Generalised Additive Models for Location Scale and Shape

Past, Present and Future

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Statistical Modelling Society

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The statistical modelling philosophy

Statistical modelling is the art of using statistical reasoning to build a parsimonious models for a better understanding of the phenomena of interest.

- get data
- build a model
- interpretate/predict

The statistical modelling principals

- Any model is a simplification of reality therefore no model is correct but some of them are useful
- Occam's Razor which states *'entities should not be multiplied beyond necessity'* or KISS (Keep It Simple Stupid)
- Far better an approximate answer to the right question, which is often vague, than an exact answer to the wrong question, which can always be made precise. – John W. Tukey
- *"no matter how beautiful your theory/model, no matter how clever you are or what your name is, if it disagrees with experiment"/data, "it's wrong"* (Richard Feynman)
- Test all the time your assumptions (there is no free meal)
- Try different models and choose the most appropriate for the data (have a data scientist attitude).

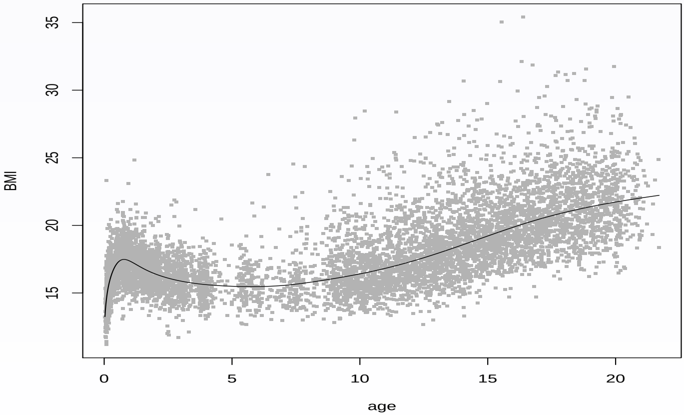
The Dutch boys data

`BMI` : the BMI of 7294 boys

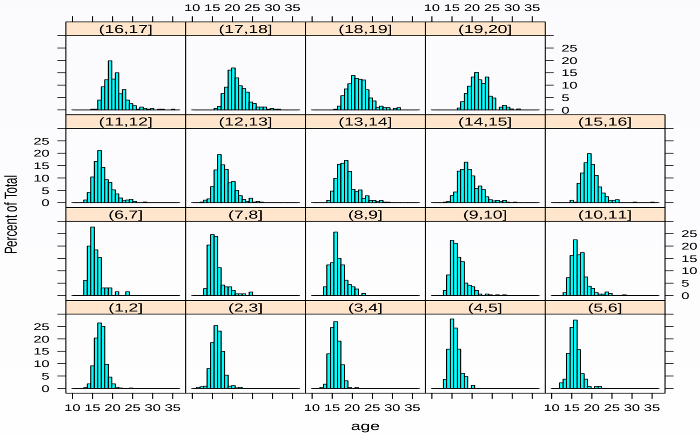
`age` : the age in years

Source: van Buuren and Fredriks (2001)

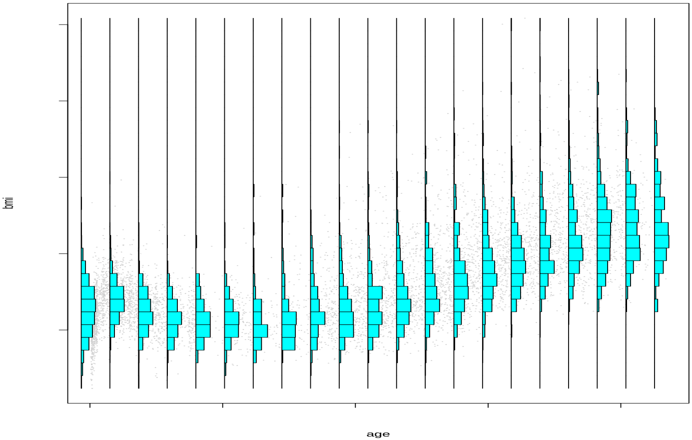
The Dutch boys data: statistical challenges



