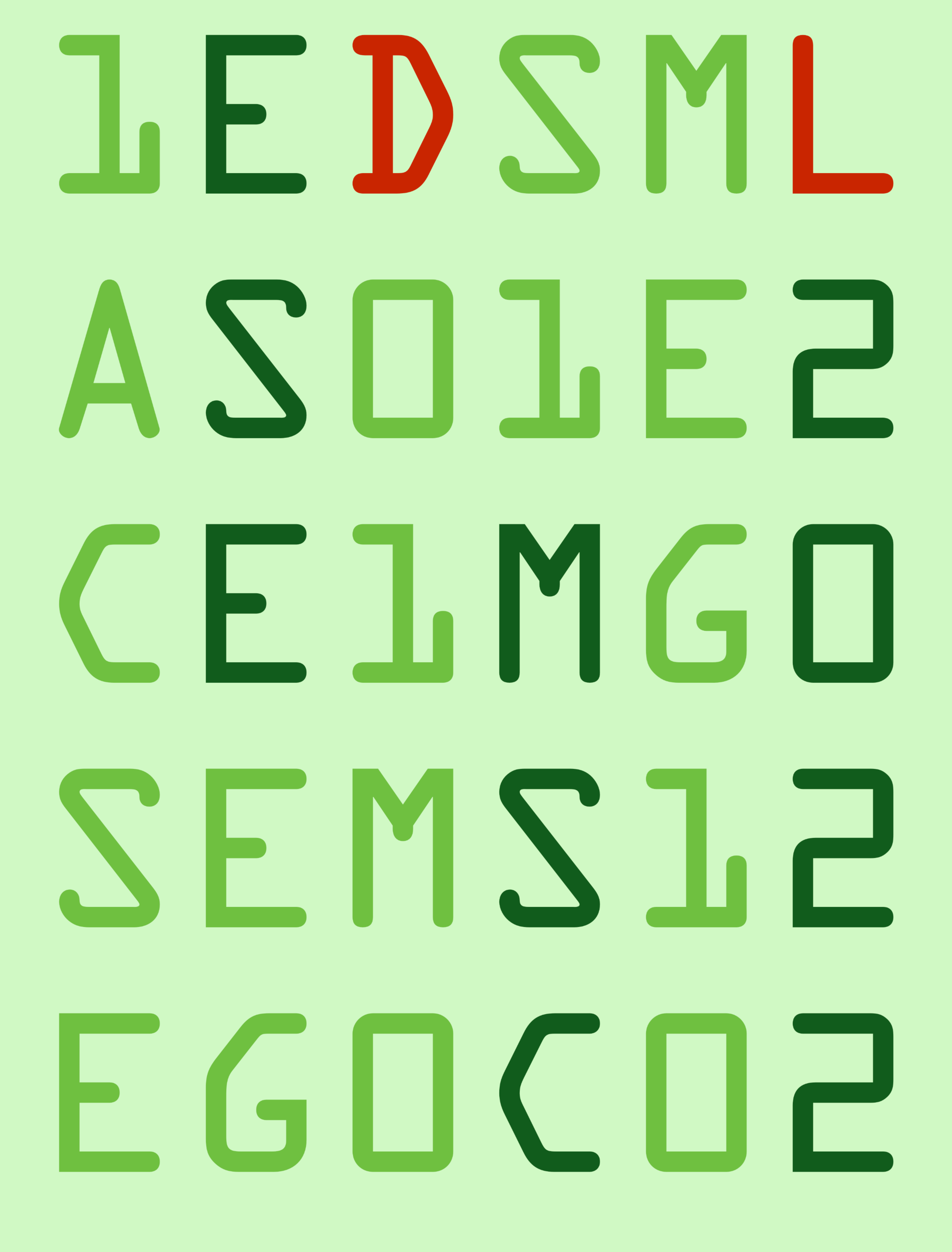


Recurrent Neural Networks and Long Short-Term Memory

Carlos Cueto

RNNs and LSTMs

1. Recurrent Neural Networks (RNNs)
2. Long Short-Term Memory (LSTM) networks

1. Recurrent Neural Networks (RNNs)

2. Long Short-Term Memory (LSTM) networks

Examples of sequential data:

