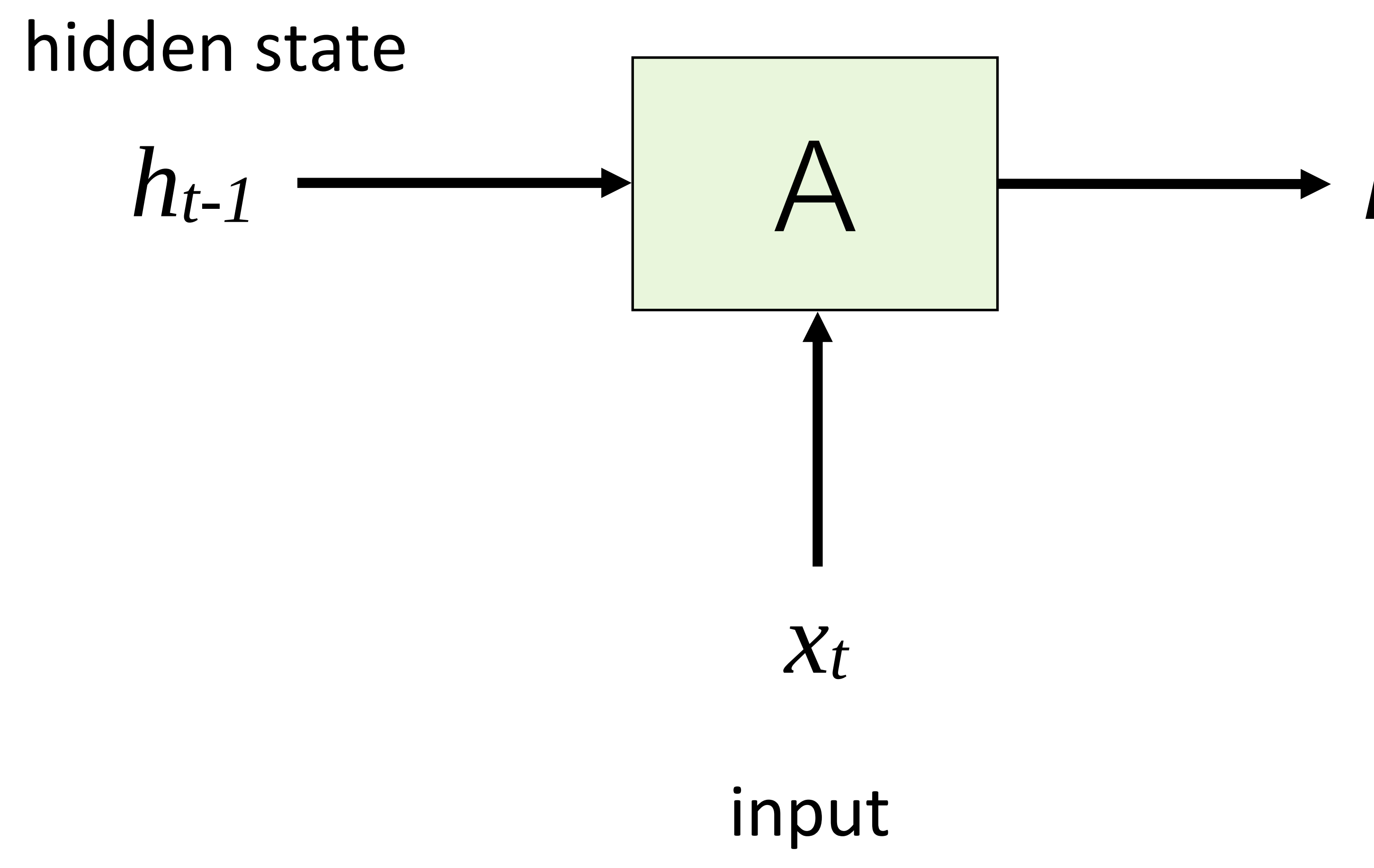
And we hot-encode the dictionary:

$$h = \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\\0\end{pmatrix} \quad ! = \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\0\\1\end{pmatrix} \quad \cdots$$

RNNs

How does a basic RNN cell work?

$$y_t = W_{hy} h_t$$ output



$$h_t = A(h_{t-1}, x_t) = \tanh(W_{hh} h_{t-1} + W_{xh} x_t)$$ hidden state

RNNs

How does training work? Back to our simple example:

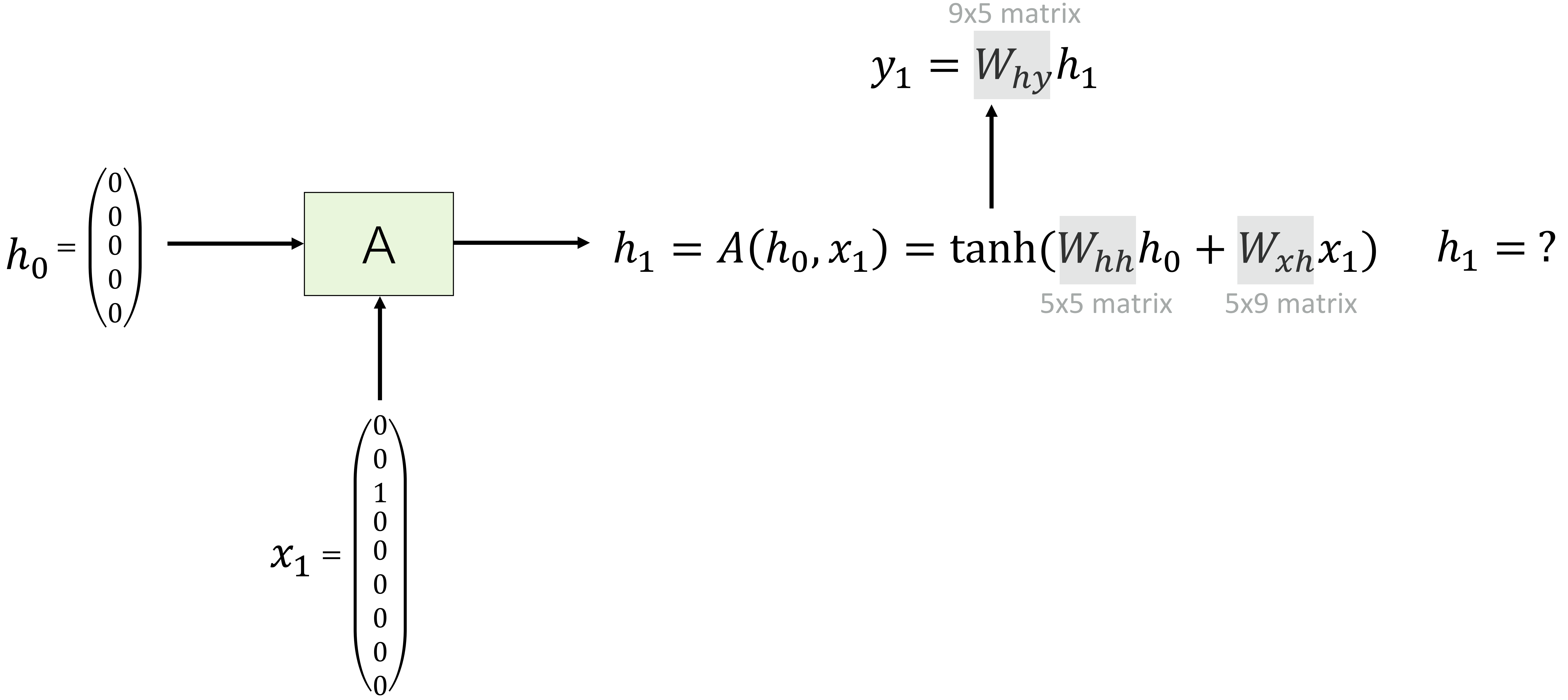
We define the size of our hidden state tensor h (hyperparameter) to be 5, and start with our first input character (‘h’):

$$\begin{tikzcd}
 & & y_1 = W_{hy}h_1 \\
h_0 = \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix} \arrow[r] & \boxed{A} \arrow[r] & h_1 = A(h_0, x_1) = \tanh(W_{hh}h_0 + W_{xh}x_1) \arrow[u] \\
 & x_1 = \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\\0\end{pmatrix} \arrow[u] &
\end{tikzcd}$$

RNNs

How does training work? Back to our simple example:

We define the size of our hidden state tensor h (hyperparameter) to be 5, and start with our first input character ('h'):



$$y_1 = \underbrace{W_{hy}}_{\text{9x5 matrix}} h_1$$

$$h_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad x_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

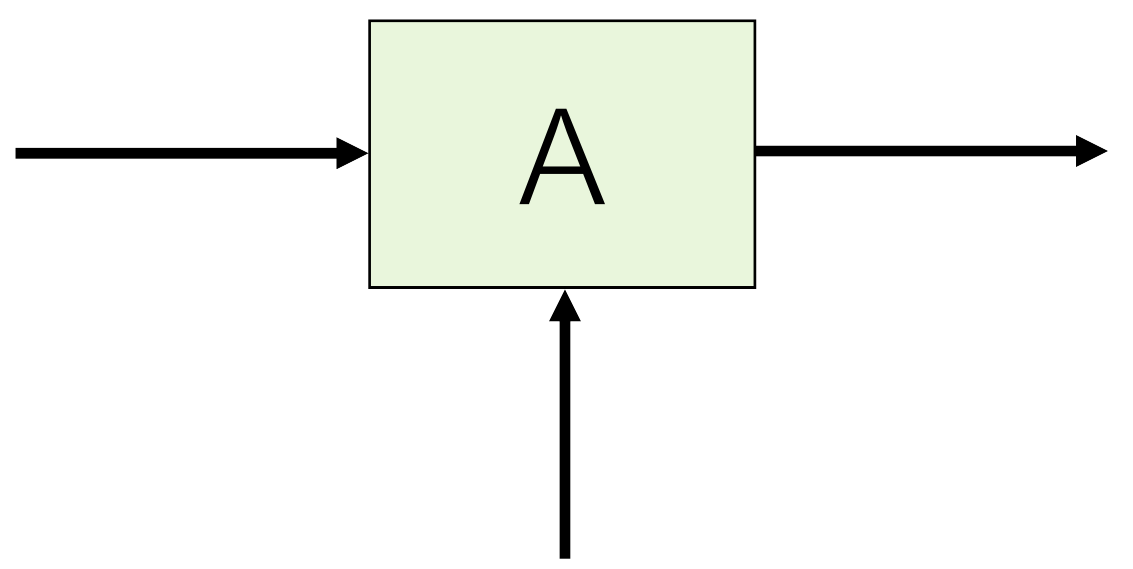
$$h_1 = A(h_0, x_1) = \tanh(\underbrace{W_{hh}}_{\text{5x5 matrix}} h_0 + \underbrace{W_{xh}}_{\text{5x9 matrix}} x_1) \qquad h_1 = ?$$

RNNs

How does training work? Back to our simple example:

We define the size of our hidden state tensor h (hyperparameter) to be 5, and start with our first input character (‘h’):

$$h_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



$$x_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

9x5 matrix

$$y_1 = W_{hy} h_1$$

$$h_1 = A(h_0, x_1) = \tanh(\underbrace{W_{hh}}_{\text{5x5 matrix}} h_0 + \underbrace{W_{xh}}_{\text{5x9 matrix}} x_1)$$

$$h_1 = \begin{pmatrix} W_{xh}^{1,3} \\ W_{xh}^{2,3} \\ W_{xh}^{3,3} \\ W_{xh}^{4,3} \\ W_{xh}^{5,3} \end{pmatrix}$$

And I will use y_1 as the prediction for the next x

RNNs

How does training work? The loss function:

$$loss =$$

$$y_1 = W_{hy} h_1 \longrightarrow Cross\text{-}entropy\ (\ Softmax\ (y_1)\ ,\ \ x_2\ )$$

Loss function corresponding to the first predicted letter

remember: hot-encoded

RNNs

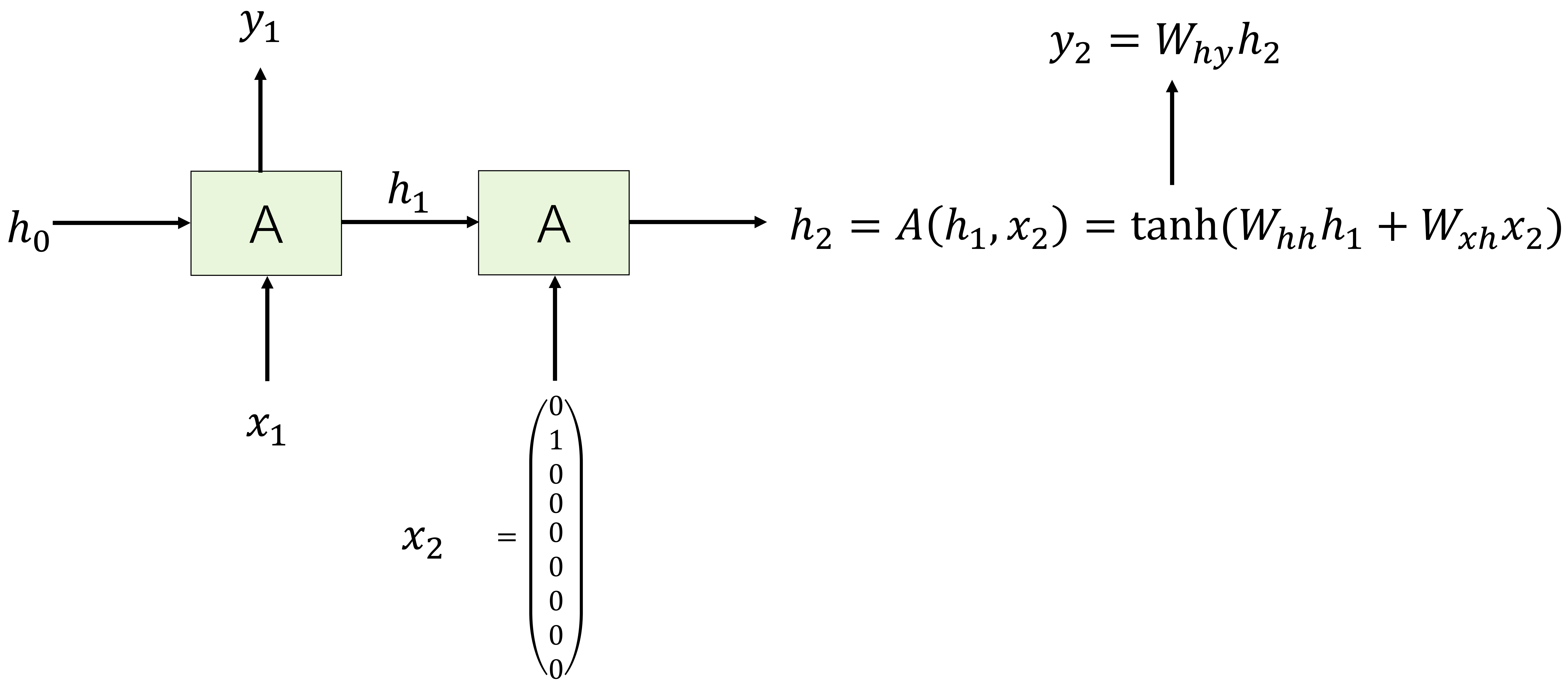
How does training work for the next characters? Back to our previous example:

$$\begin{tikzcd}
 & & y_1 = W_{hy} h_1 \\
h_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \arrow[r] & \boxed{A} \arrow[r] & h_1 = A(h_0, x_1) = \tanh(W_{hh} h_0 + W_{xh} x_1) \arrow[u] \\
 & x_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \arrow[u] &
\end{tikzcd}$$

RNNs

How does training work for the next characters? Back to our previous example:

We concatenate the **same** cell after the output of our previous one:



y_1
$y_2 = W_{hy}h_2$
h_0
A
h_1
A
$h_2 = A(h_1, x_2) = \tanh(W_{hh}h_1 + W_{xh}x_2)$
x_1
$x_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

RNNs

How does training work? The loss function:

$$loss =$$

$$y_1 = W_{hy}h_1 \longrightarrow Cross\text{-}entropy\ (\ Softmax\ (y_1)\ ,\ \ x_2\ )$$

$$+$$

$$Cross\text{-}entropy\ (\ Softmax\ (y_2)\ ,\ \ x_3\ )$$

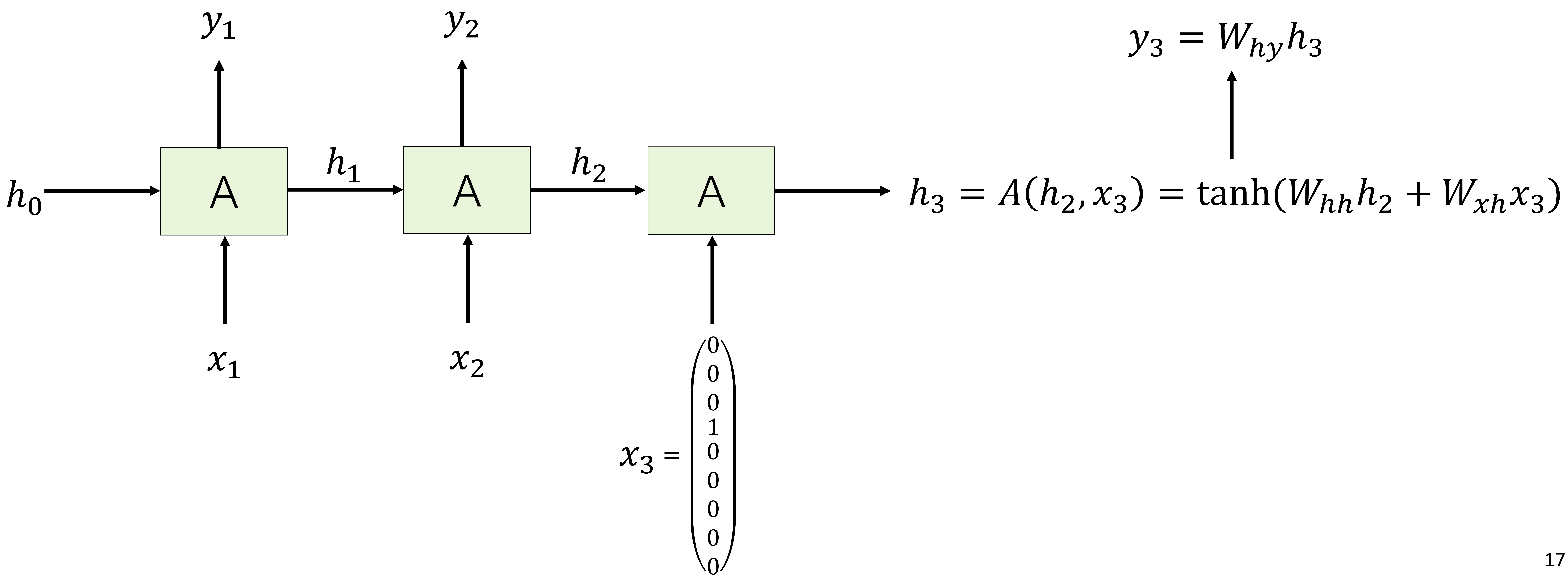
Loss function corresponding to the second predicted letter

remember: hot-encoded

RNNs

How does training work for the next characters? Back to our previous example:

We concatenate the **same** cell after the output of our previous one:



y_1
y_2
$y_3 = W_{hy}h_3$
h_0
A
h_1
A
h_2
A
$h_3 = A(h_2, x_3) = \tanh(W_{hh}h_2 + W_{xh}x_3)$
x_1
x_2
$x_3 = \begin{pmatrix}0\\0\\0\\1\\0\\0\\0\\0\\0\end{pmatrix}$

