# Package 'gamlss.dist'

March 8, 2022

Type Package

**Title** Distributions for Generalized Additive Models for Location Scale and Shape

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Date 2022-03-08

**Description** A set of distributions which can be used for modelling the response variables in Generalized Additive Models for Location Scale and Shape, Rigby and Stasinopoulos (2005), <doi:10.1111/j.1467-9876.2005.00510.x>. The distributions can be continuous, discrete or mixed distributions. Extra distributions can be created, by transforming, any continuous distribution defined on the real line, to a distribution defined on ranges 0 to infinity or 0 to 1, by using a "log" or a "logit' transformation respectively.

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URL https://www.gamlss.com/

**Depends** R (>= 3.5.0), MASS, graphics, stats, methods, grDevices

Repository CRAN

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 ${\tt gamlss.dist-package} \qquad \textit{Distributions for Generalized Additive Models for Location Scale and Shape}$ 

# **Description**

A set of distributions which can be used for modelling the response variables in Generalized Additive Models for Location Scale and Shape, Rigby and Stasinopoulos (2005), <doi:10.1111/j.1467-9876.2005.00510.x>. The distributions can be continuous, discrete or mixed distributions. Extra distributions can be created, by transforming, any continuous distribution defined on the real line, to a distribution defined on ranges 0 to infinity or 0 to 1, by using a "log" or a "logit' transformation respectively.

#### **Details**

# The DESCRIPTION file:

Package: gamlss.dist Type: Package

Title: Distributions for Generalized Additive Models for Location Scale and Shape

Version: 6.0-3 Date: 2022-03-08

Authors@R: c(person("Mikis", "Stasinopoulos", role = c("aut", "cre", "cph"), email = "d.stasinopoulos@londonmet.ac.uk")

Description: A set of distributions which can be used for modelling the response variables in Generalized Additive Models

License: GPL-2 | GPL-3

URL: https://www.gamlss.com/

Depends: R (>= 3.5.0), MASS, graphics, stats, methods, grDevices

Repository: CRAN

Author: Mikis Stasinopoulos [aut, cre, cph], Robert Rigby [aut], Calliope Akantziliotou [ctb], Vlasios Voudouris [ctb],

Maintainer: Mikis Stasinopoulos <d.stasinopoulos@londonmet.ac.uk>

# Index of help topics:

BB Beta Binomial Distribution For Fitting a GAMLSS

Model

BCCG Box-Cox Cole and Green distribution (or Box-Cox

normal) for fitting a GAMLSS

BCPE Box-Cox Power Exponential distribution for

fitting a GAMLSS

BCT Box-Cox t distribution for fitting a GAMLSS
BE The beta distribution for fitting a GAMLSS
BEINF The beta inflated distribution for fitting a

The beta initated distribution for fitting

GAMLSS

BEOI The one-inflated beta distribution for fitting

a GAMLSS

BEZI The zero-inflated beta distribution for fitting

a GAMLSS

ΒI Binomial distribution for fitting a GAMLSS **BNB** Beta Negative Binomial distribution for fitting

a GAMLSS

DBI The Double binomial distribution

DBURR12 The Discrete Burr type XII distribution for

fitting a GAMLSS model

DEL The Delaporte distribution for fitting a GAMLSS

model

The Double Poisson distribution DP0

EGB2 The exponential generalized Beta type 2

distribution for fitting a GAMLSS

EXP Exponential distribution for fitting a GAMLSS

GΑ Gamma distribution for fitting a GAMLSS

GAF The Gamma distribution family

GB1 The generalized Beta type 1 distribution for

fitting a GAMLSS

GB2 The generalized Beta type 2 and generalized

Pareto distributions for fitting a GAMLSS

**GEOM** Geometric distribution for fitting a GAMLSS

model

GG Generalized Gamma distribution for fitting a

**GAMLSS** 

GIG Generalized Inverse Gaussian distribution for

fitting a GAMLSS

GP0 The generalised Poisson distribution

The generalized t distribution for fitting a GΤ

**GAMLSS** 

GU The Gumbel distribution for fitting a GAMLSS Inverse Gaussian distribution for fitting a ΙG

**GAMLSS** 

**IGAMMA** Inverse Gamma distribution for fitting a GAMLSS JSU The Johnson's Su distribution for fitting a

**GAMLSS** 

JSUo The original Johnson's Su distribution for

fitting a GAMLSS

LG Logarithmic and zero adjusted logarithmic

> distributions for fitting a GAMLSS model Log Normal distribution for fitting in GAMLSS

LNO L0 Logistic distribution for fitting a GAMLSS LOGITNO Logit Normal distribution for fitting in GAMLSS LQN0 Normal distribution with a specific mean and

variance relationship for fitting a GAMLSS

model

MN3 Multinomial distribution in GAMLSS

NBF Negative Binomial Family distribution for

fitting a GAMLSS

NBI Negative Binomial type I distribution for

fitting a GAMLSS

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NBII Negative Binomial type II distribution for

fitting a GAMLSS

NET Normal Exponential t distribution (NET) for

fitting a GAMLSS

NO Normal distribution for fitting a GAMLSS NO2 Normal distribution (with variance as sigma

parameter) for fitting a GAMLSS

NOF Normal distribution family for fitting a GAMLSS PARETO2 Pareto distributions for fitting in GAMLSS PΕ Power Exponential distribution for fitting a

**GAMLSS** 

PIG The Poisson-inverse Gaussian distribution for

fitting a GAMLSS model

P0 Poisson distribution for fitting a GAMLSS model RG The Reverse Gumbel distribution for fitting a

GAMLSS

**RGE** Reverse generalized extreme family distribution

for fitting a GAMLSS

SEP The Skew Power exponential (SEP) distribution

for fitting a GAMLSS

SEP1 The Skew Power exponential type 1-4

distribution for fitting a GAMLSS

SHASH The Sinh-Arcsinh (SHASH) distribution for

fitting a GAMLSS

SI The Sichel dustribution for fitting a GAMLSS

model

SICHEL The Sichel distribution for fitting a GAMLSS

SIMPLEX The simplex distribution for fitting a GAMLSS SN1

Skew Normal Type 1 distribution for fitting a

**GAMLSS** 

SN2 Skew Normal Type 2 distribution for fitting a

**GAMLSS** 

ST1 The skew t distributions, type 1 to 5 TF t family distribution for fitting a GAMLSS WARING Waring distribution for fitting a GAMLSS model WFT Weibull distribution for fitting a GAMLSS WEI2 A specific parameterization of the Weibull

distribution for fitting a GAMLSS

WEI3 A specific parameterization of the Weibull

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YULE Yule distribution for fitting a GAMLSS model Zero inflated and zero adjusted Binomial ZABB

distribution for fitting in GAMLSS

Zero inflated and zero adjusted Binomial ZABI

distribution for fitting in GAMLSS

The zero adjusted Gamma distribution for ZAGA

fitting a GAMLSS model

ZAIG The zero adjusted Inverse Gaussian distribution

for fitting a GAMLSS model

ZANBI Zero inflated and zero adjusted negative

binomial distributions for fitting a GAMLSS

model

ZAP Zero adjusted poisson distribution for fitting

a GAMLSS model

ZIP Zero inflated poisson distribution for fitting

a GAMLSS model

ZIP2 Zero inflated poisson distribution for fitting

a GAMLSS model

ZIPF The zipf and zero adjusted zipf distributions

for fitting a GAMLSS model

checklink Set the Right Link Function for Specified

Parameter and Distribution

count\_1\_31 A set of functions to plot gamlss.family

distributions

exGAUS The ex-Gaussian distribution

flexDist Non-parametric pdf from limited information

data

gamlss.dist-package Distributions for Generalized Additive Models

for Location Scale and Shape

gamlss.family Family Objects for fitting a GAMLSS model gen.Family Functions to generate log and logit

distributions from existing continuous

gamlss.family distributions

hazardFun Hazard functions for gamlss.family

distributions

momentSK Sample and theoretical Moment and Centile

Skewness and Kurtosis Functions

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Maintainer: Mikis Stasinopoulos <d.stasinopoulos@londonmet.ac.uk>

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/

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9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
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#### See Also

```
gamlss.family
```

# **Examples**

Beta Binomial Distribution For Fitting a GAMLSS Model

Description

This function defines the beta binomial distribution, a two parameter distribution, for a gamlss. family object to be used in a GAMLSS fitting using the function gamlss()

#### Usage

BB

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# Arguments

mu.link Defines the mu.link, with "logit" link as the default for the mu parameter. Other links are "probit" and "cloglog" (complementary log-log) sigma.link Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse", "identity" and "sqrt" vector of positive probabilities mu sigma the dispersion parameter vector of binomial denominators hd vector of probabilities vector of quantiles x,q number of random values to return log, log.p logical; if TRUE, probabilities p are given as log(p) logical; if TRUE (default), probabilities are  $P[X \le x]$ , otherwise, P[X > x]lower.tail fast a logical variable if fast=TRUE the dBB function is used in the calculation of the inverse c.d.f function. This is faster to the default fast=FALSE, where the pBB{} is used, but not always consistent with the results obtained from pBB(), for example if  $p \leftarrow pBB(c(0,1,2,3,4,5), mu=.5, sigma=1, bd=5)$  do not ensure that qBB(p, mu=.5, sigma=1, bd=5) will be c(0,1,2,3,4,5)

#### **Details**

Definition file for beta binomial distribution.

$$f(y|\mu,\sigma) = \frac{\Gamma(n+1)}{\Gamma(y+1)\Gamma(n-y+1)} \frac{\Gamma(\frac{1}{\sigma})\Gamma(y+\frac{\mu}{\sigma})\Gamma[n+\frac{(1-\mu)}{\sigma}-y]}{\Gamma(n+\frac{1}{\sigma})\Gamma(\frac{\mu}{\sigma})\Gamma(\frac{1-\mu}{\sigma})}$$

for  $y=0,1,2,\ldots,n,0<\mu<1$  and  $\sigma>0$ . For  $\mu=0.5$  and  $\sigma=0.5$  the distribution is uniform.

#### Value

Returns a gamlss.family object which can be used to fit a Beta Binomial distribution in the gamlss() function.

# Warning

The functions pBB and qBB are calculated using a laborious procedure so they are relatively slow.

#### Note

The response variable should be a matrix containing two columns, the first with the count of successes and the second with the count of failures. The parameter mu represents a probability parameter with limits  $0<\mu<1$ .  $n\mu$  is the mean of the distribution where n is the binomial denominator.  $\{n\mu(1-\mu)[1+(n-1)\sigma/(\sigma+1)]\}^{0.5}$  is the standard deviation of the Beta Binomial distribution. Hence  $\sigma$  is a dispersion type parameter

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#### Author(s)

Mikis Stasinopoulos, Bob Rigby and Kalliope Akantziliotou

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family, BI,
```

# **Examples**

```
# BB()# gives information about the default links for the Beta Binomial distribution
#plot the pdf
plot(function(y) dBB(y, mu = .5, sigma = 1, bd =40), from=0, to=40, n=40+1, type="h")
#calculate the cdf and plotting it
ppBB <- pBB(seq(from=0, to=40), mu=.2 , sigma=3, bd=40)
plot(0:40,ppBB, type="h")
#calculating quantiles and plotting them
qqBB \leftarrow qBB(ppBB, mu=.2, sigma=3, bd=40)
plot(qqBB~ ppBB)
# when the argument fast is useful
p \leftarrow pBB(c(0,1,2,3,4,5), mu=.01, sigma=1, bd=5)
qBB(p, mu=.01 , sigma=1, bd=5, fast=TRUE)
# 0 1 1 2 3 5
qBB(p, mu=.01 , sigma=1, bd=5, fast=FALSE)
# 0 1 2 3 4 5
# generate random sample
tN <- table(Ni <- rBB(1000, mu=.2, sigma=1, bd=20))
r <- barplot(tN, col='lightblue')</pre>
# fitting a model
# library(gamlss)
#data(aep)
# fits a Beta-Binomial model
#h<-gamlss(y~ward+loglos+year, sigma.formula=~year+ward, family=BB, data=aep)</pre>
```

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| BCCG | Box-Cox Cole and Green distribution (or Box-Cox normal) for fitting |
|------|---------------------------------------------------------------------|
|      | a GAMLSS                                                            |

# **Description**

The function BCCG defines the Box-Cox Cole and Green distribution (Box-Cox normal), a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dBCCG, pBCCG, qBCCG and rBCCG define the density, distribution function, quantile function and random generation for the specific parameterization of the Box-Cox Cole and Green distribution. [The function BCCGuntr() is the original version of the function suitable only for the untruncated Box-Cox Cole and Green distribution See Cole and Green (1992) and Rigby and Stasinopoulos (2003a,2003b) for details. The function BCCGo is identical to BCCG but with log link for mu.

# Usage

```
BCCG(mu.link = "identity", sigma.link = "log", nu.link = "identity")
BCCGo(mu.link = "log", sigma.link = "log", nu.link = "identity")
BCCGuntr(mu.link = "identity", sigma.link = "log", nu.link = "identity")
dBCCG(x, mu = 1, sigma = 0.1, nu = 1, log = FALSE)
pBCCG(q, mu = 1, sigma = 0.1, nu = 1, lower.tail = TRUE, log.p = FALSE)
qBCCG(p, mu = 1, sigma = 0.1, nu = 1, lower.tail = TRUE, log.p = FALSE)
rBCCG(x, mu = 1, sigma = 0.1, nu = 1)
dBCCGo(x, mu = 1, sigma = 0.1, nu = 1, log = FALSE)
pBCCGo(q, mu = 1, sigma = 0.1, nu = 1, lower.tail = TRUE, log.p = FALSE)
qBCCGo(p, mu = 1, sigma = 0.1, nu = 1, lower.tail = TRUE, log.p = FALSE)
rBCCGo(n, mu = 1, sigma = 0.1, nu = 1)
```

# **Arguments**

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter, other links are "inverse", "log" and "own"       |
|------------|---------------------------------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter, other links are "inverse", "identity" and "own" |
| nu.link    | Defines the nu.link, with "identity" link as the default for the nu parameter, other links are "inverse", "log" and "own"       |
| x,q        | vector of quantiles                                                                                                             |
| mu         | vector of location parameter values                                                                                             |
| sigma      | vector of scale parameter values                                                                                                |
| nu         | vector of skewness parameter values                                                                                             |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                                                          |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                              |
| р          | vector of probabilities.                                                                                                        |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required                                      |
|            |                                                                                                                                 |

#### **Details**

The probability distribution function of the untrucated Box-Cox Cole and Green distribution, BCCGuntr, is defined as

$$f(y|\mu,\sigma,\nu) = \frac{1}{\sqrt{2\pi}\sigma} \frac{y^{\nu-1}}{\mu^{\nu}} \exp(-\frac{z^2}{2})$$

where if  $\nu \neq 0$  then  $z = [(y/\mu)^{\nu} - 1]/(\nu\sigma)$  else  $z = \log(y/\mu)/\sigma$ , for y > 0,  $\mu > 0$ ,  $\sigma > 0$  and  $\nu = (-\infty, +\infty)$ .

The Box-Cox Cole and Green distribution, BCCG, adjusts the above density  $f(y|\mu, \sigma, \nu)$  for the truncation resulting from the condition y>0. See Rigby and Stasinopoulos (2003a,2003b) for details.

# Value

BCCG() returns a gamlss.family object which can be used to fit a Cole and Green distribution in the gamlss() function. dBCCG() gives the density, pBCCG() gives the distribution function, qBCCG() gives the quantile function, and rBCCG() generates random deviates.

#### Warning

The BCCGuntr distribution may be unsuitable for some combinations of the parameters (mainly for large  $\sigma$ ) where the integrating constant is less than 0.99. A warning will be given if this is the case. The BCCG distribution is suitable for all combinations of the distributional parameters within their range [i.e.  $\mu > 0$ ,  $\sigma > 0$ ,  $\nu = (-\infty, +\infty)$ ]

#### Note

 $\mu$  is the median of the distribution  $\sigma$  is approximately the coefficient of variation (for small values of  $\sigma$ ), and  $\nu$  controls the skewness.

The BCCG distribution is suitable for all combinations of the parameters within their ranges [i.e.  $\mu > 0, \sigma > 0$ , and  $\nu = (-\infty, \infty)$ ]

# Author(s)

Mikis Stasinopoulos, Bob Rigby and Kalliope Akantziliotou

# References

Cole, T. J. and Green, P. J. (1992) Smoothing reference centile curves: the LMS method and penalized likelihood, *Statist. Med.* **11**, 1305–1319

Rigby, R. A. and Stasinopoulos, D. M. (2004). Smooth centile curves for skew and kurtotic data modelled using the Box-Cox Power Exponential distribution. *Statistics in Medicine*, **23**: 3053-3076.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R.A. Stasinopoulos, D.M. (2006). Using the Box-Cox *t* distribution in GAMLSS to mode skewnees and and kurtosis. *Statistical Modelling*, 6(3):209. doi: 10.1191/1471082X06st122oa

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Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/9780429298547 An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family, BCPE, BCT
```

# **Examples**

```
BCCG() # gives information about the default links for the Cole and Green distribution
# library(gamlss)
#data(abdom)
#h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=BCCG, data=abdom)
#plot(h)
plot(function(x) dBCCG(x, mu=5,sigma=.5,nu=-1), 0.0, 20,
    main = "The BCCG density mu=5,sigma=.5,nu=-1")
plot(function(x) pBCCG(x, mu=5,sigma=.5,nu=-1), 0.0, 20,
    main = "The BCCG cdf mu=5, sigma=.5, nu=-1")</pre>
```

BCPE

Box-Cox Power Exponential distribution for fitting a GAMLSS

# Description

This function defines the Box-Cox Power Exponential distribution, a four parameter distribution, for a gamlss.family object to be used for a GAMLSS fitting using the function gamlss(). The functions dBCPE, pBCPE, qBCPE and rBCPE define the density, distribution function, quantile function and random generation for the Box-Cox Power Exponential distribution. The function checkBCPE can be used, typically when a BCPE model is fitted, to check whether there exit a turning point of the distribution close to zero. It give the number of values of the response below their minimum turning point and also the maximum probability of the lower tail below minimum turning point. [The function Biventer() is the original version of the function suitable only for the untruncated BCPE distribution.] See Rigby and Stasinopoulos (2003) for details. The function BCPEo is identical to BCPE but with log link for mu.

# Usage

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# **Arguments**

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter. Other links are "inverse", "log" and "own"       |
|------------|---------------------------------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse", "identity" and "own" |
| nu.link    | Defines the nu.link, with "identity" link as the default for the nu parameter. Other links are "inverse", "log" and "own"       |
| tau.link   | Defines the tau.link, with "log" link as the default for the tau parameter. Other links are "logshifted", "identity" and "own"  |
| x,q        | vector of quantiles                                                                                                             |
| mu         | vector of location parameter values                                                                                             |
| sigma      | vector of scale parameter values                                                                                                |
| nu         | vector of nu parameter values                                                                                                   |
| tau        | vector of tau parameter values                                                                                                  |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                                                          |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                              |
| р          | vector of probabilities.                                                                                                        |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required                                      |
| obj        | a gamlss BCPE family object                                                                                                     |
|            | for extra arguments                                                                                                             |

# **Details**

The probability density function of the untrucated Box Cox Power Exponential distribution, (BCPE.untr), is defined as

$$f(y|\mu,\sigma,\nu,\tau) = \frac{y^{\nu-1}\tau\exp\left[-\frac{1}{2}\left|\frac{z}{c}\right|^{\tau}\right]}{\mu^{\nu}\sigma c^{2(1+1/\tau)}\Gamma\left(\frac{1}{\tau}\right)}$$

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where 
$$c = [2^{(-2/\tau)}\Gamma(1/\tau)/\Gamma(3/\tau)]^{0.5}$$
, where if  $\nu \neq 0$  then  $z = [(y/\mu)^{\nu} - 1]/(\nu\sigma)$  else  $z = \log(y/\mu)/\sigma$ , for  $y > 0$ ,  $\mu > 0$ ,  $\sigma > 0$ ,  $\nu = (-\infty, +\infty)$  and  $\tau > 0$ .

The Box-Cox Power Exponential, BCPE, adjusts the above density  $f(y|\mu, \sigma, \nu, \tau)$  for the truncation resulting from the condition y > 0. See Rigby and Stasinopoulos (2003) for details.

#### Value

BCPE() returns a gamlss.family object which can be used to fit a Box Cox Power Exponential distribution in the gamlss() function. dBCPE() gives the density, pBCPE() gives the distribution function, qBCPE() gives the quantile function, and rBCPE() generates random deviates.

# Warning

The BCPE.untr distribution may be unsuitable for some combinations of the parameters (mainly for large  $\sigma$ ) where the integrating constant is less than 0.99. A warning will be given if this is the case.

The BCPE distribution is suitable for all combinations of the parameters within their ranges [i.e.  $\mu > 0, \sigma > 0, \nu = (-\infty, \infty)$  and  $\tau > 0$ ]

#### Note

 $\mu$ , is the median of the distribution,  $\sigma$  is approximately the coefficient of variation (for small  $\sigma$  and moderate nu>0),  $\nu$  controls the skewness and  $\tau$  the kurtosis of the distribution

#### Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A. and Stasinopoulos, D. M. (2004). Smooth centile curves for skew and kurtotic data modelled using the Box-Cox Power Exponential distribution. *Statistics in Medicine*, **23**: 3053-3076.

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

#### See Also

```
gamlss.family, BCT
```

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# **Examples**

```
# BCPE() #
# library(gamlss)
# data(abdom)
#h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=BCPE, data=abdom)
#plot(h)
plot(function(x)dBCPE(x, mu=5,sigma=.5,nu=1, tau=3), 0.0, 15,
main = "The BCPE density mu=5,sigma=.5,nu=1, tau=3")
plot(function(x) pBCPE(x, mu=5,sigma=.5,nu=1, tau=3"), 0.0, 15,
main = "The BCPE cdf mu=5, sigma=.5, nu=1, tau=3")</pre>
```

**BCT** 

Box-Cox t distribution for fitting a GAMLSS

# Description

The function BCT() defines the Box-Cox t distribution, a four parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dBCT, pBCT, qBCT and rBCT define the density, distribution function, quantile function and random generation for the Box-Cox t distribution. [The function BCTuntr() is the original version of the function suitable only for the untruncated BCT distribution]. See Rigby and Stasinopoulos (2003) for details. The function BCT is identical to BCT but with log link for mu.

#### Usage

# **Arguments**

mu.link Defines the mu.link, with "identity" link as the default for the mu parameter.

Other links are "inverse", "log" and "own"

sigma.link Defines the sigma.link, with "log" link as the default for the sigma parameter.

Other links are "inverse", "identity", "own"

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| nu.link    | Defines the nu.link, with "identity" link as the default for the nu parameter. Other links are "inverse", "log", "own"      |
|------------|-----------------------------------------------------------------------------------------------------------------------------|
| tau.link   | Defines the tau.link, with "log" link as the default for the tau parameter. Other links are "inverse", "identity" and "own" |
| x,q        | vector of quantiles                                                                                                         |
| mu         | vector of location parameter values                                                                                         |
| sigma      | vector of scale parameter values                                                                                            |
| nu         | vector of nu parameter values                                                                                               |
| tau        | vector of tau parameter values                                                                                              |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                                                      |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                          |
| р          | vector of probabilities.                                                                                                    |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required                                  |

#### **Details**

The probability density function of the untruncated Box-Cox t distribution, BCTuntr, is given by

$$f(y|\mu,\sigma,\nu,\tau) = \frac{y^{\nu-1}}{\mu^{\nu}\sigma} \frac{\Gamma[(\tau+1)/2]}{\Gamma(1/2)\Gamma(\tau/2)\tau^{0.5}} [1 + (1/\tau)z^2]^{-(\tau+1)/2}$$

where if  $\nu \neq 0$  then  $z = [(y/\mu)^{\nu} - 1]/(\nu\sigma)$  else  $z = \log(y/\mu)/\sigma$ , for y > 0,  $\mu > 0$ ,  $\sigma > 0$ ,  $\nu = (-\infty, +\infty)$  and  $\tau > 0$ .

The Box-Cox t distribution, BCT, adjusts the above density  $f(y|\mu, \sigma, \nu, \tau)$  for the truncation resulting from the condition y > 0. See Rigby and Stasinopoulos (2003) for details.

# Value

BCT() returns a gamlss.family object which can be used to fit a Box Cox-t distribution in the gamlss() function. dBCT() gives the density, pBCT() gives the distribution function, qBCT() gives the quantile function, and rBCT() generates random deviates.

# Warning

The use BCTuntr distribution may be unsuitable for some combinations of the parameters (mainly for large  $\sigma$ ) where the integrating constant is less than 0.99. A warning will be given if this is the case.

The BCT distribution is suitable for all combinations of the parameters within their ranges [i.e.  $\mu > 0, \sigma > 0, \nu = (-\infty, \infty)$  and  $\tau > 0$  ]

#### Note

 $\mu$  is the median of the distribution,  $\sigma(\frac{\tau}{\tau-2})^{0.5}$  is approximate the coefficient of variation (for small  $\sigma$  and moderate nu>0 and moderate or large  $\tau$ ),  $\nu$  controls the skewness and  $\tau$  the kurtosis of the distribution

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#### Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

#### See Also

```
gamlss.family, BCPE, BCCG
```

# **Examples**

```
BCT() # gives information about the default links for the Box Cox t distribution
# library(gamlss)
#data(abdom)
#h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=BCT, data=abdom) #
#plot(h)
plot(function(x)dBCT(x, mu=5,sigma=.5,nu=1, tau=2), 0.0, 20,
    main = "The BCT density mu=5,sigma=.5,nu=1, tau=2")
plot(function(x) pBCT(x, mu=5,sigma=.5,nu=1, tau=2), 0.0, 20,
    main = "The BCT cdf mu=5, sigma=.5, nu=1, tau=2")</pre>
```

The beta distribution for fitting a GAMLSS

# Description

The functions BE() and BEo() define the beta distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). BE() has mean equal to the parameter mu and sigma as scale parameter, see below. BEo() is the original parameterizations of the beta distribution as in dbeta() with shape1=mu and shape2=sigma. The functions dBE and dBEo, pBE and pBEo, qBE and qBEo and finally rBE and rBE define the density, distribution function, quantile function and random generation for the BE and BEo parameterizations respectively of the beta distribution.

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# Usage

```
BE(mu.link = "logit", sigma.link = "logit")
dBE(x, mu = 0.5, sigma = 0.2, log = FALSE)
pBE(q, mu = 0.5, sigma = 0.2, lower.tail = TRUE, log.p = FALSE)
qBE(p, mu = 0.5, sigma = 0.2, lower.tail = TRUE, log.p = FALSE)
rBE(n, mu = 0.5, sigma = 0.2)
BEo(mu.link = "log", sigma.link = "log")
dBEo(x, mu = 0.5, sigma = 0.2, log = FALSE)
pBEo(q, mu = 0.5, sigma = 0.2, lower.tail = TRUE, log.p = FALSE)
qBEo(p, mu = 0.5, sigma = 0.2, lower.tail = TRUE, log.p = FALSE)
```

# **Arguments**

| the mu link function with default logit                                                    |
|--------------------------------------------------------------------------------------------|
| the sigma link function with default logit                                                 |
| vector of quantiles                                                                        |
| vector of location parameter values                                                        |
| vector of scale parameter values                                                           |
| logical; if TRUE, probabilities p are given as log(p).                                     |
| logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| vector of probabilities.                                                                   |
| number of observations. If $length(n) > 1$ , the length is taken to be the number required |
|                                                                                            |

#### **Details**

The original beta distribution is given as

$$f(y|\alpha,\beta) = \frac{1}{B(\alpha,\beta)} y^{\alpha-1} (1-y)^{\beta-1}$$

for  $y=(0,1), \, \alpha>0$  and  $\beta>0$ . In the gamlss implementation of BEo  $\alpha=\mu$  and  $\beta>\sigma$ . The reparametrization in the function BE() is  $\mu=\frac{\alpha}{\alpha+\beta}$  and  $\sigma=(\frac{1}{\alpha+\beta+1)^{1/2}}$  for  $\mu=(0,1)$  and  $\sigma=(0,1)$ . The expected value of y is  $\mu$  and the variance is  $\sigma^2\mu*(1-\mu)$ .

# Value

BE() and BEo() return a gamlss.family object which can be used to fit a beta distribution in the gamlss() function.

#### Note

Note that for BE, mu is the mean and sigma a scale parameter contributing to the variance of y

# Author(s)

Bob Rigby and Mikis Stasinopoulos

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M., Rigby R.A. and Akantziliotou C. (2006) Instructions on how to use the GAMLSS package in R. Accompanying documentation in the current GAMLSS help files, (see also https://www.gamlss.com/).

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

# See Also

```
gamlss.family, BE, LOGITNO, GB1, BEINF
```

# **Examples**

```
BE()# gives information about the default links for the beta distribution
dat1<-rBE(100, mu=.3, sigma=.5)
hist(dat1)
#library(gamlss)
# mod1<-gamlss(dat1~1,family=BE) # fits a constant for mu and sigma
#fitted(mod1)[1]
#fitted(mod1,"sigma")[1]
plot(function(y) dBE(y, mu=.1 ,sigma=.5), 0.001, .999)
plot(function(y) pBE(y, mu=.1 ,sigma=.5), 0.001, 0.999)
plot(function(y) qBE(y, mu=.1 ,sigma=.5), 0.001, 0.999)
plot(function(y) qBE(y, mu=.1 ,sigma=.5), 0.001, 0.999)
plot(function(y) qBE(y, mu=.1 ,sigma=.5, lower.tail=FALSE), 0.001, .999)
dat2<-rBEo(100, mu=1, sigma=2)
#mod2<-gamlss(dat2~1,family=BEo) # fits a constant for mu and sigma
#fitted(mod2)[1]
#fitted(mod2,"sigma")[1]</pre>
```

**BEINF** 

The beta inflated distribution for fitting a GAMLSS

#### **Description**

The function BEINF() defines the beta inflated distribution, a four parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The beta inflated is similar to the beta but allows zeros and ones as values for the response variable. The two extra parameters model the probabilities at zero and one.

The functions BEINF0() and BEINF1() are three parameter beta inflated distributions allowing zeros or ones only at the response respectively. BEINF0() and BEINF1() are re-parameterize versions of the distributions BEZI and BEOI contributed to gamlss by Raydonal Ospina (see Ospina and Ferrari (2010)).

The functions dBEINF, pBEINF, qBEINF and rBEINF define the density, distribution function, quantile function and random generation for the BEINF parametrization of the beta inflated distribution.

The functions dBEINF0, pBEINF0, qBEINF0 and rBEINF0 define the density, distribution function, quantile function and random generation for the BEINF0 parametrization of the beta inflated at zero distribution.

The functions dBEINF1, pBEINF1, qBEINF1 and rBEINF1 define the density, distribution function, quantile function and random generation for the BEINF1 parametrization of the beta inflated at one distribution.

plotBEINF, plotBEINF0 and plotBEINF1 can be used to plot the distributions. meanBEINF0 and meanBEINF1 calculates the expected value of the response for a fitted model.

#### Usage

```
BEINF(mu.link = "logit", sigma.link = "logit", nu.link = "log",
      tau.link = "log")
BEINF0(mu.link = "logit", sigma.link = "logit", nu.link = "log")
BEINF1(mu.link = "logit", sigma.link = "logit", nu.link = "log")
dBEINF(x, mu = 0.5, sigma = 0.1, nu = 0.1, tau = 0.1,
       log = FALSE)
dBEINF0(x, mu = 0.5, sigma = 0.1, nu = 0.1, log = FALSE)
dBEINF1(x, mu = 0.5, sigma = 0.1, nu = 0.1, log = FALSE)
pBEINF(q, mu = 0.5, sigma = 0.1, nu = 0.1, tau = 0.1,
       lower.tail = TRUE, log.p = FALSE)
pBEINF0(q, mu = 0.5, sigma = 0.1, nu = 0.1,
       lower.tail = TRUE, log.p = FALSE)
pBEINF1(q, mu = 0.5, sigma = 0.1, nu = 0.1,
       lower.tail = TRUE, log.p = FALSE)
qBEINF(p, mu = 0.5, sigma = 0.1, nu = 0.1, tau = 0.1,
       lower.tail = TRUE, log.p = FALSE)
qBEINF0(p, mu = 0.5, sigma = 0.1, nu = 0.1, tau = 0.1,
        lower.tail = TRUE, log.p = FALSE)
qBEINF1(p, mu = 0.5, sigma = 0.1, nu = 0.1,
       lower.tail = TRUE, log.p = FALSE)
rBEINF(n, mu = 0.5, sigma = 0.1, nu = 0.1, tau = 0.1)
rBEINF0(n, mu = 0.5, sigma = 0.1, nu = 0.1)
rBEINF1(n, mu = 0.5, sigma = 0.1, nu = 0.1)
plotBEINF(mu = 0.5, sigma = 0.5, nu = 0.5, tau = 0.5,
          from = 0.001, to = 0.999, n = 101, ...)
```

# **Arguments**

| mu.link    | the mu link function with default logit                                                    |
|------------|--------------------------------------------------------------------------------------------|
| sigma.link | the sigma link function with default logit                                                 |
| nu.link    | the nu link function with default log                                                      |
| tau.link   | the tau link function with default log                                                     |
| x,q        | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| nu         | vector of parameter values modelling the probability at zero                               |
| tau        | vector of parameter values modelling the probability at one                                |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| р          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |
| from       | where to start plotting the distribution from                                              |
| to         | up to where to plot the distribution                                                       |
| obj        | a fitted BEINF object                                                                      |
|            | other graphical parameters for plotting                                                    |

# **Details**

The beta inflated distribution is given as

$$f(y)=p_0$$
 if (y=0) 
$$f(y)=p_1$$
 if (y=1) 
$$f(y|\alpha,\beta)=\frac{1}{B(\alpha,\beta)}y^{\alpha-1}(1-y)^{\beta-1}$$

otherwise

for 
$$y=(0,1),\, \alpha>0$$
 and  $\beta>0$ . The parametrization in the function BEINF() is  $\mu=\frac{\alpha}{\alpha+\beta}$  and  $\sigma=\frac{1}{\alpha+\beta+1}$  for  $\mu=(0,1)$  and  $\sigma=(0,1)$  and  $\nu=\frac{p_0}{p_2},\, \tau=\frac{p_1}{p_2}$  where  $p_2=1-p_0-p_1$ .

#### Value

returns a gamlss.family object which can be used to fit a beta inflated distribution in the gamlss() function. ...