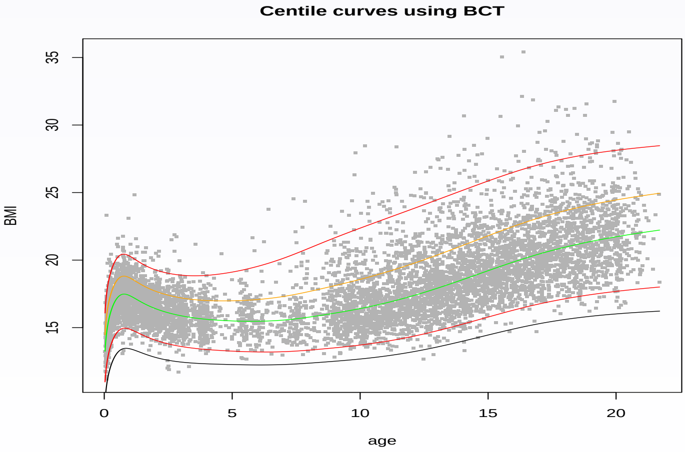
The Dutch boys data: Histograms by age



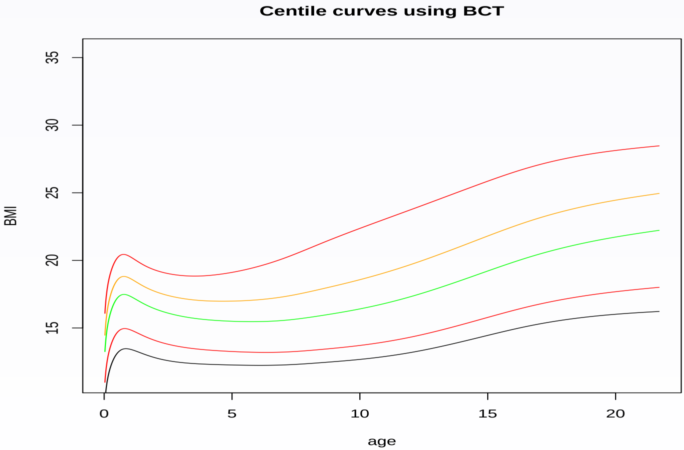
The Dutch boys data: Conditional histograms by age



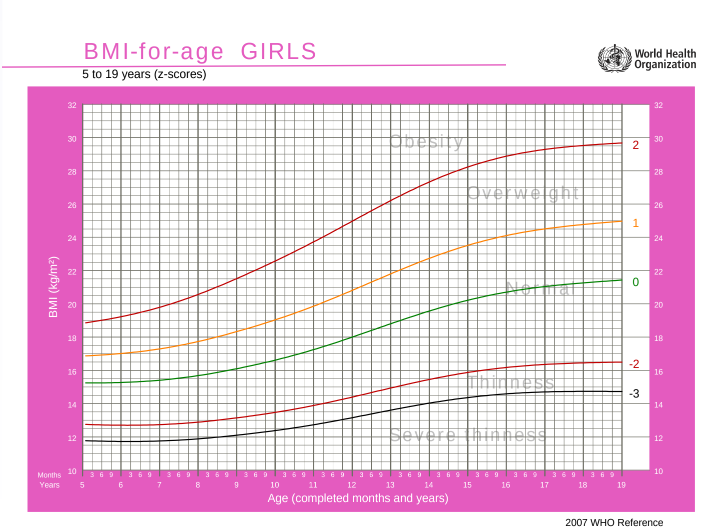
The Dutch boys data: centile estimation



The Dutch boys data: centiles



World Health Organisation Child Growth Standards: Girls



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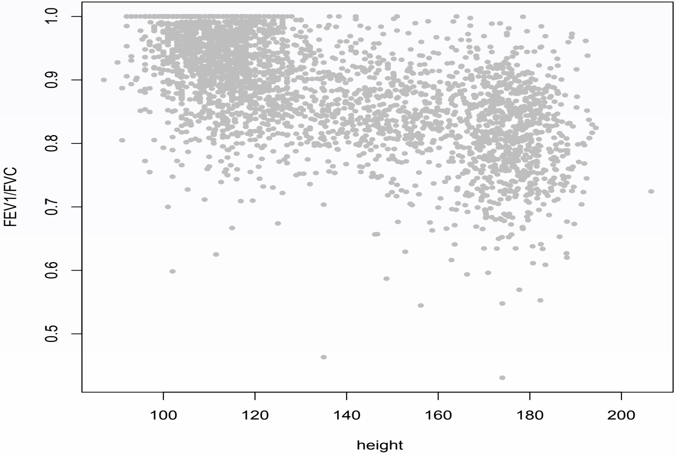
3164 male observations of lung function data