RNNs

How does training work? The loss function:

$$loss =$$

$$y_1 = W_{hy}h_1 \longrightarrow Cross\text{-}entropy\ (\ Softmax\ (y_1)\ ,\ \ x_2\ )$$

$$+$$

$$Cross\text{-}entropy\ (\ Softmax\ (y_2)\ ,\ \ x_3\ )$$

$$+$$

$$Cross\text{-}entropy\ (\ Softmax\ (y_3)\ ,\ \ x_4\ )$$

$$+\ \ldots$$

We repeat the process for all the characters in our input and add their loss contributions

Loss function corresponding to the third predicted letter

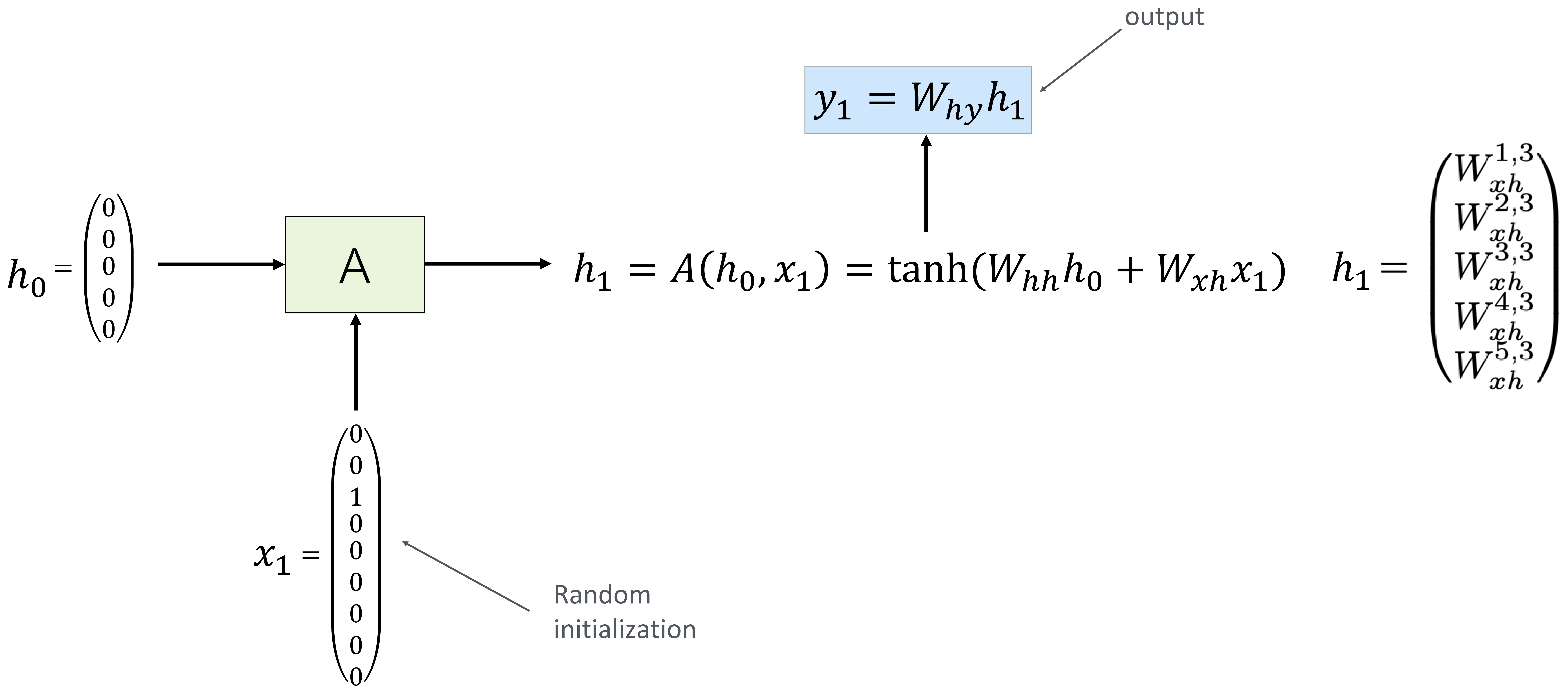
remember: hot-encoded

And then use gradient descent to train our weights in the matrices

RNNs

Text generation using a trained network

Now we are going to use our trained RNN to generate some text:



$$y_1 = W_{hy} h_1$$

$$h_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$h_1 = A(h_0, x_1) = \tanh(W_{hh} h_0 + W_{xh} x_1) \quad h_1 = \begin{pmatrix} W_{xh}^{1,3} \\ W_{xh}^{2,3} \\ W_{xh}^{3,3} \\ W_{xh}^{4,3} \\ W_{xh}^{5,3} \end{pmatrix}$$

$$x_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

RNNs

Text generation using a trained network

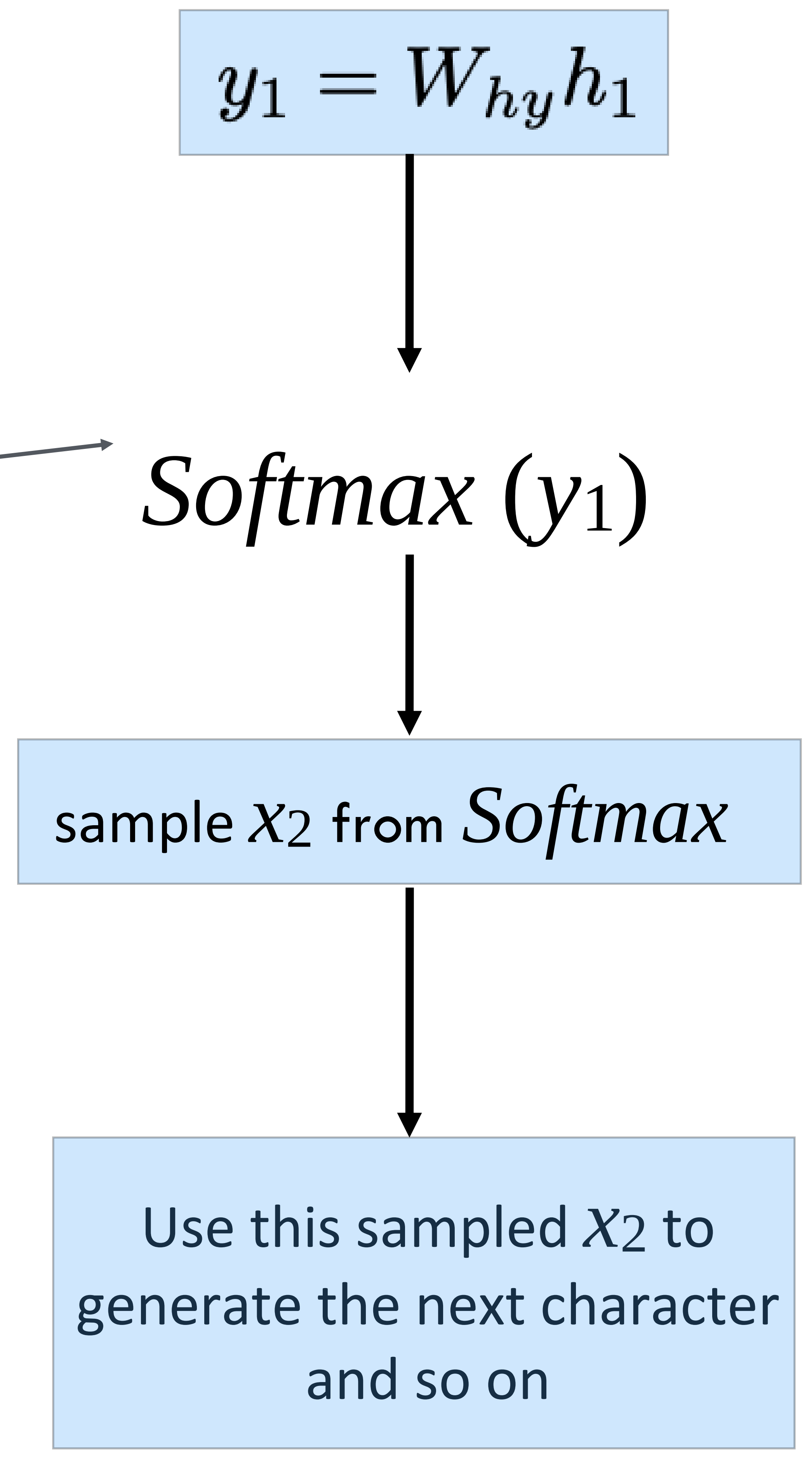
Now we are going to use our trained RNN to generate some text:

$$\begin{tikzcd}
\boxed{y_1 = W_{hy} h_1} \arrow[d] \\
\mathit{Softmax}\,(y_1) \arrow[d] \\
\boxed{\text{sample } x_2 \text{ from } \mathit{Softmax}} \arrow[d] \\
\boxed{\text{Use this sampled } x_2 \text{ to generate the next character and so on}}
\end{tikzcd}$$

RNNs

Text generation using a trained network

Now we are going to use our trained RNN to generate some text:



$$y_1 = W_{hy}h_1$$

$$Softmax\ (y_1)$$

sample x_2 from $Softmax$

Use this sampled x_2 to generate the next character and so on

Now we are not training, but we still want to compute the Softmax

Here we sample values! There is some degree of randomness which generates different text every time

Recap of training an RNN

1. Take first character x_1, combine with current hidden state h_0 (its dimension is a hyperparameter) to derive h_1, calculate associated output vector y_1 (of size equal to vocabulary), transform y_1 into Softmax vector, calculate cross-entropy with second character x_2 of the sequence.
2. Take the second character as input to repeat the step 1. and calculate loss function again using the value of the third character of the sequence, and so on until we reach the end of the sequence.
3. Add up all the above loss functions.
4. Do backpropagation to calculate gradients according to each trainable parameter.
5. Modify parameters by gradient descent.
6. Move to next sequence of p characters, until end of training set is reached. Size of p is a hyperparameter too!

Recap of generating text with an RNN

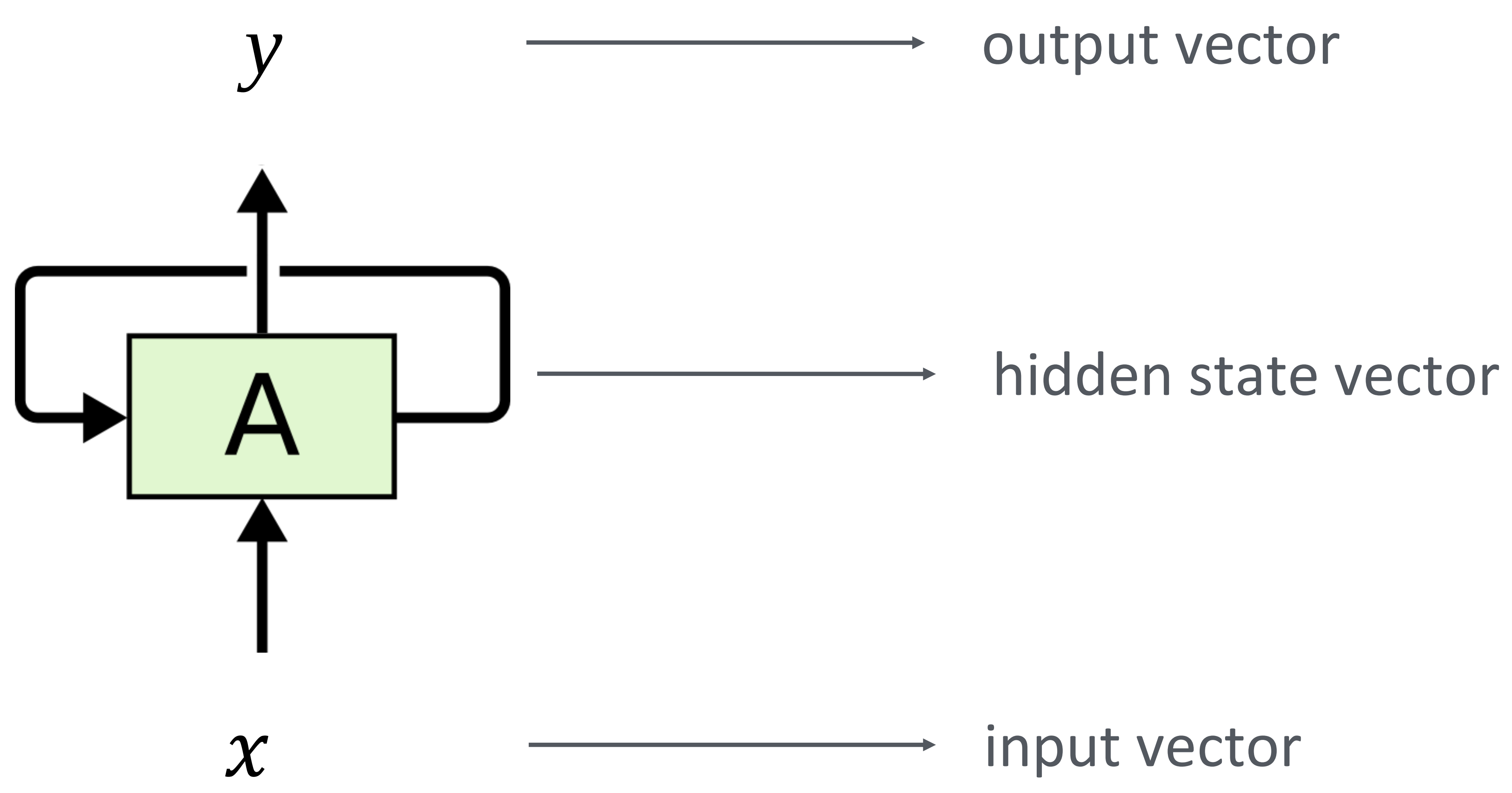
Generate a new sequence of n characters:

1. Randomly sample first character x_1.
2. Combine with initial (zero) hidden state vector value h_0 to derive new vector h_1.
3. Calculate associated output vector y_1 of size equal to vocabulary size.
4. Transform y_1 into Softmax vector.
5. Sample new character x_2 based on Softmax probability.
6. Continue until the required number of characters have been generated.

If we were using words instead of characters, the dictionary would be much bigger

RNNs

Synthetic notation for RNNs:



RNNs

Simple implementation in python:

https://gist.github.com/karpathy/d4dee566867f8291f086

And after a few hours of training, we start to get some decent writing:

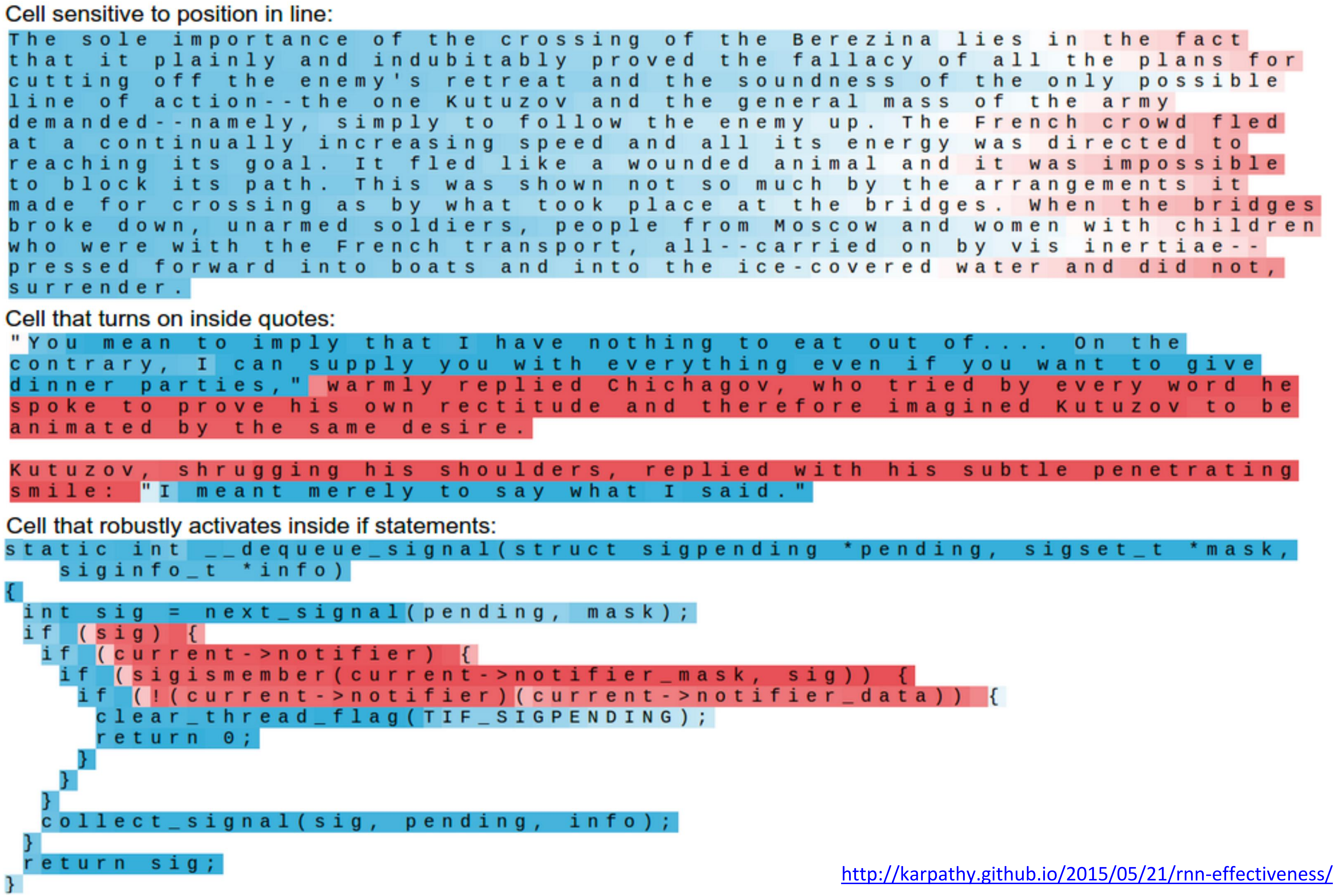
```
VIOLA:
Why, Salisbury must find his flesh and thought
That which I am not aps, not a man and in fire,
To show the reining of the raven and the wars
To grace my hand reproach within, and not a fair are hand,
That Caesar and my goodly father's world;
When I was heaven of presence and our fleets,
We spare with hours, but cut thy council I am great,
Murdered and by thy master's ready there
My power to give thee but so much as hell:
Some service in the noble bondman here,
Would show him to her wine.

KING LEAR:
O, if you were a feeble sight, the courtesy of your law,
Your sight and several breath, will wear the gods
With his heads, and my hands are wonder'd at the deeds,
So drop upon your lordship's head, and your opinion
Shall be against your honour.
```

http://karpathy.github.io/2015/05/21/rnn-effectiveness/

RNNs

Analysis of the role of h



Cell sensitive to position in line:

The sole importance of the crossing of the Berezina lies in the fact that it plainly and indubitably proved the fallacy of all the plans for cutting off the enemy's retreat and the soundness of the only possible line of action--the one Kutuzov and the general mass of the army demanded--namely, simply to follow the enemy up. The French crowd fled at a continually increasing speed and all its energy was directed to reaching its goal. It fled like a wounded animal and it was impossible to block its path. This was shown not so much by the arrangements it made for crossing as by what took place at the bridges. When the bridges broke down, unarmed soldiers, people from Moscow and women with children who were with the French transport, all--carried on by vis inertiae--pressed forward into boats and into the ice-covered water and did not, surrender.

Cell that turns on inside quotes:

"You mean to imply that I have nothing to eat out of.... On the contrary, I can supply you with everything even if you want to give dinner parties," warmly replied Chichagov, who tried by every word he spoke to prove his own rectitude and therefore imagined Kutuzov to be animated by the same desire.

Kutuzov, shrugging his shoulders, replied with his subtle penetrating smile: "I meant merely to say what I said."