#### Author(s)

Bob Rigby and Mikis Stasinopoulos

#### References

Ospina R. and Ferrari S. L. P. (2010) Inflated beta distributions, Statistical Papers, 23, 111-126.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC,doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

#### See Also

```
gamlss.family, BE, BEo, BEZI, BEOI
```

# **Examples**

```
BEINF()# gives information about the default links for the beta inflated distribution
BEINF0()
BEINF1()
# plotting the distributions
op<-par(mfrow=c(2,2))
plotBEINF( mu = .5 , sigma= .5, nu = 0.5, tau = 0.5, from = 0, to=1, n = 101)
plotBEINF0( mu = .5 , sigma = .5, nu = 0.5, from = 0, to = 1, n = 101)
plotBEINF1( mu = .5 , sigma=.5, nu = 0.5, from = 0.001, to=1, n = 101)
curve(dBE(x, mu = .5, sigma = .5), 0.01, 0.999)
par(op)
# plotting the cdf
op<-par(mfrow=c(2,2))
plotBEINF( mu =.5 , sigma=.5, nu = 0.5, tau = 0.5, from = 0, to=1, n = 101, main="BEINF")
plotBEINF0( mu = .5 , sigma=.5, nu = 0.5, from = 0, to=1, n = 101, main="BEINF0")
plotBEINF1( mu = .5 , sigma= .5, nu = 0.5, from = 0.001, to=1, n = 101, main="BEINF1")
curve(dBE(x, mu =.5, sigma=.5), 0.01, 0.999, main="BE")
par(op)
op<-par(mfrow=c(2,2))
plotBEINF( mu =.5 , sigma=.5, nu = 0.5, tau = 0.5, from = 0, to=1, n = 101, main="BEINF")
```

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```
plotBEINF0( mu = .5 , sigma= .5, nu = 0.5, from = 0, to=1, n = 101, main="BEINF0")
plotBEINF1( mu = .5 , sigma= .5, nu = 0.5, from = 0.001, to=1, n = 101, main="BEINF1")
curve(dBE(x, mu =.5, sigma=.5), 0.01, 0.999, main="BE")
par(op)
op<-par(mfrow=c(2,2))
curve( pBEINF(x, mu=.5 ,sigma=.5, nu = 0.5, tau = 0.5,), 0, 1, ylim=c(0,1), main="BEINF" )
curve(pBEINF0(x, mu=.5, sigma=.5, nu = 0.5), 0, 1, ylim=c(0,1), main="BEINF0")
curve(pBEINF1(x, mu=.5, sigma=.5, nu = 0.5), 0, 1, ylim=c(0,1), main="BEINF1")
         pBE(x, mu=.5, sigma=.5), .001, .99, ylim=c(0,1), main="BE")
curve(
par(op)
op<-par(mfrow=c(2,2))
curve(qBEINF(x, mu=.5 ,sigma=.5, nu = 0.5, tau = 0.5), .01, .99, main="BEINF")
curve(qBEINF0(x, mu=.5, sigma=.5, nu = 0.5), .01, .99, main="BEINF0")
curve(qBEINF1(x, mu=.5 ,sigma=.5, nu = 0.5), .01, .99, main="BEINF1" )
curve(qBE(x, mu=.5 ,sigma=.5), .01, .99 , main="BE")
par(op)
#-----
op<-par(mfrow=c(2,2))
hist(rBEINF(200, mu=.5, sigma=.5, nu = 0.5, tau = 0.5))
hist(rBEINF0(200, mu=.5, sigma=.5, nu = 0.5))
hist(rBEINF1(200, mu=.5, sigma=.5, nu = 0.5))
hist(rBE(200, mu=.5 ,sigma=.5))
par(op)
# fit a model to the data
# library(gamlss)
#m1<-gamlss(dat~1,family=BEINF)</pre>
#meanBEINF(m1)[1]
```

BEOI

The one-inflated beta distribution for fitting a GAMLSS

# **Description**

The function BEOI() defines the one-inflated beta distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The one-inflated beta is similar to the beta distribution but allows ones as y values. This distribution is an extension of the beta distribution using a parameterization of the beta law that is indexed by mean and precision parameters (Ferrari and Cribari-Neto, 2004). The extra parameter models the probability at one. The functions dBEOI, pBEOI, qBEOI and rBEOI define the density, distribution function, quantile function and random generation for the BEOI parameterization of the one-inflated beta distribution. plotBEOI can be used to plot the distribution. meanBEOI calculates the expected value of the response for a fitted model.

# Usage

```
BEOI(mu.link = "logit", sigma.link = "log", nu.link = "logit")
```

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# **Arguments**

| mu.link    | the mu link function with default logit                                                    |
|------------|--------------------------------------------------------------------------------------------|
| sigma.link | the sigma link function with default log                                                   |
| nu.link    | the nu link function with default logit                                                    |
| x,q        | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of precision parameter values                                                       |
| nu         | vector of parameter values modelling the probability at one                                |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| p          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |
| from       | where to start plotting the distribution from                                              |
| to         | up to where to plot the distribution                                                       |
| obj        | a fitted BEOI object                                                                       |
|            | other graphical parameters for plotting                                                    |

# **Details**

The one-inflated beta distribution is given as

$$f(y)=\nu$$
 if  $(y=1)$  
$$f(y|\mu,\sigma)=(1-\nu)\frac{\Gamma(\sigma)}{\Gamma(\mu\sigma)\Gamma((1-\mu)\sigma)}y^{\mu\sigma}(1-y)^{((1-\mu)\sigma)-1}$$
 if  $y=(0,1).$  The parameters satisfy  $0<\mu<0$ ,  $\sigma>0$  and  $0<\nu<1$ . Here  $E(y)=\nu+(1-\nu)\mu$  and  $Var(y)=(1-\nu)\frac{\mu(1-\mu)}{\sigma+1}+\nu(1-\nu)(1-\mu)^2$ .

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#### Value

returns a gamlss.family object which can be used to fit a one-inflated beta distribution in the gamlss() function.

#### Note

This work is part of my PhD project at the University of Sao Paulo under the supervion of Professor Silvia Ferrari. My thesis is concerned with regression modelling of rates and proportions with excess of zeros and/or ones

# Author(s)

Raydonal Ospina, Department of Statistics, University of Sao Paulo, Brazil. <rusp:</pre>

# References

Ferrari, S.L.P., Cribari-Neto, F. (2004). Beta regression for modelling rates and proportions. *Journal of Applied Statistics*, **31** (1), 799-815.

Ospina R. and Ferrari S. L. P. (2010) Inflated beta distributions, Statistical Papers, 23, 111-126.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape (with discussion). *Applied Statistics*, **54** (3), 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

# See Also

```
gamlss.family, BEOI
```

# **Examples**

```
BEOI()# gives information about the default links for the BEOI distribution # plotting the distribution plotBEOI( mu =0.5 , sigma=5, nu = 0.1, from = 0.001, to=1, n = 101) # plotting the cdf plot(function(y) pBEOI(y, mu=.5 ,sigma=5, nu=0.1), 0.001, 0.999) # plotting the inverse cdf plot(function(y) qBEOI(y, mu=.5 ,sigma=5, nu=0.1), 0.001, 0.999) # generate random numbers dat<-rBEOI(100, mu=.5, sigma=5, nu=0.1)
```

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```
# fit a model to the data.
# library(gamlss)
#mod1<-gamlss(dat~1,sigma.formula=~1, nu.formula=~1, family=BEOI)
#fitted(mod1)[1]
#summary(mod1)
#fitted(mod1,"mu")[1] #fitted mu
#fitted(mod1,"sigma")[1] #fitted sigma
#fitted(mod1,"nu")[1] #fitted nu
#meanBEOI(mod1)[1] # expected value of the response</pre>
```

**BEZI** 

The zero-inflated beta distribution for fitting a GAMLSS

# Description

The function BEZI() defines the zero-inflated beta distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The zero-inflated beta is similar to the beta distribution but allows zeros as y values. This distribution is an extension of the beta distribution using a parameterization of the beta law that is indexed by mean and precision parameters (Ferrari and Cribari-Neto, 2004). The extra parameter models the probability at zero. The functions dBEZI, pBEZI, qBEZI and rBEZI define the density, distribution function, quantile function and random generation for the BEZI parameterization of the zero-inflated beta distribution. plotBEZI can be used to plot the distribution. meanBEZI calculates the expected value of the response for a fitted model.

# Usage

```
BEZI(mu.link = "logit", sigma.link = "log", nu.link = "logit")

dBEZI(x, mu = 0.5, sigma = 1, nu = 0.1, log = FALSE)

pBEZI(q, mu = 0.5, sigma = 1, nu = 0.1, lower.tail = TRUE, log.p = FALSE)

qBEZI(p, mu = 0.5, sigma = 1, nu = 0.1, lower.tail = TRUE, log.p = FALSE)

rBEZI(n, mu = 0.5, sigma = 1, nu = 0.1)

plotBEZI(mu = .5, sigma = 1, nu = 0.1, from = 0, to = 0.999, n = 101, ...)

meanBEZI(obj)
```

#### **Arguments**

```
mu.link the mu link function with default logit
sigma.link the sigma link function with default log
```

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| nu.link    | the nu link function with default logit                                                    |
|------------|--------------------------------------------------------------------------------------------|
| x,q        | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of precision parameter values                                                       |
| nu         | vector of parameter values modelling the probability at zero                               |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| р          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |
| from       | where to start plotting the distribution from                                              |
| to         | up to where to plot the distribution                                                       |
| obj        | a fitted BEZI object                                                                       |
|            | other graphical parameters for plotting                                                    |

#### **Details**

The zero-inflated beta distribution is given as

$$f(y)=\nu$$
 if  $(y=0)$  
$$f(y|\mu,\sigma)=(1-\nu)\frac{\Gamma(\sigma)}{\Gamma(\mu\sigma)\Gamma((1-\mu)\sigma)}y^{\mu\sigma}(1-y)^{((1-\mu)\sigma)-1}$$
 if  $y=(0,1)$ . The parameters satisfy  $0<\mu<0,\,\sigma>0$  and  $0<\nu<1$ . Here  $E(y)=(1-\nu)\mu$  and  $Var(y)=(1-\nu)\frac{\mu(1-\mu)}{\sigma+1}+\nu(1-\nu)\mu^2$ .

# Value

returns a gamlss.family object which can be used to fit a zero-inflated beta distribution in the gamlss() function.

#### Note

This work is part of my PhD project at the University of Sao Paulo under the supervion of Professor Silvia Ferrari. My thesis is concerned with regression modelling of rates and proportions with excess of zeros and/or ones

# Author(s)

Raydonal Ospina, Department of Statistics, University of Sao Paulo, Brazil. <rusp:\text{rospina@ime.usp.br>}

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#### References

Ferrari, S.L.P., Cribari-Neto, F. (2004). Beta regression for modelling rates and proportions. *Journal of Applied Statistics*, **31** (1), 799-815.

Ospina R. and Ferrari S. L. P. (2010) Inflated beta distributions, Statistical Papers, 23, 111-126.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape (with discussion). *Applied Statistics*, **54** (3), 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family, BEZI
```

# **Examples**

```
BEZI()# gives information about the default links for the BEZI distribution
# plotting the distribution
plotBEZI( mu =0.5 , sigma=5, nu = 0.1, from = 0, to=0.99, n = 101)
# plotting the cdf
plot(function(y) pBEZI(y, mu=.5, sigma=5, nu=0.1), 0, 0.999)
# plotting the inverse cdf
plot(function(y) qBEZI(y, mu=.5 ,sigma=5, nu=0.1), 0, 0.999)
# generate random numbers
dat<-rBEZI(100, mu=.5, sigma=5, nu=0.1)
# fit a model to the data. Tits a constant for mu, sigma and nu
# library(gamlss)
#mod1<-gamlss(dat~1,sigma.formula=~1, nu.formula=~1, family=BEZI)</pre>
#fitted(mod1)[1]
#summary(mod1)
#fitted(mod1,"mu")[1]
                              #fitted mu
#fitted(mod1, "sigma")[1]
                              #fitted sigma
#fitted(mod1,"nu")[1]
                              #fitted nu
#meanBEZI(mod1)[1] # expected value of the response
```

30 BI

Description

ΒI

# The BI() function defines the binomial distribution, a one parameter family distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dBI, pBI, qBI and rBI define the density, distribution function, quantile function and random generation for the binomial, BI(), distribution.

# Usage

```
BI(mu.link = "logit")
dBI(x, bd = 1, mu = 0.5, log = FALSE)
pBI(q, bd = 1, mu = 0.5, lower.tail = TRUE, log.p = FALSE)
qBI(p, bd = 1, mu = 0.5, lower.tail = TRUE, log.p = FALSE)
rBI(n, bd = 1, mu = 0.5)
```

# **Arguments**

| mu.link    | Defines the mu.link, with "logit" link as the default for the mu parameter. Other links are "probit" and "cloglog" (complementary log-log) |
|------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| X          | vector of (non-negative integer) quantiles                                                                                                 |
| mu         | vector of positive probabilities                                                                                                           |
| bd         | vector of binomial denominators                                                                                                            |
| р          | vector of probabilities                                                                                                                    |
| q          | vector of quantiles                                                                                                                        |
| n          | number of random values to return                                                                                                          |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                                                                      |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                                         |

# **Details**

Definition file for binomial distribution.

$$f(y|\mu) = \frac{\Gamma(n+1)}{\Gamma(y+1)\Gamma(n-y+1)} \mu^y (1-\mu)^{(n-y)}$$

for y = 0, 1, 2, ..., n and  $0 < \mu < 1$ .

# Value

returns a gamlss.family object which can be used to fit a binomial distribution in the gamlss() function.

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#### Note

The response variable should be a matrix containing two columns, the first with the count of successes and the second with the count of failures. The parameter mu represents a probability parameter with limits  $0 < \mu < 1$ .  $n\mu$  is the mean of the distribution where n is the binomial denominator.

#### Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

# See Also

```
gamlss.family, ZABI, ZIBI
```

# **Examples**

```
BI()# gives information about the default links for the Binomial distribution # data(aep)
# library(gamlss)
# h<-gamlss(y~ward+loglos+year, family=BI, data=aep)
# plot of the binomial distribution
curve(dBI(x, mu = .5, bd=10), from=0, to=10, n=10+1, type="h")
tN <- table(Ni <- rBI(1000, mu=.2, bd=10))
r <- barplot(tN, col='lightblue')
```

32 BNB

#### **Description**

The BNB() function defines the beta negative binomial distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss().

The functions dBNB, pBNB, qBNB and rBNB define the density, distribution function, quantile function and random generation for the beta negative binomial distribution, BNB().

The functions ZABNB() and ZIBNB() are the zero adjusted (hurdle) and zero inflated versions of the beta negative binomial distribution, respectively. That is four parameter distributions.

The functions dZABNB, dZIBNB, pZABNB,pZIBNB, qZABNB qZIBNB rZABNB and rZIBNB define the probability, cumulative, quantile and random generation functions for the zero adjusted and zero inflated beta negative binomial distributions, ZABNB(), ZIBNB(), respectively.

# Usage

```
BNB(mu.link = "log", sigma.link = "log", nu.link = "log")
dBNB(x, mu = 1, sigma = 1, nu = 1, log = FALSE)
pBNB(q, mu = 1, sigma = 1, nu = 1, lower.tail = TRUE, log.p = FALSE)
qBNB(p, mu = 1, sigma = 1, nu = 1, lower.tail = TRUE, log.p = FALSE,
    max.value = 10000)
rBNB(n, mu = 1, sigma = 1, nu = 1, max.value = 10000)
ZABNB(mu.link = "log", sigma.link = "log", nu.link = "log",
      tau.link = "logit")
dZABNB(x, mu = 1, sigma = 1, nu = 1, tau = 0.1, log = FALSE)
pZABNB(q, mu = 1, sigma = 1, nu = 1, tau = 0.1, lower.tail = TRUE,
       log.p = FALSE)
qZABNB(p, mu = 1, sigma = 1, nu = 1, tau = 0.1, lower.tail = TRUE,
       log.p = FALSE, max.value = 10000)
rZABNB(n, mu = 1, sigma = 1, nu = 1, tau = 0.1, max.value = 10000)
ZIBNB(mu.link = "log", sigma.link = "log", nu.link = "log",
      tau.link = "logit")
dZIBNB(x, mu = 1, sigma = 1, nu = 1, tau = 0.1, log = FALSE)
pZIBNB(q, mu = 1, sigma = 1, nu = 1, tau = 0.1, lower.tail = TRUE,
       log.p = FALSE)
qZIBNB(p, mu = 1, sigma = 1, nu = 1, tau = 0.1, lower.tail = TRUE,
       log.p = FALSE, max.value = 10000)
rZIBNB(n, mu = 1, sigma = 1, nu = 1, tau = 0.1, max.value = 10000)
```

# **Arguments**

```
mu.link The link function for mu
sigma.link The link function for sigma
nu.link The link function for nu
tau.link The link function for tau
x vector of (non-negative integer)
mu vector of positive means
```

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sigma vector of positive dispersion parameter vector of a positive parameter nu vector of probabilities tau logical; if TRUE, probabilities p are given as log(p) log, log.p lower.tail logical; if TRUE (default), probabilities are  $P[X \le x]$ , otherwise, P[X > x]vector of probabilities vector of quantiles q n number of random values to return max.value a constant, set to the default value of 10000 for how far the algorithm should look for q

#### **Details**

The probability function of the BNB is

$$P(Y=y|\mu,\sigma,\nu) = \frac{\Gamma(y+\nu^{-1}) \ B(y+\mu\sigma^{-1}\nu,\sigma^{-1}+\nu^{-1}+1)}{\Gamma(y+1) \ \Gamma(\nu^{-1}) \ B(\mu\sigma^{-1}\nu,\sigma^{-1}+1)}$$

for  $y = 0, 1, 2, 3, ..., \mu > 0, \sigma > 0$  and  $\nu > 0$ .

The distribution has mean  $\mu$ .

#### Value

returns a gamlss.family object which can be used to fit a Poisson distribution in the gamlss() function.

# Author(s)

Bob Rigby and Mikis Stasinopoulos <d.stasinopoulos@londonmrt.ac.uk>

# References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M., Rigby R.A. and Akantziliotou C. (2006) Instructions on how to use the GAMLSS package in R. Accompanying documentation in the current GAMLSS help files, (see also https://www.gamlss.com/).

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

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# See Also

```
NBI, NBII
```

#### **Examples**

```
BNB()
        # gives information about the default links for the beta negative binomial
# plotting the distribution
plot(function(y) dBNB(y, mu = 10, sigma = 0.5, nu=2), from=0, to=40, n=40+1, type="h")
# creating random variables and plot them
tN <- table(Ni <- rBNB(1000, mu=5, sigma=0.5, nu=2))
r <- barplot(tN, col='lightblue')</pre>
ZABNB()
ZIBNB()
# plotting the distribution
plot(function(y) dZABNB(y, mu = 10, sigma = 0.5, nu=2, tau=.1),
     from=0, to=40, n=40+1, type="h")
plot(function(y) dZIBNB(y, mu = 10, sigma = 0.5, nu=2, tau=.1),
     from=0, to=40, n=40+1, type="h")
## Not run:
library(gamlss)
data(species)
species <- transform(species, x=log(lake))</pre>
m6 <- gamlss(fish~ pb(x), sigma.fo=~1, data=species, family=BNB)</pre>
## End(Not run)
```

checklink

Set the Right Link Function for Specified Parameter and Distribution

# Description

This function is used within the distribution family specification of a GAMLSS model to define the right link for each of the parameters of the distribution. This function should not be called by the user unless he/she specify a new distribution family or wishes to change existing link functions in the parameters.

#### **Usage**

```
checklink(which.link = NULL, which.dist = NULL, link = NULL, link.List = NULL)
```

# **Arguments**

```
which.link which parameter link e.g. which.link="mu.link"
which.dist which distribution family e.g. which.dist="Cole.Green"
link a repetition of which.link e.g. link=substitute(mu.link)
link.List what link function are required e.g. link.List=c("inverse", "log", "identity")
```

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#### Value

Defines the right link for each parameter

#### Author(s)

Calliope Akantziliotou

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC,doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

#### See Also

```
gamlss.family
```

count\_1\_31

A set of functions to plot gamlss.family distributions

# **Description**

Those functions are used in the distribution book of gamlss, see Rigby et. al 2019.

# Usage

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```
legend.where=c("left","right", "center"))
contR_2_12(family = "NO", mu = c(0, -1, 1), sigma = c(1, 0.5, 2),
          cols=c(gray(.1),gray(.2),gray(.3)),
          ltype = c(1, 2, 3), maxy = 7,
          no.points = 201, y.axis.lim = 1.1,
          cex.axis = 1.5, cex.all = 1.5,
          legend.cex=1, legend.x="topleft" )
contR_3_11(family = "PE", mu = 0, sigma = 1, nu = c(1, 2, 3),
          cols=c(gray(.1),gray(.2),gray(.3)), maxy = 7, no.points = 201,
          ltype = c(1, 2, 3), y.axis.lim = 1.1, cex.axis = 1.5,
          cex.all = 1.5, legend.cex=1, legend.x="topleft")
contR_4_13(family = "SEP3", mu = 0, sigma = 1, nu = c(0.5, 1, 2),
         tau = c(1, 2, 5), cols=c(gray(.1),gray(.2),gray(.3)), maxy = 7,
         no.points = 201, ltype = c(1, 2, 3),
         y.axis.lim = 1.1, cex.axis = 1.5, cex.all = 1.5,
         legend.cex=1, legend.x="topleft",
         legend.where=c("left","right"))
contRplus_2_11(family = GA, mu = 1, sigma = c(0.1, 0.6, 1),
          cols=c(gray(.1),gray(.2),gray(.3)),
          maxy = 4, no.points = 201,
          y.axis.lim = 1.1, ltype = c(1, 2, 3),
          cex.axis = 1.5, cex.all = 1.5,
          legend.cex=1, legend.x="topright")
contRplus_3_13(family = "BCCG", mu = 1, sigma = c(0.15, 0.2, 0.5),
         nu = c(-2, 0, 4),
          cols=c(gray(.1),gray(.2),gray(.3)),
          maxy = 4, ltype = c(1, 2, 3),
          no.points = 201, y.axis.lim = 1.1,
          cex.axis = 1.5, cex.all = 1.5,
          legend.cex=1, legend.x="topright",
          legend.where=c("left","right"))
contRplus_4_33(family = BCT, mu = 1, sigma = c(0.15, 0.2, 0.5),
          nu = c(-4, 0, 2), tau = c(100, 5, 1),
          cols=c(gray(.1),gray(.2),gray(.3)),
          maxy = 4, ltype = c(1, 2, 3),
          no.points = 201, y.axis.lim = 1.1,
          cex.axis = 1.5, cex.all = 1.5,
          legend.cex=1, legend.x="topright",
          legend.where=c("left","right"))
contR01_2_13(family = "BE", mu = c(0.2, 0.5, 0.8), sigma = c(0.2, 0.5, 0.8),
            cols=c(gray(.1),gray(.2),gray(.3)),
```

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```
ltype = c(1, 2, 3), maxy = 7, no.points = 201,
            y.axis.lim = 1.1, maxYlim = 10,
            cex.axis = 1.5, cex.all = 1.5,
            legend.cex=1, legend.x="topright",
            legend.where=c("left","right", "center"))
contR01_4_33(family = GB1, mu = c(0.5), sigma = c(0.2, 0.5, 0.7),
            nu = c(1, 2, 5), tau = c(0.5, 1, 2),
             cols=c(gray(.1),gray(.2),gray(.3)),
             maxy = 0.999, ltype = c(1, 2, 3),
             no.points = 201, y.axis.lim = 1.1,
             maxYlim = 10, cex.axis = 1.5, cex.all = 1.5,
              legend.cex=1, legend.x="topright",
              legend.where=c("left","right", "center"))
count_1_31(family = P0, mu = c(1, 2, 5), miny = 0, maxy = 10,
           cex.axis = 1.2, cex.all = 1.5)
count_1_2(family = P0, mu = c(1, 2, 5, 10), miny = 0,
           maxy = 20, cex.axis = 1.2, cex.all = 1.5)
count_2_32(family = NBI, mu = c(0.5, 1, 5), sigma = c(0.1, 2),
           miny = 0, maxy = 10, cex.axis = 1.5, cex.all = 1.5)
count_2_32R(family = NBI, mu = c(1, 2), sigma = c(0.1, 1, 2),
           miny = 0, maxy = 10, cex.axis = 1.5, cex.all = 1.5)
count_2_33(family = NBI, mu = c(0.1, 1, 2), sigma = c(0.5, 1, 2),
          miny = 0, maxy = 10, cex.axis = 1.5, cex.all = 1.5)
count_3_32(family = SICHEL, mu = c(1, 5, 10), sigma = c(0.5, 1),
           nu = c(-0.5, 0.5), miny = 0, maxy = 10, cex.axis = 1.5,
           cex.all = 1.5, cols = c("darkgray", "black"), spacing = 0.2,
           legend.cex=1, legend.x="topright",
                  legend.where=c("left","right"))
count_3_3(family = SICHEL, mu = c(1, 5, 10), sigma = c(0.5, 1, 2),
           nu = c(-0.5, 0.5, 1), miny = 0, maxy = 10, cex.axis = 1.5,
           cex.all = 1.5, cols = c("darkgray", "black"), spacing = 0.3,
           legend.cex=1, legend.x="topright",
           legend.where=c("left","right", "center"))
```

# Arguments

family a gamlss family distribution
mu the mu parameter values
sigma The sigma parameter values

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nu the nu parameter values the tau parameter values tau bd the binomial denominator miny minimal value for the y axis maxy maximal value for the y axis

cex.axis the size of the letters in the two axes cex.all the overall size of all plotting characters

cols colours

spacing spacing between plots The type of lines used ltype

the number of points in the curve no.points the maximum value for the y axis y.axis.lim the maximum permissible value for Y

legend.cex the size of the legend

legend.x where in the figure to put the legend

legend.where where in the whole plot to put the legend

#### **Details**

maxYlim

Th function plot different types of continuous and discrete distributions: i) contR: continuous distribution defined on minus infinity to plus infinity, ii) contRplus: continuous distribution defined from zero to plus infinity, iii) contR01: continuous distribution defined from zero to 1, iv) bimom binomial type discrete distributions, v) count count type discrete distributions.

The first number after the first underline in the name of the function indicates the number of parameters in the distribution. The two numbers after the second underline indicate how may rows and columns are in the plot.

## Value

The result is a plot

#### Note

more notes

# Author(s)

Mikis Stasinopoulos, Robert Rigby, Gillian Heller, Fernada De Bastiani

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#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family
```

#### **Examples**

```
count_1_31()
```

DBI

The Double binomial distribution

# Description

The function DBI() defines the double binomial distribution, a two parameters distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dDBI, pDBI, qDBI and rDBI define the density, distribution function, quantile function and random generation for the double binomial, DBI(), distribution. The function GetBI\_C calculates numerically the constant of proportionality needed for the pdf to sum up to 1.

# Usage

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# **Arguments**

| mu.link    | the link function for mu with default log                                          |
|------------|------------------------------------------------------------------------------------|
| sigma.link | the link function for sigma with default log                                       |
| x,q        | vector of (non-negative integer) quantiles                                         |
| bd         | vector of binomial denominator                                                     |
| p          | vector of probabilities                                                            |
| mu         | the mu parameter                                                                   |
| sigma      | the sigma parameter                                                                |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$ |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                              |
| n          | how many random values to generate                                                 |
|            |                                                                                    |

#### **Details**

The definition for the Double Poisson distribution first introduced by Efron (1986) is:

$$p_{Y}(y|n,\mu,\sigma) = frac1C(n,\mu,\sigma) \frac{\Gamma(n+1)}{\Gamma(y+1)\Gamma(n-y+1)} \frac{y^{y} (n-y)^{n-y}}{n^{n}} \frac{n^{n/\sigma} \mu^{y/\sigma} (1-\mu)^{(n-y)/\sigma}}{y^{y/\sigma} (n-y)^{(n-y)/\sigma}}$$

for  $y=0,1,2,\ldots,\infty,\,\mu>0$  and  $\sigma>0$  where C is the constant of proportinality which is calculated numerically using the function GetBI\_C().

## Value

The function DBI returns a gamlss.family object which can be used to fit a double binomial distribution in the gamlss() function.

#### Author(s)

Mikis Stasinopoulos, Bob Rigby, Marco Enea and Fernanda de Bastiani

## References

Efron, B., 1986. Double exponential families and their use in generalized linear Regression. Journal of the American Statistical Association 81 (395), 709-721.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

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# See Also

BI.BB

# **Examples**

```
DBI()
x <- 0:20
# underdispersed DBI
plot(x, dDBI(x, mu=.5, sigma=.2, bd=20), type="h", col="green", lwd=2)
# binomial
lines(x+0.1, dDBI(x, mu=.5, sigma=1, bd=20), type="h", col="black", lwd=2)
# overdispersed DBI
lines(x+.2, dDBI(x, mu=.5, sigma=2, bd=20), type="h", col="red", lwd=2)</pre>
```

DBURR12

The Discrete Burr type XII distribution for fitting a GAMLSS model

# **Description**

The DBURR12() function defines the discrete Burr type XII distribution, a three parameter discrete distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dDBURR12(), pDBURR12(), qDBURR12() and rDBURR12() define the density, distribution function, quantile function and random generation for the discrete Burr type XII DBURR12(), distribution.

# Usage

# **Arguments**

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter       |
|------------|--------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link    | Defines the nu.link, with "log" link as the default for the nu parameter       |
| x          | vector of (non-negative integer) quantiles                                     |
| p          | vector of probabilities                                                        |
| q          | vector of quantiles                                                            |
| mu         | vector of positive mu                                                          |
| sigma      | vector of positive dispersion parameter sigma                                  |

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nu vector of nu log, log.p logical; if TRUE, probabilities p are given as log(p) lower.tail logical; if TRUE (default), probabilities are  $P[X \le x]$ , otherwise, P[X > x] number of random values to return

#### **Details**

The probability function of the discrete Burr XII distribution is given by

$$f(y|\mu,\sigma,\nu) = (1 + (y/\mu)^{\sigma})^{\nu} - (1 + ((y+1)/\mu)^{\sigma})^{\nu}$$

for 
$$y = 0, 1, 2, ..., \infty, \mu > 0$$
,  $\sigma > 0$  and  $\mu > 0$ .

Note that the above parametrization is different from Para and Jan (2016).

# Value

The function DBURR12() Returns a gamlss.family object which can be used to fit a discrete Burr XII distribution in the gamlss() function.

#### Note

The parameters of the distributions are highly correlated so the argument of gamlss method=mixed(10,100) may have to be used.

The distribution can be under/over dispersed and also with long tails.

# Author(s)

Rigby, R. A., Stasinopoulos D. M., Fernanda De Bastiani.

# References

Para, B. A. and Jan, T. R. (2016). On discrete three parameter Burr type XII and discrete Lomax distributions and their applications to model count data from medical science. *Biometrics and Biostatistics International Journal*, **54**, part 3, pp 507-554.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **4**, pp 1-15.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

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# See Also

```
gamlss.family, DPO
```

# **Examples**

```
DBURR12()#
#plot the pdf using plot
plot(function(y) dDBURR12(y, mu=10, sigma=1, nu=1), from=0, to=100, n=100+1, type="h") # pdf
# plot the cdf
plot(seq(from=0,to=100),pDBURR12(seq(from=0,to=100), mu=10, sigma=1, nu=1), type="h") # cdf
# generate random sample
tN <- table(Ni <- rDBURR12(100, mu=5, sigma=1, nu=1))
r <- barplot(tN, col='lightblue')</pre>
```

DEL

The Delaporte distribution for fitting a GAMLSS model

# Description

The DEL() function defines the Delaporte distribution, a three parameter discrete distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dDEL, pDEL, qDEL and rDEL define the density, distribution function, quantile function and random generation for the Delaporte DEL(), distribution.

# Usage

# Arguments

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter       |
|------------|--------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link    | Defines the nu.link, with "logit" link as the default for the nu parameter     |
| X          | vector of (non-negative integer) quantiles                                     |
| mu         | vector of positive mu                                                          |
| sigma      | vector of positive dispersion parameter                                        |
| nu         | vector of nu                                                                   |
| p          | vector of probabilities                                                        |
| q          | vector of quantiles                                                            |

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| n          | number of random values to return                                                         |
|------------|-------------------------------------------------------------------------------------------|
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$        |
| max.value  | a constant, set to the default value of 10000 for how far the algorithm should look for q |

# **Details**

The probability function of the Delaporte distribution is given by

$$f(y|\mu, \sigma, \nu) = \frac{e^{-\mu\nu}}{\Gamma(1/\sigma)} \left[ 1 + \mu\sigma(1-\nu) \right]^{-1/\sigma} S$$

where

$$S = \sum_{j=0}^{y} \binom{y}{j} \frac{\mu^{y} \nu^{y-j}}{y!} \left[ \mu + \frac{1}{\sigma(1-\nu)} \right]^{-j} \Gamma\left(\frac{1}{\sigma} + j\right)$$

for  $y=0,1,2,...,\infty$  where  $\mu>0$ ,  $\sigma>0$  and  $0<\nu<1$ . This distribution is a parametrization of the distribution given by Wimmer and Altmann (1999) p 515-516 where  $\alpha=\mu\nu$ ,  $k=1/\sigma$  and  $\rho=[1+\mu\sigma(1-\nu)]^{-1}$ 

#### Value

Returns a gamlss.family object which can be used to fit a Delaporte distribution in the gamlss() function.

# Note

The mean of Y is given by  $E(Y) = \mu$  and the variance by  $V(Y) = \mu + \mu^2 \sigma (1 - \nu)^2$ .

#### Author(s)

Rigby, R. A., Stasinopoulos D. M. and Marco Enea

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC,doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

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Wimmer, G. and Altmann, G (1999). *Thesaurus of univariate discrete probability distributions* . Stamn Verlag, Essen, Germany

(see also https://www.gamlss.com/).

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# See Also

```
gamlss.family, SI, SICHEL
```

# **Examples**

```
DEL()# gives information about the default links for the Delaporte distribution #plot the pdf using plot plot(function(y) dDEL(y, mu=10, sigma=1, nu=.5), from=0, to=100, n=100+1, type="h") # pdf # plot the cdf plot(seq(from=0,to=100),pDEL(seq(from=0,to=100), mu=10, sigma=1, nu=0.5), type="h") # cdf # generate random sample tN <- table(Ni <- rDEL(100, mu=10, sigma=1, nu=0.5)) r <- barplot(tN, col='lightblue') # fit a model to the data # libary(gamlss) # gamlss(Ni~1,family=DEL, control=gamlss.control(n.cyc=50))
```

DPO

The Double Poisson distribution

#### **Description**

The function DPO() defines the double Poisson distribution, a two parameters distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dDPO, pDPO, qDPO and rPO define the density, distribution function, quantile function and random generation for the double Poisson, DPO(), distribution. The function get\_C() calculates numericaly the constant of proportionality needed for the pdf to sum up to 1.

# Usage

# **Arguments**

```
mu.link the link function for mu with default log
sigma.link the link function for sigma with default log
x, q vector of (non-negative integer) quantiles
p vector of probabilities
mu the mu parameter
sigma the sigma parameter
```

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| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                       |
|------------|----------------------------------------------------------------------------------------------------------|
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                                    |
| max.value  | a constant, set to the default value of 10000 for how far the algorithm should look for $\boldsymbol{q}$ |
| n          | how many random values to generate                                                                       |

#### **Details**

The definition for the Double Poisson distribution first introduced by Efron (1986) is:

$$f(y|\mu,\sigma) = \left(\frac{1}{\sigma}\right)^{1/2} e^{-\mu/\sigma} \left(\frac{e^{-y}y^y}{y!}\right) \left(\frac{e\mu}{y}\right)^{y/\sigma} C$$

for  $y=0,1,2,\ldots,\infty, \, \mu>0$  and  $\sigma>0$  where C is the constant of proportinality which is calculated numerically using the function get\_C.

#### Value

The function DPO returns a gamlss.family object which can be used to fit a double Poisson distribution in the gamlss() function.

#### Note

The distributons calculates the constant of proportionality numerically therefore it can be slow for large data

# Author(s)

Mikis Stasinopoulos, Bob Rigby and Marco Enea

#### References

Efron, B., 1986. Double exponential families and their use in generalized linear Regression. Journal of the American Statistical Association 81 (395), 709-721.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

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#### See Also

P0

#### **Examples**

```
DPO()
# overdisperse DPO
x <- 0:20
plot(x, dDPO(x, mu=5, sigma=3), type="h", col="red")
# underdisperse DPO
plot(x, dDPO(x, mu=5, sigma=.3), type="h", col="red")
# generate random sample
Y <- rDPO(100,5,.5)
plot(table(Y))
points(0:20, 100*dDPO(0:20, mu=5, sigma=.5)+0.2, col="red")
# fit a model to the data
# library(gamlss)
# gamlss(Y~1, family=DPO)</pre>
```

EGB2

The exponential generalized Beta type 2 distribution for fitting a GAMLSS

# **Description**

This function defines the generalized t distribution, a four parameter distribution. The response variable is in the range from minus infinity to plus infinity. The functions dEGB2, pEGB2, qEGB2 and rEGB2 define the density, distribution function, quantile function and random generation for the generalized beta type 2 distribution.