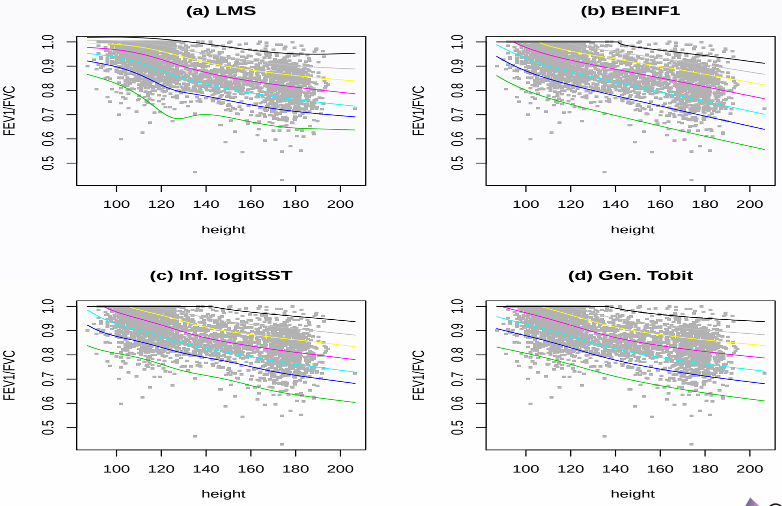
The lung function data

$Y = FEV_1/FVC$: the Spirometric lung function an established index for diagnosing airway obstruction (3164 male)

`height` : the height in cm

Source: Stanojevic et al. 2009

The lung function data: fitted centile curves



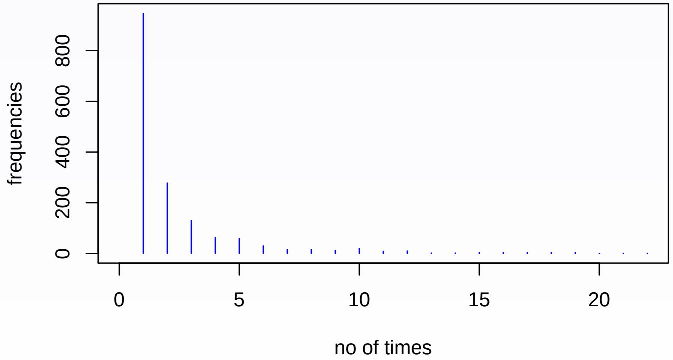
A stylometric application

64 observations

- `word` : is the number of times a word appears in a single text
- `freq` : the number of different words which occur exactly `word` times in the text

Source: Prof. Mario Cortina-Borja

The stylometric data

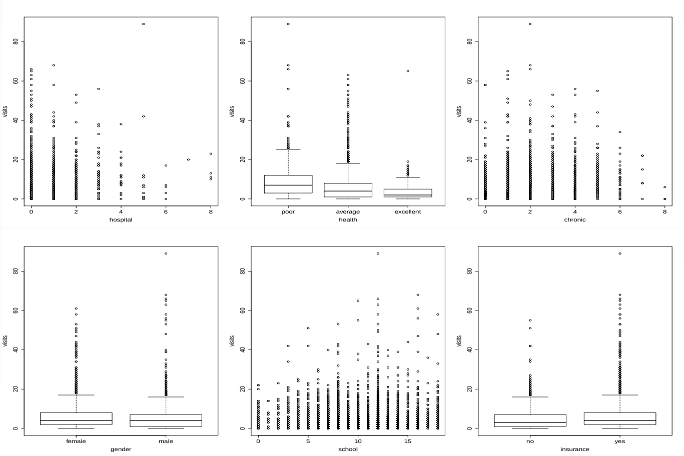


The number of physician office visit

- `visits`: number of physician office visits,
- `hospital`: number of hospital stays,
- `health`: health status: a factor indicating whether self-perceived health is poor, average (reference category) or excellent,
- `chronic`: number of chronic conditions,
- `gender`: a factor indicating gender,
- `school`: number of years of education,
- `insurance`: a factor indicating whether the individual is covered by private insurance.

Data in AER package in R

The number of physician office visit



What we need for modelling the above data?

We need

- flexible distributions for the response variable
- to be able to deal with heterogeneity in the data
- to be able to model skewness and kurtosis
- to be able to model overdispersion, excess of zeros and long tails in count data
- We need modelling all the parameters of the distributions
- flexible functions to model the relationship between the parameter of the distribution and the explanatory variables

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Historical development

Important events in the creation of the GAMLSS models

	Linear model (Gauss, 1809) Go to LM
1972	Generalised Linear Models (Nelder and Wedderburn) Go to GLM
1990	Generalised Additive Models (Hastie and Tibshirani) Go to GAM
2005	Generalised Additive Models for Location Scale and Shape (GAMLSS) (Rigby and Stasinopoulos).