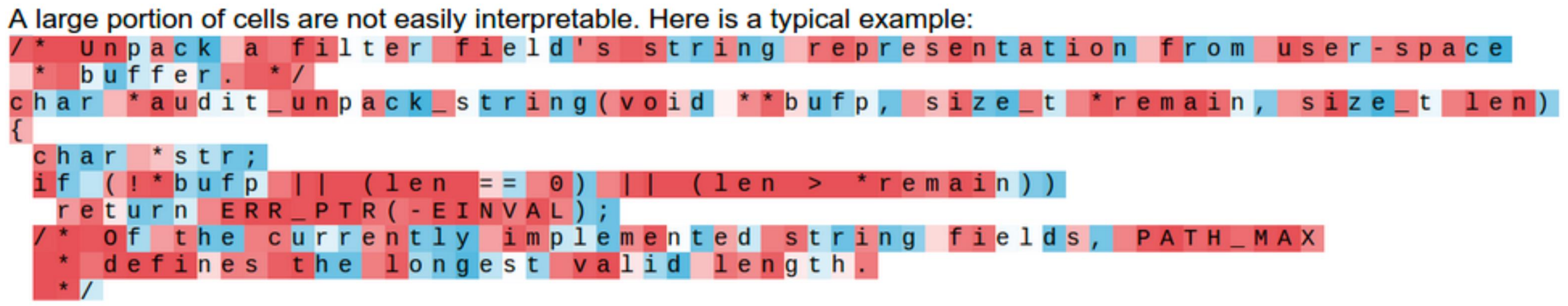
Cell that robustly activates inside if statements:

```
static int __dequeue_signal(struct sigpending *pending, sigset_t *mask,
   siginfo_t *info)
{
 int sig = next_signal(pending, mask);
 if (sig) {
  if (current->notifier) {
   if (sigismember(current->notifier_mask, sig)) {
    if (!(current->notifier)(current->notifier_data)) {
     clear_thread_flag(TIF_SIGPENDING);
     return 0;
    }
   }
  }
  collect_signal(sig, pending, info);
 }
 return sig;
}
```

http://karpathy.github.io/2015/05/21/rnn-effectiveness/

RNNs

Analysis of the role of h



A large portion of cells are not easily interpretable. Here is a typical example:

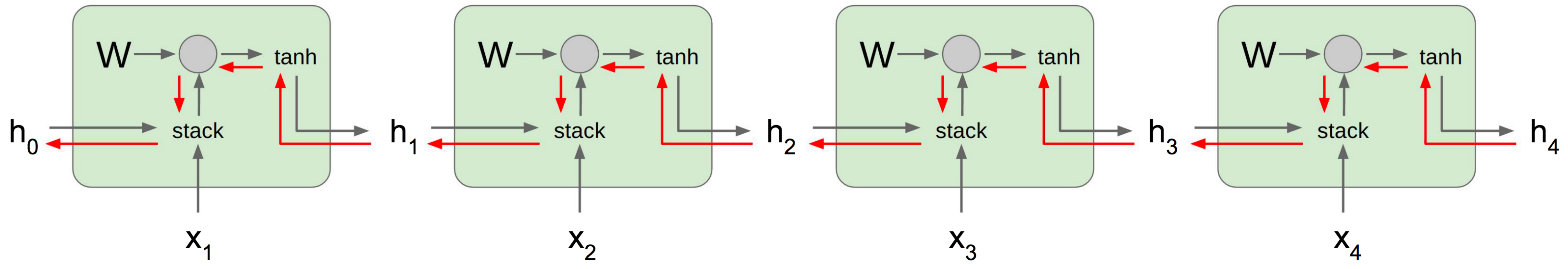
```
/* Unpack a filter field's string representation from user-space
 * buffer. */
char *audit_unpack_string(void **bufp, size_t *remain, size_t len)
{
 char *str;
 if (!*bufp || (len == 0) || (len > *remain))
  return ERR_PTR(-EINVAL);
 /* Of the currently implemented string fields, PATH_MAX
  * defines the longest valid length.
  */
```

In their study, Karpathy et al found that only ~5% of the h positions were interpretable

http://karpathy.github.io/2015/05/21/rnn-effectiveness/

RNNs

Backpropagation through RNNs



Computing gradients involves many factors of W and repeated tanh activations and that creates problems:

- Exploding gradients: can be solved by gradient clipping
- Vanishing gradients: requires changes in the RNN architecture

the derivative of tanh is in the range (0, 1) and we apply it multiple times using the chain rule in backprop

RNNs

Issues with RNNs

- Exploding and vanishing gradients
- Lack of long term memory

RNNs

Issues with RNNs

- Exploding and vanishing gradients
- Lack of long term memory

$\longrightarrow$ LSTMs to the rescue!

RNNs and LSTMs

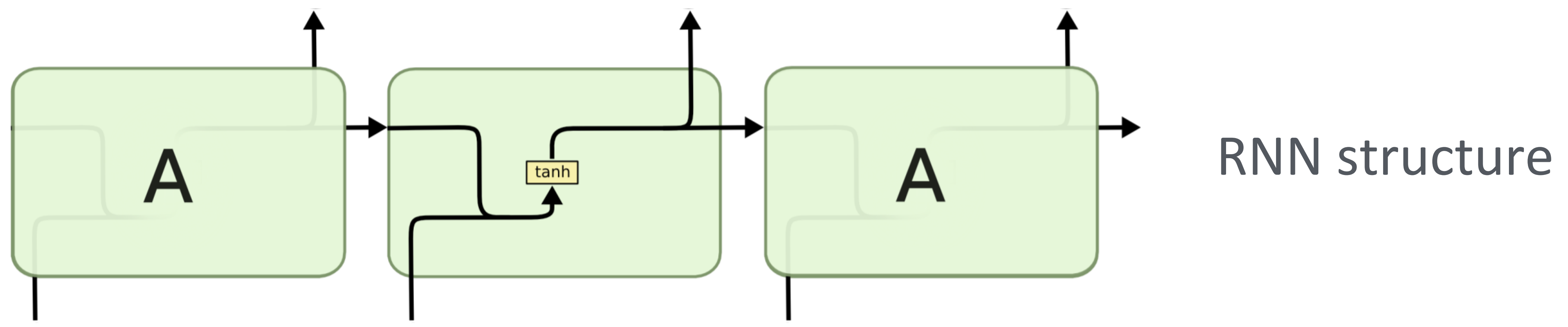
1. Recurrent Neural Networks (RNNs)

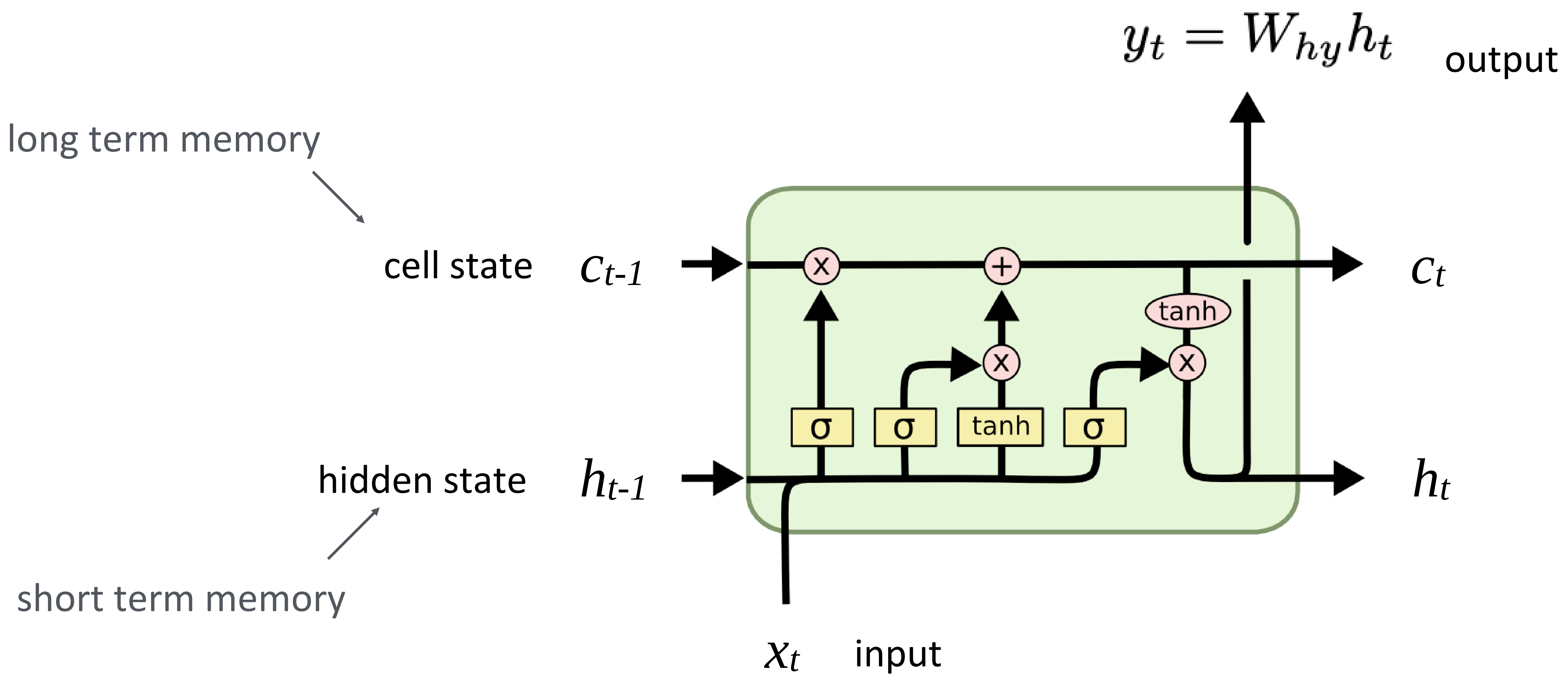
2. **Long Short-Term Memory (LSTM) networks**

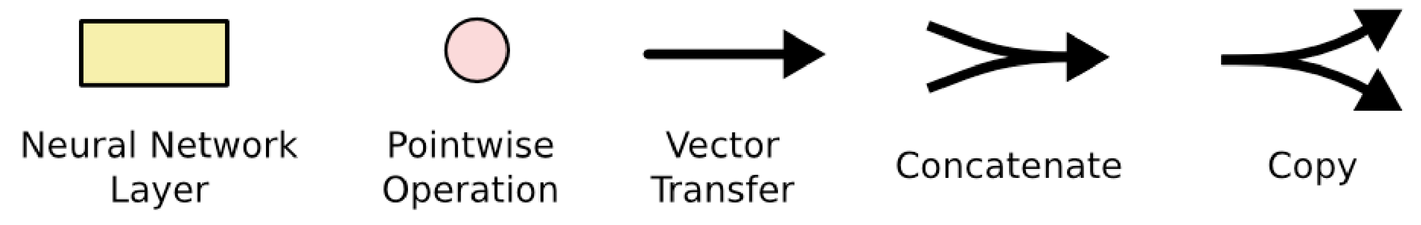
3. Text representation

LSTMs

Basic structure of an LSTM:



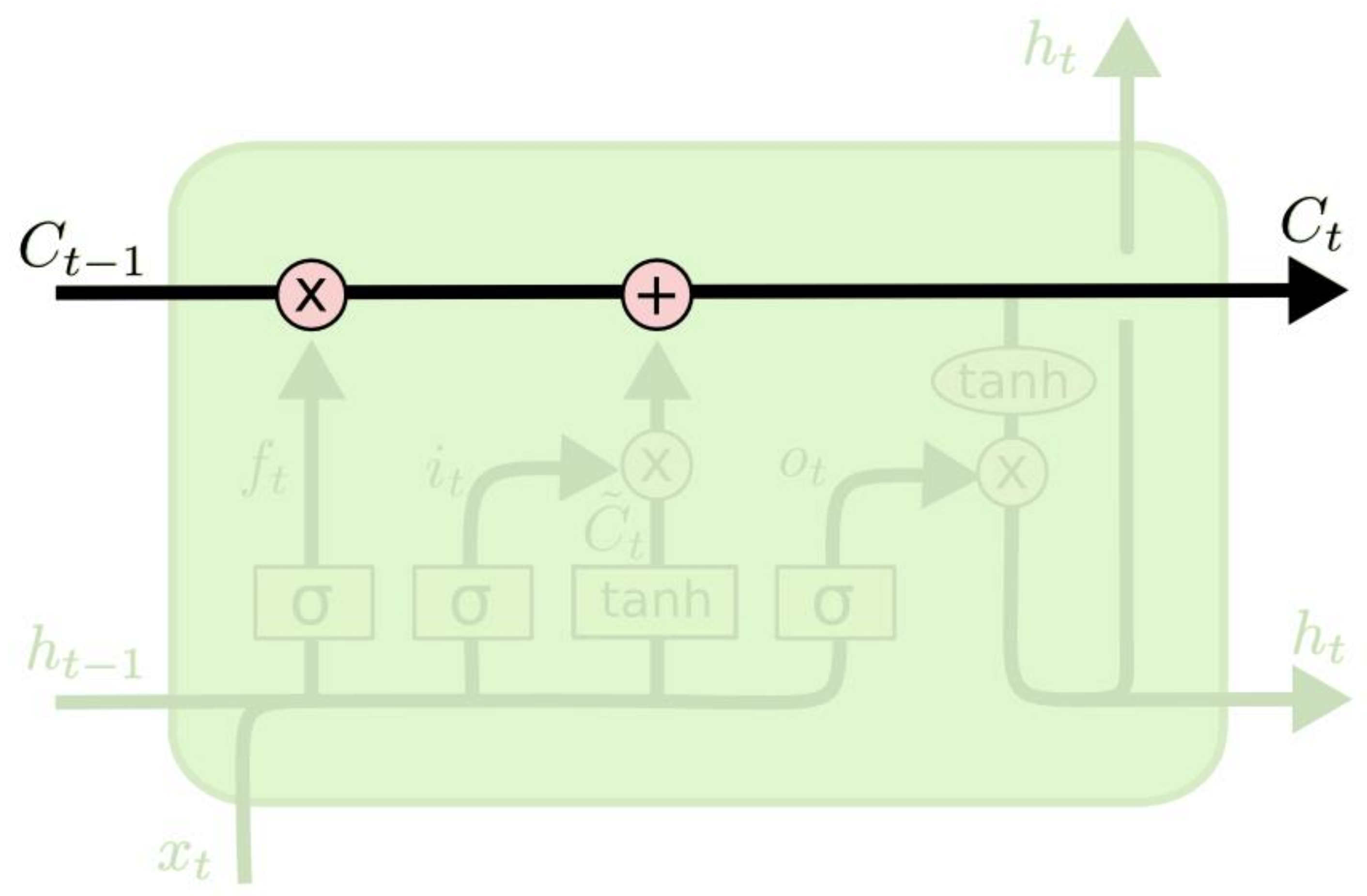


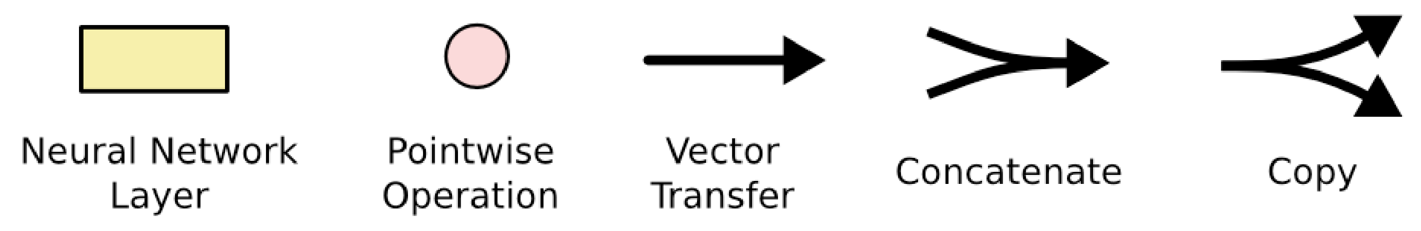


https://colah.github.io/posts/2015-08-Understanding-LSTMs/

LSTMs

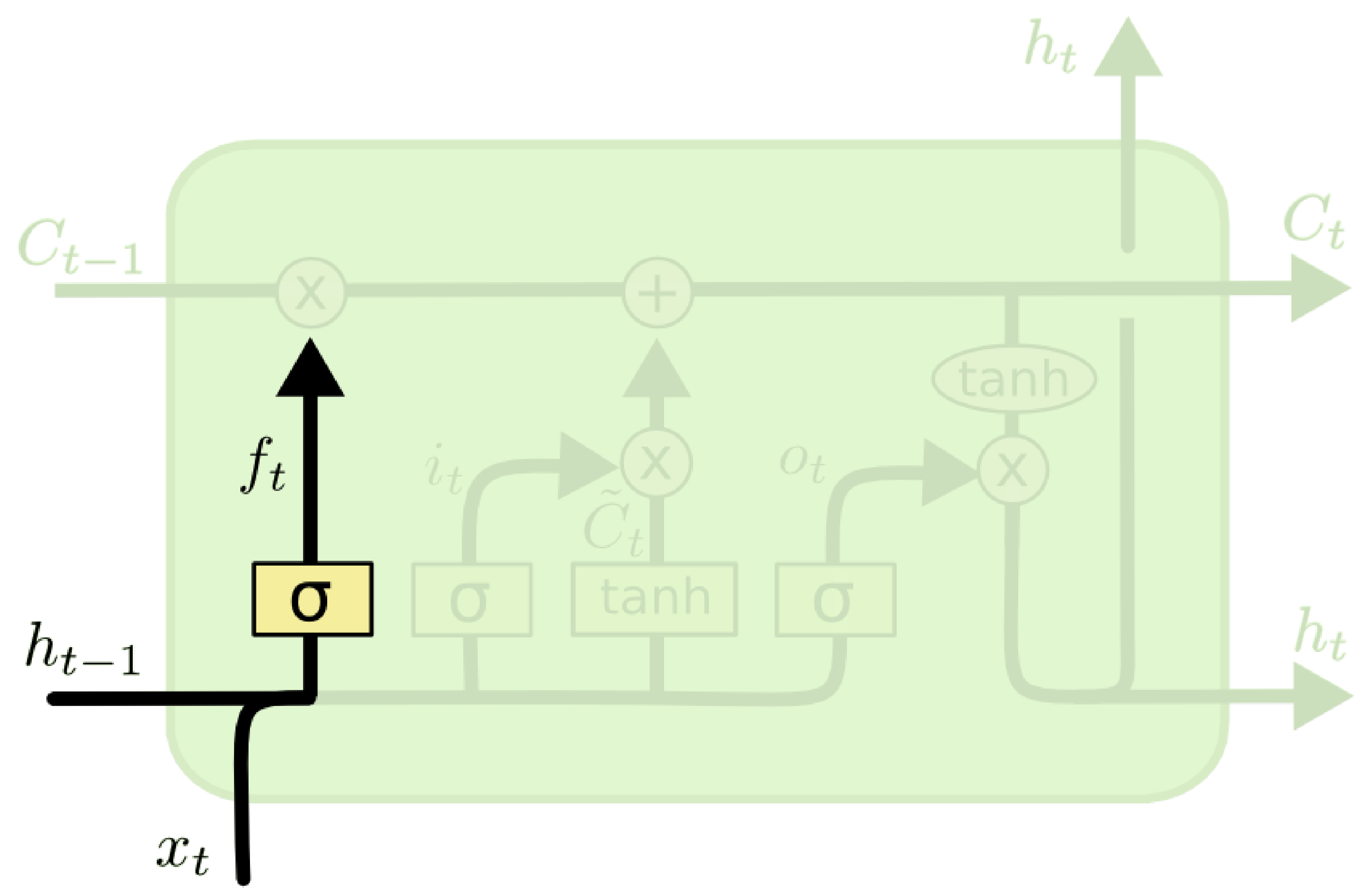
The core idea behind LSTMs: the **cell state** directly connects the whole chain of cells.