# Usage

# **Arguments**

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter.  |
|------------|---------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. |
| nu.link    | Defines the nu.link, with "log" link as the default for the nu parameter.       |
| tau.link   | Defines the tau.link, with "log" link as the default for the tau parameter.     |

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| x,q        | vector of quantiles                                                                        |
|------------|--------------------------------------------------------------------------------------------|
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| nu         | vector of skewness nu parameter values                                                     |
| tau        | vector of kurtosis tau parameter values                                                    |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| р          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

#### **Details**

The probability density function of the Generalized Beta type 2, (GB2), is defined as

$$f(y|\mu, \sigma \nu, \tau) = e^{\nu z} \{ |\sigma| B(\nu, \tau) [1 + e^z]^{\nu + \tau} \}^{-1}$$

for  $-\infty < y < \infty$ , where  $z = (y - \mu)/\sigma$  and  $-\infty < \mu < \infty, -\infty < \sigma < \infty, \nu > 0$  and  $\tau > 0$ , McDonald and Xu (1995).

## Value

EGB2() returns a gamlss.family object which can be used to fit the EGB2 distribution in the gamlss() function. dEGB2() gives the density, pEGB2() gives the distribution function, qEGB2() gives the quantile function, and rEGB2() generates random deviates.

# Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC,doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

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# See Also

```
gamlss.family, JSU, BCT
```

# **Examples**

```
EGB2()
y<- rEGB2(200, mu=5, sigma=2, nu=1, tau=4)
library(MASS)
truehist(y)
fx<-dEGB2(seq(min(y), 20, length=200), mu=5, sigma=2, nu=1, tau=4)
lines(seq(min(y), 20, length=200), fx)
# something funny here
# library(gamlss)
# histDist(y, family=EGB2, n.cyc=60)
integrate(function(x) x*dEGB2(x=x, mu=5, sigma=2, nu=1, tau=4), -Inf, Inf)
curve(dEGB2(x, mu=5 ,sigma=2, nu=1, tau=4), -10, 10, main = "The EGB2 density
             mu=5, sigma=2, nu=1, tau=4")
```

exGAUS

The ex-Gaussian distribution

# **Description**

The ex-Gaussian distribution is often used by psychologists to model response time (RT). It is defined by adding two random variables, one from a normal distribution and the other from an exponential. The parameters mu and sigma are the mean and standard deviation from the normal distribution variable while the parameter nu is the mean of the exponential variable. The functions dexGAUS, pexGAUS, qexGAUS and rexGAUS define the density, distribution function, quantile function and random generation for the ex-Gaussian distribution.

# Usage

```
exGAUS(mu.link = "identity", sigma.link = "log", nu.link = "log")
dexGAUS(x, mu = 5, sigma = 1, nu = 1, log = FALSE)
pexGAUS(q, mu = 5, sigma = 1, nu = 1, lower.tail = TRUE, log.p = FALSE)
qexGAUS(p, mu = 5, sigma = 1, nu = 1, lower.tail = TRUE, log.p = FALSE)
rexGAUS(n, mu = 5, sigma = 1, nu = 1, ...)
```

# **Arguments**

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter.                                                                             |
|------------|------------------------------------------------------------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter.                                                                            |
| nu.link    | Defines the nu.link, with "log" link as the default for the nu parameter. Other links are "inverse", "identity", "logshifted" (shifted from one) and "own" |
| x,q        | vector of quantiles                                                                                                                                        |
| mu         | vector of mu parameter values                                                                                                                              |

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| sigma      | vector of scale parameter values                                                           |
|------------|--------------------------------------------------------------------------------------------|
| nu         | vector of nu parameter values                                                              |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| р          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |
|            | for extra arguments                                                                        |

#### **Details**

The probability density function of the ex-Gaussian distribution, (exGAUS), is defined as

$$f(y|\mu,\sigma,\nu) = \frac{1}{\nu} e^{\frac{\mu-y}{\nu} + \frac{\sigma^2}{2\nu^2}} \Phi(\frac{y-\mu}{\sigma} - \frac{\sigma}{\nu})$$

where  $\Phi$  is the cdf of the standard normal distribution, for  $-\infty < y < \infty, -\infty < \mu < \infty, \sigma > 0$  and  $\nu > 0$ .

#### Value

exGAUS() returns a gamlss.family object which can be used to fit ex-Gaussian distribution in the gamlss() function. dexGAUS() gives the density, pexGAUS() gives the distribution function, qexGAUS() gives the quantile function, and rexGAUS() generates random deviates.

# Note

The mean of the ex-Gaussian is  $\mu + \nu$  and the variance is  $\sigma^2 + \nu^2$ .

#### Author(s)

Mikis Stasinopoulos and Bob Rigby

#### References

Cousineau, D. Brown, S. and Heathecote A. (2004) Fitting distributions using maximum likelihood: Methods and packages, *Behavior Research Methods, Instruments and Computers*, **46**, 742-756.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC,doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

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# See Also

```
gamlss.family, BCCG, GA, IG LNO
```

# **Examples**

```
exGAUS() #
y<- rexGAUS(100, mu=300, nu=100, sigma=35)
hist(y)
# library(gamlss)
# m1<-gamlss(y~1, family=exGAUS)
# plot(m1)
curve(dexGAUS(x, mu=300, sigma=35, nu=100), 100, 600,
    main = "The ex-GAUS density mu=300, sigma=35, nu=100")
plot(function(x) pexGAUS(x, mu=300, sigma=35, nu=100), 100, 600,
    main = "The ex-GAUS cdf mu=300, sigma=35, nu=100")</pre>
```

EXP

Exponential distribution for fitting a GAMLSS

# **Description**

The function EXP defines the exponential distribution, a one parameter distribution for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The mu parameter represents the mean of the distribution. The functions dEXP, pEXP, qEXP and rEXP define the density, distribution function, quantile function and random generation for the specific parameterization of the exponential distribution defined by function EXP.

# Usage

```
EXP(mu.link ="log")
dEXP(x, mu = 1, log = FALSE)
pEXP(q, mu = 1, lower.tail = TRUE, log.p = FALSE)
qEXP(p, mu = 1, lower.tail = TRUE, log.p = FALSE)
rEXP(n, mu = 1)
```

# Arguments

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter, other links are "inverse" and "identity" |
|------------|--------------------------------------------------------------------------------------------------------------------|
| x,q        | vector of quantiles                                                                                                |
| mu         | vector of location parameter values                                                                                |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                                             |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                 |
| p          | vector of probabilities                                                                                            |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required                         |

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# **Details**

The specific parameterization of the exponential distribution used in EXP is

$$f(y|\mu) = \frac{1}{\mu} \exp\left\{-\frac{y}{\mu}\right\}$$

, for y>0,  $\mu > 0$ .

#### Value

EXP() returns a gamlss.family object which can be used to fit an exponential distribution in the gamlss() function. dEXP() gives the density, pEXP() gives the distribution function, qEXP() gives the quantile function, and rEXP() generates random deviates.

# Author(s)

Mikis Stasinopoulos, Bob Rigby and Nicoleta Motpan

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC,doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family
```

# **Examples**

```
y<-rEXP(1000,mu=1) # generates 1000 random observations
hist(y)
# library(gamlss)
# histDist(y, family=EXP)</pre>
```

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| 1100 parametric paj from timiteti information data | flexDist | Non-parametric pdf from limited information data |
|----------------------------------------------------|----------|--------------------------------------------------|
|----------------------------------------------------|----------|--------------------------------------------------|

# Description

This is an attempt to create a distribution function if the only existing information is the quantiles or expectiles of the distribution.

# Usage

```
flexDist(quantiles = list(values=c(-1.96,0,1.96), prob=c(0.05, .50, 0.95)),
        expectiles = list(), lambda = 10,
        kappa = 10, delta = 1e-07, order = 3, n.iter = 200,
        plot = TRUE, no.inter = 100, lower = NULL,
        upper = NULL, perc.quant = 0.3, ...)
```

# **Arguments**

| quantiles  | a list with components values and prob                                         |
|------------|--------------------------------------------------------------------------------|
| expectiles | a list with components values and prob                                         |
| lambda     | smoothing parameter for the log-pdf                                            |
| kappa      | smoothing parameter for log concavity                                          |
| delta      | smoothing parameter for ridge penalty                                          |
| order      | the order of the penalty for log-pdf                                           |
| n.iter     | maximum number of iterations                                                   |
| plot       | whether to plot the result                                                     |
| no.inter   | How many discrete probabilities to evaluate                                    |
| lower      | the lower value of the x                                                       |
| upper      | the upper value of the x                                                       |
| perc.quant | how far from the quantile should go out to define the limit of x if not set by |
|            | lower or upper                                                                 |
|            | additional arguments                                                           |

# Value

# Returns a list with components

| pdf  | the hights of the fitted pdf, the sum of it multiplied by the Dx should add up to 1 i.e. $sum(object$pdf*diff(object$x)[1])$ |
|------|------------------------------------------------------------------------------------------------------------------------------|
| cdf  | the fitted cdf                                                                                                               |
| X    | the values of x where the discretise distribution is defined                                                                 |
| pFun | the cdf of the fitted non-parametric distribution                                                                            |
| qFun | the inverse cdf function of the fitted non-parametric distribution                                                           |
| rFun | a function to generate a random sample from the fitted non-parametric distribu-                                              |
|      | tion                                                                                                                         |

GA

#### Author(s)

Mikis Stasinopoulos, Paul Eilers, Bob Rigby and Vlasios Voudouris

#### References

Eilers, P. H. C., Voudouris, V., Rigby R. A., Stasinopoulos D. M. (2012) Estimation of nonparametric density from sparse summary information, under review.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), Appl. Statist., 54, part 3, pp 507-554.

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Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. Journal of Statistical Software, Vol. 23, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

#### See Also

histSmo

# **Examples**

Gamma distribution for fitting a GAMLSS

# GA

# **Description**

The function GA defines the gamma distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The parameterization used has the mean of the distribution equal to  $\mu$  and the variance equal to  $\sigma^2\mu^2$ . The functions dGA, pGA, qGA and rGA define the density, distribution function, quantile function and random generation for the specific parameterization of the gamma distribution defined by function GA.

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# Usage

```
GA(mu.link = "log", sigma.link ="log")
dGA(x, mu = 1, sigma = 1, log = FALSE)
pGA(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qGA(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rGA(n, mu = 1, sigma = 1)
```

# **Arguments**

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter, other links are "inverse", "identity" ans "own"         |
|------------|-----------------------------------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter, other link is the "inverse", "identity" and "own" |
| x,q        | vector of quantiles                                                                                                               |
| mu         | vector of location parameter values                                                                                               |
| sigma      | vector of scale parameter values                                                                                                  |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                                                            |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                                |
| р          | vector of probabilities.                                                                                                          |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required                                        |

#### **Details**

The specific parameterization of the gamma distribution used in GA is

$$f(y|\mu,\sigma) = \frac{y^{(1/\sigma^2 - 1)} \exp[-y/(\sigma^2 \mu)]}{(\sigma^2 \mu)^{(1/\sigma^2)} \Gamma(1/\sigma^2)}$$

for y > 0,  $\mu > 0$  and  $\sigma > 0$ .

# Value

GA() returns a gamlss.family object which can be used to fit a gamma distribution in the gamlss() function. dGA() gives the density, pGA() gives the distribution function, qGA() gives the quantile function, and rGA() generates random deviates. The latest functions are based on the equivalent R functions for gamma distribution.

# Note

 $\mu$  is the mean of the distribution in GA. In the function GA,  $\sigma$  is the square root of the usual dispersion parameter for a GLM gamma model. Hence  $\sigma\mu$  is the standard deviation of the distribution defined in GA.

# Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

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#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC,doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

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```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family
```

# **Examples**

```
GA()# gives information about the default links for the gamma distribution # dat<-rgamma(100, shape=1, scale=10) # generates 100 random observations # fit a gamlss model # gamlss(dat~1,family=GA) # fits a constant for each parameter mu and sigma of the gamma distribution newdata<-rGA(1000,mu=1,sigma=1) # generates 1000 random observations hist(newdata) rm(dat,newdata)
```

GAF

The Gamma distribution family

# **Description**

The function GAF() defines a gamma distribution family, which has three parameters. This is not the generalised gamma distribution which is called GG. The third parameter here is to model the mean and variance relationship. The distribution can be fitted using the function gamlss(). The mean of GAF is equal to mu. The variance is equal to sigma^2\*mu^nu so the standard deviation is sigma\*mu^(nu/2). The function is design for cases where the variance is proportional to a power of the mean. This is an instance of the Taylor's power low, see Enki et al. (2017). The functions dGAF, pGAF, qGAF and rGAF define the density, distribution function, quantile function and random generation for the GAF parametrization of the gamma family.

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# Usage

```
GAF(mu.link = "log", sigma.link = "log", nu.link = "identity")
dGAF(x, mu = 1, sigma = 1, nu = 2, log = FALSE)
pGAF(q, mu = 1, sigma = 1, nu = 2, lower.tail = TRUE,
    log.p = FALSE)
qGAF(p, mu = 1, sigma = 1, nu = 2, lower.tail = TRUE,
    log.p = FALSE)
rGAF(n, mu = 1, sigma = 1, nu = 2)
```

# Arguments

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter              |
|------------|--------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter             |
| nu.link    | Defines the nu.link with "identity" link as the default for the nu parameter               |
| x,q        | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| nu         | vector of power parameter values                                                           |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| p          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

#### **Details**

The parametrization of the gamma family given in the function GAF() is:

$$f(y|\mu, \sigma_1) = \frac{y^{(1/\sigma_1^2 - 1)} \exp[-y/(\sigma_1^2 \mu)]}{(\sigma_1^2 \mu)^{(1/\sigma^2)} \Gamma(1/\sigma^2)}$$

for y > 0,  $\mu > 0$  where  $\sigma_1 = \sigma \mu^{(\nu/1 - 1)}$ .

# Value

GAF() returns a gamlss.family object which can be used to fit the gamma family in the gamlss() function.

#### Note

For the function GAF(),  $\mu$  is the mean and  $\sigma\mu^{\nu/2}$  is the standard deviation of the gamma family. The GAF is design for fitting regression type models where the variance is proportional to a power of the mean.

Note that because the high correlation between the sigma and the nu parameter the mixed() method should be used in the fitting.

#### Author(s)

Mikis Stasinopoulos, Robert Rigby and Fernanda De Bastiani

#### References

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Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

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## See Also

```
gamlss.family, GA, GG
```

# **Examples**

gamlss.family

Family Objects for fitting a GAMLSS model

## **Description**

GAMLSS families are the current available distributions that can be fitted using the gamlss() function.

# Usage

```
gamlss.family(object,...)
as.gamlss.family(object)
as.family(object)
## S3 method for class 'gamlss.family'
print(x,...)
```

# Arguments

object a gamlss family object e.g. BCT x a gamlss family object e.g. BCT

... further arguments passed to or from other methods.

# **Details**

There are several distributions available for the response variable in the gamlss function. The following table display their names and their abbreviations in R. Note that the different distributions can be fitted using their R abbreviations (and optionally excluding the brackets) i.e. family=BI(), family=BI are equivalent.

| Distributions                       | R names   | No of parameters |
|-------------------------------------|-----------|------------------|
| Beta                                | BE()      | 2                |
| Beta Binomial                       | BB()      | 2                |
| Beta negative binomial              | BNB()     | 3                |
| Beta one inflated                   | BEOI()    | 3                |
| Beta zero inflated                  | BEZI()    | 3                |
| Beta inflated                       | BEINF()   | 4                |
| Binomial                            | BI()      | 1                |
| Box-Cox Cole and Green              | BCCG()    | 3                |
| Box-Cox Power Exponential           | BCPE()    | 4                |
| Box-Cox-t                           | BCT()     | 4                |
| Delaport                            | DEL()     | 3                |
| Discrete Burr XII                   | DBURR12() | 3                |
| Double Poisson                      | DPO()     | 2                |
| Double binomial                     | DBI()     | 2                |
| Exponential                         | EXP()     | 1                |
| Exponential Gaussian                | exGAUS()  | 3                |
| Exponential generalized Beta type 2 | EGB2()    | 4                |
| Gamma                               | GA()      | 2                |
| Generalized Beta type 1             | GB1()     | 4                |
| Generalized Beta type 2             | GB2()     | 4                |
| Generalized Gamma                   | GG()      | 3                |
| Generalized Inverse Gaussian        | GIG()     | 3                |
| Generalized t                       | GT()      | 4                |
| Geometric                           | GEOM()    | 1                |
| Geometric (original)                | GEOMo()   | 1                |
| Gumbel                              | GU()      | 2                |
| Inverse Gamma                       | IGAMMA()  | 2                |

| Inverse Gaussian               | IG()          | 2           |
|--------------------------------|---------------|-------------|
| Johnson's SU                   | JSU()         | 4           |
| Logarithmic                    | LG()          | 1           |
| Logistic                       | LO()          | 2           |
| Logit-Normal                   | LOGITNO()     | 2           |
| log-Normal                     | LOGNO()       | 2           |
| log-Normal (Box-Cox)           | LNO()         | 3 (1 fixed) |
| Negative Binomial type I       | NBI()         | 2           |
| Negative Binomial type II      | NBII()        | 2           |
| Negative Binomial family       | NBF()         | 3           |
| Normal Exponential t           | NET()         | 4 (2 fixed) |
| Normal                         | NO()          | 2           |
| Normal Family                  | NOF()         | 3 (1 fixed) |
| Normal Linear Quadratic        | LQNO()        | 2           |
| Pareto type 2                  | PARETO2()     | 2           |
| Pareto type 2 original         | PARETO2o()    | 2           |
| Power Exponential              | PE()          | 3           |
| Power Exponential type 2       | PE2()         | 3           |
| Poison                         | PO()          | 1           |
| Poisson inverse Gaussian       | PIG()         | 2           |
| Reverse generalized extreme    | RGE()         | 3           |
| Reverse Gumbel                 | RG()          | 2           |
| Skew Power Exponential type 1  | SEP1()        | 4           |
| Skew Power Exponential type 2  | SEP2()        | 4           |
| Skew Power Exponential type 3  | SEP3()        | 4           |
| Skew Power Exponential type 4  | SEP4()        | 4           |
| Shash                          | SHASH()       | 4           |
| Shash original                 | SHASHo()      | 4           |
| Shash original 2               | SHASH()       | 4           |
| Sichel (original)              | SI()          | 3           |
| Sichel (mu as the maen)        | SICHEL()      | 3           |
| Simplex                        | SIMPLEX()     | 2           |
| Skew t type 1                  | ST1()         | 3           |
| Skew t type 2                  | ST2()         | 3           |
| Skew t type 2<br>Skew t type 3 | ST3()         | 3           |
| Skew t type 3 Skew t type 4    |               | 3           |
| Skew t type 4 Skew t type 5    | ST4()         | 3           |
| t-distribution                 | ST5()<br>TF() | 3           |
|                                |               |             |
| Waring                         | WARING()      | 1           |
| Weibull                        | WEI()         | 2           |
| Weibull (PH parameterization)  | WEI2()        | 2           |
| Weibull (mu as mean)           | WEI3()        | 2           |
| Yule                           | YULE()        | 1           |
| Zero adjusted binomial         | ZABI()        | 2           |
| Zero adjusted beta neg. bin.   | ZABNB()       | 4           |
| Zero adjusted IG               | ZAIG()        | 2           |
| Zero adjusted logarithmic      | ZALG()        | 2           |
| Zero adjusted neg. bin.        | ZANBI()       | 3           |

| Zero adjusted poisson        | ZAP()              | 2 |
|------------------------------|--------------------|---|
| Zero adjusted Sichel         | ZASICHEL()         | 4 |
| Zero adjusted Zipf           | ZAZIPF()           | 2 |
| Zero inflated binomial       | ZIBI()             | 2 |
| Zero inflated beta neg. bin. | ZIBNB()            | 4 |
| Zero inflated neg. bin.      | ZINBI()            | 3 |
| Zero inflated poisson        | ZIP()              | 2 |
| Zero inf. poiss.(mu as mean) | ZIP2()             | 2 |
| Zero inflated PIG            | <pre>ZIPIG()</pre> | 3 |
| Zero inflated Sichel         | ZISICHEL()         | 4 |
| Zipf                         | ZIPF()             | 1 |

Note that some of the distributions are in the package gamlss.dist. The parameters of the distributions are in order, mu for location, sigma for scale (or dispersion), and nu and tau for shape. More specifically for the BCCG family mu is the median, sigma approximately the coefficient of variation, and nu the skewness parameter. The parameters for BCPE distribution have the same interpretation with the extra fourth parameter tau modelling the kurtosis of the distribution. The parameters for BCT have the same interpretation except that  $\sigma[(\tau/(\tau-2))^{0.5}]$  is approximately the coefficient of variation.

All of the distribution in the above list are also provided with the corresponding d, p, q and r functions for density (pdf), distribution function (cdf), quantile function and random generation function respectively, (see individual distribution for details).

# Value

The above GAMLSS families return an object which is of type gamlss. family. This object is used to define the family in the gamlss() fit.

#### Note

More distributions will be documented in later GAMLSS releases. Further user defined distributions can be incorporate relatively easy, see, for example, the help documentation accompanying the gamlss library.

# Author(s)

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#### References

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Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

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# See Also

BE,BB,BEINF,BI,LNO,BCT, BCPE,BCCG, GA,GU,JSU,IG,LO, NBI,NBII,NO,PE,PO, RG,PIG,TF,WEI,WEI2, ZIP

# **Examples**

```
normal<-NO(mu.link="log", sigma.link="log")
normal</pre>
```

GB1

The generalized Beta type 1 distribution for fitting a GAMLSS

# **Description**

This function defines the generalized beta type 1 distribution, a four parameter distribution. The function GB1 creates a gamlss.family object which can be used to fit the distribution using the function gamlss(). Note the range of the response variable is from zero to one. The functions dGB1, GB1, qGB1 and rGB1 define the density, distribution function, quantile function and random generation for the generalized beta type 1 distribution.

# Usage

# Arguments

| mu.link    | Defines the mu. 11nk, with "identity" link as the default for the mu parameter.                 |
|------------|-------------------------------------------------------------------------------------------------|
| sigma.link | Defines the ${\tt sigma.link},$ with "log" link as the default for the ${\tt sigma}$ parameter. |
| nu.link    | Defines the nu.link, with "log" link as the default for the nu parameter.                       |
| tau.link   | Defines the tau.link, with "log" link as the default for the tau parameter.                     |
| x,q        | vector of quantiles                                                                             |
| mu         | vector of location parameter values                                                             |

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| sigma      | vector of scale parameter values                                                           |
|------------|--------------------------------------------------------------------------------------------|
| nu         | vector of skewness nu parameter values                                                     |
| tau        | vector of kurtosis tau parameter values                                                    |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| р          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

#### **Details**

The probability density function of the Generalized Beta type 1, (GB1), is defined as

$$f(y|\mu,\sigma\,\nu,\tau) = \frac{\tau\nu^{\beta}y^{\tau\alpha-1}(1-y^{\tau})^{\beta-1}}{B(\alpha,\beta)[\nu+(1-\nu)y^{\tau}]^{\alpha+\beta}}$$

where 0 < y < 1,  $\alpha = \mu(1-\sigma^2)/\sigma^2$  and  $\beta = (1-\mu)(1-\sigma^2)/\sigma^2$ , and  $\alpha > 0$ ,  $\beta > 0$ . Note the  $\mu = \alpha/(\alpha+\beta)$ ,  $\sigma = (\alpha+\beta+1)^{-1/2}$ .

# Value

GB1() returns a gamlss.family object which can be used to fit the GB1 distribution in the gamlss() function. dGB1() gives the density, pGB1() gives the distribution function, qGB1() gives the quantile function, and rGB1() generates random deviates.

# Warning

The qSHASH and rSHASH are slow since they are relying on golden section for finding the quantiles

# Author(s)

Bob Rigby and Mikis Stasinopoulos

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC,doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

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#### See Also

```
gamlss.family, JSU, BCT
```

# **Examples**

GB2

The generalized Beta type 2 and generalized Pareto distributions for fitting a GAMLSS

#### **Description**

This function defines the generalized beta type 2 distribution, a four parameter distribution. The function GB2 creates a gamlss.family object which can be used to fit the distribution using the function gamlss(). The response variable is in the range from zero to infinity. The functions dGB2, GB2, qGB2 and rGB2 define the density, distribution function, quantile function and random generation for the generalized beta type 2 distribution. The generalised Pareto GP distribution is defined by setting the parameters sigma and nu of the GB2 distribution to 1.

# Usage

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# **Arguments**

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter.                     |
|------------|----------------------------------------------------------------------------------------------------|
| sigma.link | Defines the $\verb"sigma.1"$ ink, with "log" link as the default for the $\verb"sigma"$ parameter. |
| nu.link    | Defines the nu.link, with "log" link as the default for the nu parameter.                          |
| tau.link   | Defines the tau.link, with "log" link as the default for the tau parameter.                        |
| x,q        | vector of quantiles                                                                                |
| mu         | vector of location parameter values                                                                |
| sigma      | vector of scale parameter values                                                                   |
| nu         | vector of skewness nu parameter values                                                             |
| tau        | vector of kurtosis tau parameter values                                                            |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                             |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                 |
| р          | vector of probabilities.                                                                           |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required         |
|            |                                                                                                    |

# **Details**

The probability density function of the Generalized Beta type 2, (GB2), is defined as

$$f(y|\mu, \sigma \nu, \tau) = |\sigma| y^{\sigma \nu - 1} \{ \mu^{\sigma \nu} B(\nu, \tau) [1 + (y/\mu)^{\sigma}]^{\nu + \tau} \}^{-1}$$

where y > 0,  $\mu > 0$ ,  $-\infty < \sigma < \infty$ ,  $\nu > 0$  and  $\tau > 0$ .

# Value

GB2() returns a gamlss.family object which can be used to fit the GB2 distribution in the gamlss() function. dGB2() gives the density, pGB2() gives the distribution function, qGB2() gives the quantile function, and rGB2() generates random deviates.

# Warning

The qSHASH and rSHASH are slow since they are relying on golden section for finding the quantiles

# Author(s)

Bob Rigby and Mikis Stasinopoulos

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#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

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```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family, JSU, BCT
```

# **Examples**

gen.Family

Functions to generate log and logit distributions from existing continuous gamlss.family distributions

# **Description**

There are five functions here. Only the functions Family and gen. Family should be used (see details).

# Usage

```
Family.d(family = "NO", type = c("log", "logit"), ...)
Family.p(family = "NO", type = c("log", "logit"), ...)
Family.q(family = "NO", type = c("log", "logit"), ...)
Family.r(family = "NO", type = c("log", "logit"), ...)
Family(family = "NO", type = c("log", "logit"), local = TRUE, ...)
gen.Family(family = "NO", type = c("log", "logit"), ...)
```

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# Arguments

| family | a continuous gamlss.family distribution                       |
|--------|---------------------------------------------------------------|
| type   | the type of transformation only "log" and "logit" are allowed |
| local  | It is TRUE if is called within gamlss() otherwise is FALSE    |
|        | for passing extra arguments                                   |

#### **Details**

The function gen.Family creates the standard d,p,q,r functions for the distribution plus the fitting gamlss.family. For example gen.Family("NO","logit") will generate the functions dlogitNO(), plogitNO(), qlogitNO(), rlogitNO() and dlogitNO(). The latest function can be used in family argument of gamlss() to fit a logic-Normal distribution i.e. family=logitNO. The same fitting can be achieved by using family=Family("NO","logit"). Here the required dlogitNO(), plogitNO() and logitNO() functions are generated locally within the gamlss() environment.

#### Value

The function gen. Family returns the d, p, q r functions plus the fitting function.

#### Author(s)

Mikis Stasinopoulos and Bob Rigby

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

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```
(see also https://www.gamlss.com/).
```

# **Examples**

```
# generating a log t distribution
gen.Family("TF")
# plotting the d, p, q, and r functions
op<-par(mfrow=c(2,2))
curve(dlogTF(x, mu=0), 0, 10)
curve(plogTF(x, mu=0), 0, 10)
curve(qlogTF(x, mu=0), 0, 1)</pre>
```

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```
Y<- rlogTF(200)
hist(Y)
par(op)
# different mu
curve(dlogTF(x, mu=-1, sigma=1, nu=10), 0, 5, ylim=c(0,1))
curve(dlogTF(x, mu=0, sigma=1, nu=10), 0, 5, add=TRUE, col="red", lty=2)
curve(dlogTF(x, mu=1, sigma=1, nu=10), 0, 5, add=TRUE, col="blue", lty=3)
# different sigma
curve(dlogTF(x, mu=0, sigma=.5, nu=10), 0, 5, ylim=c(0,1))
curve(dlogTF(x, mu=0, sigma=1, nu=10), 0, 5, add=TRUE, col="red", lty=2)
curve(dlogTF(x, mu=0, sigma=2, nu=10), 0, 5, add=TRUE, col="blue", lty=3)
# different degrees of freedom nu
curve(dlogTF(x, mu=0, sigma=1, nu=1), 0, 5, ylim=c(0,.8), n = 1001)
curve(dlogTF(x, mu=0, sigma=1, nu=2), 0, 5, add=TRUE, col="red", lty=2)
curve(dlogTF(x, mu=0, sigma=1, nu=5), 0, 5, add=TRUE, col="blue", lty=3)
# generating a logit t distribution
gen.Family("TF", "logit")
# plotting the d, p, q, and r functions
op<-par(mfrow=c(2,2))</pre>
curve(dlogitTF(x, mu=0), 0, 1)
curve(plogitTF(x, mu=0), 0, 1)
curve(qlogitTF(x, mu=0), 0, 1)
abline(v=1)
Y<- rlogitTF(200)
hist(Y)
par(op)
# different mu
curve(dlogitTF(x, mu=-2, sigma=1, nu=10), 0, 1, ylim=c(0,5))
curve(dlogitTF(x, mu=0, sigma=1, nu=10), 0, 1, add=TRUE, col="red", lty=2)
curve(dlogitTF(x, mu=2, sigma=1, nu=10), 0, 1, add=TRUE, col="blue", lty=3)
# different sigma
curve(dlogitTF(x, mu=0, sigma=1, nu=10), 0, 1, ylim=c(0,2.5))
curve(dlogitTF(x, mu=0, sigma=2, nu=10), 0, 1, add=TRUE, col="red", lty=2)
curve(dlogitTF(x, mu=0, sigma=.7, nu=10), 0, 1, add=TRUE, col="blue", lty=3)
# different degrees of freedom nu
curve(dlogitTF(x, mu=0, sigma=1, nu=1), 0, 1, ylim=c(0,1.6))
curve(dlogitTF(x, mu=0, sigma=1, nu=2), 0, 1, add=TRUE, col="red", lty=2)
curve(dlogitTF(x, mu=0, sigma=1, nu=5), 0, 1, add=TRUE, col="blue", lty=3)
```

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# **Description**

The functions GEOMo() and GEOM() define two parametrizations of the geometric distribution. The geometric distribution is a one parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The mean of GEOM() is equal to the parameter mu. The functions dGEOM, pGEOM and rGEOM define the density, distribution function, quantile function and random generation for the GEOM parameterization of the Geometric distribution.

# Usage

```
GEOM(mu.link = "log")
dGEOM(x, mu = 2, log = FALSE)
pGEOM(q, mu = 2, lower.tail = TRUE, log.p = FALSE)
qGEOM(p, mu = 2, lower.tail = TRUE, log.p = FALSE)
rGEOM(n, mu = 2)
GEOMo(mu.link = "logit")
dGEOMo(x, mu = 0.5, log = FALSE)
pGEOMo(q, mu = 0.5, lower.tail = TRUE, log.p = FALSE)
qGEOMo(p, mu = 0.5, lower.tail = TRUE, log.p = FALSE)
rGEOMo(n, mu = 0.5)
```

# **Arguments**

| mu.link    | Defines the mu.link, with log link as the default for the mu parameter                     |
|------------|--------------------------------------------------------------------------------------------|
| x, q       | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                      |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise $P[X > x]$          |
| р          | vector of probabilities                                                                    |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

# Details

The parameterization of the original geometric distribution in the function GE is

$$f(y|\mu) = (1-\mu)^y \mu$$

for 
$$y >= 0$$
 and  $\mu > 0$ .

The parameterization of the geometric distribution in the function GEOM is

$$f(y|\mu) = \mu^y/(\mu+1)^{y+1}$$

where for y >= 0 and  $\mu > 0$ .

# Value

returns a gamlss.family object which can be used to fit a Geometric distribution in the gamlss() function.

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#### Author(s)

Fiona McElduff, Bob Rigby and Mikis Stasinopoulos.

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Johnson, N. L., Kemp, A. W., and Kotz, S. (2005). Univariate discrete distributions. Wiley.

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family
```

## **Examples**

```
par(mfrow=c(2,2))
y < -seq(0, 20, 1)
plot(y, dGEOM(y), type="h")
q < - seq(0, 20, 1)
plot(q, pGEOM(q), type="h")
p<-seq(0.0001,0.999,0.05)
plot(p , qGEOM(p), type="s")
dat <- rGEOM(100)
hist(dat)
#summary(gamlss(dat~1, family=GEOM))
par(mfrow=c(2,2))
y < -seq(0, 20, 1)
plot(y, dGEOMo(y), type="h")
q < - seq(0, 20, 1)
plot(q, pGEOMo(q), type="h")
p<-seq(0.0001,0.999,0.05)
plot(p , qGEOMo(p), type="s")
dat <- rGEOMo(100)
hist(dat)
#summary(gamlss(dat~1, family="GE"))
```

GG

# **Description**

The function GG defines the generalized gamma distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The parameterization used has the mean of the distribution equal to mu and the variance equal to  $(\sigma^2)(\mu^2)$ . The functions dGG, pGG, qGG and rGG define the density, distribution function, quantile function and random generation for the specific parameterization of the generalized gamma distribution defined by function GG.

# Usage

# **Arguments**

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter, other links are "inverse" and "identity"        |
|------------|---------------------------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter, other links are "inverse" and "identity"  |
| nu.link    | Defines the nu.link, with "identity" link as the default for the sigma parameter, other links are $1/nu^2$ and " $\log$ " |
| x,q        | vector of quantiles                                                                                                       |
| mu         | vector of location parameter values                                                                                       |
| sigma      | vector of scale parameter values                                                                                          |
| nu         | vector of shape parameter values                                                                                          |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                                                    |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                        |
| р          | vector of probabilities                                                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required                                |

GG

#### **Details**

The specific parameterization of the generalized gamma distribution used in GG is

$$f(y|\mu, \sigma, \nu) = \frac{\theta^{\theta} z^{\theta} \nu e^{(-\theta z)}}{(\Gamma(\theta)y)}$$

where  $z=(y/\mu)^{\nu}$ ,  $\theta=1/(\sigma^2|\nu|^2)$  for y>0,  $\mu>0$ ,  $\sigma>0$  and  $-\infty<\nu<+\infty$ . Note that for  $\nu=0$  the distribution is log normal.