Generalised Additive Model for Location Scale and Shape

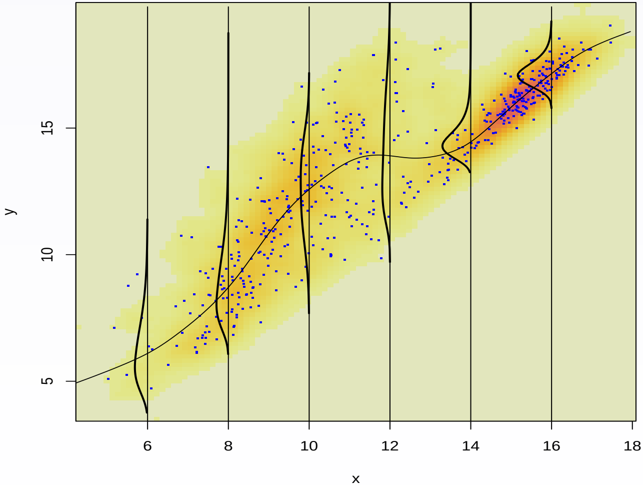
Generalised Additive Model for Location Scale and Shape Rigby and Stasinopoulos (2005)

$$\mathbf{y} \sim D(\boldsymbol{\mu}, \boldsymbol{\sigma}, \boldsymbol{\nu}, \boldsymbol{\tau})$$

$$\begin{aligned}
g_\mu(\boldsymbol{\mu}) &= \mathbf{X}_\mu \boldsymbol{\beta}_\mu + h_{1,\mu}(\mathbf{x}_{1,\mu}) + \ldots + h_{k,\mu}(\mathbf{x}_{k,\mu}) \\
g_\sigma(\boldsymbol{\sigma}) &= \mathbf{X}_\sigma \boldsymbol{\beta}_\sigma + h_{1,\sigma}(\mathbf{x}_{1,\sigma}) + \ldots + h_{k,\sigma}(\mathbf{x}_{k,\sigma}) \\
g_\nu(\boldsymbol{\nu}) &= \mathbf{X}_\nu \boldsymbol{\beta}_\nu + h_{1,\nu}(\mathbf{x}_{1,\nu}) + \ldots + h_{k,\nu}(\mathbf{x}_{k,\nu}) \\
g_\tau(\boldsymbol{\tau}) &= \mathbf{X}_\tau \boldsymbol{\beta}_\tau + h_{1,\tau}(\mathbf{x}_{1,\tau}) + \ldots + h_{k,\tau}(\mathbf{x}_{k,\tau})
\end{aligned}$$

where $D(\boldsymbol{\mu}, \boldsymbol{\sigma}, \boldsymbol{\nu}, \boldsymbol{\tau})$ can be any distribution and where $h_j(\mathbf{x}_j)$ are smooth functions of the X's.

GAMLSS assumptions



What is GAMLSS?

GAMLSS: are semi-parametric regression type models.

- regression type: we have many explanatory variables $\mathbf{X}$ and one response variable $\mathbf{y}$ and we believe that $\mathbf{X} \rightarrow \mathbf{y}$
- parametric: a parametric distribution assumption for the response variable,
- semi: the parameters of the distribution, as functions of explanatory variables, may involve non-parametric smoothing functions
- GAMLSS philosophy: try different models

GAMLSS is a generalisation of GLM and GAM models.

GAMLSS: Distributions

There are more than 100 explicit discrete discrete, continuous continuous, and mixed distributions, mixed, implemented as `gamlss.family` in the R including highly skew and kurtotic distributions Shapes,

- creating a new distribution is relatively easy
- truncating truncated an existing distribution
- using a censored version of an existing distribution
- mixing mixture different distributions to create a new finite mixture distribution.
- discretise discretise continuous distributions
- log or logit any continuous distribution in $(-\infty, \infty)$
- any distribution in $(0, \infty)$ can be zero adjusted to $[0, \infty)$
- any distribution in $(0, 1)$ can be inflated to $[0, 1]$