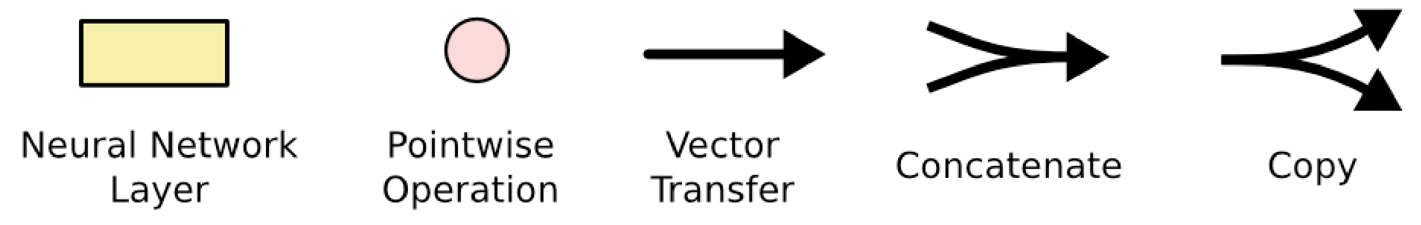
LSTMs

The **forget gate**: is there any information in the cell state that we no longer need?



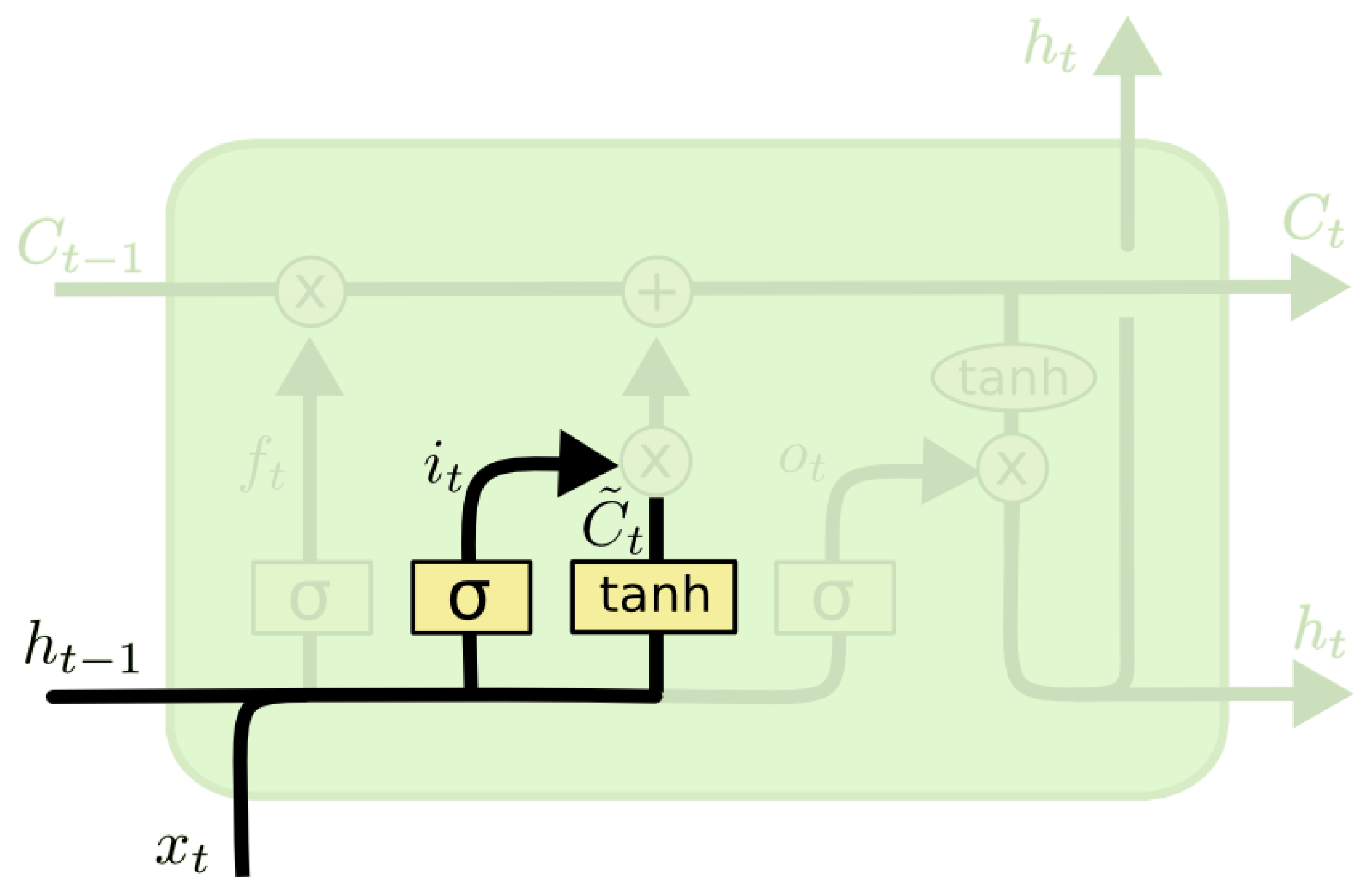
sigmoid normalises between 0 and 1

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$



LSTMs

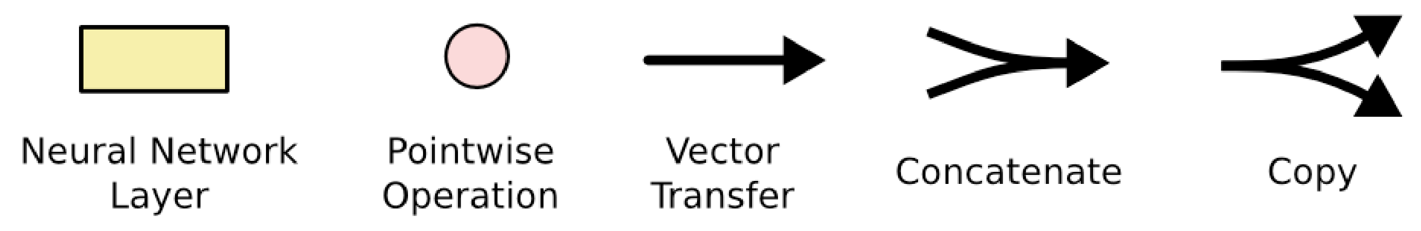
The **input gate** and the **candidate update**: what part of the input should we use to update the cell state?



$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_c \cdot [h_{t-1}, x_t] + b_c)$$

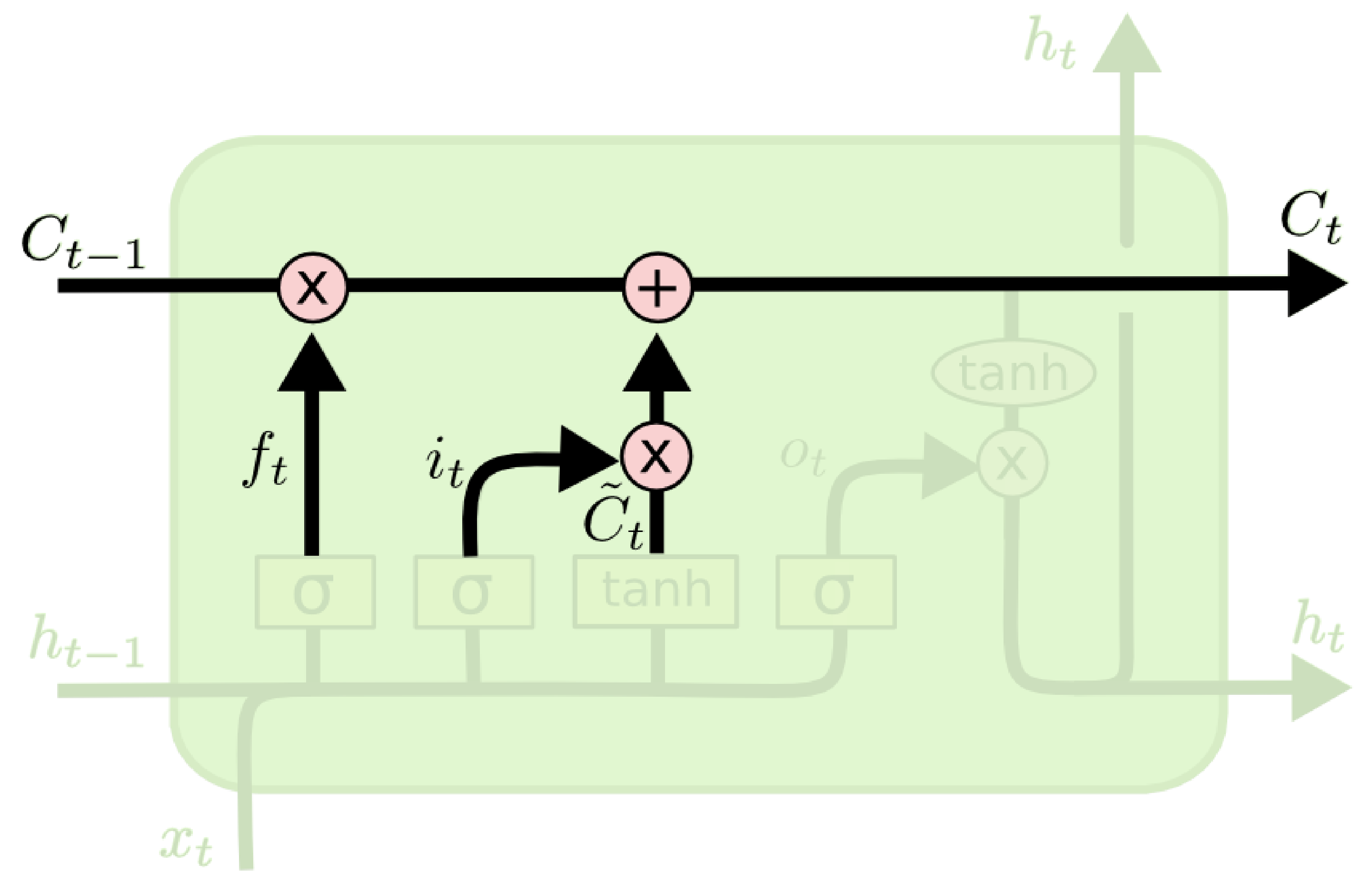
tanh normalises between -1 and 1, like cell state



https://colah.github.io/posts/2015-08-Understanding-LSTMs/

LSTMs

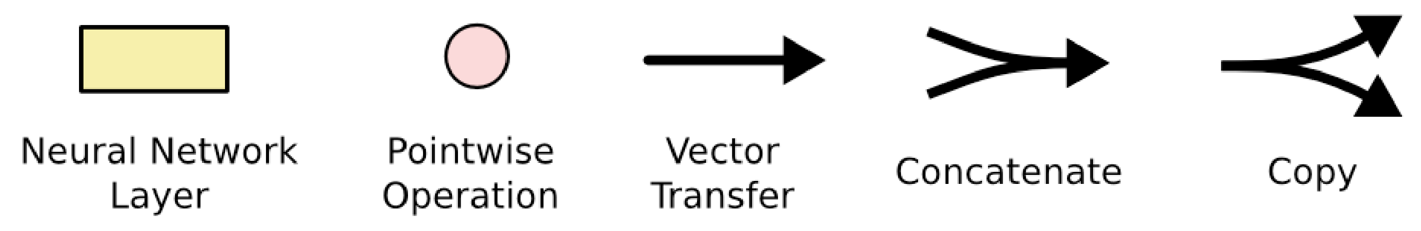
Updating the cell state:



add the gated candidate values

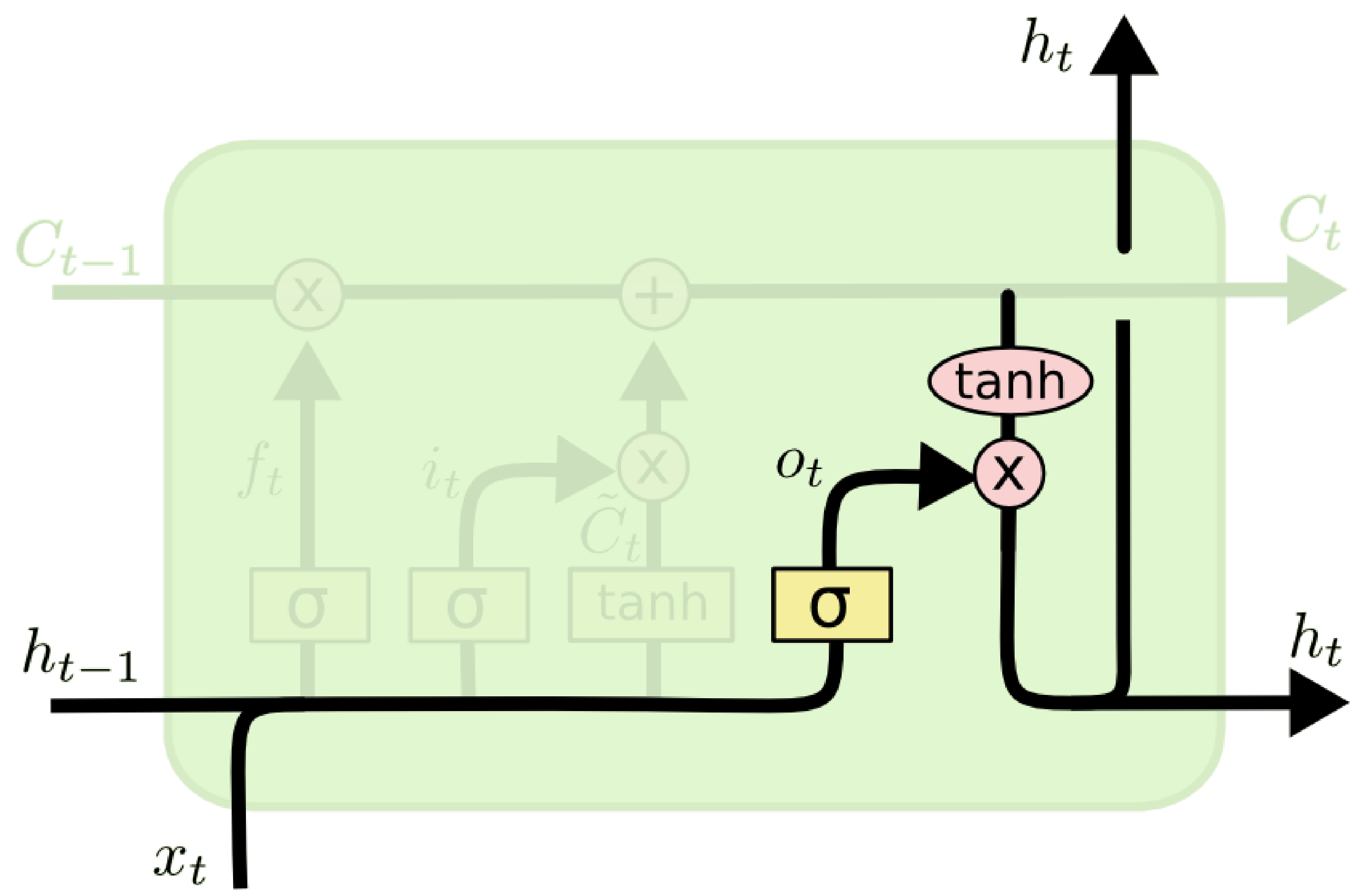
$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$

forget part of the previous cell state

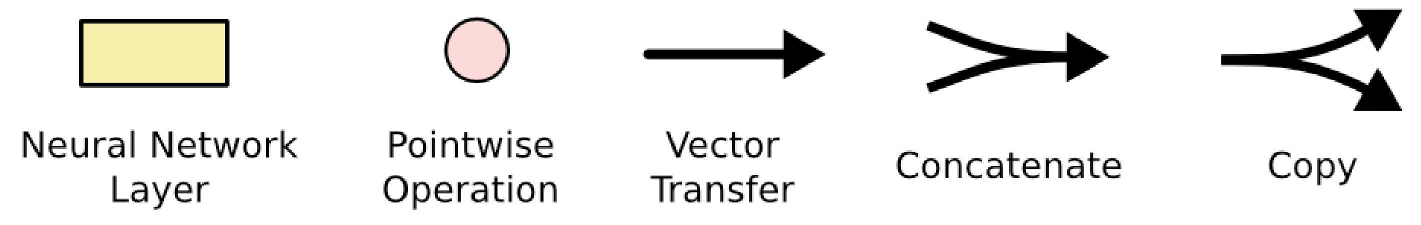


LSTMs

The **output gate**: what part of the cell state should influence the next output?



$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o)$$
$$h_t = o_t \odot \tanh(C_t)$$



LSTMs

Basic structure of an LSTM:

vanilla RNN:

$$h_t = tanh\left(W\begin{pmatrix}h_{t-1}\\x_t\end{pmatrix}\right)$$

LSTM:

$$\begin{array}{r}\text{input gate}\\\text{forget gate}\\\text{output gate}\\\text{candidate update}\end{array}\begin{pmatrix}i\\f\\o\\\tilde{c}\end{pmatrix} = \begin{pmatrix}\sigma\\\sigma\\\sigma\\tanh\end{pmatrix} W\left(\begin{pmatrix}h_{t-1}\\x_t\end{pmatrix}\right)$$

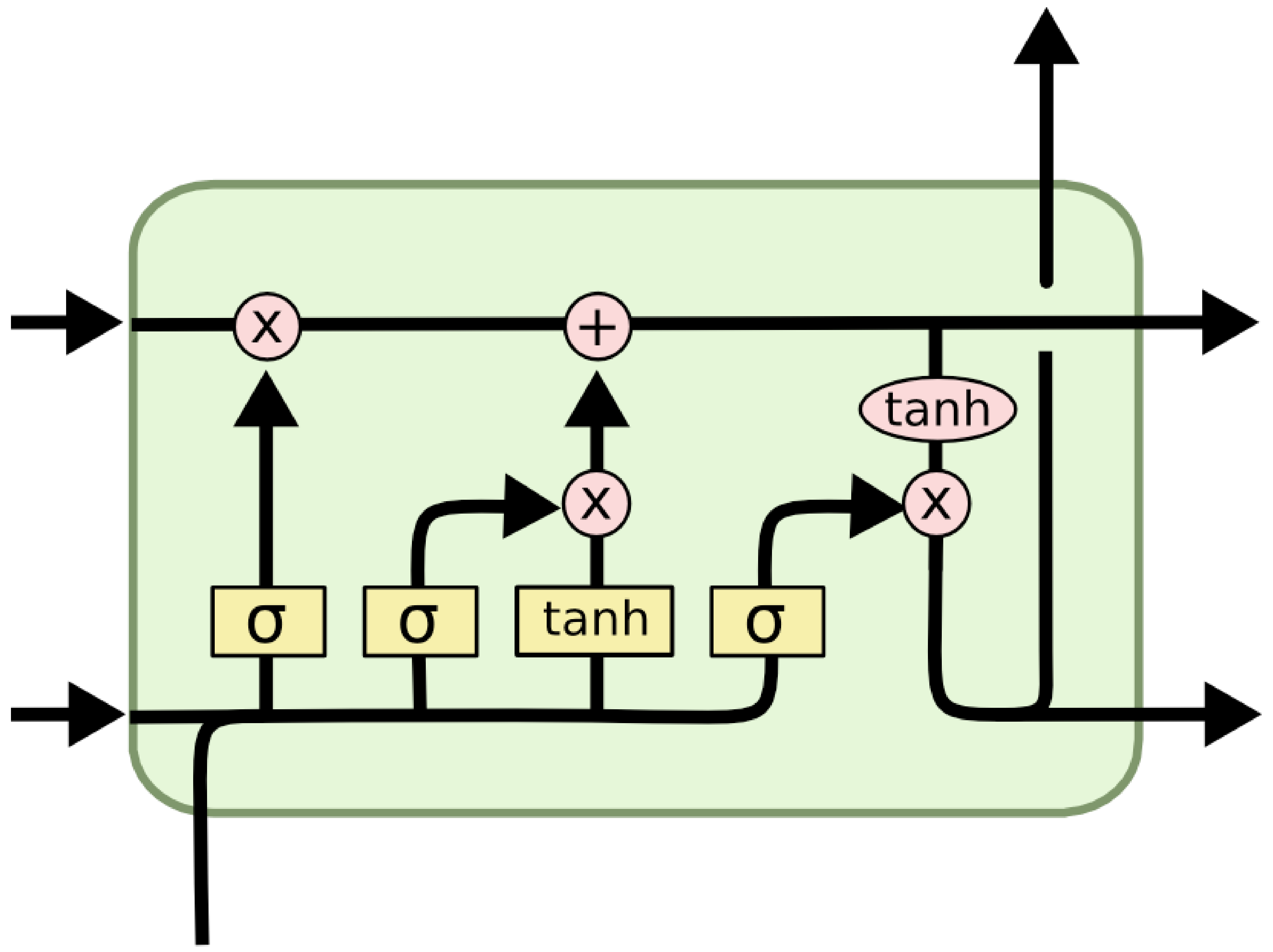
$$c_t = f \odot c_{t-1} + i \odot \tilde{c}$$

$$h_t = o \underset{\uparrow}{\odot} tanh(c_t)$$

Hadamard product (element-wise multiplication)