#### Value

GG() returns a gamlss.family object which can be used to fit a generalized gamma distribution in the gamlss() function. dGG() gives the density, pGG() gives the distribution function, qGG() gives the quantile function, and rGG() generates random deviates.

#### Author(s)

Mikis Stasinopoulos, Bob Rigby and Nicoleta Motpan

#### References

Lopatatzidis, A. and Green, P. J. (2000), Nonparametric quantile regression using the gamma distribution, unpublished.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

## See Also

```
gamlss.family, GA
```

#### **Examples**

```
y<-rGG(100,mu=1,sigma=0.1, nu=-.5) # generates 100 random observations
hist(y)
# library(gamlss)
#histDist(y, family=GG)
#m1 <-gamlss(y~1,family=GG)
#prof.dev(m1, "nu", min=-2, max=2, step=0.2)</pre>
```

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GIG

Generalized Inverse Gaussian distribution for fitting a GAMLSS

# **Description**

The function GIG defines the generalized inverse gaussian distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions DIG, pGIG, GIG and rGIG define the density, distribution function, quantile function and random generation for the specific parameterization of the generalized inverse gaussian distribution defined by function GIG.

# Usage

## **Arguments**

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter, other links are "inverse" and "identity"      |  |
|------------|-------------------------------------------------------------------------------------------------------------------------|--|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter other links are "inverse" and "identity" |  |
| nu.link    | Defines the nu.link, with "identity" link as the default for the nu parameter, other links are "inverse" and "log"      |  |
| x,q        | vector of quantiles                                                                                                     |  |
| mu         | vector of location parameter values                                                                                     |  |
| sigma      | vector of scale parameter values                                                                                        |  |
| nu         | vector of shape parameter values                                                                                        |  |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                                                  |  |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                      |  |
| р          | vector of probabilities                                                                                                 |  |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required                              |  |
|            | for extra arguments                                                                                                     |  |

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### **Details**

The specific parameterization of the generalized inverse gaussian distribution used in GIG is  $f(y|\mu,\sigma,\nu)=(\frac{c}{\mu})^{\nu}(\frac{y^{(\nu-1)}}{2K(\frac{1}{\sigma},\nu)})(\exp((\frac{-1}{2\sigma})(\frac{cy}{\mu}+\frac{\mu}{cy})))$  where  $c=\frac{K(\frac{1}{\sigma},\nu+1)}{K(\frac{1}{\sigma},\nu)}$ , for y>0,  $\mu>0$ ,  $\sigma>0$  and  $-\infty<\nu<+\infty$ .

### Value

GIG() returns a gamlss.family object which can be used to fit a generalized inverse gaussian distribution in the gamlss() function. DIG() gives the density, pGIG() gives the distribution function, GIG() gives the quantile function, and rGIG() generates random deviates.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Nicoleta Motpan

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

Jorgensen B. (1982) Statistical properties of the generalized inverse Gaussian distribution, Series: Lecture notes in statistics; 9, New York: Springer-Verlag.

```
(see also https://www.gamlss.com/).
```

### See Also

```
gamlss.family, IG
```

# **Examples**

```
y<-rGIG(100,mu=1,sigma=1, nu=-0.5) # generates 1000 random observations
hist(y)
# library(gamlss)
# histDist(y, family=GIG)</pre>
```

The generalised Poisson distribution

GP0

# **Description**

The GPO() function defines the generalised Poisson distribution, a two parameter discrete distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dGPO, pGPO, qGPO and rGPO define the density, distribution function, quantile function and random generation for the Delaporte GPO(), distribution.

## Usage

```
GPO(mu.link = "log", sigma.link = "log")

dGPO(x, mu = 1, sigma = 1, log = FALSE)

pGPO(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)

qGPO(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE, max.value = 10000)

rGPO(n, mu = 1, sigma = 1, max.value = 10000)
```

### **Arguments**

|                                                                                | mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter                                 |
|--------------------------------------------------------------------------------|------------|----------------------------------------------------------------------------------------------------------|
| sigma.link Defines the sigma.link, with "log" link as the default for the sigm |            | Defines the sigma.link, with "log" link as the default for the sigma parameter                           |
|                                                                                | X          | vector of (non-negative integer) quantiles                                                               |
|                                                                                | mu         | vector of positive mu                                                                                    |
|                                                                                | sigma      | vector of positive dispersion parameter sigma                                                            |
|                                                                                | p          | vector of probabilities                                                                                  |
|                                                                                | q          | vector of quantiles                                                                                      |
|                                                                                | n          | number of random values to return                                                                        |
|                                                                                | log, log.p | logical; if TRUE, probabilities p are given as log(p)                                                    |
|                                                                                | lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                       |
|                                                                                | max.value  | a constant, set to the default value of 10000 for how far the algorithm should look for $\boldsymbol{q}$ |
|                                                                                |            |                                                                                                          |

## **Details**

The probability function of the Generalised Poisson distribution is given by

$$P(Y = y | \mu, \sigma) = \left(\frac{\mu}{1 + \sigma\mu}\right)^y \frac{(1 + \sigma y)^{y-1}}{y!} \exp\left[\frac{-\mu(1 + \sigma y)}{1 + \sigma\mu}\right]$$

for  $y = 0, 1, 2, ..., \infty$  where  $\mu > 0$  and  $\sigma > 0$ .

GT

### Value

Returns a gamlss.family object which can be used to fit a Generalised Poisson distribution in the gamlss() function.

### Author(s)

Rigby, R. A., Stasinopoulos D. M.

### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

### See Also

```
gamlss.family, PO, DPO
```

### **Examples**

```
GPO()# gives information about the default links for the
#plot the pdf using plot
plot(function(y) dGPO(y, mu=10, sigma=1), from=0, to=100, n=100+1, type="h") # pdf
# plot the cdf
plot(seq(from=0,to=100),pGPO(seq(from=0,to=100), mu=10, sigma=1), type="h") # cdf
# generate random sample
tN <- table(Ni <- rGPO(100, mu=5, sigma=1))
r <- barplot(tN, col='lightblue')</pre>
```

GT

The generalized t distribution for fitting a GAMLSS

## **Description**

This function defines the generalized t distribution, a four parameter distribution, for a gamlss.family object to be used for a GAMLSS fitting using the function gamlss(). The functions dGT, pGT, qGT and rGT define the density, distribution function, quantile function and random generation for the generalized t distribution.

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### Usage

```
GT(mu.link = "identity", sigma.link = "log", nu.link = "log",
    tau.link = "log")

dGT(x, mu = 0, sigma = 1, nu = 3, tau = 1.5, log = FALSE)

pGT(q, mu = 0, sigma = 1, nu = 3, tau = 1.5, lower.tail = TRUE,
    log.p = FALSE)

qGT(p, mu = 0, sigma = 1, nu = 3, tau = 1.5, lower.tail = TRUE,
    log.p = FALSE)

rGT(n, mu = 0, sigma = 1, nu = 3, tau = 1.5)
```

### **Arguments**

| mu.link                                                                          | Defines the mu.link, with "identity" link as the default for the mu parameter.             |  |
|----------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|--|
| sigma.link                                                                       | Defines the sigma.link, with "log" link as the default for the sigma parameter.            |  |
| nu.link Defines the nu.link, with "log" link as the default for the nu parameter |                                                                                            |  |
| tau.link                                                                         | Defines the tau.link, with "log" link as the default for the tau parameter.                |  |
| x,q                                                                              | x,q vector of quantiles                                                                    |  |
| mu                                                                               | vector of location parameter values                                                        |  |
| sigma vector of scale parameter values                                           |                                                                                            |  |
| nu                                                                               | vector of skewness nu parameter values                                                     |  |
| tau                                                                              | vector of kurtosis tau parameter values                                                    |  |
| log, log.p                                                                       | logical; if TRUE, probabilities p are given as log(p).                                     |  |
| lower.tail                                                                       | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |  |
| р                                                                                | vector of probabilities.                                                                   |  |
| n                                                                                | number of observations. If $length(n) > 1$ , the length is taken to be the number required |  |

### **Details**

The probability density function of the generalized t distribution, (GT), , is defined as

$$f(y|\mu, \sigma \nu, \tau) = \tau \left\{ 2\sigma \nu^{1/\tau} B\left(\frac{1}{\tau}, \nu\right) \left[1 + |z|^{\tau}/\nu\right]^{\nu + 1/\tau} \right\}^{-1}$$

where 
$$-\infty < y < \infty$$
,  $z = (y - \mu)/\sigma \mu = (-\infty, +\infty)$ ,  $\sigma > 0$ ,  $\nu > 0$  and  $\tau > 0$ .

## Value

GT() returns a gamlss.family object which can be used to fit the GT distribution in the gamlss() function. dGT() gives the density, pGT() gives the distribution function, qGT() gives the quantile function, and rGT() generates random deviates.

## Warning

The qGT and rGT are slow since they are relying on optimization for finding the quantiles

GU

### Author(s)

Bob Rigby and Mikis Stasinopoulos

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

```
(see also https://www.gamlss.com/).
```

### See Also

```
gamlss.family, JSU, BCT
```

# **Examples**

GU

The Gumbel distribution for fitting a GAMLSS

# **Description**

The function GU defines the Gumbel distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dGU, pGU, qGU and rGU define the density, distribution function, quantile function and random generation for the specific parameterization of the Gumbel distribution.

# Usage

```
GU(mu.link = "identity", sigma.link = "log")
dGU(x, mu = 0, sigma = 1, log = FALSE)
pGU(q, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qGU(p, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rGU(n, mu = 0, sigma = 1)
```

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### **Arguments**

mu.link Defines the mu.link, with "identity" link as the default for the mu parameter. other available link is "inverse", "log" and "own") Defines the sigma.link, with "log" link as the default for the sigma parameter, sigma.link other links are the "inverse", "identity" and "own" vector of quantiles x,q vector of location parameter values mu vector of scale parameter values sigma log, log.p logical; if TRUE, probabilities p are given as log(p). lower.tail logical; if TRUE (default), probabilities are  $P[X \le x]$ , otherwise, P[X > x]vector of probabilities. number of observations. If length(n) > 1, the length is taken to be the number required

## **Details**

The specific parameterization of the Gumbel distribution used in GU is

$$f(y|\mu,\sigma) = \frac{1}{\sigma} \exp\left\{\left(\frac{y-\mu}{\sigma}\right) - \exp\left(\frac{y-\mu}{\sigma}\right)\right\}$$

for 
$$y=(-\infty,\infty)$$
,  $\mu=(-\infty,+\infty)$  and  $\sigma>0$ .

# Value

GU() returns a gamlss.family object which can be used to fit a Gumbel distribution in the gamlss() function. dGU() gives the density, pGU() gives the distribution function, qGU() gives the quantile function, and rGU() generates random deviates.

### Note

The mean of the distribution is  $\mu - 0.57722\sigma$  and the variance is  $\pi^2 \sigma^2/6$ .

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, \doi10.18637/jss.v023.i07.

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

### See Also

```
gamlss.family, RG
```

# **Examples**

```
plot(function(x) dGU(x, mu=0,sigma=1), -6, 3,
  main = "{Gumbel density mu=0,sigma=1}")
GU()# gives information about the default links for the Gumbel distribution
dat<-rGU(100, mu=10, sigma=2) # generates 100 random observations
hist(dat)
# library(gamlss)
# gamlss(dat~1,family=GU) # fits a constant for each parameter mu and sigma</pre>
```

hazardFun

Hazard functions for gamlss.family distributions

## **Description**

The function hazardFun() takes as an argument a gamlss.family object and creates the hazard function for it. The function gen.hazard() generates a hazard function called hNAME where NAME is a gamlss.family i.e. hGA().

## Usage

```
hazardFun(family = "NO", ...)
gen.hazard(family = "NO", ...)
```

## **Arguments**

```
family a gamlss.family object
... for passing extra arguments
```

### Value

A hazard function.

# Author(s)

Mikis Stasinopoulos, Bob Rigby and Vlasios Voudouris

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### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

### See Also

```
gamlss.family
```

## **Examples**

```
gen.hazard("WEI2")
y<-seq(0,10,by=0.01)
plot(hWEI2(y, mu=1, sigma=1)~y, type="1", col="black", ylab="h(y)", ylim=c(0,2.5))
lines(hWEI2(y, mu=1, sigma=1.2)~y, col="red",lt=2,lw=2)
lines(hWEI2(y, mu=1, sigma=.5)~y, col="blue",lt=3,lw=2)</pre>
```

ΙG

Inverse Gaussian distribution for fitting a GAMLSS

## **Description**

The function IG(), or equivalently Inverse.Gaussian(), defines the inverse Gaussian distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dIG, pIG, qIG and rIG define the density, distribution function, quantile function and random generation for the specific parameterization of the Inverse Gaussian distribution defined by function IG.

## Usage

```
IG(mu.link = "log", sigma.link = "log")
dIG(x, mu = 1, sigma = 1, log = FALSE)
pIG(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qIG(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rIG(n, mu = 1, sigma = 1, ...)
```

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### **Arguments**

mu.link Defines the mu.link, with "log" link as the default for the mu parameter Defines the sigma.link, with "log" link as the default for the sigma parameter sigma.link vector of quantiles x,q vector of location parameter values vector of scale parameter values sigma log, log.p logical; if TRUE, probabilities p are given as log(p). lower.tail logical; if TRUE (default), probabilities are  $P[X \le x]$ , otherwise, P[X > x]vector of probabilities. number of observations. If length(n) > 1, the length is taken to be the number n required ... can be used to pass the uppr.limit argument to qIG

### **Details**

Definition file for inverse Gaussian distribution.

$$f(y|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2 y^3}} \exp\left\{-\frac{1}{2\mu^2\sigma^2 y} (y-\mu)^2\right\}$$

for y > 0,  $\mu > 0$  and  $\sigma > 0$ .

### Value

returns a gamlss.family object which can be used to fit a inverse Gaussian distribution in the gamlss() function.

### Note

 $\mu$  is the mean and  $\sigma^2\mu^3$  is the variance of the inverse Gaussian

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, \doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

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## See Also

```
gamlss.family, GA, GIG
```

## **Examples**

```
IG()# gives information about the default links for the normal distribution
# library(gamlss)
# data(rent)
# gamlss(R~cs(Fl),family=IG, data=rent) #
plot(function(x)dIG(x, mu=1,sigma=.5), 0.01, 6,
    main = "{Inverse Gaussian density mu=1,sigma=0.5}")
plot(function(x)pIG(x, mu=1,sigma=.5), 0.01, 6,
    main = "{Inverse Gaussian cdf mu=1,sigma=0.5}")
```

IGAMMA

Inverse Gamma distribution for fitting a GAMLSS

## **Description**

The function IGAMMA() defines the Inverse Gamma distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(), with parameters mu (the mode) and sigma. The functions dIGAMMA, pIGAMMA and rIGAMMA define the density, distribution function, quantile function and random generation for the IGAMMA parameterization of the Inverse Gamma distribution.

# Usage

```
IGAMMA(mu.link = "log", sigma.link="log")
dIGAMMA(x, mu = 1, sigma = .5, log = FALSE)
pIGAMMA(q, mu = 1, sigma = .5, lower.tail = TRUE, log.p = FALSE)
qIGAMMA(p, mu = 1, sigma = .5, lower.tail = TRUE, log.p = FALSE)
rIGAMMA(n, mu = 1, sigma = .5)
```

# Arguments

| mu.link    | nu.link Defines the mu.link, with log link as the default for the mu parameter             |  |
|------------|--------------------------------------------------------------------------------------------|--|
| sigma.link | Defines the sigma.link, with log as the default for the sigma parameter                    |  |
| x, q       | vector of quantiles                                                                        |  |
| mu         | vector of location parameter values                                                        |  |
| sigma      | vector of scale parameter values                                                           |  |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                      |  |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise $P[X > x]$          |  |
| р          | vector of probabilities                                                                    |  |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |  |

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## **Details**

The parameterization of the Inverse Gamma distribution in the function IGAMMA is

$$f(y|\mu,\sigma) = \frac{\left[\mu\left(\alpha+1\right)\right]^{\alpha}}{\Gamma(\alpha)} y^{-(\alpha+1)} \exp\left[-\frac{\mu\left(\alpha+1\right)}{y}\right]$$

where  $alpha = 1/(sigma^2)$  for y > 0, mu > 0 and sigma > 0.

### Value

returns a gamlss.family object which can be used to fit an Inverse Gamma distribution in the gamlss() function.

### Note

For the function IGAMMA(), mu is the mode of the Inverse Gamma distribution.

### Author(s)

Fiona McElduff, Bob Rigby and Mikis Stasinopoulos.

### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, 54, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. 23, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

## See Also

```
gamlss.family, GA
```

## **Examples**

```
par(mfrow=c(2,2))
y<-seq(0.2,20,0.2)
plot(y, dIGAMMA(y), type="1")
q <- seq(0.2, 20, 0.2)
plot(q, pIGAMMA(q), type="1")
p<-seq(0.0001,0.999,0.05)
plot(p, qIGAMMA(p), type="1")
dat <- rIGAMMA(50)</pre>
```

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```
hist(dat)
#summary(gamlss(dat~1, family="IGAMMA"))
```

JSU

The Johnson's Su distribution for fitting a GAMLSS

# **Description**

This function defines the , a four parameter distribution, for a gamlss.family object to be used for a GAMLSS fitting using the function gamlss(). The functions dJSU, pJSU, qJSU and rJSU define the density, distribution function, quantile function and random generation for the the Johnson's Su distribution.

## Usage

```
 JSU(mu.link = "identity", sigma.link = "log", nu.link = "identity", tau.link = "log") \\ dJSU(x, mu = 0, sigma = 1, nu = 1, tau = 1, log = FALSE) \\ pJSU(q, mu = 0, sigma = 1, nu = 1, tau = 1, lower.tail = TRUE, log.p = FALSE) \\ qJSU(p, mu = 0, sigma = 1, nu = 1, tau = 1, lower.tail = TRUE, log.p = FALSE) \\ rJSU(n, mu = 0, sigma = 1, nu = 1, tau = 1)
```

# **Arguments**

| mu.link                                                                                                                                 | Defines the mu.link, with "identity" link as the default for the mu parameter. Other links are "inverse" "log" ans "own"    |
|-----------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------|
| sigma.link Defines the sigma.link, with "log" link as the default for the sigma paramet Other links are "inverse", "identity" ans "own" |                                                                                                                             |
| nu.link                                                                                                                                 | Defines the nu.link, with "identity" link as the default for the nu parameter. Other links are "onverse", "log" and "own"   |
| tau.link                                                                                                                                | Defines the tau.link, with "log" link as the default for the tau parameter. Other links are "onverse", "identity" ans "own" |
| x,q                                                                                                                                     | vector of quantiles                                                                                                         |
| mu                                                                                                                                      | vector of location parameter values                                                                                         |
| sigma                                                                                                                                   | vector of scale parameter values                                                                                            |
| nu                                                                                                                                      | vector of skewness nu parameter values                                                                                      |
| tau                                                                                                                                     | vector of kurtosis tau parameter values                                                                                     |
| log, log.p                                                                                                                              | logical; if TRUE, probabilities p are given as log(p).                                                                      |
| lower.tail                                                                                                                              | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                          |
| р                                                                                                                                       | vector of probabilities.                                                                                                    |
| n                                                                                                                                       | number of observations. If $length(n) > 1$ , the length is taken to be the number required                                  |

## **Details**

The probability density function of the Jonhson's SU distribution, (JSU), is defined as

$$f(y|n, \mu, \sigma \nu, \tau) == \frac{1}{c\sigma} \frac{1}{\tau(z^2+1)^{\frac{1}{2}}} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}r^2\right]$$

for 
$$-\infty < y < \infty$$
,  $\mu = (-\infty, +\infty)$ ,  $\sigma > 0$ ,  $\nu = (-\infty, +\infty)$  and  $\tau > 0$ . where  $r = -\nu + \frac{1}{\tau} \sinh^{-1}(z)$ ,  $z = \frac{y - (\mu + c\sigma w^{\frac{1}{2}} \sinh \Omega)}{c\sigma}$ ,  $c = [\frac{1}{2}(w - 1)(w \cosh 2\Omega + 1)]^{\frac{1}{2}}$ ,  $w = e^{\tau^2}$  and  $\Omega = -\nu\tau$ .

This is a reparameterization of the original Johnson Su distribution, Johnson (1954), so the parameters mu and sigma are the mean and the standard deviation of the distribution. The parameter nu determines the skewness of the distribution with nu>0 indicating positive skewness and nu<0 negative. The parameter tau determines the kurtosis of the distribution. tau should be positive and most likely in the region from zero to 1. As tau goes to 0 (and for nu=0) the distribution approaches the the Normal density function. The distribution is appropriate for leptokurtic data that is data with kurtosis larger that the Normal distribution one.

### Value

JSU() returns a gamlss.family object which can be used to fit a Johnson's Su distribution in the gamlss() function. dJSU() gives the density, pJSU() gives the distribution function, qJSU() gives the quantile function, and rJSU() generates random deviates.

### Warning

The function JSU uses first derivatives square in the fitting procedure so standard errors should be interpreted with caution

## Author(s)

Bob Rigby and Mikis Stasinopoulos

# References

Johnson, N. L. (1954). Systems of frequency curves derived from the first law of Laplace., *Trabajos de Estadistica*, **5**, 283-291.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R. A. and Akantziliotou C. (2006) Instructions on how to use the GAMLSS package in R. Accompanying documentation in the current GAMLSS help files, (see also https://www.gamlss.com/).

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

## See Also

```
gamlss.family, JSUo, BCT
```

## **Examples**

```
JSU()
plot(function(x)dJSU(x, mu=0,sigma=1,nu=-1, tau=.5), -4, 4,
    main = "The JSU density mu=0,sigma=1,nu=-1, tau=.5")
plot(function(x) pJSU(x, mu=0,sigma=1,nu=-1, tau=.5"), -4, 4,
    main = "The JSU cdf mu=0, sigma=1, nu=-1, tau=.5")
# library(gamlss)
# data(abdom)
# h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=JSU, data=abdom)</pre>
```

JSUo

The original Johnson's Su distribution for fitting a GAMLSS

### **Description**

This function defines the , a four parameter distribution, for a gamlss.family object to be used for a GAMLSS fitting using the function gamlss(). The functions dJSUo, pJSUo, qJSUo and rJSUo define the density, distribution function, quantile function and random generation for the the Johnson's Su distribution.

### Usage

```
JSUo(mu.link = "identity", sigma.link = "log", nu.link = "identity", tau.link = "log")
dJSUo(x, mu = 0, sigma = 1, nu = 0, tau = 1, log = FALSE)
pJSUo(q, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE, log.p = FALSE)
qJSUo(p, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE, log.p = FALSE)
rJSUo(n, mu = 0, sigma = 1, nu = 0, tau = 1)
```

# Arguments

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter. Other links are "inverse", "log" and "own"          |
|------------|------------------------------------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter.<br>Other links are "inverse", "identity" and "own" |
| nu.link    | Defines the nu.link, with "identity" link as the default for the nu parameter. Other links are "inverse", "log" ans "own"          |
| tau.link   | Defines the tau.link, with "log" link as the default for the tau parameter. Other links are "inverse", "identity" and "own"        |

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| x,q        | vector of quantiles                                                                        |
|------------|--------------------------------------------------------------------------------------------|
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| nu         | vector of skewness nu parameter values                                                     |
| tau        | vector of kurtosis tau parameter values                                                    |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| р          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

## **Details**

The probability density function of the orininal Jonhson's SU distribution, (JSU), is defined as

$$f(y|n, \mu, \sigma \nu, \tau) = \frac{\tau}{\sigma} \frac{1}{(z^2 + 1)^{\frac{1}{2}}} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}r^2\right]$$

for 
$$-\infty < y < \infty$$
,  $\mu = (-\infty, +\infty)$ ,  $\sigma > 0$ ,  $\nu = (-\infty, +\infty)$  and  $\tau > 0$ . where  $z = \frac{(y-\mu)}{\sigma}$ ,  $r = \nu + \tau sinh^{-1}(z)$ .

# Value

JSUo() returns a gamlss.family object which can be used to fit a Johnson's Su distribution in the gamlss() function. dJSUo() gives the density, pJSUo() gives the distribution function, qJSUo() gives the quantile function, and rJSUo() generates random deviates.

# Warning

The function JSU uses first derivatives square in the fitting procedure so standard errors should be interpreted with caution. It is recomented to be used only with method=mixed(2, 20)

## Author(s)

Mikis Stasinopoulos and Bob Rigby

# References

Johnson, N. L. (1954). Systems of frequency curves derived from the first law of Laplace., *Trabajos de Estadistica*, **5**, 283-291.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

LG

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family, JSU, BCT
```

# **Examples**

LG

Logarithmic and zero adjusted logarithmic distributions for fitting a GAMLSS model

### **Description**

The function LG defines the logarithmic distribution, a one parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dLG, pLG, qLG and rLG define the density, distribution function, quantile function and random generation for the logarithmic, LG(), distribution.

The function ZALG defines the zero adjusted logarithmic distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZALG, pZALG and rZALG define the density, distribution function, quantile function and random generation for the inflated logarithmic, ZALG(), distribution.

# Usage

```
LG(mu.link = "logit")
dLG(x, mu = 0.5, log = FALSE)
pLG(q, mu = 0.5, lower.tail = TRUE, log.p = FALSE)
qLG(p, mu = 0.5, lower.tail = TRUE, log.p = FALSE, max.value = 10000)
rLG(n, mu = 0.5)
ZALG(mu.link = "logit", sigma.link = "logit")
```

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```
dZALG(x, mu = 0.5, sigma = 0.1, log = FALSE)
pZALG(q, mu = 0.5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
qZALG(p, mu = 0.5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
rZALG(n, mu = 0.5, sigma = 0.1)
```

## **Arguments**

mu.link defines the mu.link, with logit link as the default for the mu parameter defines the sigma.link, with logit link as the default for the sigma parameter sigma.link which in this case is the probability at zero. vector of (non-negative integer) Χ mu vector of positive means vector of probabilities at zero sigma vector of probabilities vector of quantiles number of random values to return log, log.p logical; if TRUE, probabilities p are given as log(p) lower.tail logical; if TRUE (default), probabilities are  $P[X \le x]$ , otherwise, P[X > x]max.value valued needed for the numerical calculation of the q-function

### **Details**

For the definition of the distributions see Rigby and Stasinopoulos (2010) below.

The parameterization of the logarithmic distribution in the function LM is

$$f(y|\mu) = \alpha \mu^y/y$$
 where for  $y >= 1$  and  $\mu > 0$  and 
$$\alpha = -[\log(1-\mu)]^{-1}$$

## Value

The function LG and ZALG return a gamlss.family object which can be used to fit a logarithmic and a zero inflated logarithmic distributions respectively in the gamlss() function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby

### References

Johnson, Norman Lloyd; Kemp, Adrienne W; Kotz, Samuel (2005). "Chapter 7: Logarithmic and Lagrangian distributions". Univariate discrete distributions (3 ed.). John Wiley & Sons. ISBN 9780471272465.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

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Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Rigby, R. A. and Stasinopoulos D. M. (2010) The gamlss.family distributions, (distributed with this package or see <a href="https://www.gamlss.com/">https://www.gamlss.com/</a>)

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

### See Also

```
gamlss.family, PO, ZAP
```

# **Examples**

```
LG()
ZAP()
# creating data and plotting them
dat <- rLG(1000, mu=.3)
   r <- barplot(table(dat), col='lightblue')
dat1 <- rZALG(1000, mu=.3, sigma=.1)
   r1 <- barplot(table(dat1), col='lightblue')</pre>
```

Log Normal distribution for fitting in GAMLSS

## **Description**

LN<sub>0</sub>

The functions LOGNO and LOGNO2 define a gamlss. family distribution to fits the log-Normal distribution. The difference between them is that while LOGNO retains the original parametrization for mu, (identical to the normal distribution NO) and therefore  $\mu=(-\infty,+\infty)$ , the function LOGNO2 use mu as the median, so  $\mu=(0,+\infty)$ .

The function LNO is more general and can fit a Box-Cox transformation to data using the gamlss() function. In the LOGNO (and LOGNO2) there are two parameters involved mu sigma, while in the LNO there are three parameters mu sigma, and the transformation parameter nu. The transformation parameter nu in LNO is a 'fixed' parameter (not estimated) and it has its default value equal to zero allowing the fitting of the log-normal distribution as in LOGNO. See the example below on how to fix nu to be a particular value. In order to estimate (or model) the parameter nu, use the gamlss.family BCCG distribution which uses a reparameterized version of the the Box-Cox transformation. The functions dLOGNO, pLOGNO, qLOGNO and rLOGNO define the density, distribution function, quantile function and random generation for the specific parameterization of the log-normal distribution.

The functions dLOGNO2, pLOGNO2, qLOGNO2 and rLOGNO2 define the density, distribution function, quantile function and random generation when mu is the median of the log-normal distribution.

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The functions dLNO, pLNO, qLNO and rLNO define the density, distribution function, quantile function and random generation for the specific parameterization of the log-normal distribution and more generally a Box-Cox transformation.

# Usage

```
LNO(mu.link = "identity", sigma.link = "log")

LOGNO(mu.link = "identity", sigma.link = "log")

LOGNO2(mu.link = "log", sigma.link = "log")

dLNO(x, mu = 1, sigma = 0.1, nu = 0, log = FALSE)

dLOGNO(x, mu = 0, sigma = 1, log = FALSE)

dLOGNO2(x, mu = 1, sigma = 1, log = FALSE)

pLNO(q, mu = 1, sigma = 0.1, nu = 0, lower.tail = TRUE, log.p = FALSE)

pLOGNO(q, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)

pLOGNO2(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)

qLNO(p, mu = 1, sigma = 0.1, nu = 0, lower.tail = TRUE, log.p = FALSE)

qLOGNO(p, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)

qLOGNO2(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)

rLNO(n, mu = 1, sigma = 0.1, nu = 0)

rLOGNO2(n, mu = 0, sigma = 1)
```

# **Arguments**

| .link Defines the mu.link, with "identity" or "log" link depending on te parametri                                              |  |
|---------------------------------------------------------------------------------------------------------------------------------|--|
| Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse", "identity" ans "own" |  |
| vector of quantiles                                                                                                             |  |
| vector of location parameter values                                                                                             |  |
| vector of scale parameter values                                                                                                |  |
| vector of shape parameter values                                                                                                |  |
| logical; if TRUE, probabilities p are given as log(p).                                                                          |  |
| logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                              |  |
| vector of probabilities.                                                                                                        |  |
| number of observations. If $length(n) > 1$ , the length is taken to be the number required                                      |  |
|                                                                                                                                 |  |

# **Details**

The probability density function in LOGNO is defined as

$$f(y|\mu,\sigma) = \frac{1}{y\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2\sigma^2}(\log(y) - \mu)^2\right]$$

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for y > 0,  $\mu = (-\infty, +\infty)$  and  $\sigma > 0$ .

The probability density function in LNO is defined as

$$f(y|\mu, \sigma, \nu) = \frac{1}{\sqrt{2\pi}\sigma} y^{\nu-1} \exp[-\frac{1}{2\sigma^2} (z-\mu)^2]$$

where if  $\nu \neq 0$   $z = (y^{\nu} - 1)/\nu$  else  $z = \log(y)$  and  $z \sim N(0, \sigma^2)$ , for y > 0,  $\mu > 0$ ,  $\sigma > 0$  and  $\nu = (-\infty, +\infty)$ .

### Value

LNO() returns a gamlss.family object which can be used to fit a log-normal distribution in the gamlss() function. dLNO() gives the density, pLNO() gives the distribution function, qLNO() gives the quantile function, and rLNO() generates random deviates.

### Warning

This is a two parameter fit for  $\mu$  and  $\sigma$  while  $\nu$  is fixed. If you wish to model  $\nu$  use the gamlss family BCCG.

#### Note

 $\mu$  is the mean of z (and also the median of y), the Box-Cox transformed variable and  $\sigma$  is the standard deviation of z and approximate the coefficient of variation of y

### Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

### References

Box, G. E. P. and Cox, D. R. (1964) An analysis of transformations (with discussion), *J. R. Statist. Soc.* B., **26**, 211–252

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

gamlss.family, BCCG

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## **Examples**

```
LOGNO()# gives information about the default links for the log normal distribution
LOGNO2()
LNO()# gives information about the default links for the Box Cox distribution
# plotting the d, p, q, and r functions
op<-par(mfrow=c(2,2))</pre>
curve(dLOGNO(x, mu=0), 0, 10)
curve(pLOGNO(x, mu=0), 0, 10)
curve(qLOGNO(x, mu=0), 0, 1)
Y<- rLOGNO(200)
hist(Y)
par(op)
# plotting the d, p, q, and r functions
op<-par(mfrow=c(2,2))
curve(dLOGNO2(x, mu=1), 0, 10)
curve(pLOGNO2(x, mu=1), 0, 10)
curve(qLOGNO2(x, mu=1), 0, 1)
Y<- rLOGNO(200)
hist(Y)
par(op)
# library(gamlss)
# data(abdom)
\# h1<-gamlss(y~cs(x), family=LOGNO, data=abdom)#fits the log-Normal distribution
\# h2<-gamlss(y~cs(x), family=LNO, data=abdom) \#should be identical to the one above
# to change to square root transformation, i.e. fix nu=0.5
# h3<-gamlss(y~cs(x), family=LNO, data=abdom, nu.fix=TRUE, nu.start=0.5)
```

Logistic distribution for fitting a GAMLSS

## **Description**

L0

The function LO(), or equivalently Logistic(), defines the logistic distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss()

# Usage

```
LO(mu.link = "identity", sigma.link = "log")
dLO(x, mu = 0, sigma = 1, log = FALSE)
pLO(q, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qLO(p, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rLO(n, mu = 0, sigma = 1)
```

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## Arguments

mu.link Defines the mu.link, with "identity" link as the default for the mu parameter Defines the sigma.link, with "log" link as the default for the sigma parameter sigma.link vector of quantiles x,q vector of location parameter values mu vector of scale parameter values sigma log, log.p logical; if TRUE, probabilities p are given as log(p). logical; if TRUE (default), probabilities are  $P[X \le x]$ , otherwise, P[X > x]lower.tail vector of probabilities. р number of observations. If length(n) > 1, the length is taken to be the number n required

### **Details**

Definition file for Logistic distribution.

$$f(y|\mu,\sigma) = \frac{1}{\sigma}e^{-\frac{y-\mu}{\sigma}}[1 + e^{-\frac{y-\mu}{\sigma}}]^{-2}$$

for 
$$y=(-\infty,\infty)$$
,  $\mu=(-\infty,\infty)$  and  $\sigma>0$ .

## Value

LO() returns a gamlss.family object which can be used to fit a logistic distribution in the gamlss() function. dLO() gives the density, pLO() gives the distribution function, qLO() gives the quantile function, and rLO() generates random deviates for the logistic distribution. The latest functions are based on the equivalent R functions for logistic distribution.

# Note

 $\mu$  is the mean and  $\sigma\pi/\sqrt{3}$  is the standard deviation for the logistic distribution

### Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

# References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

### See Also

```
gamlss.family, NO, TF
```

## **Examples**

```
LO()# gives information about the default links for the Logistic distribution plot(function(y) dLO(y, mu=10 ,sigma=2), 0, 20) plot(function(y) pLO(y, mu=10 ,sigma=2), 0, 20) plot(function(y) qLO(y, mu=10 ,sigma=2), 0, 1) # library(gamlss) # data(abdom) # h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=LO, data=abdom) # fits # plot(h)
```

**LOGITNO** 

Logit Normal distribution for fitting in GAMLSS

## **Description**

The functions dLOGITNO, pLOGITNO, qLOGITNO and rLOGITNO define the density, distribution function, quantile function and random generation for the logit-normal distribution. The function LOGITNO can be used for fitting the distribution in gamlss().

## Usage

```
LOGITNO(mu.link = "logit", sigma.link = "log")
dLOGITNO(x, mu = 0.5, sigma = 1, log = FALSE)
pLOGITNO(q, mu = 0.5, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qLOGITNO(p, mu = 0.5, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rLOGITNO(n, mu = 0.5, sigma = 1)
```

## **Arguments**

| mu.link    | the link function for mu                                                                   |
|------------|--------------------------------------------------------------------------------------------|
| sigma.link | the link function for sigma                                                                |
| x,q        | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| р          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

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## **Details**

The probability density function in LOGITNO is defined as

$$f(y|\mu,\sigma) = \frac{1}{y(1-y)\sqrt{2\pi}\sigma} \exp[-\frac{1}{2\sigma^2}(\log(y/(1-y)) - \log(\mu/(1-\mu))^2]$$

for 
$$0 < y > 1$$
,  $\mu = (0, 1 \text{ and } \sigma > 0$ .

#### Value

LOGITNO() returns a gamlss.family object which can be used to fit a logit-normal distribution in the gamlss() function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, \doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

## See Also

```
gamlss.family, LOGNO
```

# **Examples**

```
# plotting the d, p, q, and r functions
op<-par(mfrow=c(2,2))
curve(dLOGITNO(x), 0, 1)
curve(pLOGITNO(x), 0, 1)
curve(qLOGITNO(x), 0, 1)
Y<- rLOGITNO(200)
hist(Y)
par(op)

# plotting the d, p, q, and r functions
# sigma 3
op<-par(mfrow=c(2,2))
curve(dLOGITNO(x, sigma=3), 0, 1)</pre>
```

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```
curve(pLOGITNO(x, sigma=3), 0, 1)
curve(qLOGITNO(x, sigma=3), 0, 1)
Y<- rLOGITNO(200, sigma=3)
hist(Y)
par(op)</pre>
```

**LQNO** 

Normal distribution with a specific mean and variance relationship for fitting a GAMLSS model

# Description

The function LQNO() defines a normal distribution family, which has a specific mean and variance relationship. The distribution can be used in a GAMLSS fitting using the function gamlss(). The mean of LQNO is equal to mu. The variance is equal to mu\*(1+sigma\*mu) so the standard deviation is sqrt(mu\*(1+sigma\*mu)). The function is found useful in modelling small RNA sequencing experiments. The functions dLQNO, pLQNO, qLQNO and rLQNO define the density, distribution function, quantile function (inverse cdf) and random generation for the LQNO() parametrization of the normal distribution.