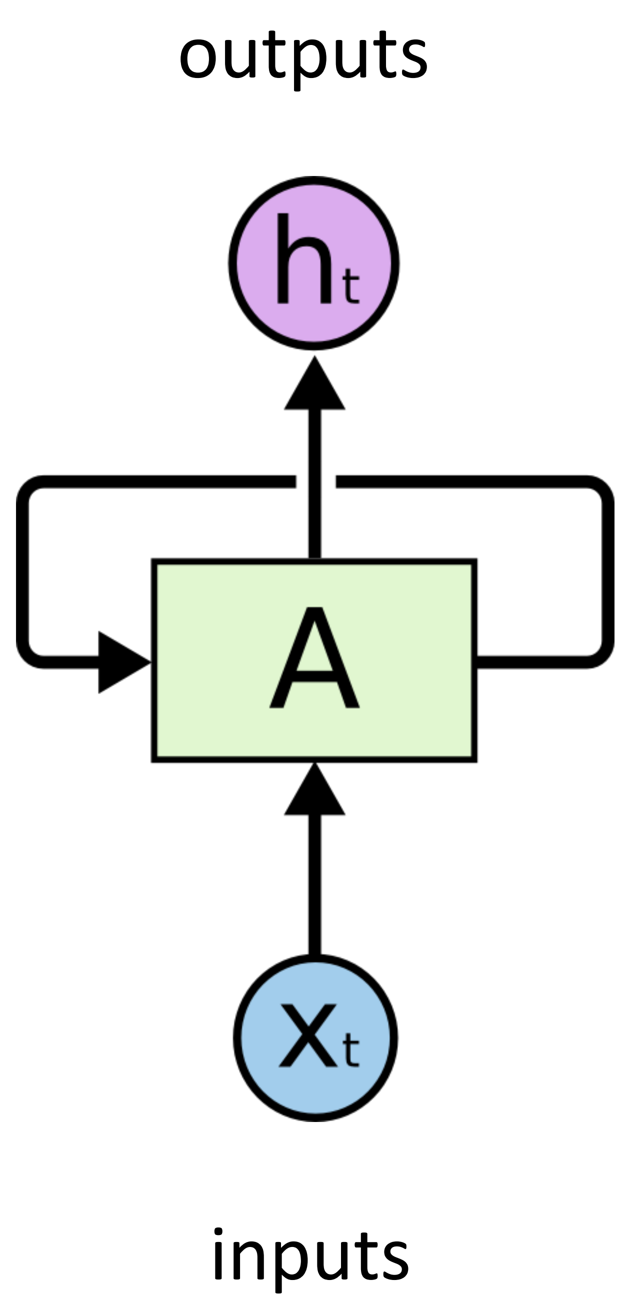
- Text considered as a sequence of words or characters
- Continuous parameter which is a function of time (e.g. stock price)
- Sequence of images in a video-clip
- Sequence of labels in a genome sequence
- … plus many others

Applications of RNNs to sequential data:

- Text generation “in the style of”
- Provide ”sentiment analysis” on a piece of text
- Automatic “speech-to-text” as in MS Teams
- Automatic foreign language translation
- Prediction of stock price on the basis of historical data
- Label sequence of images in a video-clip