LSTMs

Basic structure of an LSTM:



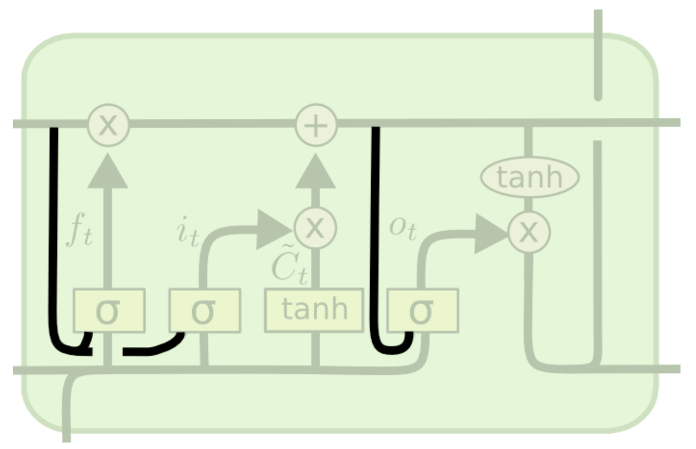
$$\begin{pmatrix} i \\ f \\ o \\ \tilde{c} \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ tanh \end{pmatrix} W \left(\begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right)$$

$$c_t = f \odot c_{t-1} + i \odot \tilde{c}$$

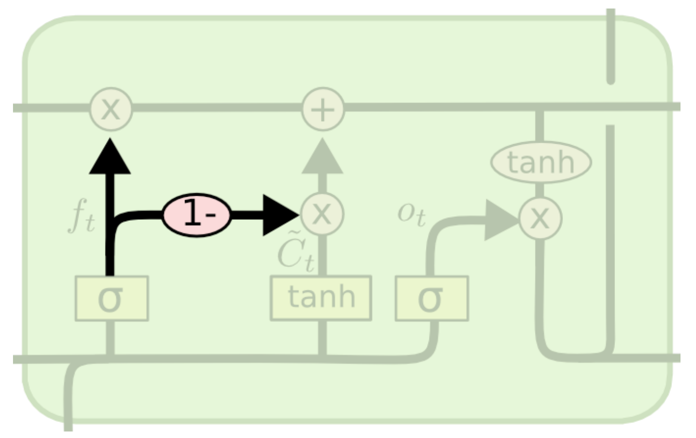
$$h_t = o \odot tanh(c_t)$$

LSTMs

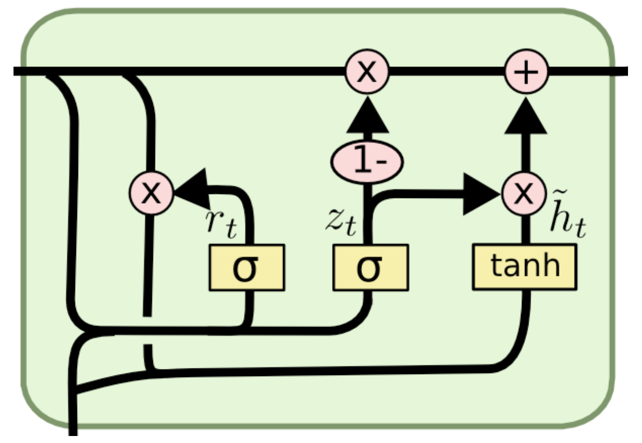
Variations on LSTMs:



peep-hole connections

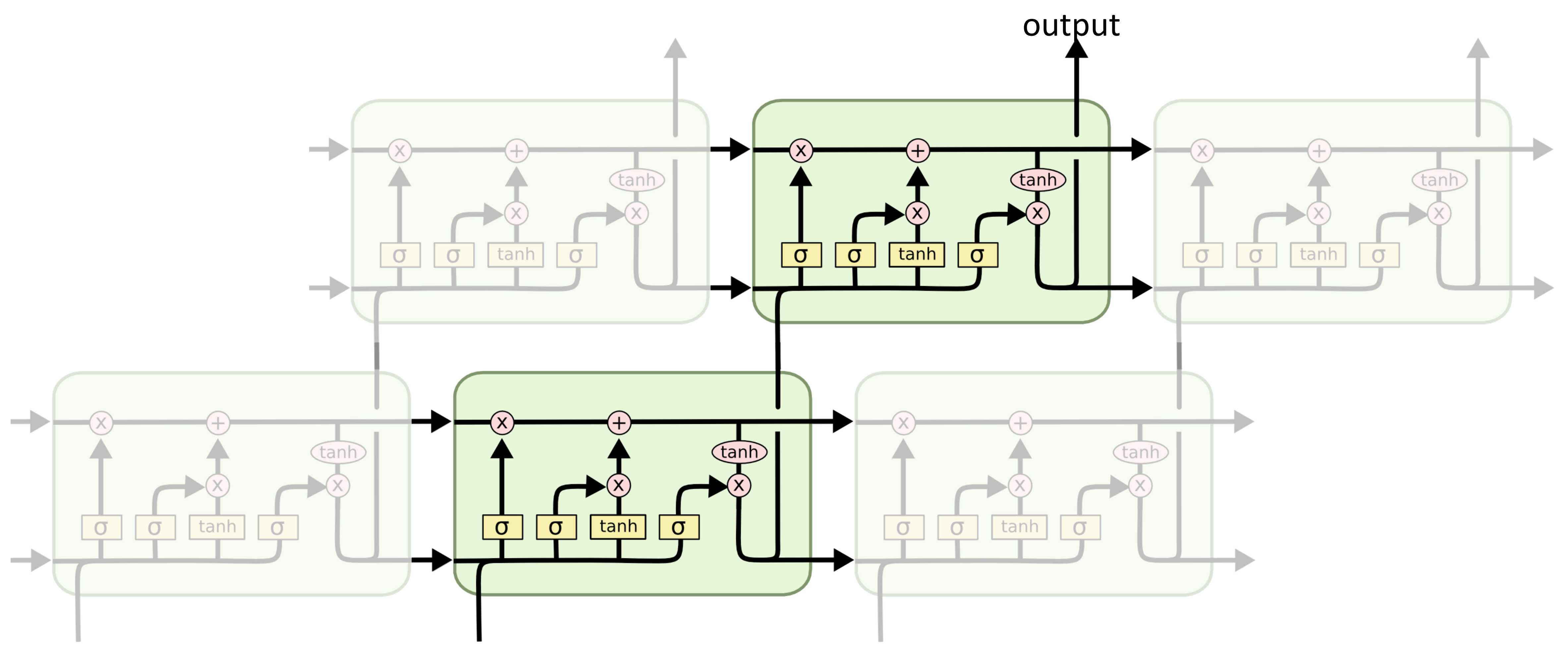


coupled forget and input gates



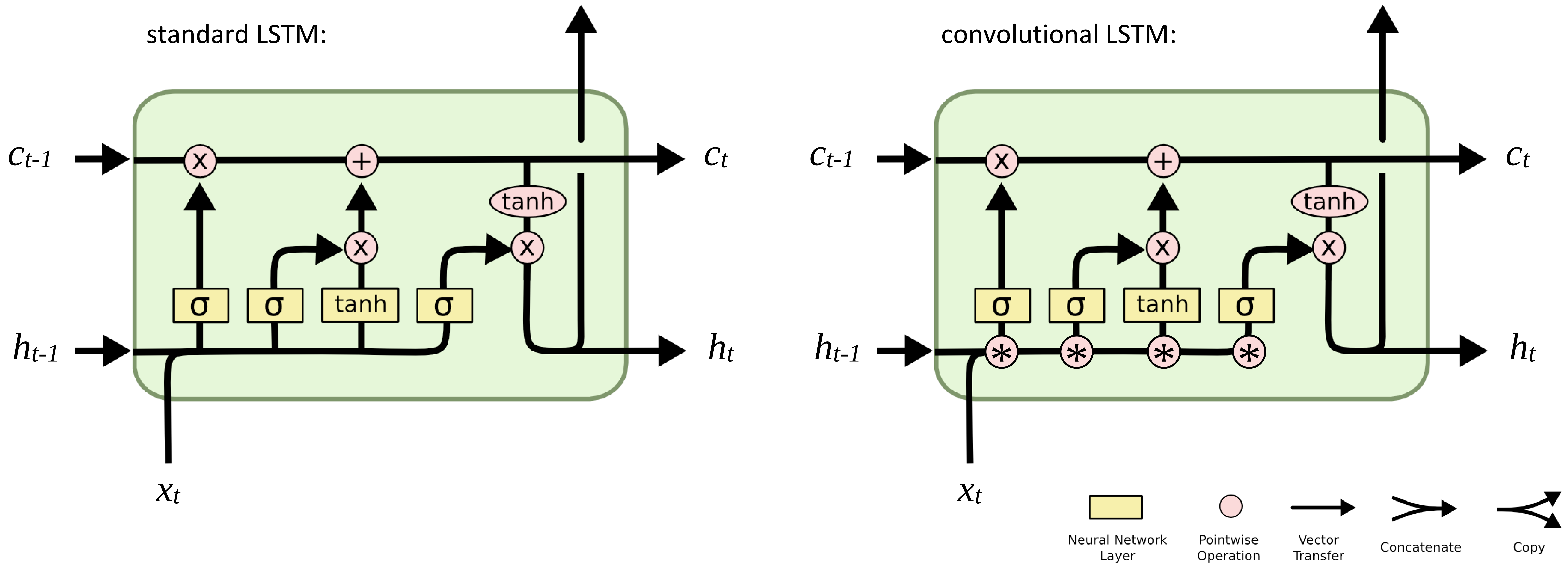
GRU (Gated Recurrent Unit), combines forget and input gates into a single update gate

stacked LSTMs



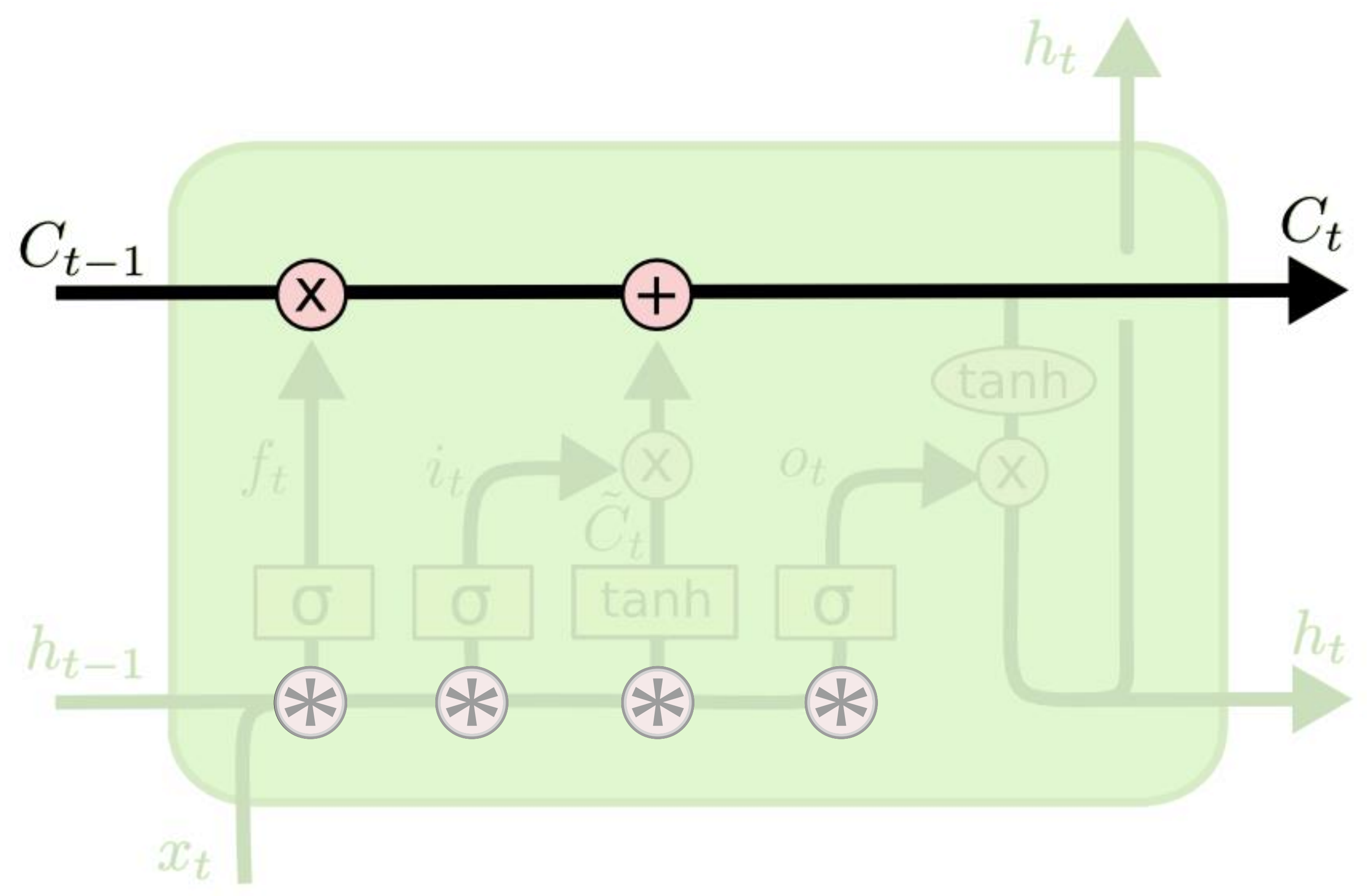
CONVOLUTIONAL LSTMS

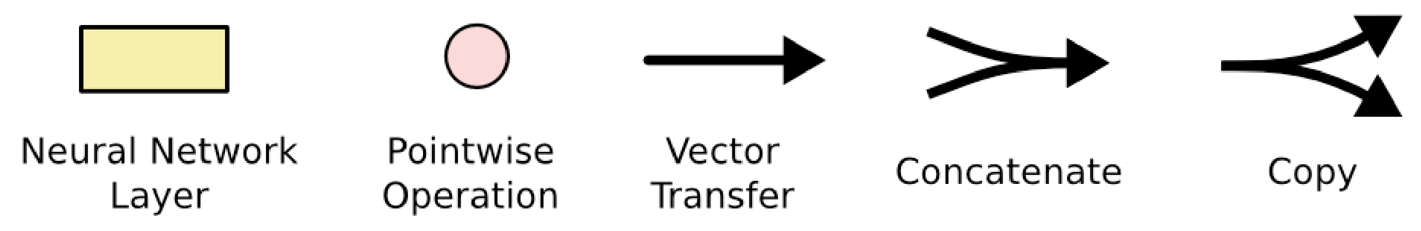
Basic structure of a convolutional LSTM:



Convolutional LSTMs

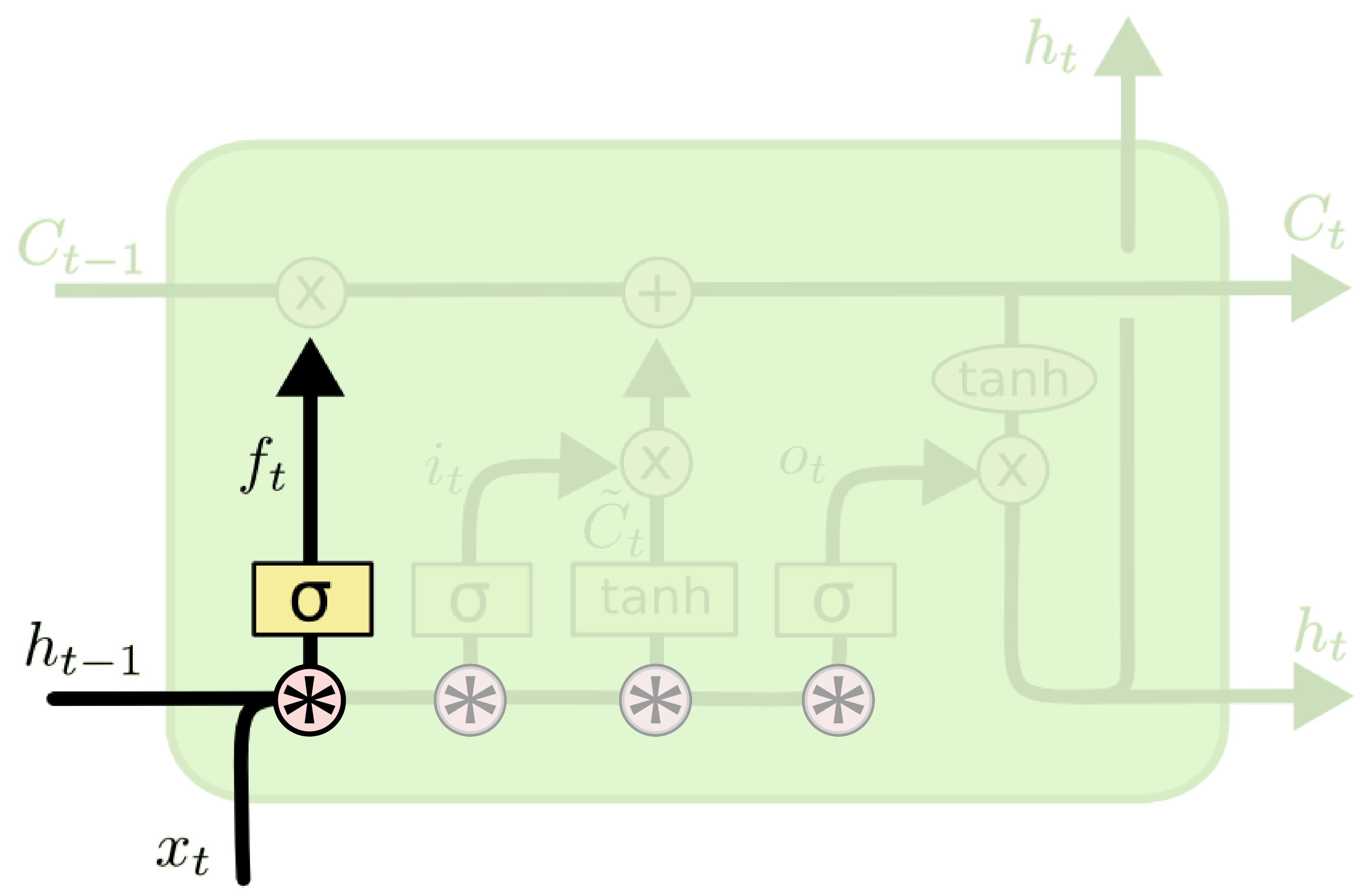
The core idea behind convolutional LSTMs is the same:





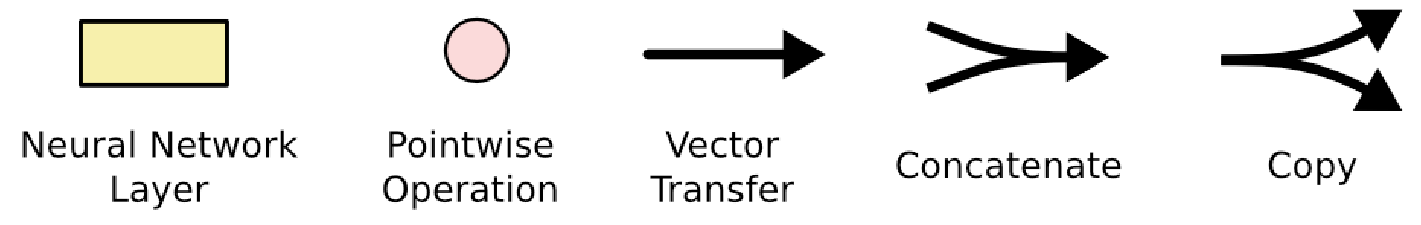
Convolutional LSTMs

The **forget gate**: is there any information in the cell state that we no longer need?