### Usage

```
LQNO(mu.link = "log", sigma.link = "log")
dLQNO(x, mu = 1, sigma = 1, log = FALSE)
pLQNO(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qLQNO(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rLQNO(n, mu = 1, sigma = 1)
```

## **Arguments**

```
mu link function with "log" as default
mu.link
sigma.link
                   mu link function with "log" as default
                   vector of quantiles
x,q
mu
                   vector of location parameter values
sigma
                   vector of scale parameter values
log, log.p
                   logical; if TRUE, probabilities p are given as log(p)
                   if TRUE (default), probabilities are P[X \le x], otherwise, P[X > x].
lower.tail
                   vector of probabilities
p
                   number of observations. If length(n) > 1, the length is taken to be the number
n
                   required
```

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### **Details**

LQNO stands for Linear Quadratic Normal Family, in which the variance is a linear quadratic function of the mean: Var(Y) = mu\*(1+sigma\*mu). This is created to facilitate the analysis of data coming from small RNA sequencing experiments, basically counts of short RNAs that one isolates from cells or biofluids such as urine, plasma or cerebrospinal fluid. Argyropoulos *et al.* (2017) showing that the LQNO distribution (and the Negative Binomial which implements the same mean-variance relationship) are highly accurate approximations to the generative models of the signals in these experiments

### Value

The function LQNO returns a gamlss.family object which can be used to fit this specific form of the normal distribution family in the gamlss() function.

### Note

The mu parameters must be positive so for the relationship Var(Y) = mu\*(1+sigma\*mu) to be valid.

### Author(s)

Christos Argyropoulos

# References

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Argyropoulos C, Etheridge A, Sakhanenko N, Galas D. (2017) Modeling bias and variation in the stochastic processes of small RNA sequencing. *Nucleic Acids Res.* 2017 Mar 27. doi: 10.1093/nar/gkx199. [Epub ahead of print] PubMed PMID: 28369495.

### See Also

NO,NO2, NOF

## **Examples**

```
LQNO()# gives information about the default links for the normal distribution # a comparison of different Normal models #m1 <- gamlss(y~pb(x), sigma.fo=~pb(x), data=abdom, family=NO(mu.link="log")) #m2 <- gamlss(y~pb(x), sigma.fo=~pb(x), data=abdom, family=LQNO) #m3 <- gamlss(y~pb(x), sigma.fo=~pb(x), data=abdom, family=NOF(mu.link="log")) #AIC(m1,m2,m3)
```

make.link.gamlss

Create a Link for GAMLSS families

# Description

The function make.link.gamlss() is used with gamlss.family distributions in package gamlss(). Given a link, it returns a link function, an inverse link function, the derivative dpar/deta where 'par' is the appropriate distribution parameter and a function for checking the domain. It differs from the usual make.link of glm() by having extra links as the logshifto1, and the own. For the use of the own link see the example bellow. show.link provides a way in which the user can identify the link functions available for each gamlss distribution. If your required link function is not available for any of the gamlss distributions you can add it in.

## Usage

```
make.link.gamlss(link)
show.link(family = "NO")
```

### **Arguments**

link character or numeric; one of "logit", "probit", "cloglog", "identity",

"log", "sqrt", "1/mu^2", "inverse", "logshifted", "logitshifted", or

number, say lambda resulting in power link  $\mu^{\lambda}$ .

family a gamlss distribution family

# Details

The own link function is added to allow the user greater flexibility. In order to used the own link function for any of the parameters of the distribution the own link should appear in the available links for this parameter. You can check this using the function show.link. If the own do not appear in the list you can create a new function for the distribution in which own is added in the list. For example the first line of the code of the binomial distribution, BI, has change from

"mstats <- checklink("mu.link", "Binomial", substitute(mu.link), c("logit", "probit", "cloglog", "log")), in version 1.0-0 of gamlss, to

"mstats <- checklink("mu.link", "Binomial", substitute(mu.link), c("logit", "probit", "cloglog", "log", "own"))

in version 1.0-1. Given that the parameter has own as an option the user needs also to define the following four new functions in order to used an own link.

- i) own.linkfun
- ii) own.linkinv
- iii) own.mu.eta and
- iv) own.valideta.

An example is given below.

Only one parameter of the distribution at a time is allowed to have its own link, (unless the same four own functions above are suitable for more that one parameter of the distribution).

Note that from **gamlss** version 1.9-0 the user can introduce its own link function by define an appropriate function, (see the example below).

### Value

For the make.link.gamlss a list with components

 $link function \ function \ (parameter)$ 

linkinv: Inverse link function function(eta)

mu.eta: Derivative function(eta) dparameter/deta

valideta: function(eta) TRUE if all of eta is in the domain of linkinv.

For the show. link a list with components the available links for the distribution parameters

### Note

For the links involving parameters as in logshifted and logitshifted the parameters can be passed in the definition of the distribution by calling the checklink function, for example in the definition of the tau parameter in BCPE distribution the following call is made: tstats <-checklink(
"tau.link", "Box Cox Power Exponential", substitute(tau.link), c("logshifted", "log", "identity"), par.link = c(1))

# Author(s)

Mikis Stasinopoulos and Bob Rigby

### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

### See Also

```
gamlss.family
```

## **Examples**

```
str(make.link.gamlss("logshiftto1"))
12<-make.link.gamlss("logshiftto1")</pre>
12$linkfun(2) # should close to zero (Note that 0.00001 is added)
12$linkfun(1-0.00001) # should be -Inf but it is large negative
#-----
# now use the own link function
# first if the distribution allows you
show.link(BI)
# seems OK now define the four own functions
# First try the probit link using the own link function
# 1: the linkfun function
own.linkfun <- function(mu) { qNO(p=mu)}</pre>
# 2: the inverse link function
own.linkinv <- function(eta) {</pre>
              thresh <- -qNO(.Machine$double.eps)</pre>
               eta <- pmin(thresh, pmax(eta, -thresh))</pre>
              pNO(eta)}
# 3: the dmu/deta function
own.mu.eta <- function(eta) pmax(dNO(eta), .Machine$double.eps)</pre>
# 4: the valideta function
own.valideta <- function(eta) TRUE
## bring the data
# library(gamlss)
#data(aep)
# fitting the model using "own"
# h1<-gamlss(y~ward+loglos+year, family=BI(mu.link="own"), data=aep)</pre>
# model h1 should be identical to the probit
# h2<-gamlss(y~ward+loglos+year, family=BI(mu.link="probit"), data=aep)</pre>
# now using a function instead of "own"
probittest <- function()</pre>
linkfun <- function(mu) { gNO(p=mu)}</pre>
linkinv <- function(eta)</pre>
              thresh <- -qNO(.Machine$double.eps)</pre>
               eta <- pmin(thresh, pmax(eta, -thresh))</pre>
              pNO(eta)
mu.eta <- function(eta) pmax(dNO(eta), .Machine$double.eps)</pre>
valideta <- function(eta) TRUE</pre>
link <- "probitTest"</pre>
structure(list(linkfun = linkfun, linkinv = linkinv, mu.eta = mu.eta,
        valideta = valideta, name = link), class = "link-gamlss")
# h3<-gamlss(y~ward+loglos+year, family=BI(mu.link=probittest()), data=aep)</pre>
# Second try the complementary log-log
# using the Gumbel distribution
own.linkfun <- function(mu) { qGU(p=mu)}</pre>
own.linkinv <- function(eta) {</pre>
              thresh <- -qGU(.Machine$double.eps)</pre>
```

```
eta <- pmin(thresh, pmax(eta, -thresh))</pre>
               pGU(eta)}
own.mu.eta <- function(eta) pmax(dGU(eta), .Machine$double.eps)</pre>
own.valideta <- function(eta) TRUE</pre>
# h1 and h2 should be identical to cloglog
# h1<-gamlss(y~ward+loglos+year, family=BI(mu.link="own"), data=aep)</pre>
# h2<-gamlss(y~ward+loglos+year, family=BI(mu.link="cloglog"), data=aep)</pre>
# note that the Gumbel distribution is negatively skew
# for a positively skew link function we can used the Reverse Gumbel
revloglog <- function()</pre>
linkfun <- function(mu) { qRG(p=mu)}</pre>
linkinv <- function(eta) {</pre>
               thresh <- -qRG(.Machine$double.eps)</pre>
                eta <- pmin(thresh, pmax(eta, -thresh))</pre>
               pRG(eta)}
mu.eta <- function(eta) pmax(dRG(eta), .Machine$double.eps)</pre>
valideta <- function(eta) TRUE</pre>
link <- "revloglog"</pre>
structure(list(linkfun = linkfun, linkinv = linkinv, mu.eta = mu.eta,
        valideta = valideta, name = link), class = "link-gamlss")
# h1<-gamlss(y~ward+loglos+year, family=BI(mu.link=revloglog()), data=aep)</pre>
# a considerable improvement in the deviance
# try a shifted logit link function from -1, 1
own.linkfun <- function(mu)</pre>
             \{ \text{ shift = } c(-1,1) \}
                log((mu-shift[1])/(shift[2]-mu))
own.linkinv <- function(eta)</pre>
             shift = c(-1,1)
             thresh <- -log(.Machine$double.eps)</pre>
                eta <- pmin(thresh, pmax(eta, -thresh))</pre>
                        shift[2]-(shift[2]-shift[1])/(1 + exp(eta))
             }
own.mu.eta <- function(eta)</pre>
        shift = c(-1,1)
             thresh <- -log(.Machine$double.eps)</pre>
                res <- rep(.Machine$double.eps, length(eta))</pre>
             res[abs(eta) < thresh] <- ((shift[2]-shift[1])*exp(eta)/(1 +</pre>
                                    exp(eta))^2)[abs(eta) < thresh]</pre>
             res
             }
own.valideta <- function(eta) TRUE
str(make.link.gamlss("own"))
12<-make.link.gamlss("own")</pre>
12$linkfun(0) # should be zero
12$linkfun(1) # should be Inf
12$linkinv(-5:5)
```

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Multinomial distribution in GAMLSS

MN3

# **Description**

The set of function presented here is useful for fitting multinomial regression within gamlss.

# Usage

```
MN3(mu.link = "log", sigma.link = "log")
MN4(mu.link = "log", sigma.link = "log", nu.link = "log")
MN5(mu.link = "log", sigma.link = "log", nu.link = "log", tau.link = "log")
MULTIN(type = "3")
fittedMN(model)
dMN3(x, mu = 1, sigma = 1, log = FALSE)
dMN4(x, mu = 1, sigma = 1, nu = 1, log = FALSE)
dMN5(x, mu = 1, sigma = 1, nu = 1, tau = 1, log = FALSE)
pMN3(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
pMN4(q, mu = 1, sigma = 1, nu = 1, lower.tail = TRUE, log.p = FALSE)
pMN5(q, mu = 1, sigma = 1, nu = 1, tau = 1, lower.tail = TRUE, log.p = FALSE)
qMN3(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qMN4(p, mu = 1, sigma = 1, nu = 1, lower.tail = TRUE, log.p = FALSE)
qMN5(p, mu = 1, sigma = 1, nu = 1, tau = 1, lower.tail = TRUE, log.p = FALSE)
rMN3(n, mu = 1, sigma = 1)
rMN4(n, mu = 1, sigma = 1, nu = 1)
rMN5(n, mu = 1, sigma = 1, nu = 1, tau = 1)
```

# **Arguments**

```
the link function for mu
mu.link
                   the link function for sigma
sigma.link
nu.link
                   the link function for nu
tau.link
                   the link function for tau
                   the x variable
Х
                   vector of quantiles
q
                   vector of probabilities
lower.tail
                   logical; if TRUE (default), probabilities are P[X \le x] otherwise, P[X > x].
                   logical; if TRUE, probabilities p are given as log(p).
log.p
                   logical; if TRUE, probabilities p are given as log(p).
log
                   the number of observations
n
```

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| mu    | the mu parameter                               |
|-------|------------------------------------------------|
| sigma | the sigma parameter                            |
| nu    | the nu parameter                               |
| tau   | the tau parameter                              |
| type  | permitted values are 2 (Binomial), 3, 4, and 5 |
| model | a gamlss multinomial fitted model              |

### **Details**

GAMLSS is in general not suitable for multinomial regression. Nevertheless multinomial regression can be fitted within GAMLSS if the response variable y has less than five categories. The function here provide the facilities to do so. The functions MN3(), MN4() and MN5() fit multinomial responses with 3, 4 and 5 categories respectively. The function MULTIN() can be used instead of codeMN3(), MN4() and MN5() by specifying the number of levels of the response. Note that MULTIN(2) will produce a binomial fit.

### Value

returns a gamlss.family object which can be used to fit a binomial distribution in the gamlss() function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Vlasios Voudouris

### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

### See Also

```
gamlss.family, BI
```

106 momentSK

## **Examples**

dMN3(3) pMN3(2) qMN3(.6) rMN3(10)

momentSK

Sample and theoretical Moment and Centile Skewness and Kurtosis Functions

# **Description**

The functions momentSK(), centileSK(), centileSkew() and centileKurt(), calculate sample statistics related to skewness and kurtosis. The function theoCentileSK() calculates the theoretical centile statistics from a given gamlss.family distribution. The plotCentileSK() plots the theoretical centile skewness and kurtosis against p (see below).

The function checkMomentSK() can be use to check (a) whether the moment skewness and kurtosis of a fitted model are modelled adequantly (the residuals of the model are used). (b) whether a given sample display skewness or kurtosis.

## Usage

```
momentSK(x, weights=NULL)
centileSK(x, cent = c(1, 25), weights=NULL)
centileSkew(x, cent = 1, weights=NULL)
centileKurt(x, cent = 1, weights=NULL)
theoCentileSK(fam = "NO", p = 0.01, ...)
plotCentileSK(fam = "NO", plotting = c("skew", "kurt", "standKurt"),
             add = FALSE, col = 1, lty = 1, lwd = 1, ylim = NULL, ...)
checkMomentSK(x, weights=NULL, add = FALSE, bootstrap = TRUE, no.bootstrap = 99,
               col.bootstrap = "lightblue", pch.bootstrap = 21,
               asCharacter = TRUE, col.point = "black", pch.point = 4,
               lwd.point = 2, text.to.show = NULL, cex.text = 1.5,
               col.text = "black", show.legend = TRUE)
checkCentileSK(x,weights=NULL, type = c("central", "tail"), add = FALSE,
              bootstrap = TRUE, no.bootstrap = 99,
              col.bootstrap = "lightblue", pch.bootstrap = 21,
              asCharacter = TRUE, col.point = "black", pch.point = 4,
              lwd.point = 2, text.to.show = NULL, cex.text = 1.5,
              col.text = "black", show.legend = TRUE)
```

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### **Arguments**

x data vector or gamlss modelweights prior weights for the x

cent the centile required

type For centile skewness and kurtosis only whether "central" (default) or "tail")

fam A gamlss distribution family

plotting what to plot

add whether to add the line to the existing plot

col the colour of the line
lty the type of the line
lwd the width of the line
ylim the y limit of the graph

p the value determining the centile skewness or kurtosis

additional arguments pass to theoCentileSK() function i.e. the values of the

distribution parameters

bootstrap whether a plot of the bootstrap skewness and kurtosis measures should be added

in the plot

no.bootstrap the number of boostrap skewness and kurtosis measures

col.bootstrap the coloue for boostraps
pch.bootstrap the point type of boostraps

asCharacter whether to plot the estimated skewness and kurtosis measure as character or as

point

col.point the colour of the skewness and kurtosis measure
pch.point the point type of the skewness and kurtosis measure

lwd.point the width of the plotted point

text.to.show to display text different from variable or model

cex.text the size of the text
col.text the colour of the text

show.legend whether to show the legent

## **Details**

Those function calculate sample moment and centile skewness and kurtosis statistics and theoretical centile values for a specific distribution.

## Value

Different functions produce different output: The function momentSK() produce:

mom. skew: sample moment skewness

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trans.mom.skew:

sample transformed moment skewness

mom.kurt: sample moment kurtosis

excess.mom.kurt:

sample excess moment kurtosis

trans.mom.kurt:

sample ransformed moment excess kurtosis

jarque.bera.test:

the value of the Jarque-bera test for testing whether skewness and excess kurtosis

are zero or not

The function centileSK() produces:

S0.25: sample centile central skewness

S0.01: sample centile tail skewness

K0.01: sample centile kurtosis

standK0.01: standardised centile kurtosis, (K0.01/3.449)

exc.K0.01: excess centile kurtosis, (K0.01-3.449)

trans.K0.01: transfored excess centile kurtosis, (exc.K0.01/(1+abs(exc.K0.01))

The function centileSkew() for a given argument p produces:

p: the value determining the centile skewness

Sp: sample centile skewness at p

The function centileKurt() for a given argument p produces:

p the value determining the centile kurtosis

Kp sample centile kurtosis at p

sKp sample standardised centile kurtosis at p

ex.Kp: sample excess centile kurtosis at p

teKp: sample transformed excess centile kurtosis at p

The function theoCentileSK for a given gamlss.family produces:

IR the interquartile range of the distribution

SIR the semi interquartile range of the distribution

S\_0.25 the central skewness of the distribution

S\_0.01: the tail skewness of the distribution

K\_0.01: the centile kurtosis of the distribution

sK\_0.01: the standardised centile kurtosis of the distribution

## Author(s)

Mikis Stasinopoulos, Bobert Rigby, Gillain Heller and Fernanda De Bastiani.

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#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

### See Also

```
gamlss.family
```

## **Examples**

```
Y <- rSEP3(1000)
momentSK(Y)
centileSK(Y)
centileSkew(Y, cent=20)
centileKurt(Y, cent=30)

theoCentileSK("BCCG", mu=2, sigma=.2, nu=2)
plotCentileSK(fam="BCCG", mu=2, sigma=.2, nu=2)
checkMomentSK(Y)
checkCentileSK(Y)
checkCentileSK(Y, type="tail")</pre>
```

NBF

Negative Binomial Family distribution for fitting a GAMLSS

## **Description**

The NBF() function defines the Negative Binomial family distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dNBF, pNBF, qNBF and rNBF define the density, distribution function, quantile function and random generation for the negative binomial family, NBF(), distribution.

The functions dZINBF, pZINBF and rZINBF define the density, distribution function, quantile function and random generation for the zero inflated negative binomial family, ZINBF(), distribution a four parameter distribution.

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### Usage

```
NBF(mu.link = "log", sigma.link = "log", nu.link = "log")
dNBF(x, mu = 1, sigma = 1, nu = 2, log = FALSE)
pNBF(q, mu = 1, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)
qNBF(p, mu = 1, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)
rNBF(n, mu = 1, sigma = 1, nu = 2)

ZINBF(mu.link = "log", sigma.link = "log", nu.link = "log", tau.link = "logit")

dZINBF(x, mu = 1, sigma = 1, nu = 2, tau = 0.1, log = FALSE)
pZINBF(q, mu = 1, sigma = 1, nu = 2, tau = 0.1, lower.tail = TRUE, log.p = FALSE)

qZINBF(p, mu = 1, sigma = 1, nu = 2, tau = 0.1, lower.tail = TRUE, log.p = FALSE)
```

## Arguments

| mu.link    | The link function for mu                                                           |
|------------|------------------------------------------------------------------------------------|
| sigma.link | The link function for sigma                                                        |
| nu.link    | The link function for nu                                                           |
| tau.link   | The link function for tau                                                          |
| X          | vector of (non-negative integer)                                                   |
| mu         | vector of positive means                                                           |
| sigma      | vector of positive dispersion parameter                                            |
| nu         | vector of power parameter                                                          |
| tau        | vector of inflation parameter                                                      |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                              |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$ |
| р          | vector of probabilities                                                            |
| q          | vector of quantiles                                                                |
| n          | number of random values to return                                                  |

## **Details**

The definition for Negative Binomial Family distribution , NBF, is similar to the Negative Binomial type I. The probability function of the NBF can be obtained by replacing  $\sigma$  with  $\sigma\mu^{\nu-2}$  where  $\nu$  is a power parameter. The distribution has mean  $\mu$  and variance  $\mu + \sigma\mu^{\nu}$ .

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### Value

returns a gamlss.family object which can be used to fit a Negative Binomial Family distribution in the gamlss() function.

## Author(s)

Bob Rigby and Mikis Stasinopoulos

#### References

Anscombe, F. J. (1950) Sampling theory of the negative binomial and logarithmic distributions, *Biometrika*, **37**, 358-382.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

#### See Also

NBI, NBII

```
NBF() # default link functions for the Negative Binomial Family
# plotting the distribution
plot(function(y) dNBF(y, mu = 10, sigma = 0.5, nu=2), from=0,
     to=40, n=40+1, type="h")
# creating random variables and plot them
tN <- table(Ni <- rNBF(1000, mu=5, sigma=0.5, nu=2))
r <- barplot(tN, col='lightblue')</pre>
# zero inflated NBF
ZINBF() # default link functions for the zero inflated NBF
# plotting the distribution
plot(function(y) dZINBF(y, mu = 10, sigma = 0.5, nu=2, tau=.1),
     from=0, to=40, n=40+1, type="h")
# creating random variables and plot them
tN <- table(Ni <- rZINBF(1000, mu=5, sigma=0.5, nu=2, tau=0.1))
r <- barplot(tN, col='lightblue')</pre>
## Not run:
library(gamlss)
data(species)
species <- transform(species, x=log(lake))</pre>
m6 <- gamlss(fish~poly(x,2), sigma.fo=~1, data=species, family=NBF,
          n.cyc=200)
fitted(m6, "nu")[1]
```

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```
## End(Not run)
```

NBI

Negative Binomial type I distribution for fitting a GAMLSS

### **Description**

The NBI() function defines the Negative Binomial type I distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dNBI, pNBI, qNBI and rNBI define the density, distribution function, quantile function and random generation for the Negative Binomial type I, NBI(), distribution.

## Usage

```
NBI(mu.link = "log", sigma.link = "log")
dNBI(x, mu = 1, sigma = 1, log = FALSE)
pNBI(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qNBI(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rNBI(n, mu = 1, sigma = 1)
```

### **Arguments**

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter           |
|------------|------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter     |
| X          | vector of (non-negative integer) quantiles                                         |
| mu         | vector of positive means                                                           |
| sigma      | vector of positive despersion parameter                                            |
| р          | vector of probabilities                                                            |
| q          | vector of quantiles                                                                |
| n          | number of random values to return                                                  |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                              |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$ |

### **Details**

Definition file for Negative Binomial type I distribution.

$$P(Y=y|\mu,\sigma) = \frac{\Gamma(y+\frac{1}{\sigma})}{\Gamma(\frac{1}{\sigma})\Gamma(y+1)} \ \left(\frac{\sigma\mu}{1+\sigma\mu}\right)^y \ \left(\frac{1}{1+\sigma\mu}\right)^{1/\sigma}$$

for  $y=0,1,2,\ldots,\infty$ ,  $\mu>0$  and  $\sigma>0$ . This parameterization is equivalent to that used by Anscombe (1950) except he used  $\alpha=1/\sigma$  instead of  $\sigma$ .

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### Value

returns a gamlss.family object which can be used to fit a Negative Binomial type I distribution in the gamlss() function.

### Warning

For values of  $\sigma < 0.0001$  the d,p,q,r functions switch to the Poisson distribution

### Note

 $\mu$  is the mean and  $(\mu + \sigma \mu^2)^{0.5}$  is the standard deviation of the Negative Binomial type I distribution (so  $\sigma$  is the dispersion parameter in the usual GLM for the negative binomial type I distribution)

#### Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

#### References

Anscombe, F. J. (1950) Sampling theory of the negative bimomial and logarithmic distributiona, *Biometrika*, **37**, 358-382.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

### See Also

```
gamlss.family, NBII, PIG, SI
```

```
NBI() # gives information about the default links for the Negative Binomial type I distribution
# plotting the distribution
plot(function(y) dNBI(y, mu = 10, sigma = 0.5), from=0, to=40, n=40+1, type="h")
# creating random variables and plot them
tN <- table(Ni <- rNBI(1000, mu=5, sigma=0.5))
r <- barplot(tN, col='lightblue')
# library(gamlss)
# data(aids)
# h<-gamlss(y~cs(x,df=7)+qrt, family=NBI, data=aids) # fits the model</pre>
```

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```
# plot(h)
# pdf.plot(family=NBI, mu=10, sigma=0.5, min=0, max=40, step=1)
```

NBII

Negative Binomial type II distribution for fitting a GAMLSS

## **Description**

The NBII() function defines the Negative Binomial type II distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dNBII, pNBII, qNBII and rNBII define the density, distribution function, quantile function and random generation for the Negative Binomial type II, NBII(), distribution.

## Usage

```
NBII(mu.link = "log", sigma.link = "log")
dNBII(x, mu = 1, sigma = 1, log = FALSE)
pNBII(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qNBII(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rNBII(n, mu = 1, sigma = 1)
```

### **Arguments**

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter           |
|------------|------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter     |
| x          | vector of (non-negative integer) quantiles                                         |
| mu         | vector of positive means                                                           |
| sigma      | vector of positive despersion parameter                                            |
| p          | vector of probabilities                                                            |
| q          | vector of quantiles                                                                |
| n          | number of random values to return                                                  |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                              |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$ |

### **Details**

Definition file for Negative Binomial type II distribution.

$$P(Y=y|\mu,\sigma) = \frac{\Gamma(y+\frac{\mu}{\sigma})\sigma^y}{\Gamma(\frac{\mu}{\sigma})\Gamma(y+1)(1+\sigma)^{y+\mu/\sigma}}$$

for  $y = 0, 1, 2, ..., \infty$ ,  $\mu > 0$  and  $\sigma > 0$ . This parameterization was used by Evans (1953) and also by Johnson *et al.* (1993) p 200.

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### Value

returns a gamlss.family object which can be used to fit a Negative Binomial type II distribution in the gamlss() function.

#### Note

 $\mu$  is the mean and  $[(1+\sigma)\mu]^{0.5}$  is the standard deviation of the Negative Binomial type II distribution, so  $\sigma$  is a dispersion parameter

### Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

### References

Evans, D. A. (1953). Experimental evidence concerning contagious distributions in ecology. *Biometrika*, **40**: 186-211.

Johnson, N. L., Kotz, S. and Kemp, A. W. (1993). *Univariate Discrete Distributions*, 2nd edn. Wiley, New York.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

### See Also

```
gamlss.family, NBI, PIG, SI
```

```
NBII() # gives information about the default links for the Negative Binomial type II distribution
# plotting the distribution
plot(function(y) dNBII(y, mu = 10, sigma = 0.5), from=0, to=40, n=40+1, type="h")
# creating random variables and plot them
tN <- table(Ni <- rNBII(1000, mu=5, sigma=0.5))
r <- barplot(tN, col='lightblue')
# library(gamlss)
# data(aids)
# h<-gamlss(y~cs(x,df=7)+qrt, family=NBII, data=aids) # fits a model
# plot(h)
# pdf.plot(family=NBII, mu=10, sigma=0.5, min=0, max=40, step=1)</pre>
```

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NET Normal Exponential t distribution (NET) for fitting a GAMLSS

## **Description**

This function defines the Power Exponential t distribution (NET), a four parameter distribution, for a gamlss.family object to be used for a GAMLSS fitting using the function gamlss(). The functions dNET, pNET define the density and distribution function the NET distribution.

# Usage

## **Arguments**

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter. Other links are "inverse", "log" and "own"       |
|------------|---------------------------------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse", "identity" and "own" |
| nu.link    | Defines the nu.link, and because nu is fixed we use "identity" link                                                             |
| tau.link   | Defines the tau.link, and because tau is fixed we use "identity" link                                                           |
| x,q        | vector of quantiles                                                                                                             |
| р          | vector of probabilities                                                                                                         |
| n          | number of observations.                                                                                                         |
| mu         | vector of location parameter values                                                                                             |
| sigma      | vector of scale parameter values                                                                                                |
| nu         | vector of nu parameter values                                                                                                   |
| tau        | vector of tau parameter values                                                                                                  |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                                                          |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                              |
|            |                                                                                                                                 |

### **Details**

The NET distribution was introduced by Rigby and Stasinopoulos (1994) as a robust distribution for a response variable with heavier tails than the normal. The NET distribution is the abbreviation of the Normal Exponential Student t distribution. The NET distribution is a four parameter continuous distribution, although in the GAMLSS implementation only the two parameters, mu and sigma, of

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the distribution are modelled with nu and tau fixed. The distribution takes its names because it is normal up to nu, Exponential from nu to tau (hence abs(nu)<=abs(tau)) and Student-t with nu\*tau-1 degrees of freedom after tau. Maximum likelihood estimator of the third and forth parameter can be obtained, using the GAMLSS functions, find.hyper or prof.dev.

### Value

NET() returns a gamlss.family object which can be used to fit a Box Cox Power Exponential distribution in the gamlss() function. dNET() gives the density, pNET() gives the distribution function.

### Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

#### References

Rigby, R. A. and Stasinopoulos, D. M. (1994), Robust fitting of an additive model for variance heterogeneity, *COMPSTAT: Proceedings in Computational Statistics*, editors:R. Dutter and W. Grossmann, pp 263-268, Physica, Heidelberg.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

SStasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

### See Also

```
gamlss.family, BCPE
```

```
NET() #
data(abdom)
plot(function(x)dNET(x, mu=0,sigma=1,nu=2, tau=3), -5, 5)
plot(function(x)pNET(x, mu=0,sigma=1,nu=2, tau=3), -5, 5)
# fit NET with nu=1 and tau=3
# library(gamlss)
#h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=NET,
# data=abdom, nu.start=2, tau.start=3)
#plot(h)</pre>
```

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Normal distribution for fitting a GAMLSS

NO

## Description

The function NO() defines the normal distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(), with mean equal to the parameter mu and sigma equal the standard deviation. The functions dNO, pNO, qNO and rNO define the density, distribution function, quantile function and random generation for the NO parameterization of the normal distribution. [A alternative parameterization with sigma equal to the variance is given in the function NO2()]

## Usage

```
NO(mu.link = "identity", sigma.link = "log")
dNO(x, mu = 0, sigma = 1, log = FALSE)
pNO(q, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qNO(p, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rNO(n, mu = 0, sigma = 1)
```

### **Arguments**

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter              |
|------------|--------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter             |
| x,q        | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| p          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

### **Details**

The parametrization of the normal distribution given in the function NO() is

$$f(y|\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}(\frac{y-\mu}{\sigma})^2\right]$$

for 
$$y=(-\infty,\infty), \mu=(-\infty,+\infty)$$
 and  $\sigma>0$ .

# Value

returns a gamlss.family object which can be used to fit a normal distribution in the gamlss() function.

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### Note

For the function NO(),  $\mu$  is the mean and  $\sigma$  is the standard deviation (not the variance) of the normal distribution.

### Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

### See Also

```
gamlss.family, NO2
```

```
NO()# gives information about the default links for the normal distribution plot(function(y) dNO(y, mu=10 ,sigma=2), 0, 20) plot(function(y) pNO(y, mu=10 ,sigma=2), 0, 20) plot(function(y) qNO(y, mu=10 ,sigma=2), 0, 1) dat<-rNO(100) hist(dat) # library(gamlss) # gamlss(dat~1,family=NO) # fits a constant for mu and sigma
```

NO2

## **Description**

The function NO2() defines the normal distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss() with mean equal to mu and variance equal to sigma. The functions dNO2, pNO2, qNO2 and rNO2 define the density, distribution function, quantile function and random generation for this specific parameterization of the normal distribution.

[A alternative parameterization with sigma as the standard deviation is given in the function NO()]

### Usage

```
NO2(mu.link = "identity", sigma.link = "log")
dNO2(x, mu = 0, sigma = 1, log = FALSE)
pNO2(q, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qNO2(p, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rNO2(n, mu = 0, sigma = 1)
```

### **Arguments**

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter              |
|------------|--------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter             |
| x,q        | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| р          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

## **Details**

The parametrization of the normal distribution given in the function NO2() is

$$f(y|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{1}{2} \frac{(y-\mu)^2}{\sigma}\right]$$

for 
$$y = (-\infty, \infty)$$
,  $\mu = (-\infty, +\infty)$  and  $\sigma > 0$ .

## Value

returns a gamlss.family object which can be used to fit a normal distribution in the gamlss() function.

#### Note

For the function NO(),  $\mu$  is the mean and  $\sigma$  is the standard deviation (not the variance) of the normal distribution. [The function NO2() defines the normal distribution with  $\sigma$  as the variance.]

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### Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

#### See Also

```
gamlss.family, NO
```

### **Examples**

```
NO()# gives information about the default links for the normal distribution dat<-rNO(100) hist(dat) plot(function(y) dNO(y, mu=10 ,sigma=2), 0, 20) plot(function(y) pNO(y, mu=10 ,sigma=2), 0, 20) plot(function(y) qNO(y, mu=10 ,sigma=2), 0, 1) # library(gamlss) # gamlss(dat~1,family=NO) # fits a constant for mu and sigma
```

NOF

Normal distribution family for fitting a GAMLSS

## Description

The function NOF() defines a normal distribution family, which has three parameters. The distribution can be used using the function gamlss(). The mean of NOF is equal to mu. The variance is equal to sigma^2\*mu^nu so the standard deviation is sigma\*mu^(nu/2). The function is design for cases where the variance is proportional to a power of the mean. This is an instance of the Taylor's power low, see Enki et al. (2017). The functions dNOF, pNOF, qNOF and rNOF define the density, distribution function, quantile function and random generation for the NOF parametrization of the normal distribution family.