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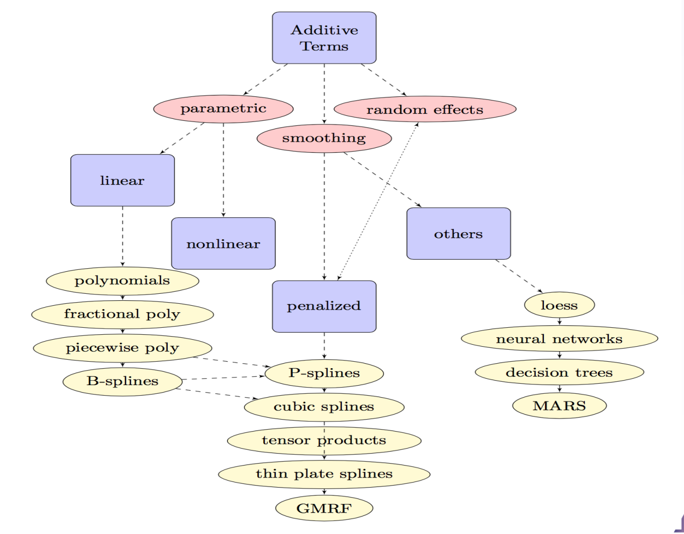
Additive Terms



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GAMLSS: R implementation

GAMLSS is implemented in series of packages in R

gamlss	the original package
gamlss.dist	for distributions
gamlss.data	for distributions
gamlss.demo	for demos
gamlss.nl	for non-linear terms
gamlss.tr	for truncated distributions
gamlss.cens	for censored (left, right or interval) response variables
gamlss.mx	for finite mixtures and random effects
gamlss.spatial	for Gaussian Markov Random Fields
gamlss.inf	for zero adjusted and inflated mixed distributions

The GAMLSS packages can be downloaded from CRAN, the R library at http://www.r-project.org/

GAMLSS components

Let $\mathcal{M} = \{\mathcal{D}, \mathcal{G}, \mathcal{T}, \boldsymbol{\lambda}\}$ represent the GAMLSS model

- $\mathcal{D}$: distribution
- $\mathcal{G}$: the link function for distributional parameters
- $\mathcal{T}$: predictor terms for ($\boldsymbol{\eta}$' s) i.e. $\boldsymbol{\eta} = \mathbf{X}\boldsymbol{\beta} + \sum_j h_j(\mathbf{x}_j)$
- $\boldsymbol{\lambda}$: the hyper-parameters

Problems, solutions and future research

- which distribution $\mathcal{D}$?
  - a book on distribution is prepared
  - a new function `chooseDist()` Go to chooseDist
  - robustify distributions
    - before fitting
    - after fitting
- which additive term for μ, σ, ν and τ?
  - all step-GAIC's are now parallel
  - possible connection of `ChooseDist()` and `stepGAIC()`
  - Machine learning techniques
    - GAMLSS boosting is well developed
    - connection to `glmnet`
- choosing the smoothing hyper parameters for terms
  - connection to **caret** package

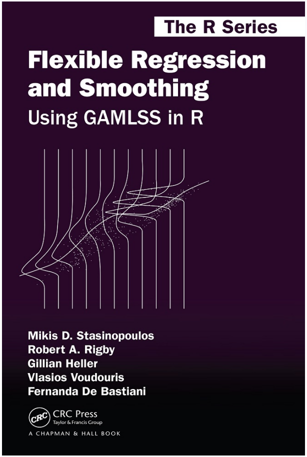
Problems, solutions and future research

- selection between different (GAMLSS or not) models
  - GAIC and diagnostics exist but more work is needed to see where the benefits of using GAMLSS are coming from
  - influential observations
- Which inferential procedure?
  - penalised likelihood
  - Bayesian, see package **BAMLSS**
  - boosting, see package **gamboostLSS**
- Forcasting
  - developing time series modelling within GAMLSS
  - distributional forecast
  - automazation

The Books

- Flexible Regression and Smoothing: Using GAMLSS in R (out in April 2017)
- Distributions for Location Scale and Shape: Using GAMLSS in R (expected in six to eight months)
- Generalized Additive Models for Location Scale and Shape: A Distributional Regression Approach. (starts in September)

The 1st Book (out in April 2017)



Conclusions

- GAMLSS is a very flexible statistical model
- It is a unified framework for univariate regression type of models
- Allows any distribution for the response variable Y
- Models all the parameters of the distribution of Y
- Allows a variety of penalised additive terms in the models for the distribution parameters
- The fitted algorithm is modular, where different components can be added easily
- it can easily introduced to students since it relies on known concepts
- It deals with overdispersion, skewness and kurtosis

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This is a collaborative work

co-authors	current collaborators
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	Raydonal Ospina
	Konstantinos Pateras
	Nicoleta Mortan