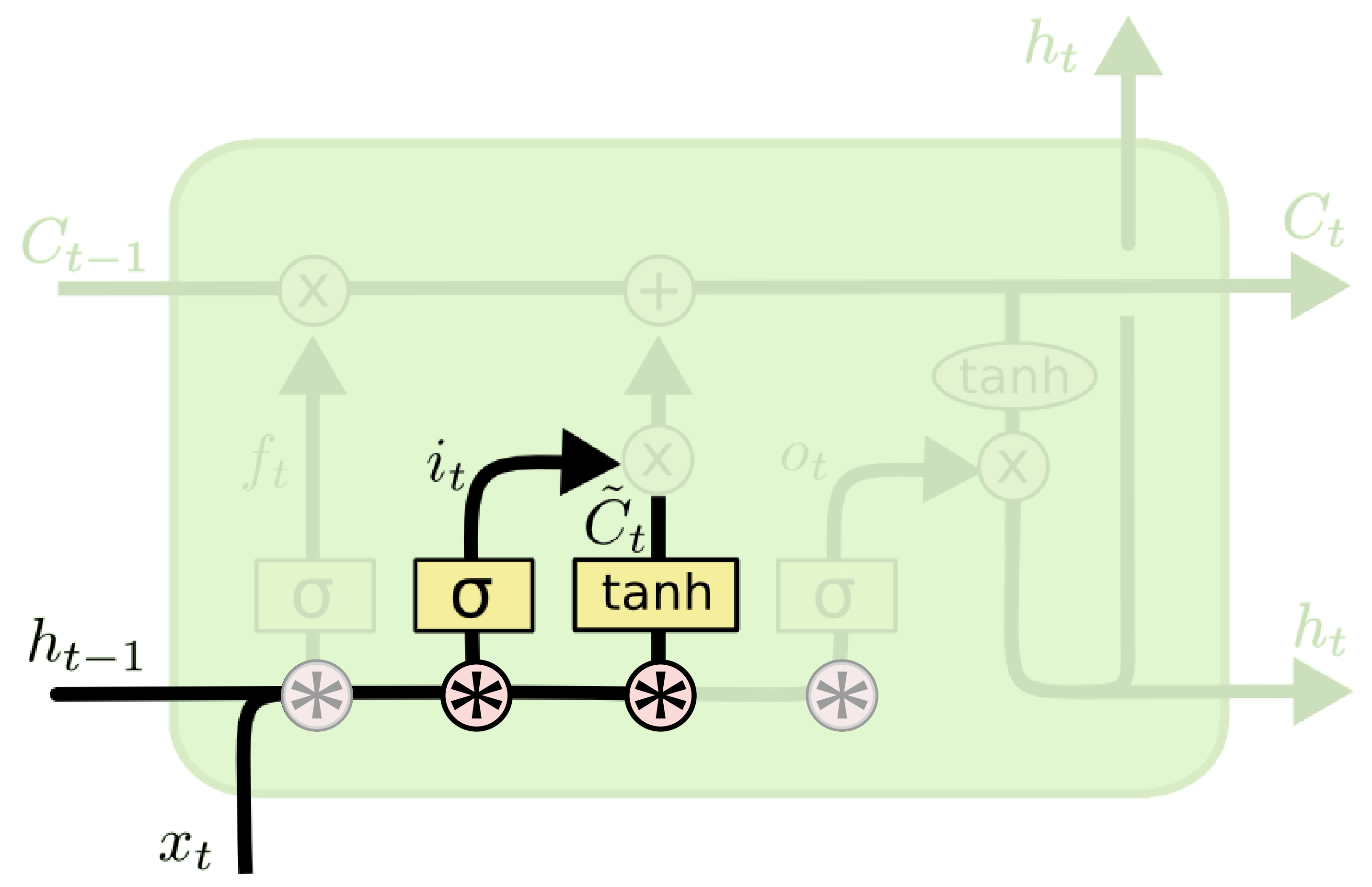
convolution operation

$$f_t = \sigma(W_f * [h_{t-1}, x_t] + b_f)$$



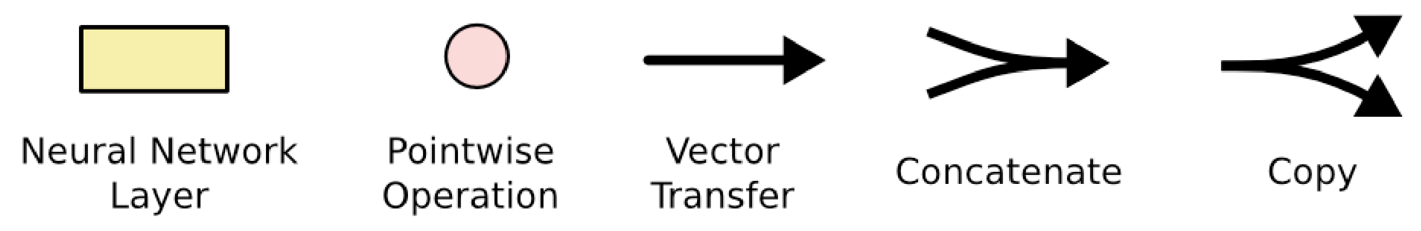
CONVOLUTIONAL LSTMS

The **input gate** and the **candidate update**: what part of the input should we use to update the cell state?



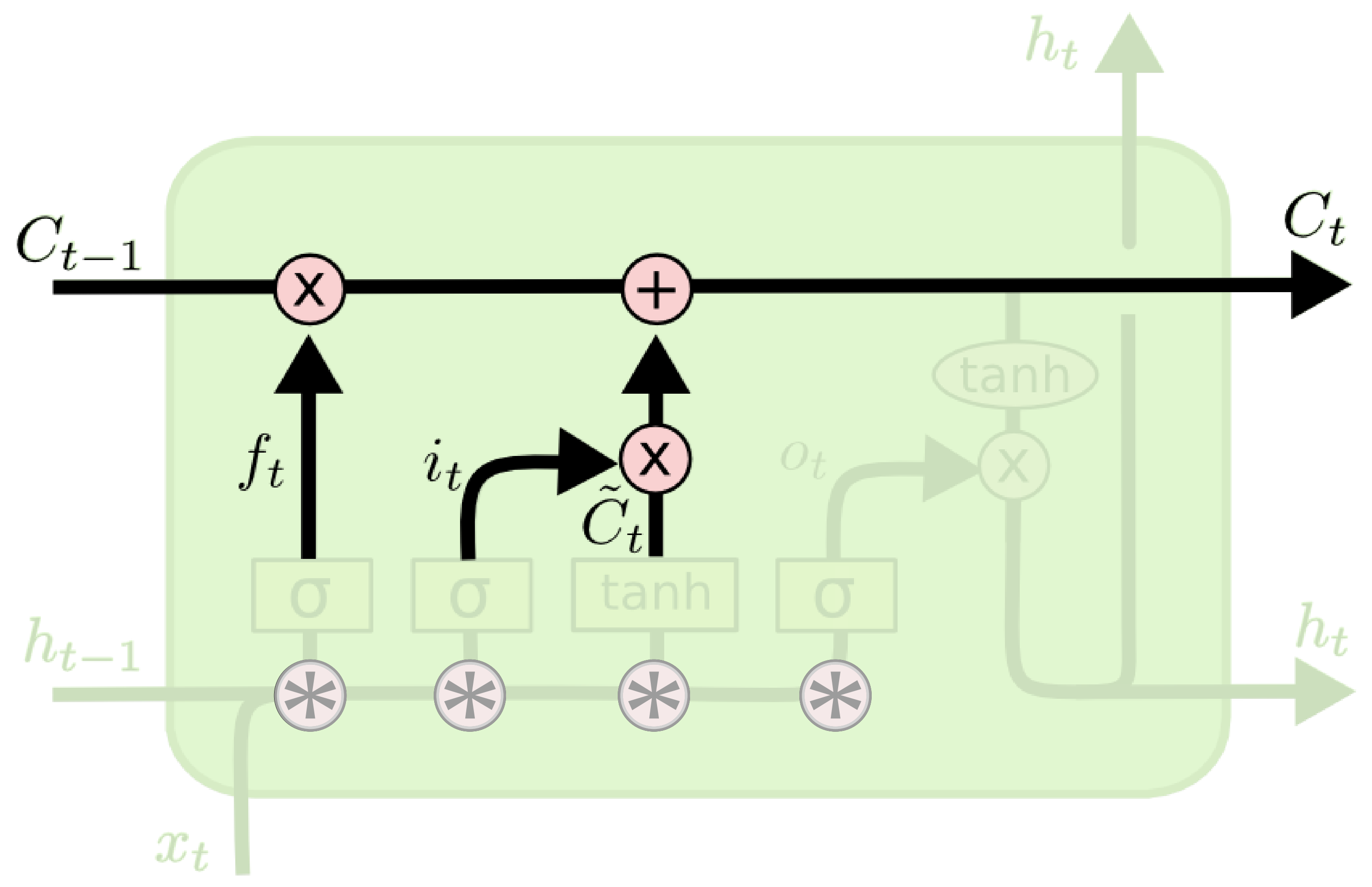
$$i_t = \sigma(W_i * [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_c * [h_{t-1}, x_t] + b_c)$$



CONVOLUTIONAL LSTMs

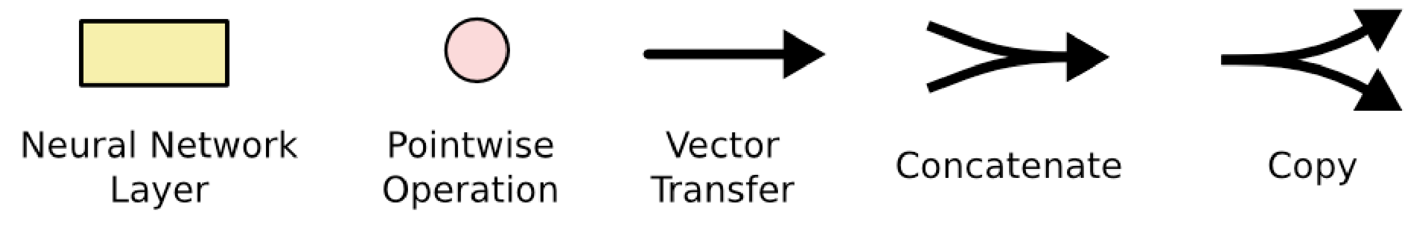
Updating the cell state:



add the gated candidate values

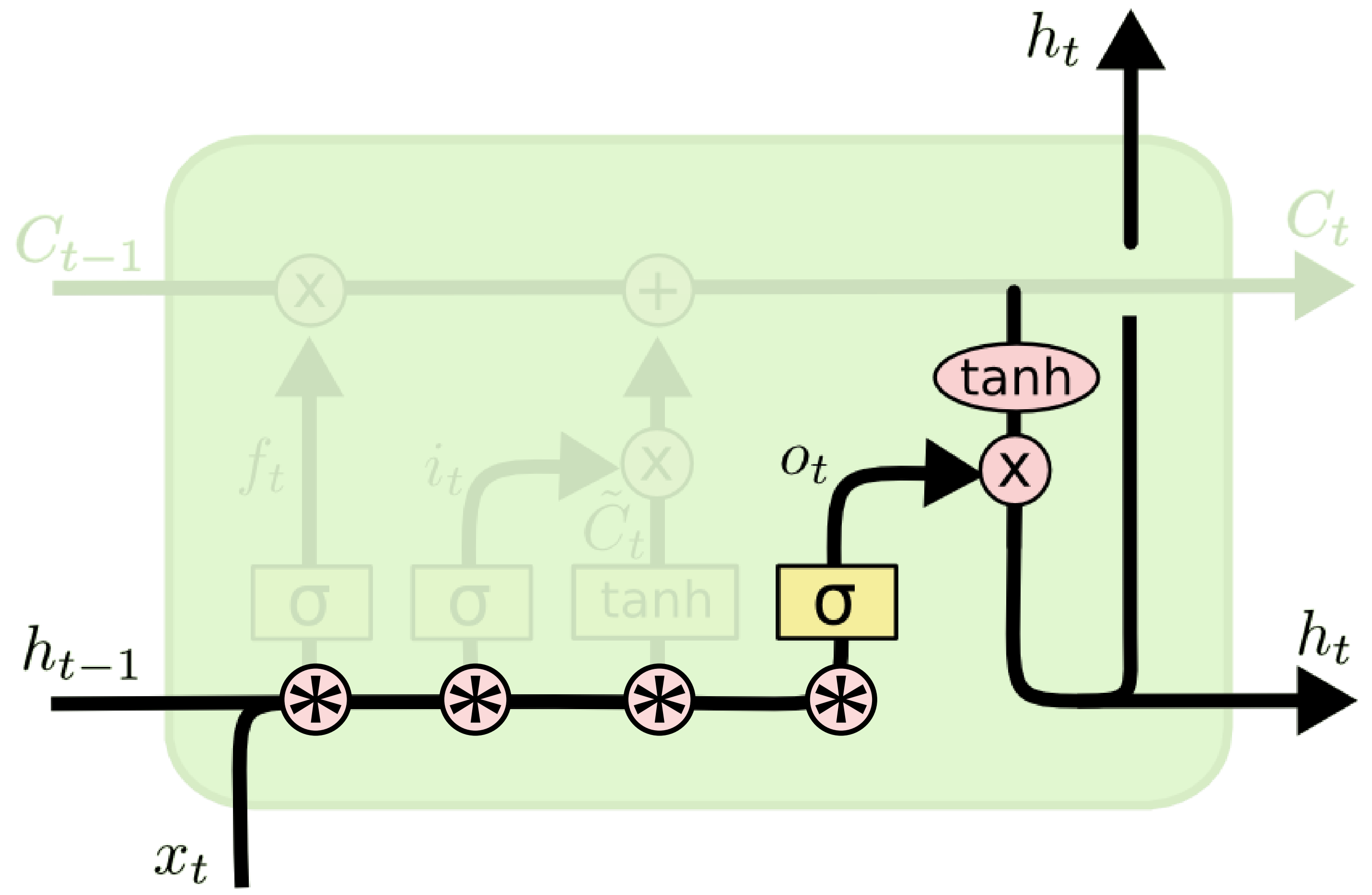
$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$

forget part of the previous cell state

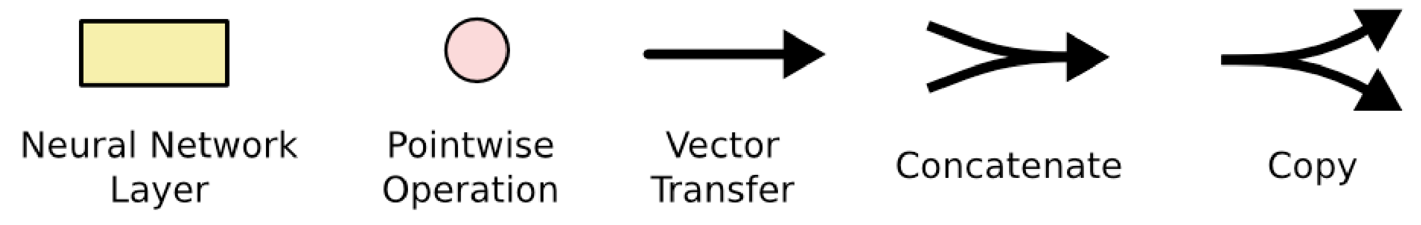


Convolutional LSTMs

The **output gate**: what part of the cell state should influence the next output?



$$o_t = \sigma(W_o * [h_{t-1}, x_t] + b_o)$$
$$h_t = o_t \odot \tanh(C_t)$$



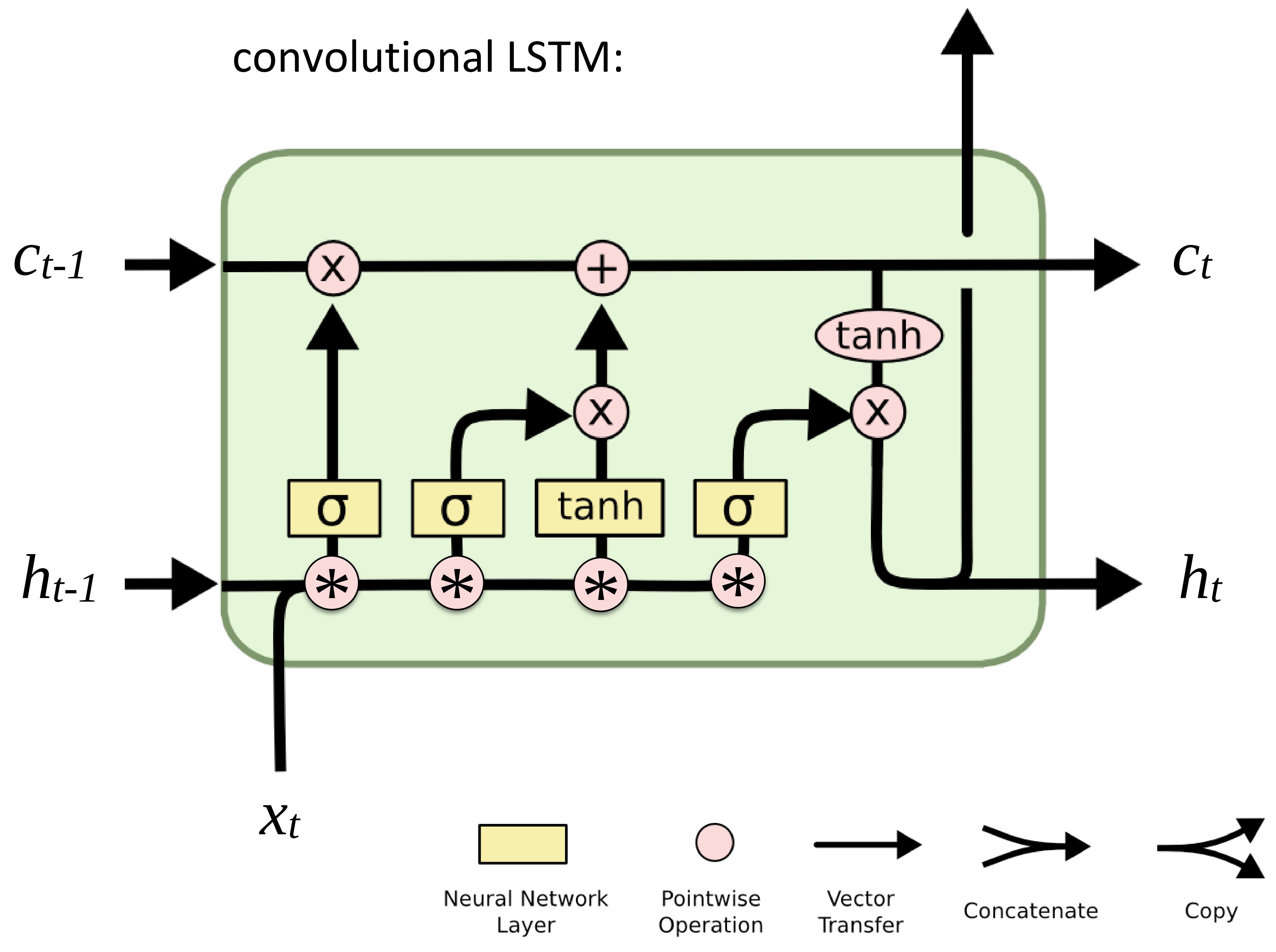
https://colah.github.io/posts/2015-08-Understanding-LSTMs/