NOF

### Usage

```
NOF(mu.link = "identity", sigma.link = "log", nu.link = "identity")

dNOF(x, mu = 0, sigma = 1, nu = 0, log = FALSE)

pNOF(q, mu = 0, sigma = 1, nu = 0, lower.tail = TRUE, log.p = FALSE)

qNOF(p, mu = 0, sigma = 1, nu = 0, lower.tail = TRUE, log.p = FALSE)

rNOF(n, mu = 0, sigma = 1, nu = 0)
```

### **Arguments**

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter              |
|------------|--------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter             |
| nu.link    | Defines the nu.link with "identity" link as the default for the nu parameter               |
| x,q        | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| nu         | vector of power parameter values                                                           |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| p          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

### **Details**

The parametrization of the normal distribution given in the function NOF() is

$$f(y|\mu, \sigma, \nu) = \frac{1}{\sqrt{2\pi}\sigma\mu^{\nu/2}} \exp\left[-\frac{1}{2}\frac{(y-\mu)^2}{\sigma^2\mu^{\nu}}\right]$$

for 
$$y=(-\infty,\infty), \mu=(-\infty,\infty), \sigma>0$$
 and  $\nu=(-\infty,+\infty)$ .

## Value

returns a gamlss. family object which can be used to fit a normal distribution family in the gamlss() function.

# Note

For the function NOF (),  $\mu$  is the mean and  $\sigma\mu^{\nu/2}$  is the standard deviation of the normal distribution family. The NOF is design for fitting regression type models where the variance is proportional to a power of the mean. Models of this type are also related to the "pseudo likelihood" models of Carroll and Rubert (1987) but here a proper likelihood is miximised.

Note that because the high correlation between the sigma and the nu parameter the mixed() method should be used in the fitting.

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### Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

### References

Davidian, M. and Carroll, R. J. (1987), Variance Function Estimation, *Journal of the American Statistical Association*, Vol. **82**, pp. 1079-1091

Enki, D G, Noufaily, A., Farrington, P., Garthwaite, P., Andrews, N. and Charlett, A. (2017) Taylor's power law and the statistical modelling of infectious disease surveillance data, Journal of the Royal Statistical Society: Series A (Statistics in Society), volume=180, number=1, pages=45-72.

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```
(see also https://www.gamlss.com/).
```

## See Also

```
gamlss.family, NO, NO2
```

```
NOF()# gives information about the default links for the normal distribution family ## Not run:

## the normal distribution, fitting a constant sigma
m1<-gamlss(y~poly(x,2), sigma.fo=~1, family=NO, data=abdom)

## the normal family, fitting a variance proportional to the mean (mu)
m2<-gamlss(y~poly(x,2), sigma.fo=~1, family=NOF, data=abdom, method=mixed(1,20))

## the normal distribution fitting the variance as a function of x
m3 <-gamlss(y~poly(x,2), sigma.fo=~x, family=NO, data=abdom, method=mixed(1,20))
GAIC(m1,m2,m3)

## End(Not run)
```

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PARETO2

Pareto distributions for fitting in GAMLSS

## **Description**

The functions PARETO() defines the one parameter Pareto distribution for y>1.

The functions PARETO1() defines the one parameter Pareto distribution for y>0.

The functions PARET001() defines the one parameter Pareto distribution for y>mu therefor requires mu to be fixed.

The functions PARETO2() and PARETO2o() define the Pareto Type 2 distribution, for y>0, a two parameter distribution, for a gamlss. family object to be used in GAMLSS fitting using the function gamlss(). The parameters are mu and sigma in both functions but the parameterasation different. The mu is identical for both PARETO2() and PARETO2o(). The sigma in PARETO2o() is the inverse of the sigma in codePARETO2() and coresponse to the usual parameter alpha of the Patreto distribution. The functions dPARETO2, pPARETO2, qPARETO2 and rPARETO2 define the density, distribution function, quantile function and random generation for the PARETO2 parameterization of the Pareto type 2 distribution while the functions dPARET020, pPARET020, qPARET020 and rPARET020 define the density, distribution function, quantile function and random generation for the original PARET020 parameterization of the Pareto type 2 distribution

### Usage

```
PARETO(mu.link = "log")
dPARETO(x, mu = 1, log = FALSE)
pPARETO(q, mu = 1, lower.tail = TRUE, log.p = FALSE)
qPARETO(p, mu = 1, lower.tail = TRUE, log.p = FALSE)
rPARETO(n, mu = 1)
PARETO1(mu.link = "log")
dPARETO1(x, mu = 1, log = FALSE)
pPARETO1(q, mu = 1, lower.tail = TRUE, log.p = FALSE)
qPARETO1(p, mu = 1, lower.tail = TRUE, log.p = FALSE)
rPARETO1(n, mu = 1)
PARETO1o(mu.link = "log", sigma.link = "log")
dPARETO1o(x, mu = 1, sigma = 0.5, log = FALSE)
pPARETO1o(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
qPARETO1o(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
rPARETO1o(n, mu = 1, sigma = 0.5)
PARETO2(mu.link = "log", sigma.link = "log")
dPARETO2(x, mu = 1, sigma = 0.5, log = FALSE)
pPARETO2(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
qPARETO2(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
rPARETO2(n, mu = 1, sigma = 0.5)
```

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```
PARETO2o(mu.link = "log", sigma.link = "log")
dPARETO2o(x, mu = 1, sigma = 0.5, log = FALSE)
pPARETO2o(q, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
qPARETO2o(p, mu = 1, sigma = 0.5, lower.tail = TRUE, log.p = FALSE)
rPARETO2o(n, mu = 1, sigma = 0.5)
```

### **Arguments**

| mu.link    | Defines the mu.link, with "'" link sa the default for the mu parameter                     |
|------------|--------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "'log" as the default for the sigma parameter                 |
| x,q        | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                      |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise $P[X > x]$          |
| p          | vector of probabilities                                                                    |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

### **Details**

The parameterization of the one parameter Pareto distribution in the function PARETO is:

$$f(y|\mu) = \mu y^{\mu+1}$$

for y > 1 and mu > 0.

The parameterization of the Pareto Type 2 distribution in the function PARETO2 is:

$$f(y|\mu,\sigma) = \frac{1}{\sigma} \mu^{\frac{1}{\sigma}} (y + mu)^{-\frac{1}{sigma+1}}$$

for y >= 0, mu > 0 and sigma > 0.

### Value

returns a gamlss.family object which can be used to fit a Pareto type 2 distribution in the gamlss() function.

## Author(s)

Fiona McElduff, Bob Rigby and Mikis Stasinopoulos

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#### References

Johnson, N., Kotz, S., and Balakrishnan, N. (1997). *Discrete Multivariate Distributions*. Wiley-Interscience, NY, USA.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

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```
(see also https://www.gamlss.com/).
```

### See Also

```
gamlss.family
```

## **Examples**

```
par(mfrow=c(2,2))
y<-seq(0.2,20,0.2)
plot(y, dPARETO2(y), type="1" , lwd=2)
q<-seq(0,20,0.2)
plot(q, pPARETO2(q), ylim=c(0,1), type="1", lwd=2)
p<-seq(0.0001,0.999,0.05)
plot(p, qPARETO2(p), type="1", lwd=2)
dat <- rPARETO2(100)
hist(rPARETO2(100), nclass=30)
#summary(gamlss(a~1, family="PARETO2"))</pre>
```

PE

Power Exponential distribution for fitting a GAMLSS

### **Description**

The functions define the Power Exponential distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dPE, pPE, qPE and rPE define the density, distribution function, quantile function and random generation for the specific parameterization of the power exponential distribution showing below. The functions dPE2, pPE2, qPE2 and rPE2 define the density, distribution function, quantile function and random generation of a standard parameterization of the power exponential distribution.

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## Usage

```
PE(mu.link = "identity", sigma.link = "log", nu.link = "log")

dPE(x, mu = 0, sigma = 1, nu = 2, log = FALSE)

pPE(q, mu = 0, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)

qPE(p, mu = 0, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)

rPE(n, mu = 0, sigma = 1, nu = 2)

PE2(mu.link = "identity", sigma.link = "log", nu.link = "log")

dPE2(x, mu = 0, sigma = 1, nu = 2, log = FALSE)

pPE2(q, mu = 0, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)

qPE2(p, mu = 0, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)

rPE2(n, mu = 0, sigma = 1, nu = 2)
```

## **Arguments**

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter              |
|------------|--------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter             |
| nu.link    | Defines the nu.link, with "log" link as the default for the nu parameter                   |
| x,q        | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| nu         | vector of kurtosis parameter                                                               |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| р          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

# Details

Power Exponential distribution (PE) is defined as

$$f(y|\mu, \sigma, \nu) = \frac{\nu \exp\left[-\left(\frac{1}{2}\right)\left|\frac{z}{c}\right|^{\nu}\right]}{\sigma c 2^{(1+1/\nu)}\Gamma\left(\frac{1}{\nu}\right)}$$

where  $c = [2^{-2/\nu}\Gamma(1/\nu)/\Gamma(3/\nu)]^{0.5}$ , for  $y = (-\infty, +\infty)$ ,  $\mu = (-\infty, +\infty)$ ,  $\sigma > 0$  and  $\nu > 0$ . This parametrization was used by Nelson (1991) and ensures  $\mu$  is the mean and  $\sigma$  is the standard deviation of y (for all parameter values of  $\mu$ ,  $\sigma$  and  $\nu$  within the rages above)

Thw Power Exponential distribution (PE2) is defined as

$$f(y|\mu, \sigma, \nu) = \frac{\nu \exp[-|z|^{\nu}]}{2\sigma\Gamma(\frac{1}{\nu})}$$

#### Value

returns a gamlss.family object which can be used to fit a Power Exponential distribution in the gamlss() function.

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### Note

 $\mu$  is the mean and  $\sigma$  is the standard deviation of the Power Exponential distribution

## Author(s)

Mikis Stasinopoulos, Bob Rigby

#### References

Nelson, D.B. (1991) Conditional heteroskedasticity in asset returns: a new approach. *Econometrica*, **57**, 347-370.

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

### See Also

```
gamlss.family, BCPE
```

```
PE()# gives information about the default links for the Power Exponential distribution # library(gamlss)
# data(abdom)
# h1<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=PE, data=abdom) # fit
# h2<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=PE2, data=abdom) # fit
# plot(h1)
# plot(h2)
# leptokurtotic
plot(function(x) dPE(x, mu=10,sigma=2,nu=1), 0.0, 20,
main = "The PE density mu=10,sigma=2,nu=1")
# platykurtotic
plot(function(x) dPE(x, mu=10,sigma=2,nu=4), 0.0, 20,
main = "The PE density mu=10,sigma=2,nu=4")
```

PIG

## **Description**

The PIG() function defines the Poisson-inverse Gaussian distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The PIG2() function is a repametrization of PIG() where mu and sigma are orthogonal see Heller *et al.* (2018).

The functions dPIG, pPIG, qPIG and rPIG define the density, distribution function, quantile function and random generation for the Poisson-inverse Gaussian PIG(), distribution. Also codedPIG2, pPIG2, qPIG2 and rPIG2 are the equivalent functions for codePIG2()

The functions ZAPIG() and ZIPIG() are the zero adjusted (hurdle) and zero inflated versions of the Poisson-inverse Gaussian distribution, respectively. That is three parameter distributions.

The functions dZAPIG, dZIPIG, pZAPIG,pZIPIG, qZAPIG qZIPIG rZAPIG and rZIPIG define the probability, cumulative, quantile and random generation functions for the zero adjusted and zero inflated Poisson Inverse Gaussian distributions, ZAPIG(), ZIPIG(), respectively.

### Usage

```
PIG(mu.link = "log", sigma.link = "log")
dPIG(x, mu = 1, sigma = 1, log = FALSE)
pPIG(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qPIG(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE,
     max.value = 10000)
rPIG(n, mu = 1, sigma = 1, max.value = 10000)
PIG2(mu.link = "log", sigma.link = "log")
dPIG2(x, mu=0.5, sigma=0.02, log = FALSE)
pPIG2(q, mu=0.5, sigma=0.02, lower.tail = TRUE, log.p = FALSE)
qPIG2(p, mu=0.5, sigma=0.02, lower.tail = TRUE, log.p = FALSE,
     max.value = 10000)
rPIG2(n, mu=0.5, sigma=0.02)
ZIPIG(mu.link = "log", sigma.link = "log", nu.link = "logit")
dZIPIG(x, mu = 1, sigma = 1, nu = 0.3, log = FALSE)
pZIPIG(q, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE)
qZIPIG(p, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE,
       max.value = 10000)
rZIPIG(n, mu = 1, sigma = 1, nu = 0.3, max.value = 10000)
ZAPIG(mu.link = "log", sigma.link = "log", nu.link = "logit")
dZAPIG(x, mu = 1, sigma = 1, nu = 0.3, log = FALSE)
pZAPIG(q, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE)
qZAPIG(p, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE,
      max.value = 10000)
```

```
rZAPIG(n, mu = 1, sigma = 1, nu = 0.3, max.value = 10000)
```

## **Arguments**

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter                                 |
|------------|----------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter                           |
| nu.link    | Defines the mu.link, with "logit" link as the default for the nu parameter                               |
| x          | vector of (non-negative integer) quantiles                                                               |
| mu         | vector of positive means                                                                                 |
| sigma      | vector of positive dispersion parameter                                                                  |
| nu         | vector of zero probability parameter                                                                     |
| р          | vector of probabilities                                                                                  |
| q          | vector of quantiles                                                                                      |
| n          | number of random values to return                                                                        |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                                    |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                       |
| max.value  | a constant, set to the default value of 10000 for how far the algorithm should look for $\boldsymbol{q}$ |
|            |                                                                                                          |

### **Details**

The probability function of the Poisson-inverse Gaussian distribution, is given by

$$f(y|\mu,\sigma) = \left(\frac{2\alpha^{\frac{1}{2}}}{\pi}\right) \frac{\mu^y e^{\frac{1}{\sigma}} K_{y-\frac{1}{2}}(\alpha)}{(\alpha\sigma)^y y!}$$

where  $\alpha^2=\frac{1}{\sigma^2}+\frac{2\mu}{\sigma}$ , for  $y=0,1,2,...,\infty$  where  $\mu>0$  and  $\sigma>0$  and  $K_\lambda(t)=\frac{1}{2}\int_0^\infty x^{\lambda-1}\exp\{-\frac{1}{2}t(x+x^{-1})\}dx$  is the modified Bessel function of the third kind. [Note that the above parameterization was used by Dean, Lawless and Willmot(1989). It is also a special case of the Sichel distribution SI() when  $\nu=-\frac{1}{2}$ .]

## Value

Returns a gamlss.family object which can be used to fit a Poisson-inverse Gaussian distribution in the gamlss() function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Marco Enea

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#### References

Dean, C., Lawless, J. F. and Willmot, G. E., A mixed poisson-inverse-Gaussian regression model, *Canadian J. Statist.*, **17**, 2, pp 171-181

Heller, G. Z., Couturier, D.L. and Heritier, S. R. (2018) Beyond mean modelling: Bias due to misspecification of dispersion in Poisson-inverse Gaussian regression *Biometrical Journal*, **2**, pp 333-342.

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#### See Also

```
gamlss.family, NBI, NBII, SI, SICHEL
```

# **Examples**

```
PIG()# gives information about the default links for the Poisson-inverse Gaussian distribution #plot the pdf using plot
plot(function(y) dPIG(y, mu=10, sigma = 1 ), from=0, to=50, n=50+1, type="h") # pdf
# plot the cdf
plot(seq(from=0,to=50),pPIG(seq(from=0,to=50), mu=10, sigma=1), type="h") # cdf
# generate random sample
tN <- table(Ni <- rPIG(100, mu=5, sigma=1))
r <- barplot(tN, col='lightblue')
# fit a model to the data
# library(gamlss)
# gamlss(Ni~1,family=PIG)
ZIPIG()
ZAPIG()
```

PO

Poisson distribution for fitting a GAMLSS model

## Description

This function PO defines the Poisson distribution, an one parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dPO, pPO, qPO and rPO define the density, distribution function, quantile function and random generation for the Poisson, PO(), distribution.

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### Usage

```
PO(mu.link = "log")
dPO(x, mu = 1, log = FALSE)
pPO(q, mu = 1, lower.tail = TRUE, log.p = FALSE)
qPO(p, mu = 1, lower.tail = TRUE, log.p = FALSE)
rPO(n, mu = 1)
```

## **Arguments**

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter           |
|------------|------------------------------------------------------------------------------------|
| x          | vector of (non-negative integer) quantiles                                         |
| mu         | vector of positive means                                                           |
| р          | vector of probabilities                                                            |
| q          | vector of quantiles                                                                |
| n          | number of random values to return                                                  |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                              |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$ |

# **Details**

Definition file for Poisson distribution.

$$f(y|\mu) = \frac{e^{-\mu}\mu^y}{\Gamma(y+1)}$$

```
for y = 0, 1, 2, ... and \mu > 0.
```

## Value

returns a gamlss.family object which can be used to fit a Poisson distribution in the gamlss() function.

### Note

 $\mu$  is the mean of the Poisson distribution

# Author(s)

Bob Rigby, Mikis Stasinopoulos, and Kalliope Akantziliotou

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

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Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

(Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

### See Also

```
gamlss.family, NBI, NBII, SI, SICHEL
```

## **Examples**

```
PO()# gives information about the default links for the Poisson distribution
# fitting data using PO()

# plotting the distribution
plot(function(y) dPO(y, mu=10), from=0, to=20, n=20+1, type="h")
# creating random variables and plot them
tN <- table(Ni <- rPO(1000, mu=5))
r <- barplot(tN, col='lightblue')
# library(gamlss)
# data(aids)
# h<-gamlss(y~cs(x,df=7)+qrt, family=PO, data=aids) # fits the constant+x+qrt model
# plot(h)
# pdf.plot(family=PO, mu=10, min=0, max=20, step=1)</pre>
```

The Reverse Gumbel distribution for fitting a GAMLSS

RG

## Description

The function RG defines the reverse Gumbel distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dRG, pRG, qRG and rRG define the density, distribution function, quantile function and random generation for the specific parameterization of the reverse Gumbel distribution.

## Usage

```
RG(mu.link = "identity", sigma.link = "log")
dRG(x, mu = 0, sigma = 1, log = FALSE)
pRG(q, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qRG(p, mu = 0, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rRG(n, mu = 0, sigma = 1)
```

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### **Arguments**

mu.link Defines the mu.link, with "identity" link as the default for the mu parameter. other available link is "inverse", "log" and "own" Defines the sigma.link, with "log" link as the default for the sigma parameter, sigma.link other links are the "inverse", "identity" and "own" x,q vector of quantiles vector of location parameter values mu vector of scale parameter values sigma logical; if TRUE, probabilities p are given as log(p). log, log.p lower.tail logical; if TRUE (default), probabilities are  $P[X \le x]$ , otherwise, P[X > x]vector of probabilities. number of observations. If length(n) > 1, the length is taken to be the number n required

### **Details**

The specific parameterization of the reverse Gumbel distribution used in RG is

$$f(y|\mu,\sigma) = \frac{1}{\sigma} \exp\left\{-\left(\frac{y-\mu}{\sigma}\right) - \exp\left[-\frac{(y-\mu)}{\sigma}\right]\right\}$$

for 
$$y=(-\infty,\infty)$$
,  $\mu=(-\infty,+\infty)$  and  $\sigma>0$ .

## Value

RG() returns a gamlss.family object which can be used to fit a Gumbel distribution in the gamlss() function. dRG() gives the density, pGU() gives the distribution function, qRG() gives the quantile function, and rRG() generates random deviates.

### Note

The mean of the distribution is  $\mu + 0.57722\sigma$  and the variance is  $\pi^2 \sigma^2/6$ .

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

## See Also

```
gamlss.family
```

## **Examples**

```
plot(function(x) dRG(x, mu=0,sigma=1), -3, 6,
  main = "{Reverse Gumbel density mu=0,sigma=1}")
RG()# gives information about the default links for the Gumbel distribution
dat<-rRG(100, mu=10, sigma=2) # generates 100 random observations
# library(gamlss)
# gamlss(dat~1,family=RG) # fits a constant for each parameter mu and sigma</pre>
```

**RGE** 

Reverse generalized extreme family distribution for fitting a GAMLSS

## **Description**

The function RGE defines the reverse generalized extreme family distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dRGE, pRGE, qRGE and rRGE define the density, distribution function, quantile function and random generation for the specific parameterization of the reverse generalized extreme distribution given in details below.

# Usage

```
RGE(mu.link = "identity", sigma.link = "log", nu.link = "log")

dRGE(x, mu = 1, sigma = 0.1, nu = 1, log = FALSE)

pRGE(q, mu = 1, sigma = 0.1, nu = 1, lower.tail = TRUE, log.p = FALSE)

qRGE(p, mu = 1, sigma = 0.1, nu = 1, lower.tail = TRUE, log.p = FALSE)

rRGE(n, mu = 1, sigma = 0.1, nu = 1)
```

## **Arguments**

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter  |
|------------|--------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link    | Defines the nu.link, with "log" link as the default for the nu parameter       |
| x,q        | vector of quantiles                                                            |
| mu         | vector of location parameter values                                            |
| sigma      | vector of scale parameter values                                               |

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nu vector of the shape parameter values
log, log.p logical; if TRUE, probabilities p are given as log(p).
lower.tail logical; if TRUE (default), probabilities are P[X <= x], otherwise, P[X > x]
p vector of probabilities.
n number of observations. If length(n) > 1, the length is taken to be the number required

### **Details**

Definition file for reverse generalized extreme family distribution.

The probability density function of the generalized extreme value distribution is obtained from Johnson *et al.* (1995), Volume 2, p76, equation (22.184) [where  $(\xi, \theta, \gamma) \longrightarrow (\mu, \sigma, \nu)$ ].

The probability density function of the reverse generalized extreme value distribution is then obtained by replacing y by -y and  $\mu$  by  $-\mu$ .

Hence the probability density function of the reverse generalized extreme value distribution with  $\nu > 0$  is given by

$$f(y|\mu,\sigma,\nu) = \frac{1}{\sigma} \left[ 1 + \frac{\nu(y-\mu)}{\sigma} \right]^{\frac{1}{\nu}-1} S_1(y|\mu,\sigma,\nu)$$

for

$$\mu - \frac{\sigma}{\nu} < y < \infty$$

where

$$S_1(y|\mu, \sigma, \nu) = \exp\left\{-\left[1 + \frac{\nu(y-\mu)}{\sigma}\right]^{\frac{1}{\nu}}\right\}$$

and where  $-\infty < \mu < y + \frac{\sigma}{\nu}$ ,  $\sigma > 0$  and  $\nu > 0$ . Note that only the case  $\nu > 0$  is allowed here. The reverse generalized extreme value distribution is denoted as  $RGE(\mu, \sigma, \nu)$  or as Reverse Generalized. Extreme. Family  $(\mu, \sigma, \nu)$ .

Note the the above distribution is a reparameterization of the three parameter Weibull distribution given by

$$f(y|\alpha_1, \alpha_2, \alpha_3) = \frac{\alpha_3}{\alpha_2} \left[ \frac{y - \alpha_1}{\alpha_2} \right]^{\alpha_3 - 1} \exp\left[ -\left( \frac{y - \alpha_1}{\alpha_2} \right)^{\alpha_3} \right]$$

given by setting  $\alpha_1 = \mu - \sigma/\nu$ ,  $\alpha_2 = \sigma/\nu$ ,  $\alpha_3 = 1/\nu$ .

## Value

RGE() returns a gamlss.family object which can be used to fit a reverse generalized extreme distribution in the gamlss() function. dRGE() gives the density, pRGE() gives the distribution function, qRGE() gives the quantile function, and rRGE() generates random deviates.

### Note

This distribution is very difficult to fit because the y values depends on the parameter values. The RS() and CG() algorithms are not appropriate for this type of problem.

### Author(s)

Bob Rigby, Mikis Stasinopoulos and Kalliope Akantziliotou

### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

## See Also

```
gamlss.family
```

### **Examples**

```
RGE()# default links for the reverse generalized extreme family distribution newdata<-rRGE(100,mu=0,sigma=1,nu=5) # generates 100 random observations # library(gamlss) # gamlss(newdata~1, family=RGE, method=mixed(5,50)) # difficult to converse
```

SEP

The Skew Power exponential (SEP) distribution for fitting a GAMLSS

# Description

This function defines the Skew Power exponential (SEP) distribution, a four parameter distribution, for a gamlss.family object to be used for a GAMLSS fitting using the function gamlss(). The functions dSEP, pSEP, qSEP and rSEP define the density, distribution function, quantile function and random generation for the Skew Power exponential (SEP) distribution.

### Usage

```
SEP(mu.link = "identity", sigma.link = "log", nu.link = "identity",
    tau.link = "log")

dSEP(x, mu = 0, sigma = 1, nu = 0, tau = 2, log = FALSE)
pSEP(q, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE,
    log.p = FALSE)
qSEP(p, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE,
    log.p = FALSE, lower.limit = mu - 5 * sigma,
    upper.limit = mu + 5 * sigma)
rSEP(n, mu = 0, sigma = 1, nu = 0, tau = 2)
```

### **Arguments**

| mu.link     | Defines the mu.link, with "identity" link as the default for the mu parameter. Other links are " $1/mu^2$ " and "log"    |
|-------------|--------------------------------------------------------------------------------------------------------------------------|
| sigma.link  | Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse" and "identity" |
| nu.link     | Defines the nu.link, with "identity" link as the default for the nu parameter. Other links are " $1/nu^2$ " and "log"    |
| tau.link    | Defines the tau.link, with "log" link as the default for the tau parameter. Other links are " $1/tau^2$ ", and "identity |
| x,q         | vector of quantiles                                                                                                      |
| mu          | vector of location parameter values                                                                                      |
| sigma       | vector of scale parameter values                                                                                         |
| nu          | vector of skewness nu parameter values                                                                                   |
| tau         | vector of kurtosis tau parameter values                                                                                  |
| log, log.p  | logical; if TRUE, probabilities p are given as log(p).                                                                   |
| lower.tail  | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                       |
| р           | vector of probabilities.                                                                                                 |
| n           | number of observations. If $length(n) > 1$ , the length is taken to be the number required                               |
| lower.limit | lower limit for the golden search to find quantiles from probabilities                                                   |
| upper.limit | upper limit for the golden search to find quantiles from probabilities                                                   |

## **Details**

The probability density function of the Skew Power exponential distribution, (SEP), is defined as

$$f(y|n,\mu,\sigma\,\nu,\tau) == \frac{z}{\sigma} \Phi(\omega) \ f_{EP}(z,0,1,\tau)$$

for  $-\infty < y < \infty$ ,  $\mu = (-\infty, +\infty)$ ,  $\sigma > 0$ ,  $\nu = (-\infty, +\infty)$  and  $\tau > 0$ . where  $z = \frac{y-\mu}{\sigma}$ ,  $\omega = sign(z)|z|^{\tau/2}\nu\sqrt{2/\tau}$  and  $f_{EP}(z,0,1,\tau)$  is the pdf of an Exponential Power distribution.

### Value

SEP() returns a gamlss.family object which can be used to fit the SEP distribution in the gamlss() function. dSEP() gives the density, pSEP() gives the distribution function, qSEP() gives the quantile function, and rSEP() generates random deviates.

## Warning

The qSEP and rSEP are slow since they are relying on golden section for finding the quantiles

### Author(s)

Bob Rigby and Mikis Stasinopoulos

#### References

Diciccio, T. J. and Mondi A. C. (2004). Inferential Aspects of the Skew Exponential Power distribution., *JASA*, **99**, 439-450.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family, JSU, BCT
```

```
SEP() #
plot(function(x)dSEP(x, mu=0,sigma=1, nu=1, tau=2), -5, 5,
    main = "The SEP density mu=0,sigma=1,nu=1, tau=2")
plot(function(x) pSEP(x, mu=0,sigma=1,nu=1, tau=2), -5, 5,
    main = "The BCPE cdf mu=0, sigma=1, nu=1, tau=2")
dat <- rSEP(100,mu=10,sigma=1,nu=-1,tau=1.5)
# library(gamlss)
# gamlss(dat~1,family=SEP, control=gamlss.control(n.cyc=30))</pre>
```

SEP1

The Skew Power exponential type 1-4 distribution for fitting a GAMLSS

### **Description**

These functions define the Skew Power exponential type 1 to 4 distributions. All of them are four parameter distributions and can be used to fit a GAMLSS model. The functions dSEP1, dSEP2, dSEP3 and dSEP4 define the probability distribution functions, the functions pSEP1, pSEP2, pSEP3 and pSEP4 define the cumulative distribution functions the functions qSEP1, qSEP2, qSEP3 and qSEP4 define the inverse cumulative distribution functions and the functions rSEP1, rSEP2, rSEP3 and rSEP4 define the random generation for the Skew exponential power distributions.

### **Usage**

```
SEP1(mu.link = "identity", sigma.link = "log", nu.link = "identity",
    tau.link = "log")
dSEP1(x, mu = 0, sigma = 1, nu = 0, tau = 2, log = FALSE)
pSEP1(q, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE,
    log.p = FALSE)
qSEP1(p, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE,
    log.p = FALSE)
rSEP1(n, mu = 0, sigma = 1, nu = 0, tau = 2)
SEP2(mu.link = "identity", sigma.link = "log", nu.link = "identity",
     tau.link = "log")
dSEP2(x, mu = 0, sigma = 1, nu = 0, tau = 2, log = FALSE)
pSEP2(q, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE,
     log.p = FALSE)
qSEP2(p, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE,
     log.p = FALSE
rSEP2(n, mu = 0, sigma = 1, nu = 0, tau = 2)
SEP3(mu.link = "identity", sigma.link = "log", nu.link = "log",
     tau.link = "log")
dSEP3(x, mu = 0, sigma = 1, nu = 2, tau = 2, log = FALSE)
pSEP3(q, mu = 0, sigma = 1, nu = 2, tau = 2, lower.tail = TRUE,
     log.p = FALSE)
qSEP3(p, mu = 0, sigma = 1, nu = 2, tau = 2, lower.tail = TRUE,
     log.p = FALSE)
SEP4(mu.link = "identity", sigma.link = "log", nu.link = "log",
     tau.link = "log")
dSEP4(x, mu = 0, sigma = 1, nu = 2, tau = 2, log = FALSE)
pSEP4(q, mu = 0, sigma = 1, nu = 2, tau = 2, lower.tail = TRUE,
     log.p = FALSE
qSEP4(p, mu = 0, sigma = 1, nu = 2, tau = 2, lower.tail = TRUE,
```

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$$log.p = FALSE$$
)
 $rSEP4(n, mu = 0, sigma = 1, nu = 2, tau = 2)$ 

## **Arguments**

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter. Other links are "inverse" and "log"       |
|------------|--------------------------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse" and "identity" |
| nu.link    | Defines the nu.link, with "log" link as the default for the nu parameter. Other links are "identity" and "inverse"       |
| tau.link   | Defines the tau.link, with "log" link as the default for the tau parameter. Other links are "inverse", and "identity     |
| x,q        | vector of quantiles                                                                                                      |
| mu         | vector of location parameter values                                                                                      |
| sigma      | vector of scale parameter values                                                                                         |
| nu         | vector of skewness nu parameter values                                                                                   |
| tau        | vector of kurtosis tau parameter values                                                                                  |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                                                   |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                       |
| р          | vector of probabilities.                                                                                                 |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required                               |

## **Details**

The probability density function of the Skew Power exponential distribution type 2, (SEP2), is defined as

$$f_Y(y|\mu,\sigma\nu,\tau) = \frac{\nu}{\sigma(1+\nu^2)2^{1/\tau}\Gamma(1+1/\tau)} \left\{ \exp\left(-\frac{1}{2} \left| \frac{\nu(y-\mu)}{\sigma} \right|^{\tau}\right) I(y<\mu) + \exp\left(-\frac{1}{2} \left| \frac{(y-\mu)}{\sigma\nu} \right|^{\tau}\right) I(y\geq\mu) \right\}$$

for 
$$-\infty < y < \infty$$
,  $\mu = (-\infty, +\infty)$ ,  $\sigma > 0$ ,  $\nu > 0$ ) and  $\tau > 0$ .

## Value

SEP2() returns a gamlss.family object which can be used to fit the SEP2 distribution in the gamlss() function. dSEP2() gives the density, pSEP2() gives the distribution function, qSEP2() gives the quantile function, and rSEP2() generates random deviates.

## Author(s)

Bob Rigby and Mikis Stasinopoulos

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#### References

Fernadez C., Osiewalski J. and Steel M.F.J.(1995) Modelling and inference with v-spherical distributions. *JASA*, **90**, pp 1331-1340.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

### See Also

```
gamlss.family, SEP
```

### **Examples**

SHASH

The Sinh-Arcsinh (SHASH) distribution for fitting a GAMLSS

### **Description**

The Sinh-Arcsinh (SHASH) distribution is a four parameter distribution, for a gamlss.family object to be used for a GAMLSS fitting using the function gamlss(). The functions dSHASH, pSHASH, qSHASH and rSHASH define the density, distribution function, quantile function and random generation for the Sinh-Arcsinh (SHASH) distribution.

There are 3 different SHASH distributions implemented in GAMLSS.

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### Usage

```
SHASH(mu.link = "identity", sigma.link = "log", nu.link = "log",
     tau.link = "log")
dSHASH(x, mu = 0, sigma = 1, nu = 0.5, tau = 0.5, log = FALSE)
pSHASH(q, mu = 0, sigma = 1, nu = 0.5, tau = 0.5, lower.tail = TRUE,
     log.p = FALSE)
qSHASH(p, mu = 0, sigma = 1, nu = 0.5, tau = 0.5, lower.tail = TRUE,
     log.p = FALSE)
rSHASH(n, mu = 0, sigma = 1, nu = 0.5, tau = 0.5)
SHASHo(mu.link = "identity", sigma.link = "log", nu.link = "identity",
      tau.link = "log")
dSHASHo(x, mu = 0, sigma = 1, nu = 0, tau = 1, log = FALSE)
pSHASHo(q, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE,
     log.p = FALSE)
qSHASHo(p, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE,
     log.p = FALSE)
rSHASHo(n, mu = 0, sigma = 1, nu = 0, tau = 1)
SHASHo2(mu.link = "identity", sigma.link = "log", nu.link = "identity",
      tau.link = "log")
dSHASHo2(x, mu = 0, sigma = 1, nu = 0, tau = 1, log = FALSE)
pSHASHo2(q, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE,
     log.p = FALSE)
qSHASHo2(p, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE,
     log.p = FALSE)
rSHASHo2(n, mu = 0, sigma = 1, nu = 0, tau = 1)
```

## **Arguments**

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter.             |
|------------|--------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter.            |
| nu.link    | Defines the nu.link, with "log" link as the default for the nu parameter.                  |
| tau.link   | Defines the tau.link, with "log" link as the default for the tau parameter.                |
| x,q        | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| nu         | vector of skewness nu parameter values                                                     |
| tau        | vector of kurtosis tau parameter values                                                    |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| p          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

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### **Details**

The probability density function of the Sinh-Arcsinh distribution, (SHASH), Jones (2005), is defined as

$$f(y|\mu,\sigma\,\nu,\tau) = \frac{c}{\sqrt{2\pi}\sigma(1+z^2)^{1/2}}e^{-r^2/2}$$

where

$$r = \frac{1}{2} \left\{ \exp \left[ \tau \sinh^{-1}(z) \right] - \exp \left[ -\nu \sinh^{-1}(z) \right] \right\}$$

and

$$c = \frac{1}{2} \left\{ \tau \exp \left[ \tau \sinh^{-1}(z) \right] + \nu \exp \left[ -\nu \sinh^{-1}(z) \right] \right\}$$

and 
$$z = (y - \mu)/\sigma$$
 for  $-\infty < y < \infty$ ,  $\mu = (-\infty, +\infty)$ ,  $\sigma > 0$ ,  $\nu > 0$  and  $\tau > 0$ .