For more GAMLSS

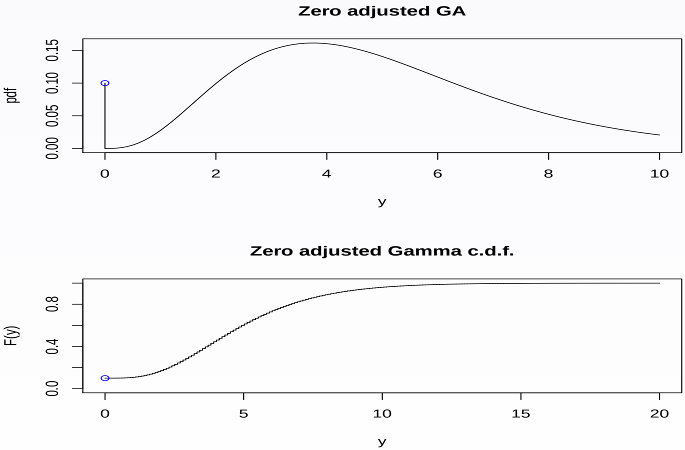
the END

for more information see

www.gamlss.org

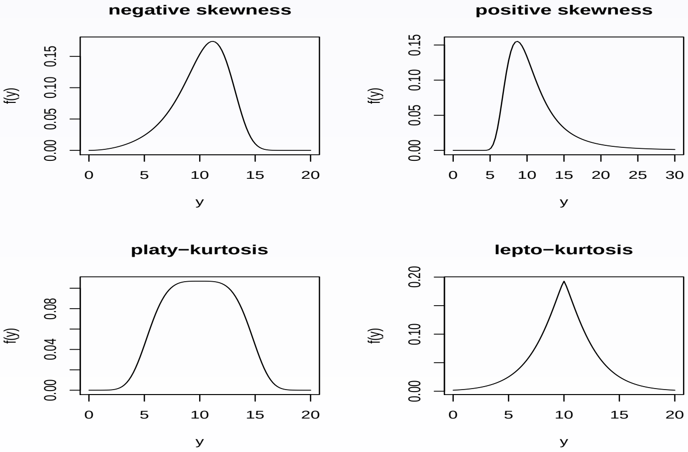
Example of mixed distribution distributions

Go back Distributions



Continuous distributions: different shapes

Go back Distributions



Continuous distributions: different types

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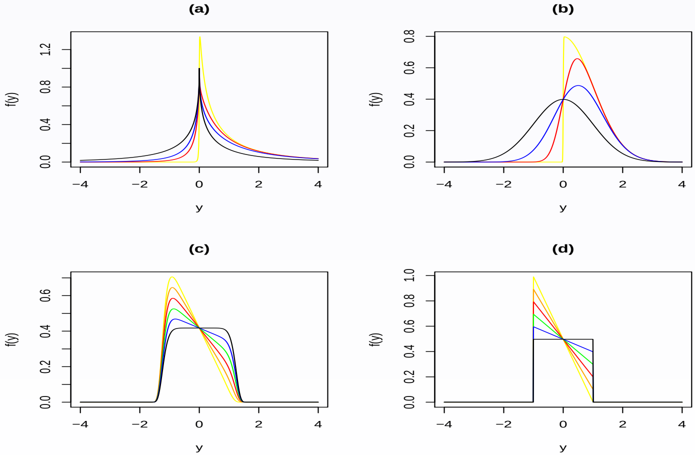
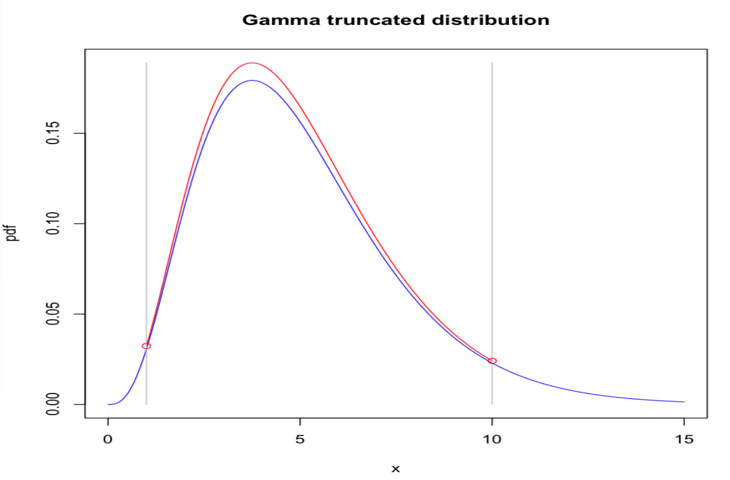