Convolutional LSTMs

Basic structure of a convolutional LSTM:

$$f_t = \sigma(W_f * [h_{t-1}, x_t] + b_f)$$
$$i_t = \sigma(W_i * [h_{t-1}, x_t] + b_i)$$
$$\tilde{C}_t = \tanh(W_c * [h_{t-1}, x_t] + b_c)$$
$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$
$$o_t = \sigma(W_o * [h_{t-1}, x_t] + b_o)$$
$$h_t = o_t \odot \tanh(C_t)$$

convolutional LSTM:



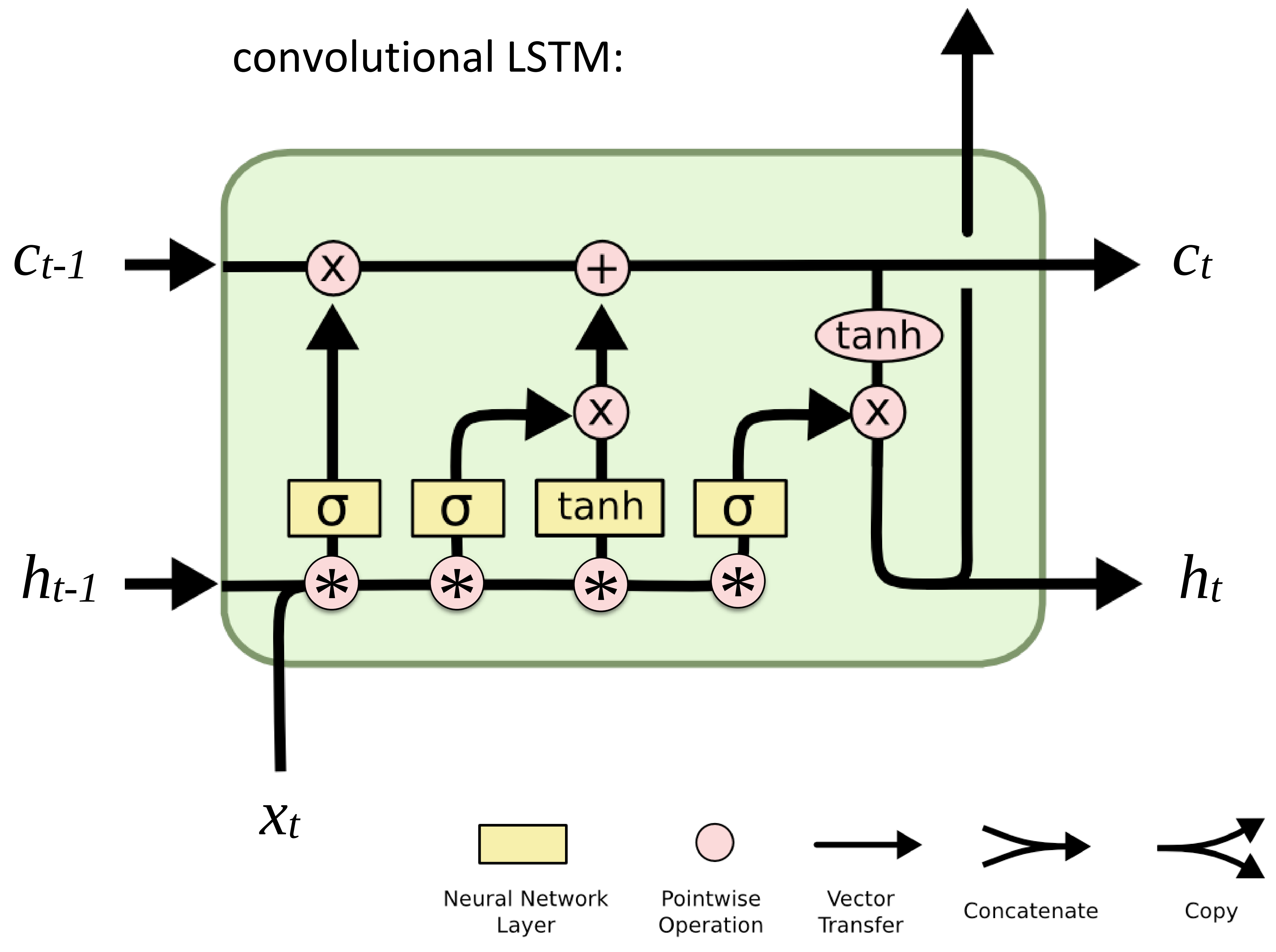
https://papers.nips.cc/paper/2015/hash/07563a3fe3bbe7e3ba84431ad9d055af-Abstract.html

Convolutional LSTMs

Basic structure of a convolutional LSTM (with peep-hole connections):

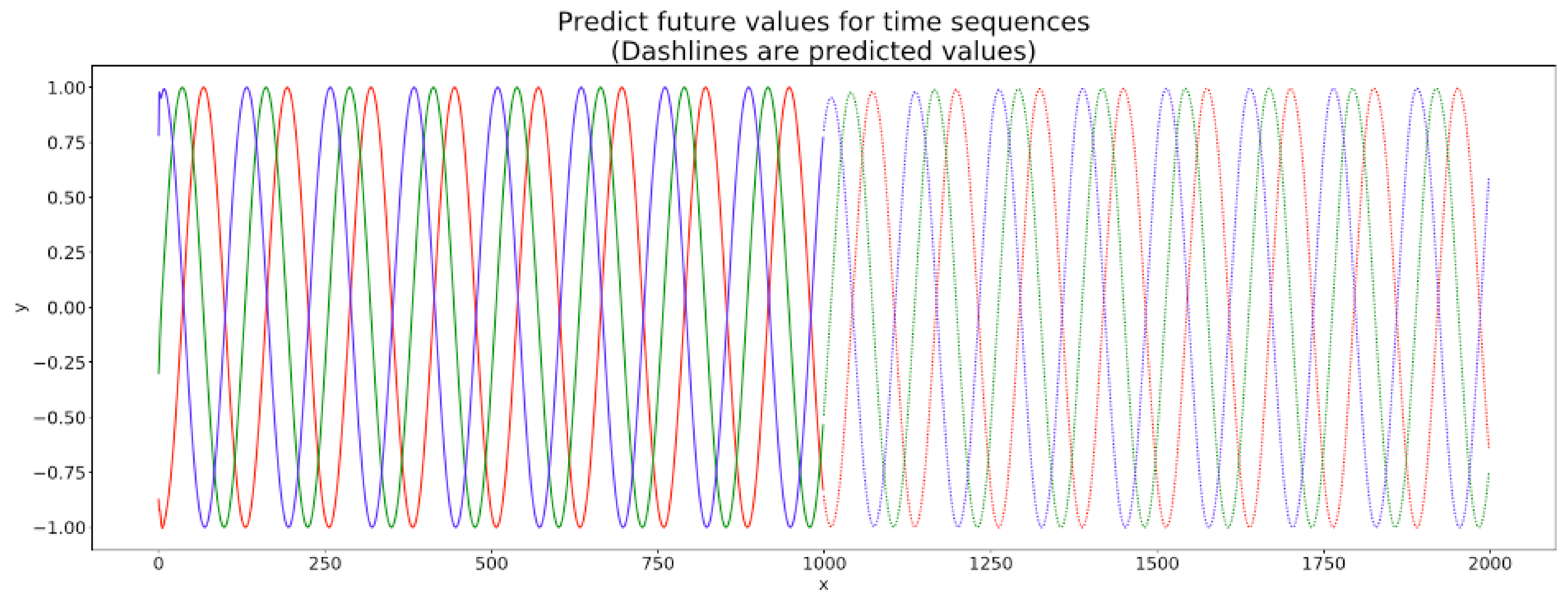
$$f_t = \sigma(W_f * [h_{t-1}, x_t] + W_{cf} \odot C_{t-1} + b_f)$$
$$i_t = \sigma(W_i * [h_{t-1}, x_t] + W_{ci} \odot C_{t-1} + b_i)$$
$$\tilde{C}_t = \tanh(W_c * [h_{t-1}, x_t] + b_c)$$
$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$
$$o_t = \sigma(W_o * [h_{t-1}, x_t] + W_{co} \odot C_{t-1} + b_o)$$
$$h_t = o_t \odot \tanh(C_t)$$

convolutional LSTM:

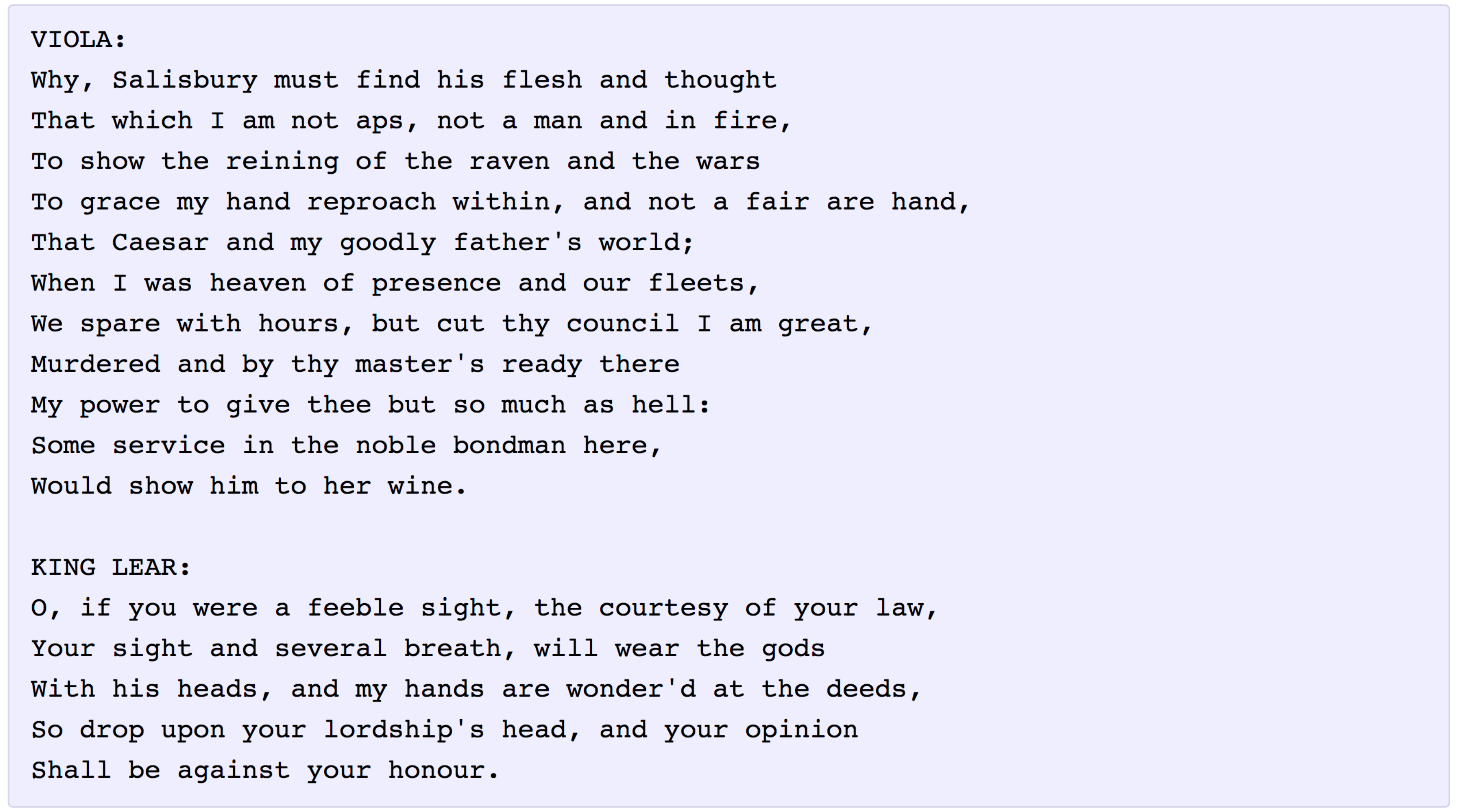


LSTMs

Examples of application:



https://github.com/pytorch/examples/tree/master/time_sequence_prediction



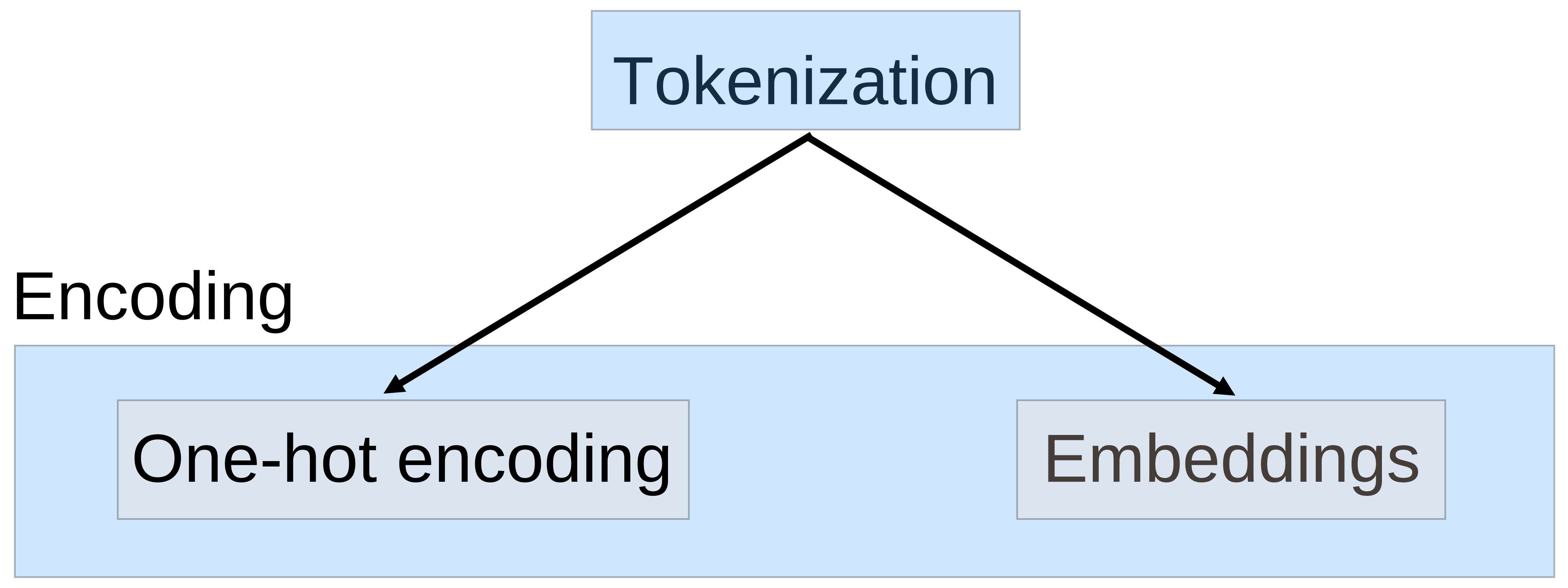
```
VIOLA:
Why, Salisbury must find his flesh and thought
That which I am not aps, not a man and in fire,
To show the reining of the raven and the wars
To grace my hand reproach within, and not a fair are hand,
That Caesar and my goodly father's world;
When I was heaven of presence and our fleets,
We spare with hours, but cut thy council I am great,
Murdered and by thy master's ready there
My power to give thee but so much as hell:
Some service in the noble bondman here,
Would show him to her wine.

KING LEAR:
O, if you were a feeble sight, the courtesy of your law,
Your sight and several breath, will wear the gods
With his heads, and my hands are wonder'd at the deeds,
So drop upon your lordship's head, and your opinion
Shall be against your honour.
```

http://karpathy.github.io/2015/05/21/rnn-effectiveness/

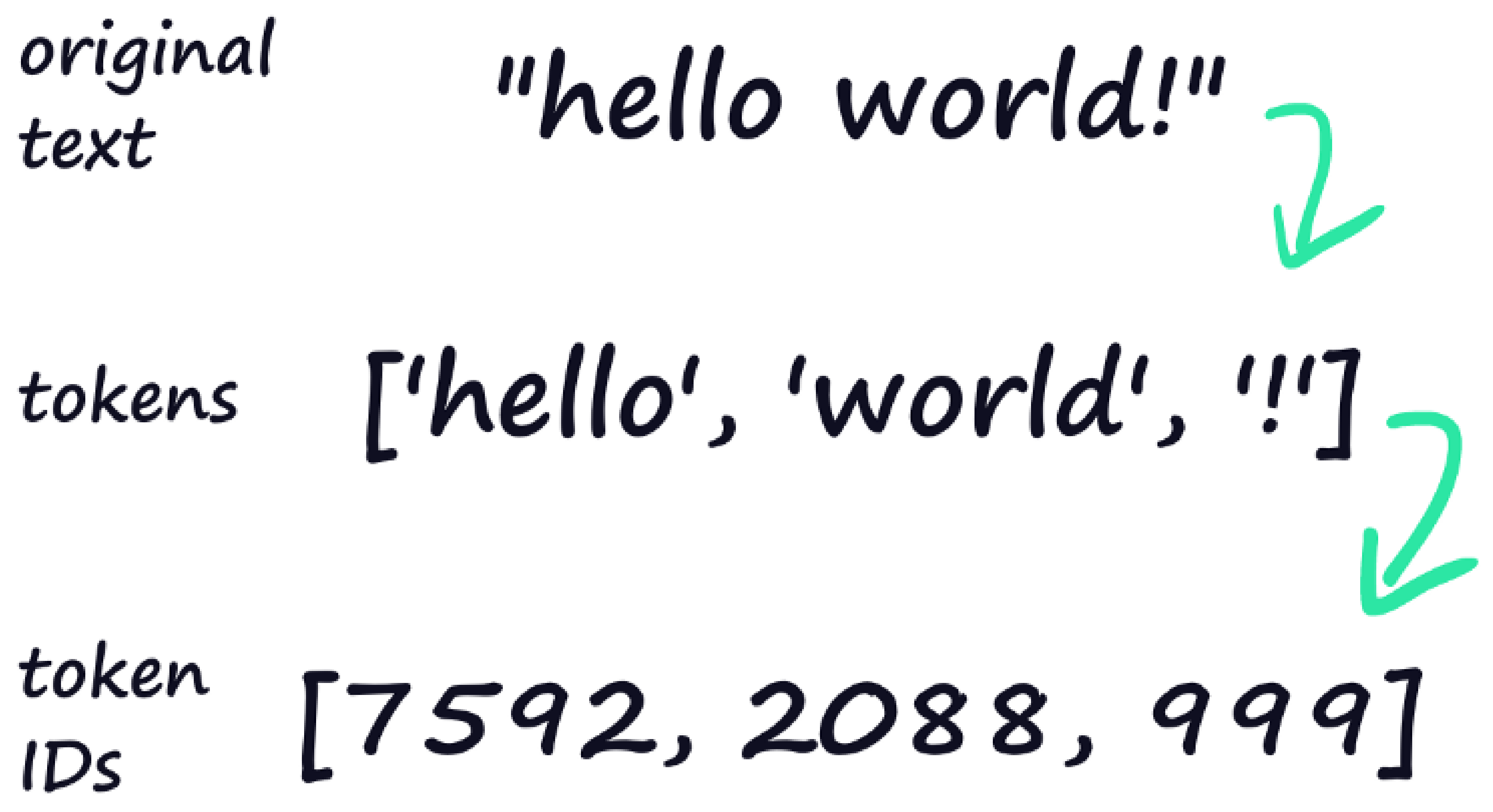
TEXT REPRESENTATION

Text datasets need to be transformed into numerical form to be used by our networks:



TOKENIZATION

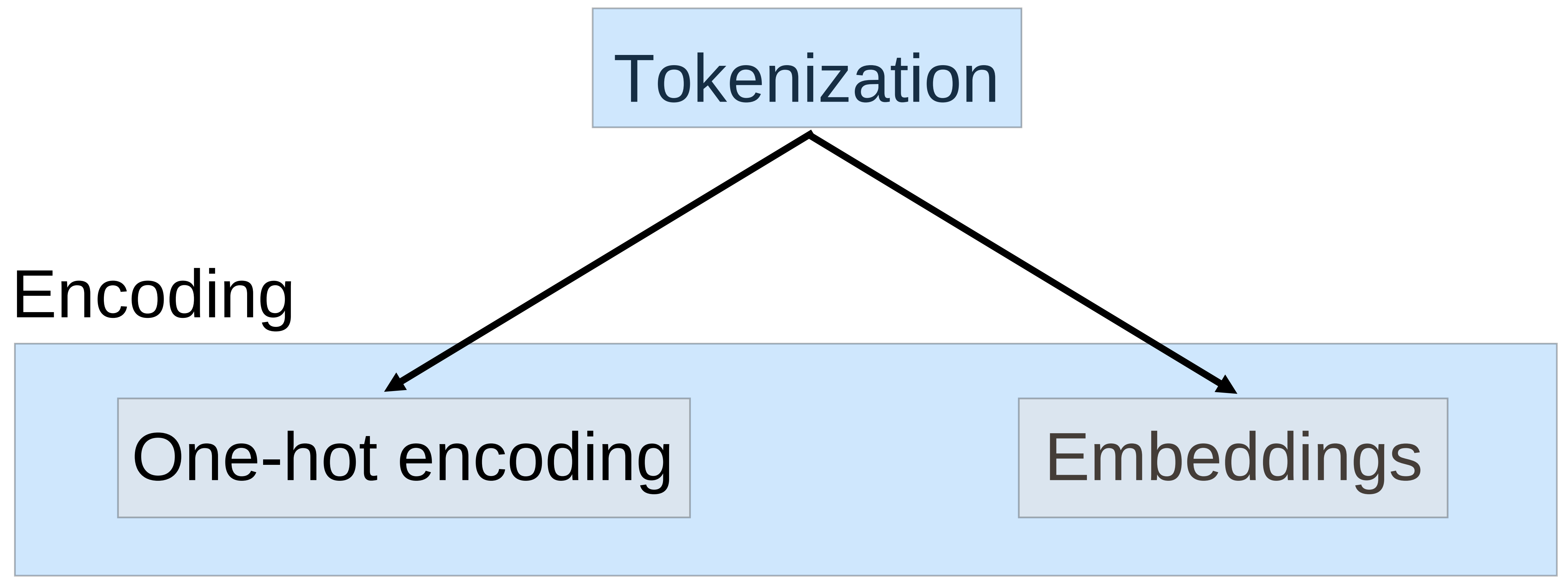
Tokenization is the process of encoding a string of text into numerical integer IDs:



TEXT REPRESENTATION

Text datasets need to be transformed into numerical form to be used by our networks:

Once tokenized, we need to encode tokens into vectors:



ONE-HOT ENCODING

With hot encoding we transform numerical integer IDs into vectors of 1s and 0s:

token IDs [7592, 2088, 999]

$$\begin{matrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{matrix}$$

ONE-HOT ENCODING

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token IDs [7592, 2088, 999]

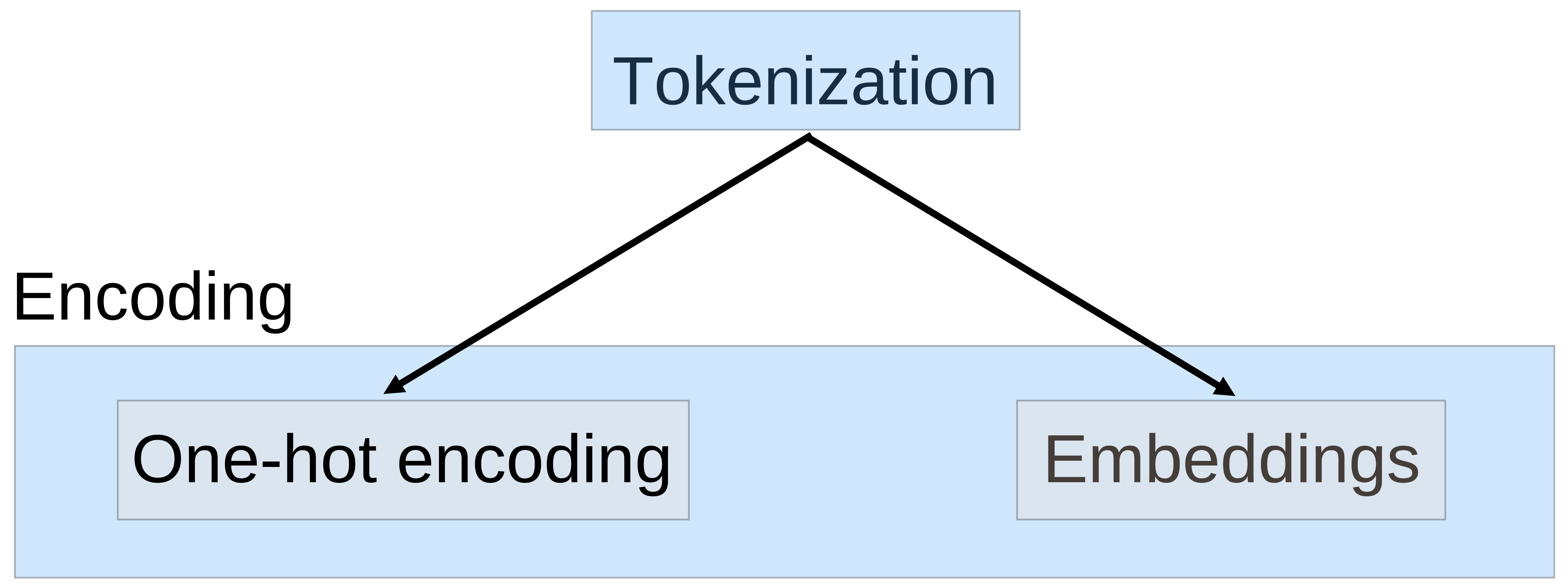
$$\begin{matrix}0&0&0\\1&0&0\\0&0&0\\\vdots&\vdots&\vdots\\0&1&0\\0&0&0\\0&0&1\end{matrix}$$

When using words instead of characters, the dictionary would be huge!

TEXT REPRESENTATION

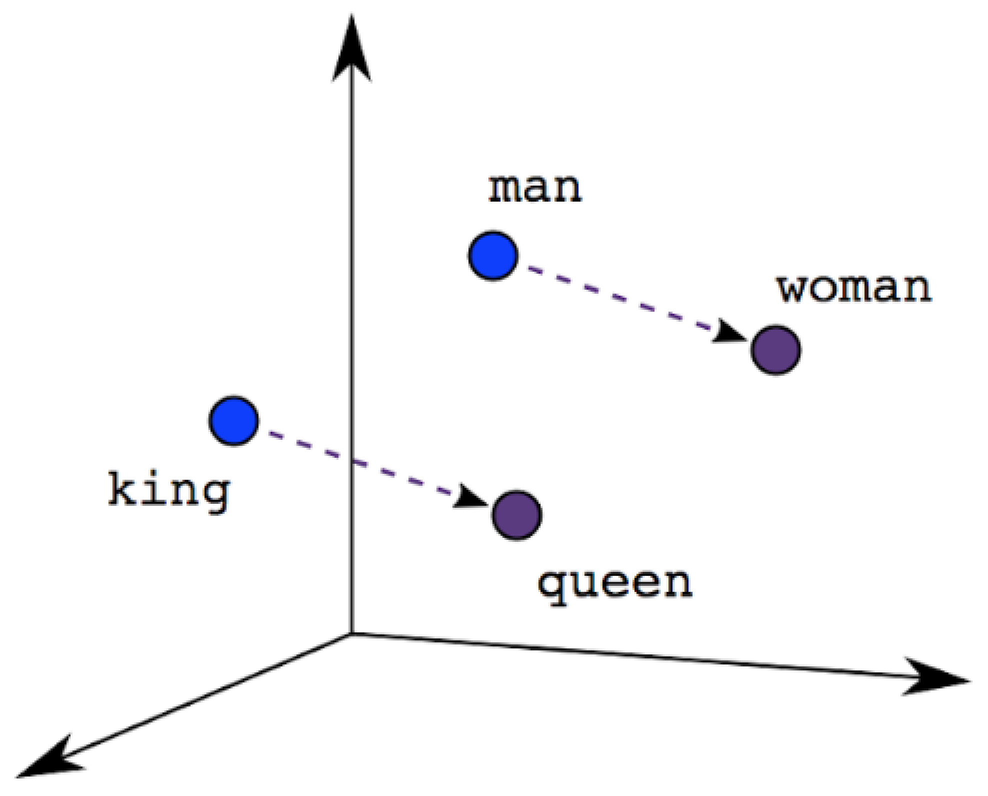
Text datasets need to be transformed into numerical form to be used by our networks:

To deal with large dictionaries, we need smaller but representation denser vectors:

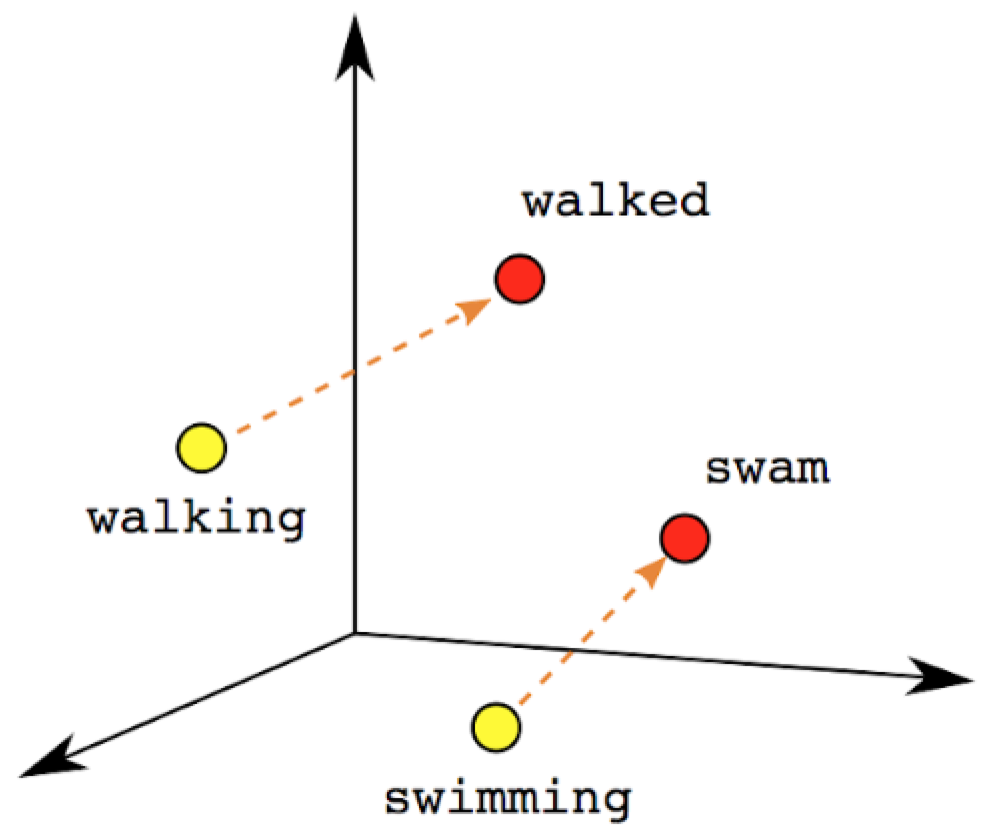


EMBEDDINGS

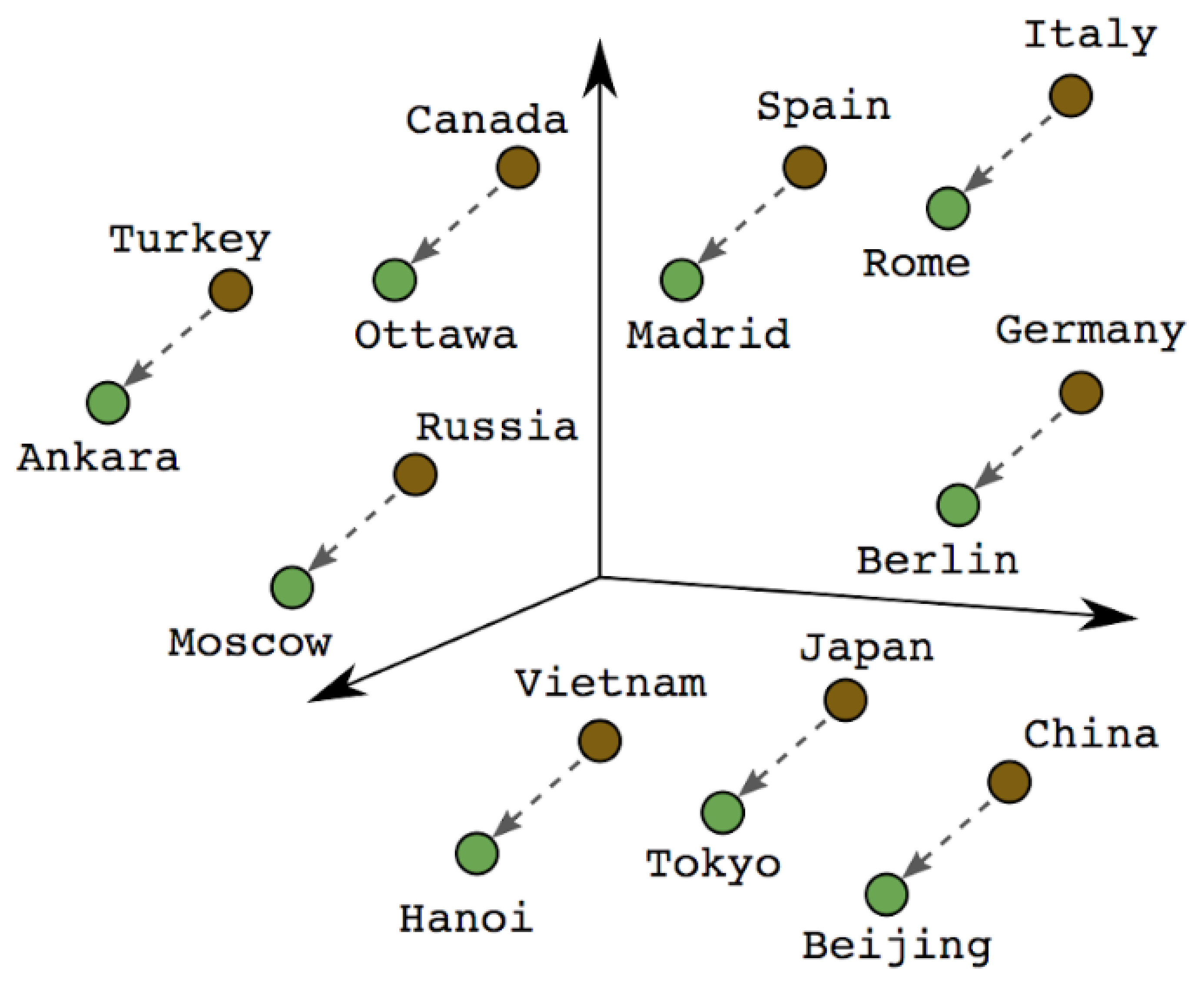
Words have meaning and relations, so we can represent word tokens in a dense vector space:



Male-Female



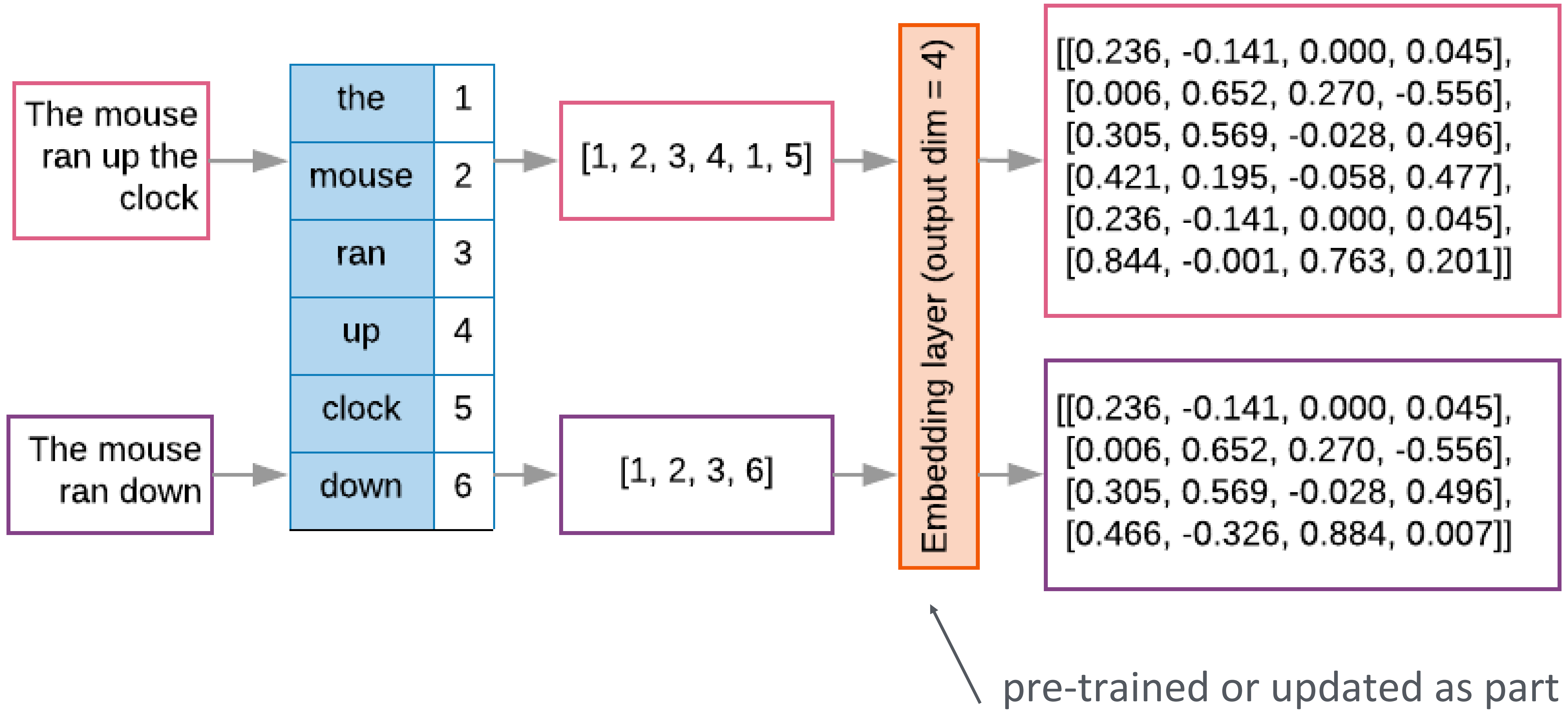
Verb Tense



Country-Capital

EMBEDDINGS

Words have meaning and relations, so we can represent word tokens in a dense vector space:



pre-trained or updated as part of our network training

The basis of successful natural language processing (NLP) models like GPT-3 or BERT.

Summary

- Recurrent Neural Networks (RNNs) are the basic approach for sequential data.
- RNNs suffer from exploding and vanishing gradients and only preserve short-term memory.
- Long Short-Term Memory (LSTM) networks address these two issues.
- Before text can be processed by neural networks it needs to be tokenized into unique IDs.
- Tokenized text can be encoded using one-hot encoding (appropriate for small dictionaries) or using embeddings (more efficient for larger dictionaries).