Class 7 (Monday 5 December)

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These tasks are designed to be worked on in the practical class on Monday 5 December.

LU for a tridiagonal matrix

In Friday's lecture, we computed the LU factorisation of a dense matrix. The code we wrote during Friday's lecture can be found at this link. In today's class, we are going to compute the LU decomposition of a tridiagonal matrix.

We will use the following n by n matrix:

$$\mathrm{A}=\begin{pmatrix} a_0 & -1 & 0 & 0 & \cdots & 0 \\ -1 & a_1 & -1 & 0 & \cdots & 0 \\ 0 & -1 & a_2 & -1 & \cdots & 0 \\ 0 & 0 & -1 & a_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a_{n-1} \end{pmatrix},$$

where a_0 to a_{n-1} are random decimal values between 5 and 10.

Write a function that takes n as an input and returns the matrix A stored in a sparse format of your choice.

Using the code we wrote in Friday's lecture as a template, write a function that computes the LU decomposition of A, and returns the factors L and U in a sparse format of your choice. Due to the structure of the matrix, you should not need to do any permuting of the rows.

For a range of values of n, compute the LU decomposition using your function and measure the time this takes. Convert each matrix to a dense matrix and compute the LU decomposition using the code we wrote in Friday's lecture, timing this too. Plot these timings on log-log axes. What do you notice?

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