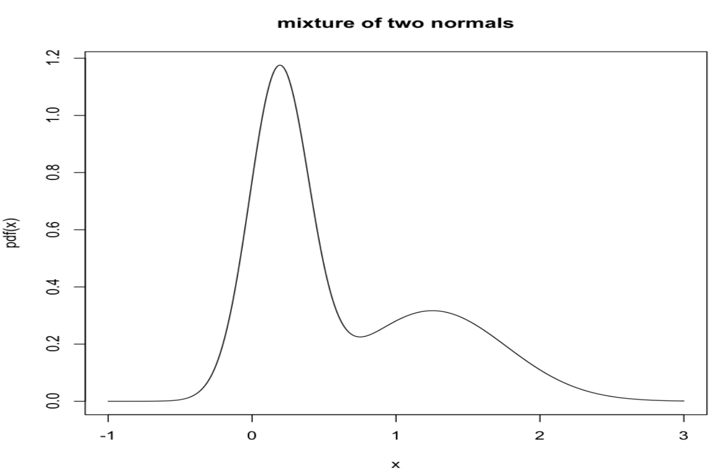
Continuous distributions: different types

Go back Distributions



Continuous distributions: different types

Go back Distributions



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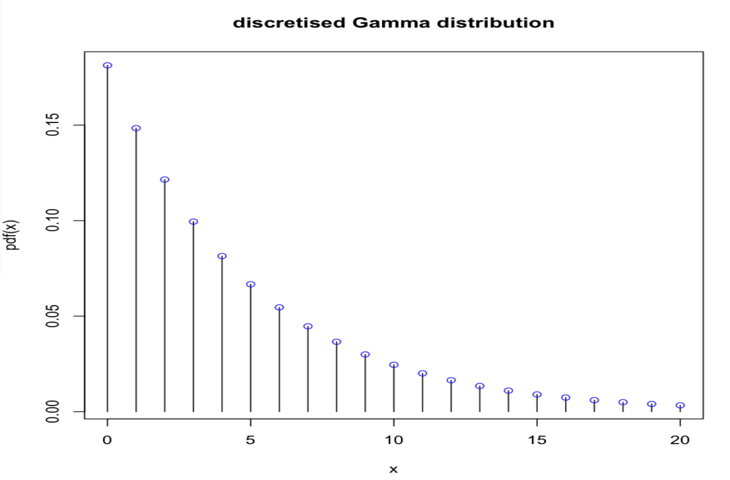
Continuous distributions: different types

Go back Distributions



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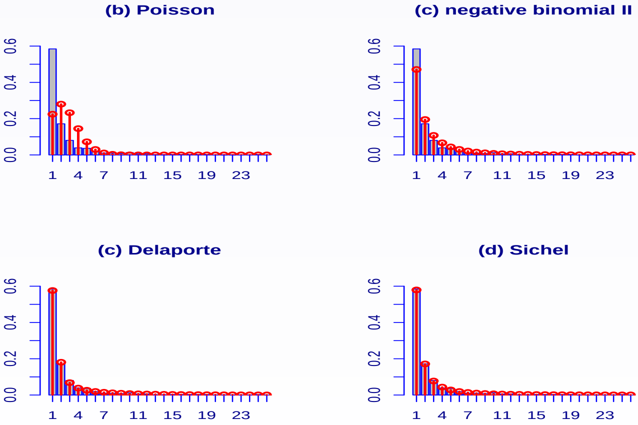
The stylometric data, Go back to distributions



Choose Distribution

Go back

```
mf <- chooseDist(m1, type="count")
```

	2	3.84	8.39
PO	35959.23	35973.95	36010.35
GEOM	24402.77	24417.49	24453.89
GEOMo	24402.77	24417.49	24453.89
...	...	...	...
ZASICHEL	24469.07	24502.19	24584.09
ZINBF.1	24207.21	24240.33	24322.23
ZIBNB	24107.94	24141.06	24222.96
ZISICHEL	24196.25	24229.37	24311.27

```
getOrder(mf)
```

The linear model

Linear Model, Gauss

$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ where $\boldsymbol{\epsilon} \sim NO(\mathbf{0}, \sigma^2 \mathbf{I})$

The model can be also written as:

$\mathbf{y} \sim NO(\boldsymbol{\mu}, \sigma^2 \mathbf{I})$ where $\boldsymbol{\mu} = \mathbf{X}\boldsymbol{\beta}$