The parameters  $\mu$  and  $\sigma$  are the location and scale of the distribution. The parameter  $\nu$  determines the left hand tail of the distribution with  $\nu>1$  indicating a lighter tail than the normal and  $\nu<1$  heavier tail than the normal. The parameter  $\tau$  determines the right hand tail of the distribution in the same way.

The second form of the Sinh-Arcsinh distribution can be found in Jones and Pewsey (2009, p.2) denoted by SHASHo and the probability density function is defined as,

$$f(y|\mu, \sigma, \nu, \tau) = \frac{\tau}{\sigma} \frac{c}{\sqrt{2\pi}} \frac{1}{2\sqrt{1+z^2}} \exp(-\frac{r^2}{2})$$

where

$$r = \sinh(\tau \arcsin(z) - \nu)$$

and

$$c = \cosh(\tau \arcsin(z) - \nu)$$

and 
$$z = (y - \mu)/\sigma$$
 for  $-\infty < y < \infty$ ,  $\mu = (-\infty, +\infty)$ ,  $\sigma > 0$ ,  $\nu = (-\infty, +\infty)$  and  $\tau > 0$ .

The third form of the Sinh-Arcsinh distribution (Jones and Pewsey, 2009, p.8) divides the distribution by sigma for the density of the unstandardized variable. This distribution is denoted by SHASHo2 and has pdf

$$f(y|\mu, \sigma, \nu, \tau) = \frac{c}{\sigma} \frac{\tau}{\sqrt{2\pi}} \frac{1}{\sqrt{1+z^2}} - \exp{-\frac{r^2}{2}}$$

where  $z=(y-\mu)/(\sigma\tau)$ , with r and c as for the pdf of the SHASHo distribution, for  $-\infty < y < \infty$ ,  $\mu=(-\infty,+\infty), \sigma>0, \nu=(-\infty,+\infty)$  and  $\tau>0$ .

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#### Value

SHASH() returns a gamlss.family object which can be used to fit the SHASH distribution in the gamlss() function. dSHASH() gives the density, pSHASH() gives the distribution function, qSHASH() gives the quantile function, and rSHASH() generates random deviates.

## Warning

The qSHASH and rSHASH are slow since they are relying on golden section for finding the quantiles

## Author(s)

Bob Rigby, Mikis Stasinopoulos and Fiona McElduff

#### References

Jones, M. C. (2006) p 546-547 in the discussion of Rigby, R. A. and Stasinopoulos D. M. (2005) *Appl. Statist.*, **54**, part 3.

Jones and Pewsey (2009) Sinh-arcsinh distributions. *Biometrika*. **96**(4), pp. 761?780.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

## See Also

```
gamlss.family, JSU, BCT
```

#### **Examples**

```
SHASH() #
plot(function(x)dSHASH(x, mu=0,sigma=1, nu=1, tau=2), -5, 5,
main = "The SHASH density mu=0,sigma=1,nu=1, tau=2")
plot(function(x) pSHASH(x, mu=0,sigma=1,nu=1, tau=2), -5, 5,
main = "The BCPE cdf mu=0, sigma=1, nu=1, tau=2")
dat<-rSHASH(100,mu=10,sigma=1,nu=1,tau=1.5)
hist(dat)
# library(gamlss)
# gamlss(dat~1,family=SHASH, control=gamlss.control(n.cyc=30))</pre>
```

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# Description

SI

The SI() function defines the Sichel distribution, a three parameter discrete distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dSI, pSI, qSI and rSI define the density, distribution function, quantile function and random generation for the Sichel SI(), distribution.

## Usage

## Arguments

mu link

| mu.11nk    | Defines the mu. 11nk, with log link as the default for the mu parameter                             |
|------------|-----------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter                      |
| nu.link    | Defines the nu.link, with "identity" link as the default for the nu parameter                       |
| x          | vector of (non-negative integer) quantiles                                                          |
| mu         | vector of positive mu                                                                               |
| sigma      | vector of positive despersion parameter                                                             |
| nu         | vector of nu                                                                                        |
| р          | vector of probabilities                                                                             |
| q          | vector of quantiles                                                                                 |
| n          | number of random values to return                                                                   |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                               |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                  |
| max.value  | a constant, set to the default value of $10000\ \text{for how}$ far the algorithm should look for q |
| У          | the y variable. The function tofyS() should be not used on its own.                                 |
| what       | take values 1 or 2, for function tofyS().                                                           |
|            |                                                                                                     |

Defines the mullink with "log" link as the default for the mulnarameter

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#### **Details**

The probability function of the Sichel distribution is given by

$$f(y|\mu, \sigma, \nu) = \frac{\mu^y K_{y+\nu}(\alpha)}{(\alpha\sigma)^{y+\nu} y! K_{\nu}(\frac{1}{\sigma})}$$

where  $\alpha^2=\frac{1}{\sigma^2}+\frac{2\mu}{\sigma}$ , for  $y=0,1,2,...,\infty$  where  $\mu>0$ ,  $\sigma>0$  and  $-\infty<\nu<\infty$  and  $K_\lambda(t)=\frac{1}{2}\int_0^\infty x^{\lambda-1}\exp\{-\frac{1}{2}t(x+x^{-1})\}dx$  is the modified Bessel function of the third kind. Note that the above parameterization is different from Stein, Zucchini and Juritz (1988) who use the above probability function but treat  $\mu$ ,  $\alpha$  and  $\nu$  as the parameters. Note that  $\sigma=[(\mu^2+\alpha^2)^{\frac{1}{2}}-\mu]^{-1}$ .

#### Value

Returns a gamlss.family object which can be used to fit a Sichel distribution in the gamlss() function.

#### Author(s)

Akantziliotou C., Rigby, R. A., Stasinopoulos D. M. and Marco Enea

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, \doi:10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

Stein, G. Z., Zucchini, W. and Juritz, J. M. (1987). Parameter Estimation of the Sichel Distribution and its Multivariate Extension. *Journal of American Statistical Association*, **82**, 938-944.

(see also https://www.gamlss.com/).

## See Also

```
gamlss.family, PIG, NBI, NBII
```

## **Examples**

```
SI()# gives information about the default links for the Sichel distribution #plot the pdf using plot plot(function(y) dSI(y, mu=10, sigma=1, nu=1), from=0, to=100, n=100+1, type="h") # pdf # plot the cdf plot(seq(from=0,to=100),pSI(seq(from=0,to=100), mu=10, sigma=1, nu=1), type="h") # cdf # generate random sample tN <- table(Ni <- rSI(100, mu=5, sigma=1, nu=1))
```

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```
r <- barplot(tN, col='lightblue')
# fit a model to the data
# library(gamlss)
# gamlss(Ni~1,family=SI, control=gamlss.control(n.cyc=50))</pre>
```

**SICHEL** 

The Sichel distribution for fitting a GAMLSS model

## **Description**

The SICHEL() function defines the Sichel distribution, a three parameter discrete distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dSICHEL, pSICHEL and rSICHEL define the density, distribution function, quantile function and random generation for the Sichel SICHEL(), distribution. The function VSICHEL gives the variance of a fitted Sichel model.

The functions ZASICHEL() and ZISICHEL() are the zero adjusted (hurdle) and zero inflated versions of the Sichel distribution, respectively. That is four parameter distributions.

The functions dZASICHEL, dZISICHEL, pZASICHEL, pZASICHEL, qZASICHEL qZISICHEL rZASICHEL and rZISICHEL define the probability, cumulative, quantile and random generation functions for the zero adjusted and zero inflated Sichel distributions, ZASICHEL(), ZISICHEL(), respectively.

#### Usage

```
SICHEL(mu.link = "log", sigma.link = "log", nu.link = "identity")
dSICHEL(x, mu=1, sigma=1, nu=-0.5, log=FALSE)
pSICHEL(q, mu=1, sigma=1, nu=-0.5, lower.tail = TRUE,
         log.p = FALSE)
qSICHEL(p, mu=1, sigma=1, nu=-0.5, lower.tail = TRUE,
        log.p = FALSE, max.value = 10000)
rSICHEL(n, mu=1, sigma=1, nu=-0.5, max.value = 10000)
VSICHEL(obi)
tofySICHEL(y, mu, sigma, nu)
ZASICHEL(mu.link = "log", sigma.link = "log", nu.link = "identity",
         tau.link = "logit")
dZASICHEL(x, mu = 1, sigma = 1, nu = -0.5, tau = 0.1, log = FALSE)
pZASICHEL(q, mu = 1, sigma = 1, nu = -0.5, tau = 0.1,
          lower.tail = TRUE, log.p = FALSE)
qZASICHEL(p, mu = 1, sigma = 1, nu = -0.5, tau = 0.1,
          lower.tail = TRUE, log.p = FALSE, max.value = 10000)
rZASICHEL(n, mu = 1, sigma = 1, nu = -0.5, tau = 0.1,
          max.value = 10000)
ZISICHEL(mu.link = "log", sigma.link = "log", nu.link = "identity",
          tau.link = "logit")
dZISICHEL(x, mu = 1, sigma = 1, nu = -0.5, tau = 0.1, log = FALSE)
pZISICHEL(q, mu = 1, sigma = 1, nu = -0.5, tau = 0.1,
```

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## **Arguments**

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter                                 |
|------------|----------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter                           |
| nu.link    | Defines the nu.link, with "identity" link as the default for the nu parameter                            |
| tau.link   | Defines the tau.link, with "logit" link as the default for the tau parameter                             |
| x          | vector of (non-negative integer) quantiles                                                               |
| mu         | vector of positive mu                                                                                    |
| sigma      | vector of positive dispersion parameter sigma                                                            |
| nu         | vector of nu                                                                                             |
| tau        | vector of probabilities tau                                                                              |
| р          | vector of probabilities                                                                                  |
| q          | vector of quantiles                                                                                      |
| n          | number of random values to return                                                                        |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                                    |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                       |
| max.value  | a constant, set to the default value of 10000 for how far the algorithm should look for $\boldsymbol{q}$ |
| obj        | a fitted Sichel gamlss model                                                                             |
| У          | the y variable, the tofySICHEL() should not be used on its own.                                          |
|            |                                                                                                          |

## **Details**

The probability function of the Sichel distribution is given by

$$f(y|\mu,\sigma,\nu) = \frac{\mu^y K_{y+\nu}(\alpha)}{y!(\alpha\sigma)^{y+\nu} K_{\nu}(\frac{1}{\sigma})}$$

for  $y=0,1,2,...,\infty, \mu>0$  ,  $\sigma>0$  and  $-\infty<\nu<\infty$  where

$$\alpha^2 = \frac{1}{\sigma^2} + \frac{2\mu}{\sigma}$$

$$c = K_{\nu+1}(1/\sigma)/K_{\nu}(1/\sigma)$$

and  $K_{\lambda}(t)$  is the modified Bessel function of the third kind. Note that the above parametrization is different from Stein, Zucchini and Juritz (1988) who use the above probability function but treat  $\mu$ ,  $\alpha$  and  $\nu$  as the parameters.

SICHEL

#### Value

Returns a gamlss.family object which can be used to fit a Sichel distribution in the gamlss() function.

#### Note

The mean of the above Sichel distribution is  $\mu$  and the variance is  $\mu^2 \left[ \frac{2\sigma(\nu+1)}{c} + \frac{1}{c^2} - 1 \right]$ 

## Author(s)

Rigby, R. A., Stasinopoulos D. M., Akantziliotou C and Marco Enea.

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

Stein, G. Z., Zucchini, W. and Juritz, J. M. (1987). Parameter Estimation of the Sichel Distribution and its Multivariate Extension. *Journal of American Statistical Association*, **82**, 938-944.

```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family, PIG, SI
```

#### **Examples**

```
SICHEL()# gives information about the default links for the Sichel distribution #plot the pdf using plot plot(function(y) dSICHEL(y, mu=10, sigma=1, nu=1), from=0, to=100, n=100+1, type="h") # pdf # plot the cdf plot(seq(from=0,to=100),pSICHEL(seq(from=0,to=100), mu=10, sigma=1, nu=1), type="h") # cdf # generate random sample tN <- table(Ni <- rSICHEL(100, mu=5, sigma=1, nu=1)) r <- barplot(tN, col='lightblue') # fit a model to the data # library(gamlss) # gamlss(Ni~1,family=SICHEL, control=gamlss.control(n.cyc=50))
```

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SIMPLEX

The simplex distribution for fitting a GAMLSS

## **Description**

The functions SIMPLEX() define the simplex distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). SIMPLEX() has mean equal to the parameter mu and sigma as scale parameter, see below. The functions dSIMPLEX, pSIMPLEX qSIMPLEX and rSIMPLEX define the density, comulative distribution function, quantile function and random generation for the simplex distribution.

## **Usage**

```
SIMPLEX(mu.link = "logit", sigma.link = "log")
dSIMPLEX(x, mu = 0.5, sigma = 1, log = FALSE)
pSIMPLEX(q, mu = 0.5, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qSIMPLEX(p, mu = 0.5, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rSIMPLEX(n = 1, mu = 0.5, sigma = 1)
```

## Arguments

| mu.link    | the mu link function with default logit                                                    |
|------------|--------------------------------------------------------------------------------------------|
| sigma.link | the sigma link function with default log                                                   |
| x,q        | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| р          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

## **Details**

The simplex distribution is given as

$$f(y|\mu,\sigma) = \frac{1}{(2\pi\sigma^2(y(1-y))^3)^{1/2}} exp(-\frac{1}{2\sigma^2} \frac{(y-\mu)^2}{y(1-y)\mu^2(1-\mu)^2})$$

for y in (0,1),  $0 < \mu < 1$  and  $\sigma > 0$ .

## Value

SIMPLEX() returns a gamlss.family object which can be used to fit a simplex distribution in the gamlss() function.

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#### Author(s)

Bob Rigby, Mikis Stasinopoulos and Fernanda De Bastiani

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

## **Examples**

```
SIMPLEX()# default links for the simplex distribution plot(function(y) dSIMPLEX(y, mu=.5 ,sigma=1), 0.001, .999) plot(function(y) pSIMPLEX(y, mu=.5 ,sigma=1), 0.001, 0.999) plot(function(y) qSIMPLEX(y, mu=.5 ,sigma=1), 0.001, 0.999) plot(function(y) qSIMPLEX(y, mu=.5 ,sigma=1, lower.tail=FALSE), 0.001, .999)
```

SN1

Skew Normal Type 1 distribution for fitting a GAMLSS

#### **Description**

The function SN1() defines the Skew Normal Type 1 distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(), with parameters mu, sigma and nu. The functions dSN1, pSN1, qSN1 and rSN1 define the density, distribution function, quantile function and random generation for the SN1 parameterization of the Skew Normal Type 1 distribution.

#### Usage

```
SN1(mu.link = "identity", sigma.link = "log", nu.link="identity")
dSN1(x, mu = 0, sigma = 1, nu = 0, log = FALSE)
pSN1(q, mu = 0, sigma = 1, nu = 0, lower.tail = TRUE, log.p = FALSE)
qSN1(p, mu = 0, sigma = 1, nu = 0, lower.tail = TRUE, log.p = FALSE)
rSN1(n, mu = 0, sigma = 1, nu = 0)
```

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## **Arguments**

| mu.link    | Defines the mu.link, with "'identity"' links the default for the mu parameter              |
|------------|--------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "'log"' as the default for the sigma parameter                |
| nu.link    | Defines the nu.link, with "'identity"' as the default for the sigma parameter              |
| x, q       | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| nu         | vector of scale parameter values                                                           |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                      |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise $P[X > x]$          |
| р          | vector of probabilities                                                                    |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

#### **Details**

The parameterization of the Skew Normal Type 1 distribution in the function SN1 is ...

#### Value

returns a gamlss.family object which can be used to fit a Skew Normal Type 1 distribution in the gamlss() function.

## Note

This is a special case of the Skew Exponential Power type 1 distribution (SEP1)where tau=2.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Fiona McElduff

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

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## See Also

```
gamlss.family
```

## **Examples**

```
par(mfrow=c(2,2))
y<-seq(-3,3,0.2)
plot(y, dSN1(y), type="l" , lwd=2)
q<-seq(-3,3,0.2)
plot(q, pSN1(q), ylim=c(0,1), type="l", lwd=2)
p<-seq(0.0001,0.999,0.05)
plot(p, qSN1(p), type="l", lwd=2)
dat <- rSN1(100)
hist(rSN1(100), nclass=30)</pre>
```

SN2

Skew Normal Type 2 distribution for fitting a GAMLSS

## Description

The function SN2() defines the Skew Normal Type 2 distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(), with parameters mu, sigma and nu. The functions dSN2, pSN2, qSN2 and rSN2 define the density, distribution function, quantile function and random generation for the SN2 parameterization of the Skew Normal Type 2 distribution.

## Usage

```
SN2(mu.link = "identity", sigma.link = "log", nu.link = "log")
dSN2(x, mu = 0, sigma = 1, nu = 2, log = FALSE)
pSN2(q, mu = 0, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)
qSN2(p, mu = 0, sigma = 1, nu = 2, lower.tail = TRUE, log.p = FALSE)
rSN2(n, mu = 0, sigma = 1, nu = 2)
```

#### **Arguments**

| mu.link    | Defines the mu.link, with "'identity" links the default for the mu parameter |
|------------|------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "'log" as the default for the sigma parameter   |
| nu.link    | Defines the nu.link, with "'log"' as the default for the sigma parameter     |
| x,q        | vector of quantiles                                                          |
| mu         | vector of location parameter values                                          |
| sigma      | vector of scale parameter values                                             |
| nu         | vector of scale parameter values                                             |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                        |

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| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise $P[X > x]$ |
|------------|-----------------------------------------------------------------------------------|
| р          | vector of probabilities                                                           |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number |
|            | required                                                                          |

#### **Details**

The parameterization of the Skew Normal Type 2 distribution in the function SN2 is ...

## Value

returns a gamlss.family object which can be used to fit a Skew Normal Type 2 distribution in the gamlss() function.

#### Note

This is a special case of the Skew Exponential Power type 3 distribution (SEP3)where tau=2.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Fiona McElduff.

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

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Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

#### See Also

```
gamlss.family
```

## **Examples**

```
par(mfrow=c(2,2))
y<-seq(-3,3,0.2)
plot(y, dSN2(y), type="1" , lwd=2)
q<-seq(-3,3,0.2)
plot(q, pSN2(q), ylim=c(0,1), type="1", lwd=2)
p<-seq(0.0001,0.999,0.05)
plot(p, qSN2(p), type="1", lwd=2)</pre>
```

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```
dat <- rSN2(100)
hist(rSN2(100), nclass=30)</pre>
```

ST1

*The skew t distributions, type 1 to 5* 

#### **Description**

There are 5 different skew t distributions implemented in GAMLSS.

The Skew t type 1 distribution, ST1, is based on Azzalini (1986).

The skew t type 2 distribution, ST2, is based on Azzalini and Capitanio (2003).

The skew t type 3, ST3 and ST3C, distribution is based Fernande and Steel (1998). The difference between the ST3 and ST3C is that the first is written entirely in R while the second is in C.

The skew t type 4 distribution, ST4, is a spliced-shape distribution.

The skew t type 5 distribution, ST5, is Jones and Faddy (2003).

The SST is a reparametrised version of dST3 where sigma is the standard deviation of the distribution.

#### Usage

```
ST1(mu.link = "identity", sigma.link = "log", nu.link = "identity", tau.link="log")
dST1(x, mu = 0, sigma = 1, nu = 0, tau = 2, log = FALSE)
pST1(q, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE, log.p = FALSE)
qST1(p, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE, log.p = FALSE)
rST1(n, mu = 0, sigma = 1, nu = 0, tau = 2)
ST2(mu.link = "identity", sigma.link = "log", nu.link = "identity", tau.link = "log")
dST2(x, mu = 0, sigma = 1, nu = 0, tau = 2, log = FALSE)
pST2(q, mu = 0, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE, log.p = FALSE)
qST2(p, mu = 1, sigma = 1, nu = 0, tau = 2, lower.tail = TRUE, log.p = FALSE)
rST2(n, mu = 0, sigma = 1, nu = 0, tau = 2)
ST3(mu.link = "identity", sigma.link = "log", nu.link = "log", tau.link = "log")
dST3(x, mu = 0, sigma = 1, nu = 1, tau = 10, log = FALSE)
pST3(q, mu = 0, sigma = 1, nu = 1, tau = 10, lower.tail = TRUE, log.p = FALSE)
qST3(p, mu = 0, sigma = 1, nu = 1, tau = 10, lower.tail = TRUE, log.p = FALSE)
rST3(n, mu = 0, sigma = 1, nu = 1, tau = 10)
ST3C(mu.link = "identity", sigma.link = "log", nu.link = "log", tau.link = "log")
dST3C(x, mu = 0, sigma = 1, nu = 1, tau = 10, log = FALSE)
pST3C(q, mu = 0, sigma = 1, nu = 1, tau = 10, lower.tail = TRUE, log.p = FALSE)
qST3C(p, mu = 0, sigma = 1, nu = 1, tau = 10, lower.tail = TRUE, log.p = FALSE)
rST3C(n, mu = 0, sigma = 1, nu = 1, tau = 10)
SST(mu.link = "identity", sigma.link = "log", nu.link = "log",
```

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```
tau.link = "logshiftto2")

dSST(x, mu = 0, sigma = 1, nu = 0.8, tau = 7, log = FALSE)

pSST(q, mu = 0, sigma = 1, nu = 0.8, tau = 7, lower.tail = TRUE, log.p = FALSE)

qSST(p, mu = 0, sigma = 1, nu = 0.8, tau = 7, lower.tail = TRUE, log.p = FALSE)

rSST(n, mu = 0, sigma = 1, nu = 0.8, tau = 7)

ST4(mu.link = "identity", sigma.link = "log", nu.link = "log", tau.link = "log")

dST4(x, mu = 0, sigma = 1, nu = 1, tau = 10, log = FALSE)

pST4(q, mu = 0, sigma = 1, nu = 1, tau = 10, lower.tail = TRUE, log.p = FALSE)

qST4(p, mu = 0, sigma = 1, nu = 1, tau = 10, lower.tail = TRUE, log.p = FALSE)

rST4(n, mu = 0, sigma = 1, nu = 1, tau = 10)

ST5(mu.link = "identity", sigma.link = "log", nu.link = "identity", tau.link = "log")

dST5(x, mu = 0, sigma = 1, nu = 0, tau = 1, log = FALSE)

pST5(q, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE, log.p = FALSE)

qST5(p, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE, log.p = FALSE)

rST5(n, mu = 0, sigma = 1, nu = 0, tau = 1, lower.tail = TRUE, log.p = FALSE)
```

## **Arguments**

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter. Other links are " $1/mu^2$ " and "log"    |
|------------|--------------------------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter. Other links are "inverse" and "identity" |
| nu.link    | Defines the nu.link, with "identity" link as the default for the nu parameter. Other links are " $1/mu^2$ " and "log"    |
| tau.link   | Defines the nu.link, with "log" link as the default for the nu parameter. Other links are "inverse", "identity"          |
| x,q        | vector of quantiles                                                                                                      |
| mu         | vector of mu parameter values                                                                                            |
| sigma      | vector of scale parameter values                                                                                         |
| nu         | vector of nu parameter values                                                                                            |
| tau        | vector of tau parameter values                                                                                           |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                                                   |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                       |
| р          | vector of probabilities.                                                                                                 |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required                               |

#### **Details**

$$f(y|\mu,\sigma,\nu,\frac{z}{\sigma} f_{z_1}(z) F_{z_2}(w)\tau) =$$

for  $-\infty < y < \infty$ , where  $z = (y - \mu)/\sigma$ ,  $w = \nu \lambda^{1/2} z$ ,  $\lambda = (\tau + 1)/(\tau + z^2)$  and  $z_1 \sim TF(0, 1, \tau)$  and  $z_2 \sim TF(0, 1, \tau + 1)$ .

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The probability density function of the skew t distribution type q, (ST3), is defined in Chapter 10 of the GAMLSS manual.

The probability density function of the skew t distribution type q, (ST4), is defined in Chapter of the GAMLSS manual.

The probability density function of the skew t distribution type 5, (ST5), is defined as

$$f(y|\mu,\sigma,\nu,\tau) = \frac{1}{c} \left[ 1 + \frac{z}{(a+b+z^2)^{1/2}} \right]^{a+1/2} \left[ 1 - \frac{z}{(a+b+z^2)^{1/2}} \right]^{b+1/2}$$

where  $c = 2^{a+b-1}(a+b)^{1/2}B(a,b)$ , and  $B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$  and  $z = (y-\mu)/\sigma$  and  $\nu = (a-b)/\left[ab(a+b)\right]^{1/2}$  and  $\tau = 2/(a+b)$  for  $-\infty < y < \infty, -\infty < \mu < \infty, \sigma > 0, -\infty < \nu > \infty$  and  $\tau > 0$ .

## Value

The functions ST1(), ST2(), ST3(), ST4() and ST5() return a gamlss.family object which can be used to fit the skew t type 1-5 distribution in the gamlss() function.

The functions dST1(), dST2(), dST3(), dST4() and dST5() give the density functions.

The funcions pST1(), pST2(), pST3(), pST4() and pST5() give the cumulative distribution functions.

The functions qST1(), qST2(), qST3(), qST4() and qST5() give the quantile function, and rST1(), rST2(), rST3(), rST4() and rST3() generates random deviates.

#### Note

The mean of the ex-Gaussian is  $\mu + \nu$  and the variance is  $\sigma^2 + \nu^2$ .

#### Author(s)

Bob Rigby and Mikis Stasinopoulos

#### References

Azzalini A. (1986) Futher results on a class of distributions which includes the normal ones, *Statistica*, **46**, pp. 199-208.

Azzalini A. and Capitanio, A. Distributions generated by perturbation of symmetry with emphasis on a multivariate skew t-distribution, *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, **65**, pp. 367-389.

Jones, M.C. and Faddy, M. J. (2003) A skew extension of the t distribution, with applications. *Journal of the Royal Statistical Society*, Series B, **65**, pp 159-174.

Fernandez, C. and Steel, M. F. (1998) On Bayesian modeling of fat tails and skewness. *Journal of the American Statistical Association*, **93**, pp. 359-371.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

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Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

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```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family, SEP1, SHASH
```

## **Examples**

```
y<- rST5(200, mu=5, sigma=1, nu=.1)
hist(y)
curve(dST5(x, mu=30, sigma=5, nu=-1), -50, 50, main = "The ST5 density mu=30, sigma=5, nu=1")
# library(gamlss)
# m1<-gamlss(y~1, family=ST1)
# m2<-gamlss(y~1, family=ST2)
# m3<-gamlss(y~1, family=ST3)
# m4<-gamlss(y~1, family=ST4)
# m5<-gamlss(y~1, family=ST5)
# GAIC(m1, m2, m3, m4, m5)</pre>
```

t family distribution for fitting a GAMLSS

TF

## **Description**

The function TF defines the t-family distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dTF, pTF, qTF and rTF define the density, distribution function, quantile function and random generation for the specific parameterization of the t distribution given in details below, with mean equal to  $\mu$  and standard deviation equal to  $\sigma(\frac{\nu}{\nu-2})^{0.5}$  with the degrees of freedom  $\nu$  The function TF2 is a different parametrization where sigma is the standard deviation.

## Usage

```
TF(mu.link = "identity", sigma.link = "log", nu.link = "log")
dTF(x, mu = 0, sigma = 1, nu = 10, log = FALSE)
pTF(q, mu = 0, sigma = 1, nu = 10, lower.tail = TRUE, log.p = FALSE)
qTF(p, mu = 0, sigma = 1, nu = 10, lower.tail = TRUE, log.p = FALSE)
rTF(n, mu = 0, sigma = 1, nu = 10)
```

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```
TF2(mu.link = "identity", sigma.link = "log", nu.link = "logshiftto2")
dTF2(x, mu = 0, sigma = 1, nu = 10, log = FALSE)
pTF2(q, mu = 0, sigma = 1, nu = 10, lower.tail = TRUE, log.p = FALSE)
qTF2(p, mu = 0, sigma = 1, nu = 10, lower.tail = TRUE, log.p = FALSE)
rTF2(n, mu = 0, sigma = 1, nu = 10)
```

## Arguments

| mu.link    | Defines the mu.link, with "identity" link as the default for the mu parameter              |
|------------|--------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter             |
| nu.link    | Defines the nu.link, with "log" link as the default for the nu parameter                   |
| x,q        | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| nu         | vector of the degrees of freedom parameter values                                          |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| p          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |

#### **Details**

Definition file for t family distribution.

$$f(y|\mu,\sigma,\nu) = \frac{\Gamma((\nu+1)/2)}{\sigma\Gamma(1/2)\Gamma(\nu/2)\nu^{0.5}} \left[1 + \frac{(y-\mu)^2}{\nu\sigma^2}\right]^{-(\nu+1)/2}$$

 $y=(-\infty,+\infty), \ \mu=(-\infty,+\infty), \ \sigma>0$  and  $\nu>0$ . Note that  $z=(y-\mu)/\sigma$  has a standard t distribution with degrees of freedom  $\nu$ .

#### Value

TF() returns a gamlss.family object which can be used to fit a t distribution in the gamlss() function. dTF() gives the density, pTF() gives the distribution function, qTF() gives the quantile function, and rTF() generates random deviates. The latest functions are based on the equivalent R functions for gamma distribution.

#### Note

 $\mu$  is the mean and  $\sigma[\nu/(\nu-2)]^{0.5}$  is the standard deviation of the t family distribution.  $\nu>0$  is a positive real valued parameter.

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Kalliope Akantziliotou

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#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

#### See Also

```
gamlss.family
```

#### **Examples**

```
TF()# gives information about the default links for the t-family distribution
# library(gamlss)
#data(abdom)
#h<-gamlss(y~cs(x,df=3), sigma.formula=~cs(x,1), family=TF, data=abdom) # fits
#plot(h)
newdata<-rTF(1000,mu=0,sigma=1,nu=5) # generates 1000 random observations
hist(newdata)</pre>
```

WARING

Waring distribution for fitting a GAMLSS model

## Description

The function WARING() defines the Waring distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(), with mean equal to the parameter mu and scale parameter sigma. The functions dWARING, pWARING, qWARING and rWARING define the density, distribution function, quantile function and random generation for the WARING parameterization of the Waring distribution.

## Usage

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## Arguments

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter               |
|------------|----------------------------------------------------------------------------------------|
| sigma.link | Defines the ${\tt sigma.link},$ with "log" link as the default for the sigma parameter |
| x          | vector of (non-negative integer) quantiles.                                            |
| q          | vector of quantiles.                                                                   |
| р          | vector of probabilities.                                                               |
| n          | number of random values to return.                                                     |
| mu         | vector of positive mu values.                                                          |
| sigma      | vector of positive sigma values.                                                       |
| lower.tail | logical; if TRUE (default) probabilities are $P[Y \leq y]$ , otherwise, $P[Y > y]$ .   |
| log, log.p | logical; if TRUE probabilities p are given as log(p).                                  |
| max.value  | constant; generates a sequence of values for the cdf function.                         |

#### **Details**

The Waring distribution has density,

$$f(y|\mu,\sigma) = \frac{(1+\sigma)\,\Gamma(y+\frac{\mu}{\sigma})\,\Gamma(\frac{\mu+\sigma+1}{\sigma})}{\sigma\,\Gamma(y+\frac{\mu+1}{\sigma}+2)\,\Gamma(\frac{\mu}{\sigma})}$$

for y = 0, 1, 2, ..., mu > 0 and sigma > 0.

#### Value

Returns a gamlss.family object which can be used to fit a Waring distribution in the gamlss() function.

## Author(s)

Fiona McElduff, Bob Rigby and Mikis Stasinopoulos. <f.mcelduff@ich.ucl.ac.uk>

#### References

Wimmer, G. and Altmann, G. (1999) *Thesaurus of univariate discrete probability distributions*. Stamm.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

## See Also

```
gamlss.family
```

(see also https://www.gamlss.com/).

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## **Examples**

```
par(mfrow=c(2,2))
y<-seq(0,20,1)
plot(y, dWARING(y), type="h")
q <- seq(0, 20, 1)
plot(q, pWARING(q), type="h")
p<-seq(0.0001,0.999,0.05)
plot(p, qWARING(p), type="s")
dat <- rWARING(100)
hist(dat)
#summary(gamlss(dat~1, family=WARING))</pre>
```

WEI

Weibull distribution for fitting a GAMLSS

## **Description**

The function WEI can be used to define the Weibull distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). [Note that the GAMLSS function WEI2 uses a different parameterization for fitting the Weibull distribution.] The functions dWEI, pWEI, qWEI and rWEI define the density, distribution function, quantile function and random generation for the specific parameterization of the Weibul distribution.

## Usage

```
WEI(mu.link = "log", sigma.link = "log")
dWEI(x, mu = 1, sigma = 1, log = FALSE)
pWEI(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qWEI(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rWEI(n, mu = 1, sigma = 1)
```

## **Arguments**

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter, other links are "inverse", "identity" and "own"         |
|------------|-----------------------------------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter, other link is the "inverse", "identity" and "own" |
| x,q        | vector of quantiles                                                                                                               |
| mu         | vector of the mu parameter                                                                                                        |
| sigma      | vector of sigma parameter                                                                                                         |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                                                            |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                                |
| p          | vector of probabilities.                                                                                                          |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required                                        |

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#### **Details**

The parameterization of the function WEI is given by

$$f(y|\mu,\sigma) = \frac{\sigma y^{\sigma-1}}{\mu^{\sigma}} \exp\left[-\left(\frac{y}{\mu}\right)^{\sigma}\right]$$

for y>0,  $\mu>0$  and  $\sigma>0$ . The GAMLSS functions dWEI, pWEI, qWEI, and rWEI can be used to provide the pdf, the cdf, the quantiles and random generated numbers for the Weibull distribution with argument mu, and sigma. [See the GAMLSS function WEI2 for a different parameterization of the Weibull.]

#### Value

WEI() returns a gamlss.family object which can be used to fit a Weibull distribution in the gamlss() function. dWEI() gives the density, pWEI() gives the distribution function, qWEI() gives the quantile function, and rWEI() generates random deviates. The latest functions are based on the equivalent R functions for Weibull distribution.

#### Note

The mean in WEI is given by  $\mu\Gamma(\frac{1}{\sigma}+1)$  and the variance  $\mu^2\left[\Gamma(\frac{2}{\sigma}+1)-(\Gamma(\frac{1}{\sigma}+1))^2\right]$ 

## Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

(see also https://www.gamlss.com/).

#### See Also

gamlss.family, WEI2, WEI3

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## **Examples**

```
WEI()
dat<-rWEI(100, mu=10, sigma=2)
# library(gamlss)
# gamlss(dat~1, family=WEI)</pre>
```

WEI2

A specific parameterization of the Weibull distribution for fitting a GAMLSS

## **Description**

The function WEI2 can be used to define the Weibull distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). This is the parameterization of the Weibull distribution usually used in proportional hazard models and is defined in details below. [Note that the GAMLSS function WEI uses a different parameterization for fitting the Weibull distribution.] The functions dWEI2, pWEI2, qWEI2 and rWEI2 define the density, distribution function, quantile function and random generation for the specific parameterization of the Weibull distribution.

## Usage