Go back 70-80

The linear model assumptions

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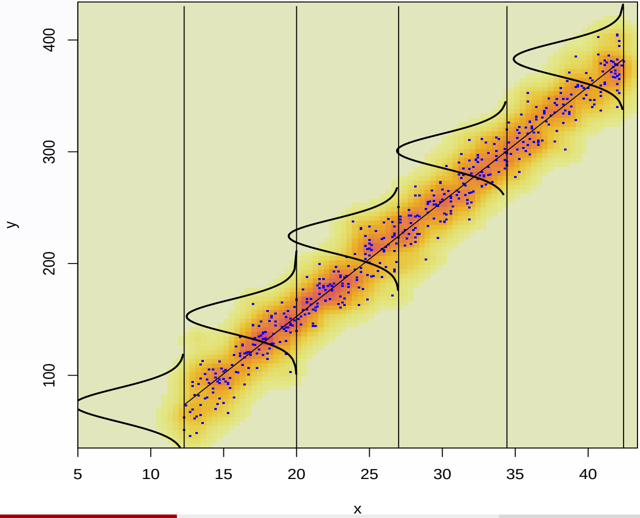

The weighted linear model assumptions

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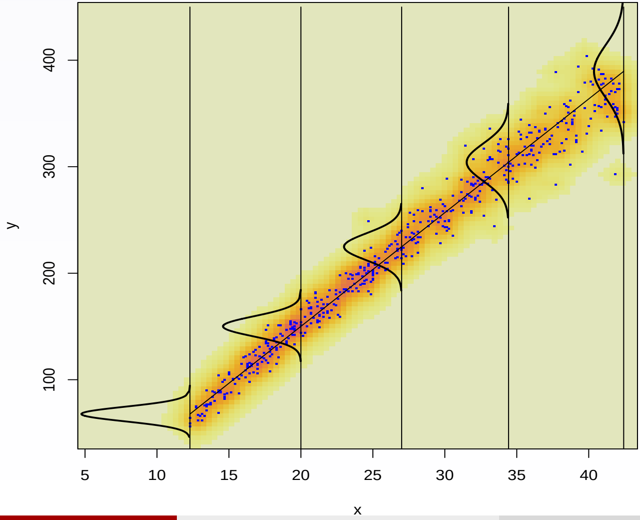

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The linear model: comments

- estimation is achieved by Least Squares or Weighted Least Squares (WLS)
- the normal distribution is important for inference
- we only modelling the mean as linear function of the explanatory variables
- One of the top ten reasons to become statistician (according to Friedman, Friedman & Amoo, 2002, Journal of Statistics Education):

"Statisticians are mean lovers".

Go back 70-80

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The generalised linear model

Generalised Linear Model, Nelder and Wedderburn(1972)

$g(\boldsymbol{\mu}) = \mathbf{X}\beta$ where $\mathbf{y} \sim \mathit{ExpFamily}(\boldsymbol{\mu}, \phi)$

where $g()$ is the link function

The exponential family

1. normal
2. Gamma
3. inverse Gaussian
4. Poisson
5. binomial

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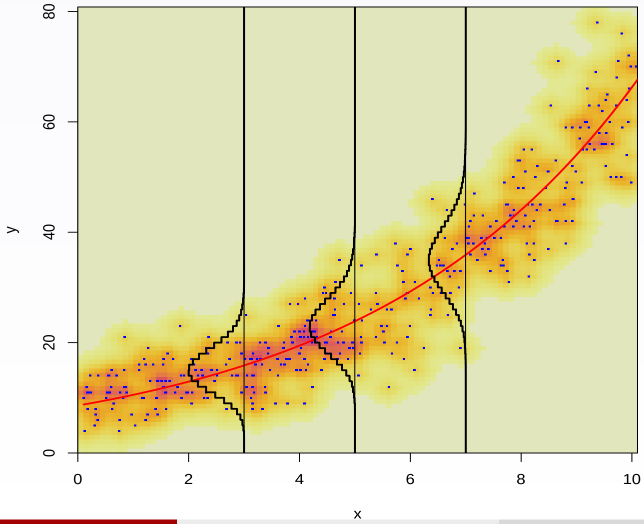
The generalised linear model

Go back 70-80 Next page



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The generalised linear model

- estimation is achieved by Iterative Re-weighted Least Squares (IRLS)
- we can model discrete response variables
- we are still "mean lovers".

Go back 70-80

The generalised additive model

Generalised additive model Hastie and Tibshirani (1990)

$\mathbf{y} \sim \mathit{ExpFamily}(\boldsymbol{\mu}, \phi)$

$g(\boldsymbol{\mu}) = \mathbf{X}\beta + h_1(\mathbf{x}_1) + ... + h_k(\mathbf{x}_k)$

where $h_j(\mathbf{x}_j)$ are smooth functions of the X's.

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