```
WEI2(mu.link = "log", sigma.link = "log")
dWEI2(x, mu = 1, sigma = 1, log = FALSE)
pWEI2(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qWEI2(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rWEI2(n, mu = 1, sigma = 1)
```

## **Arguments**

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter, other links are "inverse" and "identity"         |
|------------|----------------------------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter, other link is the "inverse" and "identity" |
| x,q        | vector of quantiles                                                                                                        |
| mu         | vector of the mu parameter values                                                                                          |
| sigma      | vector of sigma parameter values                                                                                           |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                         |
| р          | vector of probabilities.                                                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required                                 |

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## **Details**

The parameterization of the function WEI2 is given by

$$f(y|\mu,\sigma) = \sigma \mu y^{\sigma-1} e^{-\mu y^{\sigma}}$$

for y>0,  $\mu>0$  and  $\sigma>0$ . The GAMLSS functions dWEI2, pWEI2, qWEI2, and rWEI2 can be used to provide the pdf, the cdf, the quantiles and random generated numbers for the Weibull distribution with argument mu, and sigma. [See the GAMLSS function WEI for a different parameterization of the Weibull.]

#### Value

WEI2() returns a gamlss.family object which can be used to fit a Weibull distribution in the gamlss() function. dWEI2() gives the density, pWEI2() gives the distribution function, qWEI2() gives the quantile function, and rWEI2() generates random deviates. The latest functions are based on the equivalent R functions for Weibull distribution.

## Warning

In WEI2 the estimated parameters mu and sigma can be highly correlated so it is advisable to use the CG() method for fitting [as the RS() method can be very slow in this situation.]

#### Note

The mean in WEI2 is given by  $\mu^{-1/\sigma}\Gamma(\frac{1}{\sigma}+1)$  and the variance  $\mu^{-2/\sigma}(\Gamma(\frac{2}{\sigma}+1)-\left[\Gamma(\frac{1}{\sigma}+1)\right]^2)$ 

#### Author(s)

Mikis Stasinopoulos, Bob Rigby and Calliope Akantziliotou

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

#### See Also

```
gamlss.family, WEI, WEI3,
```

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## **Examples**

```
WEI2()
dat<-rWEI(100, mu=.1, sigma=2)
hist(dat)
# library(gamlss)
# gamlss(dat~1, family=WEI2, method=CG())</pre>
```

WEI3

A specific parameterization of the Weibull distribution for fitting a GAMLSS

## **Description**

The function WEI3 can be used to define the Weibull distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). This is a parameterization of the Weibull distribution where  $\mu$  is the mean of the distribution. [Note that the GAMLSS functions WEI and WEI2 use different parameterizations for fitting the Weibull distribution.] The functions dWEI3, pWEI3, qWEI3 and rWEI3 define the density, distribution function, quantile function and random generation for the specific parameterization of the Weibull distribution.

## Usage

```
WEI3(mu.link = "log", sigma.link = "log")
dWEI3(x, mu = 1, sigma = 1, log = FALSE)
pWEI3(q, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
qWEI3(p, mu = 1, sigma = 1, lower.tail = TRUE, log.p = FALSE)
rWEI3(n, mu = 1, sigma = 1)
```

## **Arguments**

| mu.link    | Defines the $mu.link$ , with "log" link as the default for the $mu$ parameter, other links are "inverse" and "identity"    |
|------------|----------------------------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter, other link is the "inverse" and "identity" |
| x,q        | vector of quantiles                                                                                                        |
| mu         | vector of the mu parameter values                                                                                          |
| sigma      | vector of sigma parameter values                                                                                           |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                         |
| р          | vector of probabilities.                                                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required                                 |

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#### **Details**

The parameterization of the function WEI3 is given by

$$f(y|\mu,\sigma) = \frac{\sigma}{\beta} \left(\frac{y}{\beta}\right)^{\sigma-1} e^{-\left(\frac{y}{\beta}\right)^{\sigma}}$$

where  $\beta=\frac{\mu}{\Gamma((1/\sigma)+1)}$  for y>0,  $\mu>0$  and  $\sigma>0$ . The GAMLSS functions dWEI3, pWEI3, qWEI3, and rWEI3 can be used to provide the pdf, the cdf, the quantiles and random generated numbers for the Weibull distribution with argument mu, and sigma. [See the GAMLSS function WEI for a different parameterization of the Weibull.]

#### Value

WEI3() returns a gamlss.family object which can be used to fit a Weibull distribution in the gamlss() function. dWEI3() gives the density, pWEI3() gives the distribution function, qWEI3() gives the quantile function, and rWEI3() generates random deviates. The latest functions are based on the equivalent R functions for Weibull distribution.

## Warning

In WEI3 the estimated parameters mu and sigma can be highly correlated so it is advisable to use the CG() method for fitting [as the RS() method can be very slow in this situation.]

#### Note

The mean in WEI3 is given by  $\mu$  and the variance  $\mu^2\left\{\Gamma(2/\sigma+1)/\left[\Gamma(1/\sigma+1)\right]^2-1\right\}$ 

#### Author(s)

Bob Rigby and Mikis Stasinopoulos

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

## (see also https://www.gamlss.com/).

## See Also

```
gamlss.family, WEI, WEI2
```

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## **Examples**

```
WEI3()
dat<-rWEI(100, mu=.1, sigma=2)
# library(gamlss)
# gamlss(dat~1, family=WEI3, method=CG())</pre>
```

YULE

Yule distribution for fitting a GAMLSS model

## Description

The function YULE defines the Yule distribution, a one parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(), with mean equal to the parameter mu. The functions dYULE, pYULE and rYULE define the density, distribution function, quantile function and random generation for the YULE parameterization of the Yule distribution.

## Usage

```
YULE(mu.link = "log")
dYULE(x, mu = 2, log = FALSE)
pYULE(q, mu = 2, lower.tail = TRUE, log.p = FALSE)
qYULE(p, mu = 2, lower.tail = TRUE, log.p = FALSE,
    max.value = 10000)
rYULE(n, mu = 2)
```

## Arguments

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter           |
|------------|------------------------------------------------------------------------------------|
| X          | vector of (non-negative integer) quantiles.                                        |
| q          | vector of quantiles.                                                               |
| р          | vector of probabilities.                                                           |
| n          | number of random values to return.                                                 |
| mu         | vector of positive mu values.                                                      |
| lower.tail | logical; if TRUE (default) probabilities are $P[Y \leq y],$ otherwise, $P[Y > y].$ |
| log, log.p | logical; if TRUE probabilities p are given as log(p).                              |
| max.value  | constant; generates a sequence of values for the cdf function.                     |

## **Details**

The Yule distribution has density

$$P(Y = y | \mu) = (\mu^{-1} + 1)B(y + 1, \mu^{-1} + 2)$$

```
for y = 0, 1, 2, ... and mu > 0.
```

ZABB

## Value

Returns a gamlss.family object which can be used to fit a Yule distribution in the gamlss() function.

## Author(s)

Fiona McElduff, Bob Rigby and Mikis Stasinopoulos.

#### References

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

Wimmer, G. and Altmann, G. (1999) Thesaurus of univariate discrete probability distributions. Stamm.

```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family
```

## **Examples**

```
par(mfrow=c(2,2))
y<-seq(0,20,1)
plot(y, dYULE(y), type="h")
q <- seq(0, 20, 1)
plot(q, pYULE(q), type="h")
p<-seq(0.0001,0.999,0.05)
plot(p , qYULE(p), type="s")
dat <- rYULE(100)
hist(dat)
#summary(gamlss(dat~1, family=YULE))</pre>
```

ZABB

Zero inflated and zero adjusted Binomial distribution for fitting in GAMLSS

## **Description**

The function ZIBB defines the zero inflated beta binomial distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZIBB, pZIBB, qZIBB and rZINN define the density, distribution function, quantile function and random generation for the zero inflated beta binomial, ZIBB, distribution.

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The function ZABB defines the zero adjusted beta binomial distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZABB, pZABB, qZABB and rZABB define the density, distribution function, quantile function and random generation for the zero inflated beta binomial, ZABB(), distribution.

## Usage

```
ZABB(mu.link = "logit", sigma.link = "log", nu.link = "logit")

ZIBB(mu.link = "logit", sigma.link = "log", nu.link = "logit")

dZIBB(x, mu = 0.5, sigma = 0.5, nu = 0.1, bd = 1, log = FALSE)

dZABB(x, mu = 0.5, sigma = 0.1, nu = 0.1, bd = 1, lower.tail = TRUE, log.p = FALSE)

pZIBB(q, mu = 0.5, sigma = 0.5, nu = 0.1, bd = 1, lower.tail = TRUE, log.p = FALSE)

pZABB(q, mu = 0.5, sigma = 0.1, nu = 0.1, bd = 1, lower.tail = TRUE, log.p = FALSE)

qZIBB(p, mu = 0.5, sigma = 0.5, nu = 0.1, bd = 1, lower.tail = TRUE, log.p = FALSE)

qZABB(p, mu = 0.5, sigma = 0.1, nu = 0.1, bd = 1, lower.tail = TRUE, log.p = FALSE)

rZIBB(n, mu = 0.5, sigma = 0.5, nu = 0.1, bd = 1)

rZABB(n, mu = 0.5, sigma = 0.1, nu = 0.1, bd = 1)
```

## Arguments

| mu.link    | Defines the mu.link, with "logit" link as the default for the mu parameter. Other links are "probit" and "cloglog" (complementary log-log)    |
|------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter.                                                               |
| nu.link    | Defines the sigma.link, with "logit" link as the default for the mu parameter. Other links are "probit" and "cloglog" (complementary log-log) |
| х          | vector of (non-negative integer) quantiles                                                                                                    |
| mu         | vector of positive probabilities                                                                                                              |
| sigma      | vector of positive dispertion parameter                                                                                                       |
| nu         | vector of positive probabilities                                                                                                              |
| bd         | vector of binomial denominators                                                                                                               |
| p          | vector of probabilities                                                                                                                       |
| q          | vector of quantiles                                                                                                                           |
| n          | number of random values to return                                                                                                             |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                                                                         |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                                            |

#### **Details**

For the definition of the distributions see Rigby and Stasinopoulos (2010) below.

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## Value

The functions ZIBB and ZABB return a gamlss.family object which can be used to fit a zero inflated or zero adjusted beta binomial distribution respectively in the gamlss() function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) Distributions for modeling location, scale, and shape: Using GAMLSS in R, Chapman and Hall/CRC. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07..

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family, NBI, NBII
```

#### **Examples**

```
ZIBB()
ZABB()
# creating data and plotting them
dat <- rZIBB(1000, mu=.5, sigma=.5, nu=0.1, bd=10)
   r <- barplot(table(dat), col='lightblue')
dat1 <- rZABB(1000, mu=.5, sigma=.2, nu=0.1, bd=10)
   r1 <- barplot(table(dat1), col='lightblue')</pre>
```

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## **Description**

The ZABI() function defines the zero adjusted binomial distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZABI, pZABI and rZABI define the density, distribution function, quantile function and random generation for the zero adjusted binomial, ZABI(), distribution.

The ZIBI() function defines the zero inflated binomial distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZIBI, pZIBI, qZIBI and rZIBI define the density, distribution function, quantile function and random generation for the zero inflated binomial, ZIBI(), distribution.

## Usage

```
ZABI(mu.link = "logit", sigma.link = "logit")
dZABI(x, bd = 1, mu = 0.5, sigma = 0.1, log = FALSE)
pZABI(q, bd = 1, mu = 0.5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
qZABI(p, bd = 1, mu = 0.5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
rZABI(n, bd = 1, mu = 0.5, sigma = 0.1)

ZIBI(mu.link = "logit", sigma.link = "logit")
dZIBI(x, bd = 1, mu = 0.5, sigma = 0.1, log = FALSE)
pZIBI(q, bd = 1, mu = 0.5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
qZIBI(p, bd = 1, mu = 0.5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
rZIBI(n, bd = 1, mu = 0.5, sigma = 0.1)
```

#### Arguments

| mu.link    | Defines the mu.link, with "logit" link as the default for the mu parameter. Other links are "probit" and "cloglog" (complementary log-log)    |
|------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "logit" link as the default for the mu parameter. Other links are "probit" and "cloglog" (complementary log-log) |
| X          | vector of (non-negative integer) quantiles                                                                                                    |
| mu         | vector of positive probabilities                                                                                                              |
| sigma      | vector of positive probabilities                                                                                                              |
| bd         | vector of binomial denominators                                                                                                               |
| р          | vector of probabilities                                                                                                                       |
| q          | vector of quantiles                                                                                                                           |
| n          | number of random values to return                                                                                                             |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                                                                         |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                                            |

## **Details**

For the definition of the distributions see Rigby and Stasinopoulos (2010) below.

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#### Value

The functions ZABI and ZIBI return a gamlss.family object which can be used to fit a binomial distribution in the gamlss() function.

#### Note

The response variable should be a matrix containing two columns, the first with the count of successes and the second with the count of failures.

## Author(s)

Mikis Stasinopoulos, Bob Rigby

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family, BI
```

## **Examples**

```
ZABI()
curve(dZABI(x, mu = .5, bd=10), from=0, to=10, n=10+1, type="h")
tN <- table(Ni <- rZABI(1000, mu=.2, sigma=.3, bd=10))
r <- barplot(tN, col='lightblue')

ZIBI()
curve(dZIBI(x, mu = .5, bd=10), from=0, to=10, n=10+1, type="h")
tN <- table(Ni <- rZIBI(1000, mu=.2, sigma=.3, bd=10))
r <- barplot(tN, col='lightblue')</pre>
```

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ZAGA

The zero adjusted Gamma distribution for fitting a GAMLSS model

## **Description**

The function ZAGA() defines the zero adjusted Gamma distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The zero adjusted Gamma distribution is similar to the Gamma distribution but allows zeros as y values. The extra parameter nu models the probabilities at zero. The functions dZAGA, pZAGA, qZAGA and rZAGA define the density, distribution function, quartile function and random generation for the ZAGA parameterization of the zero adjusted Gamma distribution. plotZAGA can be used to plot the distribution. meanZAGA calculates the expected value of the response for a fitted model.

#### **Usage**

## **Arguments**

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter                   |
|------------|--------------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter             |
| nu.link    | Defines the nu.link, with "logit" link as the default for the sigma parameter              |
| x,q        | vector of quantiles                                                                        |
| mu         | vector of location parameter values                                                        |
| sigma      | vector of scale parameter values                                                           |
| nu         | vector of probability at zero parameter values                                             |
| log, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| p          | vector of probabilities.                                                                   |
| n          | number of observations. If $length(n) > 1$ , the length is taken to be the number required |
| from       | where to start plotting the distribution from                                              |
| to         | up to where to plot the distribution                                                       |

ZAGA

obj a fitted gamlss object

main for title in the plot

... can be used to pass the uppr.limit argument to qIG

#### **Details**

The Zero adjusted GA distribution is given as

$$f(y|\mu,\sigma\nu) = \nu$$

if (y=0) 
$$f(y|\mu,\sigma,\nu) = (1-\nu) \left[ \frac{1}{(\sigma^2\mu)^{1/\sigma^2}} \; \frac{y^{\frac{1}{\sigma^2}-1} \; e^{-y/(\sigma^2\mu)}}{\Gamma(1/\sigma^2)} \right]$$

otherwise

for 
$$y = (0, \infty)$$
,  $\mu > 0$ ,  $\sigma > 0$  and  $0 < \nu < 1$ .  $E(y) = (1 - \nu)\mu$  and  $Var(y) = (1 - \nu)\mu^2(\nu + \sigma^2)$ .

## Value

The function ZAGA returns a gamlss.family object which can be used to fit a zero adjusted Gamma distribution in the gamlss() function.

## Author(s)

Bob Rigby, Mikis Stasinopoulos and Almond Stocker

## References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M., Rigby R.A. and Akantziliotou C. (2006) Instructions on how to use the GAMLSS package in R. Accompanying documentation in the current GAMLSS help files, (see also https://www.gamlss.com/).

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

(see also https://www.gamlss.com/).

## See Also

```
gamlss.family, GA, ZAIG
```

## **Examples**

```
ZAGA()# gives information about the default links for the ZAGA distribution # plotting the function

PPP <- par(mfrow=c(2,2))

plotZAGA(mu=1, sigma=.5, nu=.2, from=0,to=3)

#curve(dZAGA(x,mu=1, sigma=.5, nu=.2), 0,3) # pdf

curve(pZAGA(x,mu=1, sigma=.5, nu=.2), 0,3, ylim=c(0,1)) # cdf

curve(qZAGA(x,mu=1, sigma=.5, nu=.2), 0,.99) # inverse cdf

y<-rZAGA(100, mu=1, sigma=.5, nu=.2) # randomly generated values

hist(y)

par(PPP)

# check that the positive part sums up to .8 (since nu=0.2)

integrate(function(x) dZAGA(x,mu=1, sigma=.5, nu=.2), 0,Inf)
```

ZAIG

The zero adjusted Inverse Gaussian distribution for fitting a GAMLSS model

## **Description**

The function ZAIG() defines the zero adjusted Inverse Gaussian distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The zero adjusted Inverse Gaussian distribution is similar to the Inverse Gaussian distribution but allows zeros as y values. The extra parameter models the probabilities at zero. The functions dZAIG, pZAIG, qZAIG and rZAIG define the density, distribution function, quantile function and random generation for the ZAIG parameterization of the zero adjusted Inverse Gaussian distribution. plotZAIG can be used to plot the distribution. meanZAIG calculates the expected value of the response for a fitted model.

## Usage

#### Arguments

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter       |
|------------|--------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter |
| nu.link    | Defines the nu.link, with "logit" link as the default for the sigma parameter  |
| x,q        | vector of quantiles                                                            |
| mu         | vector of location parameter values                                            |

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| S  | igma      | vector of scale parameter values                                                           |
|----|-----------|--------------------------------------------------------------------------------------------|
| nı | u         | vector of probability at zero parameter values                                             |
| 1  | og, log.p | logical; if TRUE, probabilities p are given as log(p).                                     |
| 1  | ower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$         |
| р  |           | vector of probabilities.                                                                   |
| n  |           | number of observations. If $length(n) > 1$ , the length is taken to be the number required |
| f  | rom       | where to start plotting the distribution from                                              |
| t  | 0         | up to where to plot the distribution                                                       |
| ol | bj        | a fitted BEINF object                                                                      |
| ma | ain       | for title in the plot                                                                      |
|    |           | can be used to pass the uppr.limit argument to qIG                                         |

## **Details**

The Zero adjusted IG distribution is given as

$$f(y|\mu,\sigma\,\nu)=\nu$$
 if (y=0) 
$$f(y|\mu,\sigma,\nu)=(1-\nu)\frac{1}{\sqrt{2\pi\sigma^2y^3}}\exp(-\frac{(y-\mu)^2}{2\mu^2\sigma^2y})$$
 otherwise for  $y=(0,\infty), \mu>0, \sigma>0$  and  $0<\nu<1$ .  $E(y)=(1-\nu)\mu$  and  $Var(y)=(1-\nu)\mu^2(\nu+\mu\sigma^2)$ .

## Value

returns a gamlss.family object which can be used to fit a zero adjusted inverse Gaussian distribution in the gamlss() function.

## Author(s)

Bob Rigby and Mikis Stasinopoulos

## References

Heller, G. Stasinopoulos M and Rigby R.A. (2006) The zero-adjusted Inverse Gaussian distribution as a model for insurance claims. in *Proceedings of the 21th International Workshop on Statistial Modelling*, eds J. Hinde, J. Einbeck and J. Newell, pp 226-233, Galway, Ireland.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

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Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

#### See Also

```
gamlss.family, IG
```

## **Examples**

```
ZAIG()# gives information about the default links for the ZAIG distribution # plotting the distribution plotZAIG( mu =10 , sigma=.5, nu = 0.1, from = 0, to=10, n = 101) # plotting the cdf plot(function(y) pZAIG(y, mu=10 ,sigma=.5, nu = 0.1 ), 0, 1) # plotting the inverse cdf plot(function(y) qZAIG(y, mu=10 ,sigma=.5, nu = 0.1 ), 0.001, .99) # generate random numbers dat <- rZAIG(100,mu=10,sigma=.5, nu=.1) # fit a model to the data # library(gamlss) # m1<-gamlss(dat~1,family=ZAIG) # meanZAIG(m1)[1]
```

ZANBI

Zero inflated and zero adjusted negative binomial distributions for fitting a GAMLSS model

## **Description**

The function ZINBI defines the zero inflated negative binomial distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZINBI, pZINBI and rZINBI define the density, distribution function, quantile function and random generation for the zero inflated negative binomial, ZINBI(), distribution.

The function ZANBI defines the zero adjusted negative binomial distribution, a three parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZANBI, pZANBI and rZANBI define the density, distribution function, quantile function and random generation for the zero inflated negative binomial, ZANBI(), distribution.

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## **Usage**

```
ZINBI(mu.link = "log", sigma.link = "log", nu.link = "logit")
dZINBI(x, mu = 1, sigma = 1, nu = 0.3, log = FALSE)
pZINBI(q, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE)
qZINBI(p, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE)
rZINBI(n, mu = 1, sigma = 1, nu = 0.3)
ZANBI(mu.link = "log", sigma.link = "log", nu.link = "logit")
dZANBI(x, mu = 1, sigma = 1, nu = 0.3, log = FALSE)
pZANBI(q, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE)
qZANBI(p, mu = 1, sigma = 1, nu = 0.3, lower.tail = TRUE, log.p = FALSE)
rZANBI(n, mu = 1, sigma = 1, nu = 0.3)
```

## **Arguments**

| mu.link    | Defines the mu.link, with "log" link as the default for the mu parameter           |
|------------|------------------------------------------------------------------------------------|
| sigma.link | Defines the sigma.link, with "log" link as the default for the sigma parameter     |
| nu.link    | Defines the mu.link, with "logit" link as the default for the nu parameter         |
| x          | vector of (non-negative integer) quantiles                                         |
| mu         | vector of positive means                                                           |
| sigma      | vector of positive despersion parameter                                            |
| nu         | vector of zero probability parameter                                               |
| р          | vector of probabilities                                                            |
| q          | vector of quantiles                                                                |
| n          | number of random values to return                                                  |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                              |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$ |

## **Details**

The definition for the zero inflated Negative Binomial type I distribution and for the zero adjusted Negative Binomial type I distribution is given in Rigby and Stasinopoulos (2010) below

## Value

The functions ZINBI and ZANBI return a gamlss.family object which can be used to fit a zero inflated or zero adjusted Negative Binomial type I distribution respectively in the gamlss() function.

#### Author(s)

Mikis Stasinopoulos, Bob Rigby

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#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973

```
(see also https://www.gamlss.com/).
```

## See Also

```
gamlss.family, NBI, NBII
```

# **Examples**

```
ZINBI()
ZANBI()
# creating data and plotting them
dat <- rZINBI(1000, mu=5, sigma=.5, nu=0.1)
   r <- barplot(table(dat), col='lightblue')
dat1 <- rZANBI(1000, mu=5, sigma=.5, nu=0.1)
   r1 <- barplot(table(dat1), col='lightblue')</pre>
```

ZAP

Zero adjusted poisson distribution for fitting a GAMLSS model

## **Description**

The function ZAP defines the zero adjusted Poisson distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZAP, pZAP, qZAP and rZAP define the density, distribution function, quantile function and random generation for the inflated poisson, ZAP(), distribution.

# Usage

```
ZAP(mu.link = "log", sigma.link = "logit")

dZAP(x, mu = 5, sigma = 0.1, log = FALSE)

pZAP(q, mu = 5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)

qZAP(p, mu = 5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)

rZAP(n, mu = 5, sigma = 0.1)
```

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# Arguments

| mu.link    | defines the mu.link, with "log" link as the default for the mu parameter                                                                                                                       |
|------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| sigma.link | defines the sigma.link, with "logit" link as the default for the sigma parameter which in this case is the probability at zero. Other links are "probit" and "cloglog" (complementary log-log) |
| X          | vector of (non-negative integer)                                                                                                                                                               |
| mu         | vector of positive means                                                                                                                                                                       |
| sigma      | vector of probabilities at zero                                                                                                                                                                |
| р          | vector of probabilities                                                                                                                                                                        |
| q          | vector of quantiles                                                                                                                                                                            |
| n          | number of random values to return                                                                                                                                                              |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                                                                                                                          |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                                                                                             |

#### **Details**

For the definition of the distribution see Rigby and Stasinopoulos (2010) below.

## Value

The function ZAP returns a gamlss.family object which can be used to fit a zero inflated poisson distribution in the gamlss() function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby

#### References

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/)..

## See Also

```
gamlss.family, PO, ZIP, ZIP2, ZALG
```

## **Examples**

```
ZAP()
# creating data and plotting them
dat<-rZAP(1000, mu=5, sigma=.1)
r <- barplot(table(dat), col='lightblue')</pre>
```

ZIP

Zero inflated poisson distribution for fitting a GAMLSS model

## **Description**

The function ZIP defines the zero inflated Poisson distribution, a two parameter distribution, for a gamlss.family object to be used in GAMLSS fitting using the function gamlss(). The functions dZIP, pZIP, qZIP and rZIP define the density, distribution function, quantile function and random generation for the inflated poisson, ZIP(), distribution.

# Usage

```
ZIP(mu.link = "log", sigma.link = "logit")
dZIP(x, mu = 5, sigma = 0.1, log = FALSE)
pZIP(q, mu = 5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
qZIP(p, mu = 5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
rZIP(n, mu = 5, sigma = 0.1)
```

# **Arguments**

| mu.link    | defines the mu.link, with "log" link as the default for the mu parameter                                                                                                                       |
|------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| sigma.link | defines the sigma.link, with "logit" link as the default for the sigma parameter which in this case is the probability at zero. Other links are "probit" and "cloglog" (complementary log-log) |
| x          | vector of (non-negative integer) quantiles                                                                                                                                                     |
| mu         | vector of positive means                                                                                                                                                                       |
| sigma      | vector of probabilities at zero                                                                                                                                                                |
| р          | vector of probabilities                                                                                                                                                                        |
| q          | vector of quantiles                                                                                                                                                                            |
| n          | number of random values to return                                                                                                                                                              |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                                                                                                                          |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                                                                                             |

#### **Details**

Let Y=0 with probability  $\sigma$  and  $Y\sim Po(\mu)$  with probability  $(1-\sigma)$  the Y has a Zero inflated Poisson Distribution given by

$$f(y)=\sigma+(1-\sigma)e^{-\mu}$$
 if (y=0) 
$$f(y)=(1-\sigma)\frac{e^{-\mu}\mu^y}{y!}$$
 if (y>0) for  $y=0,1,....$ 

#### Value

returns a gamlss.family object which can be used to fit a zero inflated poisson distribution in the gamlss() function.

## Author(s)

Mikis Stasinopoulos, Bob Rigby

## References

Lambert, D. (1992), Zero-inflated Poisson Regression with an application to defects in Manufacturing, *Technometrics*, **34**, pp 1-14.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

#### See Also

```
gamlss.family, PO, ZIP2
```

# **Examples**

```
ZIP()# gives information about the default links for the normal distribution
# creating data and plotting them
dat<-rZIP(1000, mu=5, sigma=.1)
r <- barplot(table(dat), col='lightblue')
# library(gamlss)</pre>
```

```
# fit the distribution
# mod1<-gamlss(dat~1, family=ZIP)# fits a constant for mu and sigma
# fitted(mod1)[1]
# fitted(mod1, "sigma")[1]
```

ZIP2

Zero inflated poisson distribution for fitting a GAMLSS model

# **Description**

The function ZIP2 defines the zero inflated Poisson type 2 distribution, a two parameter distribution, for a gamlss. family object to be used in GAMLSS fitting using the function gamlss(). The functions dZIP2, pZIP2, qZIP2 and rZIP2 define the density, distribution function, quantile function and random generation for the inflated poisson, ZIP2(), distribution. The ZIP2 is a different parameterization of the ZIP distribution. In the ZIP2 the mu is the mean of the distribution.

# Usage

```
ZIP2(mu.link = "log", sigma.link = "logit")
dZIP2(x, mu = 5, sigma = 0.1, log = FALSE)
pZIP2(q, mu = 5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
qZIP2(p, mu = 5, sigma = 0.1, lower.tail = TRUE, log.p = FALSE)
rZIP2(n, mu = 5, sigma = 0.1)
```

# **Arguments**

| mu.link    | defines the mu.link, with "log" link as the default for the mu parameter                                                                                                                       |
|------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| sigma.link | defines the sigma.link, with "logit" link as the default for the sigma parameter which in this case is the probability at zero. Other links are "probit" and "cloglog" (complementary log-log) |
| x          | vector of (non-negative integer) quantiles                                                                                                                                                     |
| mu         | vector of positive means                                                                                                                                                                       |
| sigma      | vector of probabilities at zero                                                                                                                                                                |
| р          | vector of probabilities                                                                                                                                                                        |
| q          | vector of quantiles                                                                                                                                                                            |
| n          | number of random values to return                                                                                                                                                              |
| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                                                                                                                          |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                                                                                             |

#### **Details**

Let Y=0 with probability  $\sigma$  and  $Y \sim Po(\mu/[1-\sigma])$  with probability  $(1-\sigma)$  then Y has a Zero inflated Poisson type 2 distribution given by

$$f(y|\mu,\sigma) = \sigma + (1-\sigma)e^{-\mu/(1-\sigma)} \text{ if } y = 0$$

$$f(y|\mu,\sigma) = (1-\sigma)\frac{e^{-\mu/(1-\sigma)} \left[\mu/(1-\sigma)\right]^y}{y!} \text{ if } y = 1, 2, 3, \dots$$

The mean of the distribution in this parameterization is mu.

## Value

returns a gamlss.family object which can be used to fit a zero inflated poisson distribution in the gamlss() function.

#### Author(s)

Bob Rigby, Gillian Heller and Mikis Stasinopoulos

#### References

Lambert, D. (1992), Zero-inflated Poisson Regression with an application to defects in Manufacturing, *Technometrics*, **34**, pp 1-14.

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

Stasinopoulos D. M. Rigby R.A. (2007) Generalized additive models for location scale and shape (GAMLSS) in R. *Journal of Statistical Software*, Vol. **23**, Issue 7, Dec 2007, doi: 10.18637/jss.v023.i07.

Stasinopoulos D. M., Rigby R.A., Heller G., Voudouris V., and De Bastiani F., (2017) *Flexible Regression and Smoothing: Using GAMLSS in R*, Chapman and Hall/CRC. doi: 10.1201/b21973 (see also https://www.gamlss.com/).

## See Also

```
gamlss.family, ZIP
```

# Examples

```
ZIP2()# gives information about the default links for the normal distribution
# creating data and plotting them
dat<-rZIP2(1000, mu=5, sigma=.1)
r <- barplot(table(dat), col='lightblue')
# fit the disteibution
# library(gamlss)</pre>
```

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```
# mod1<-gamlss(dat~1, family=ZIP2)# fits a constant for mu and sigma
# fitted(mod1)[1]
# fitted(mod1,"sigma")[1]</pre>
```

ZIPF

The zipf and zero adjusted zipf distributions for fitting a GAMLSS model

#### **Description**

This function ZIPF() defines the zipf distribution, Johnson et. al., (2005), sections 11.2.20, p 527-528. The zipf distribution is an one parameter distribution with long tails (a discete version of the Pareto distribution). The function ZIPF() creates a gamlss.family object to be used in GAMLSS fitting. The functions dZIPF, pZIPF, qZIPF and rZIPF define the density, distribution function, quantile function and random generation for the zipf, ZIPF(), distribution. The function zetaP() defines the zeta function and it is based on the zeta function defined on the VGAM package of Thomas Yee, see Yee (2017).

The distribution zipf is defined on  $y=1,2,3,\ldots,\infty$ , the zero adjusted zipf permits values on  $y=,01,2,\ldots,\infty$ . The function ZAZIPF() defines the zero adjusted zipf distribution. The function ZAZIPF() creates a gamlss.family object to be used in GAMLSS fitting. The functions dZAZIPF, pZAZIPF, qZAZIPF and rZAZIPF define the density, distribution function, quantile function and random generation for the zero adjusted zipf, ZAZIPF(), distribution.

#### Usage

## **Arguments**

```
    mu.link the link function for the parameter mu with default log x,q vectors of (non-negative integer) quantiles
    p vector of probabilities
    mu vector of positive parameter
```

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| log, log.p | logical; if TRUE, probabilities p are given as log(p)                                                                                                                                                                                  |
|------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| lower.tail | logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$                                                                                                                                                     |
| n          | number of random values to return                                                                                                                                                                                                      |
| max.value  | a constant, set to the default value of 10000, It is used in the q function which numerically calculates how far the algorithm should look for q. Maybe for zipf data the values has to increase at a considerable computational cost. |
| sigma.link | the link function for the parameter aigma with default logit                                                                                                                                                                           |
| sigma      | a vector of probabilities of zero                                                                                                                                                                                                      |

#### **Details**

The probability density for the zipf distribution, ZIPF, is:

$$f(y|\mu) = \frac{y^{-(\mu+1)}}{\zeta(\mu+1)}$$

for  $y = 1, 2, ..., \infty, \mu > 0$  and where  $\zeta()$  is the (Reimann) zeta function.

The distribution has mean  $\zeta(\mu)/\zeta(\mu+1)$  and variance  $\zeta(\mu+1)\zeta(\mu-1)-[\zeta(\mu)]^2/[\zeta(\mu+1)]^2$ .

## Value

The function ZIPF() returns a gamlss.family object which can be used to fit a zipf distribution in the gamlss() function.

## Note

Because the zipf distribution has very long tails the max.value in the q and r, may need to increase.

# Author(s)

Mikis Stasinopoulos and Bob Rigby

## References

N. L. Johnson, A. W. Kemp, and S. Kotz. (2005) Univariate Discrete Distributions. Wiley, New York, 3rd edition.

Thomas W. Yee (2017). VGAM: Vector Generalized Linear and Additive Models. R package version 1.0-3. https://CRAN.R-project.org/package=VGAM

Rigby, R. A. and Stasinopoulos D. M. (2005). Generalized additive models for location, scale and shape, (with discussion), *Appl. Statist.*, **54**, part 3, pp 507-554.

Rigby, R. A., Stasinopoulos, D. M., Heller, G. Z., and De Bastiani, F. (2019) *Distributions for modeling location, scale, and shape: Using GAMLSS in R*, Chapman and Hall/CRC, doi: 10.1201/9780429298547. An older version can be found in https://www.gamlss.com/.

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# See Also

PO, LG, GEOM, YULE

# **Examples**

```
# ZIPF
par(mfrow=c(2,2))
y < -seq(1, 20, 1)
plot(y, dZIPF(y), type="h")
q < - seq(1, 20, 1)
plot(q, pZIPF(q), type="h")
p<-seq(0.0001,0.999,0.05)
plot(p , qZIPF(p), type="s")
dat <- rZIPF(100)</pre>
hist(dat)
# ZAZIPF
y < -seq(0, 20, 1)
plot(y, dZAZIPF(y, mu=.9, sigma=.1), type="h")
q <- seq(1, 20, 1)
plot(q, pZAZIPF(q, mu=.9, sigma=.1), type="h")
p<-seq(0.0001,0.999,0.05)
plot(p, qZAZIPF(p, mu=.9, sigma=.1), type="s")
dat <- rZAZIPF(100, mu=.9, sigma=.1)</pre>
hist(dat)
```

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