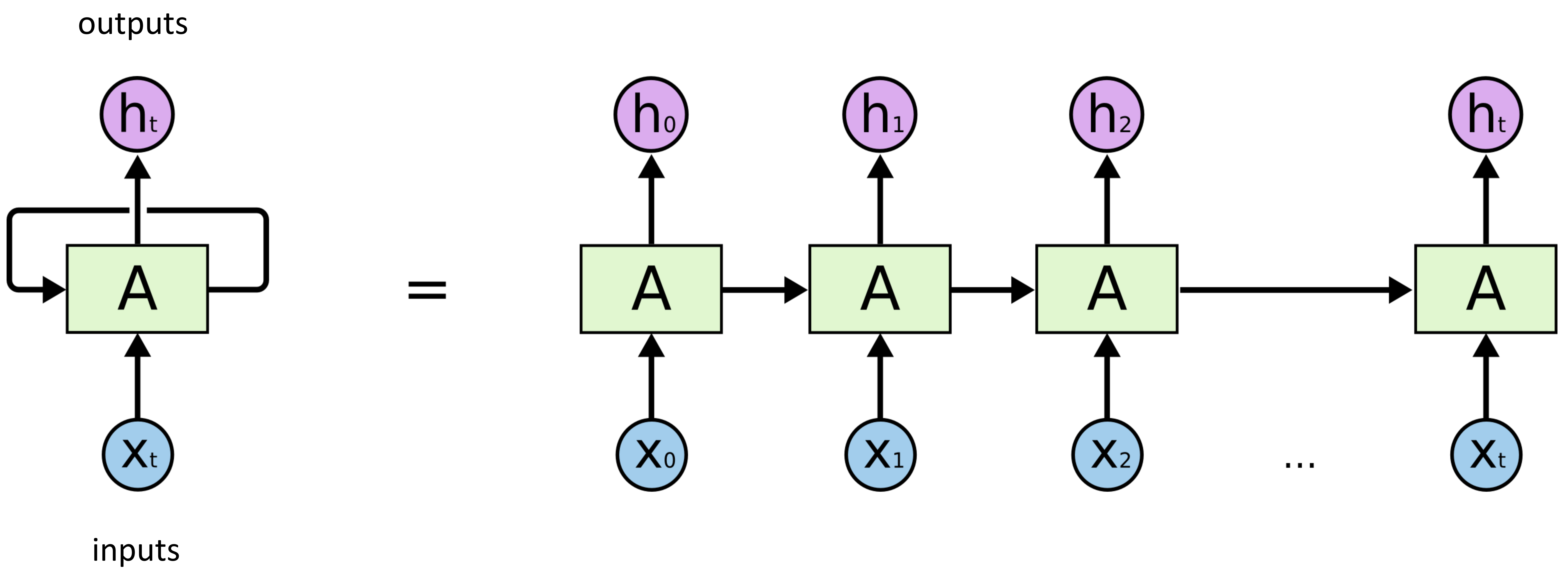
RNNs

Basic structure of RNNs:



RNNs

Basic structure of RNNs:



Simple (unrealistic) example:

Example:
Based on a (usually long) text used for training, we wish to generate “similar” text on a character by character basis.

Given a training string: `Hello world!`

We identify the dictionary associated with the input (ignoring capitalisation) contains 9 characters: `(d,e,h,l,o,r,w, ,!)`

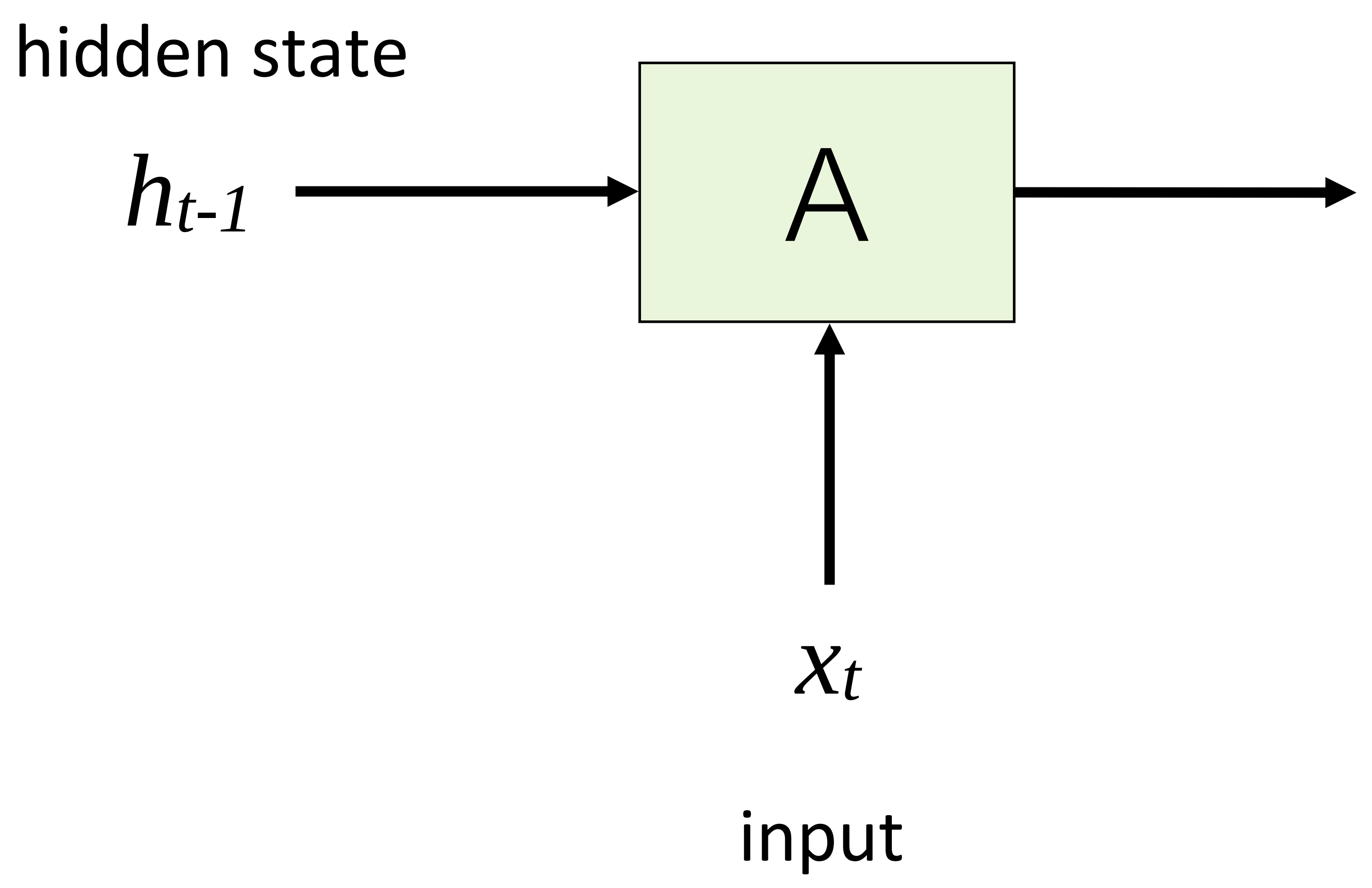
And we hot-encode the dictionary:

$$h = \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\\0\\0\end{pmatrix} \quad ! = \begin{pmatrix}0\\0\\0\\0\\0\\0\\0\\0\\1\end{pmatrix} \quad \cdots$$

RNNs

How does a basic RNN cell work?

$$y_t = W_{hy} h_t$$ output



$$h_t = A(h_{t-1}, x_t) = \tanh(W_{hh} h_{t-1} + W_{xh} x_t)$$ hidden state

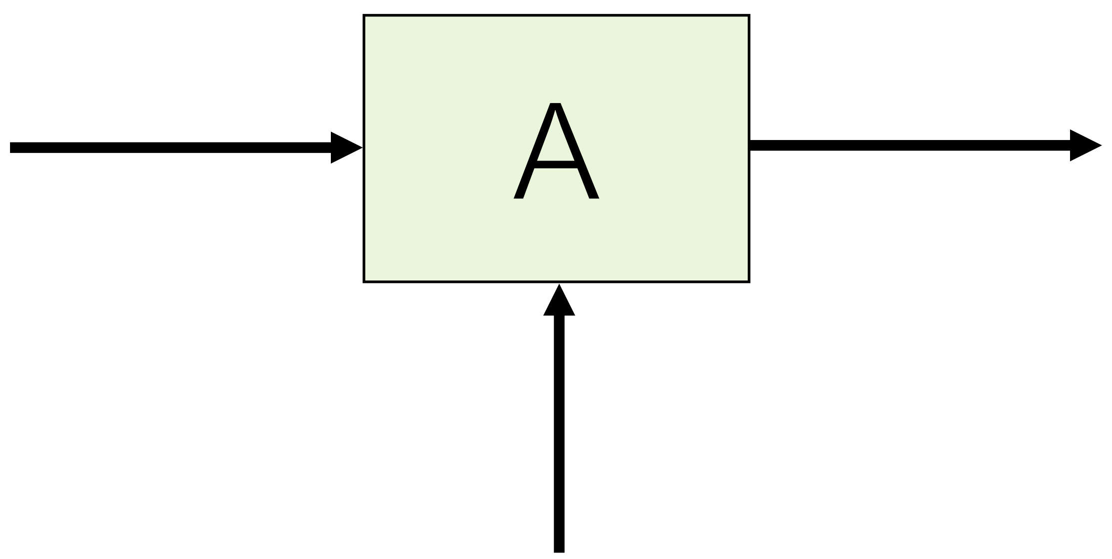
RNNs

How does training work? Back to our simple example:

We define the size of our hidden state vector h (hyperparameter) to be 5, and start with our first input character ('h'):

$$y_1 = W_{hy}h_1$$

$$h_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



$$h_1 = A(h_0, x_1) = \tanh(W_{hh}h_0 + W_{xh}x_1)$$

$$x_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

RNNs

How does training work? Back to our simple example:

We define the size of our hidden state tensor h (hyperparameter) to be 5, and start with our first input character (‘h’):

9x5 matrix

$$y_1 = W_{hy} h_1$$

$$h_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

A

$$h_1 = A(h_0, x_1) = \tanh(W_{hh} h_0 + W_{xh} x_1)$$

5x5 matrix 5x9 matrix

$$x_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

1EDSML
ASO1E2
CE1MGO
SEMS12
EGOCO2

How does training work? The loss function:

$$loss =$$

$$y_1 = W_{hy} h_1 \longrightarrow Cross\text{-}entropy\ (\ Softmax\ (y_1)\ ,\ x_2\ )$$

Loss function corresponding to the first predicted letter

remember: hot-encoded

RNNs

How does training work for the next characters? Back to our previous example:

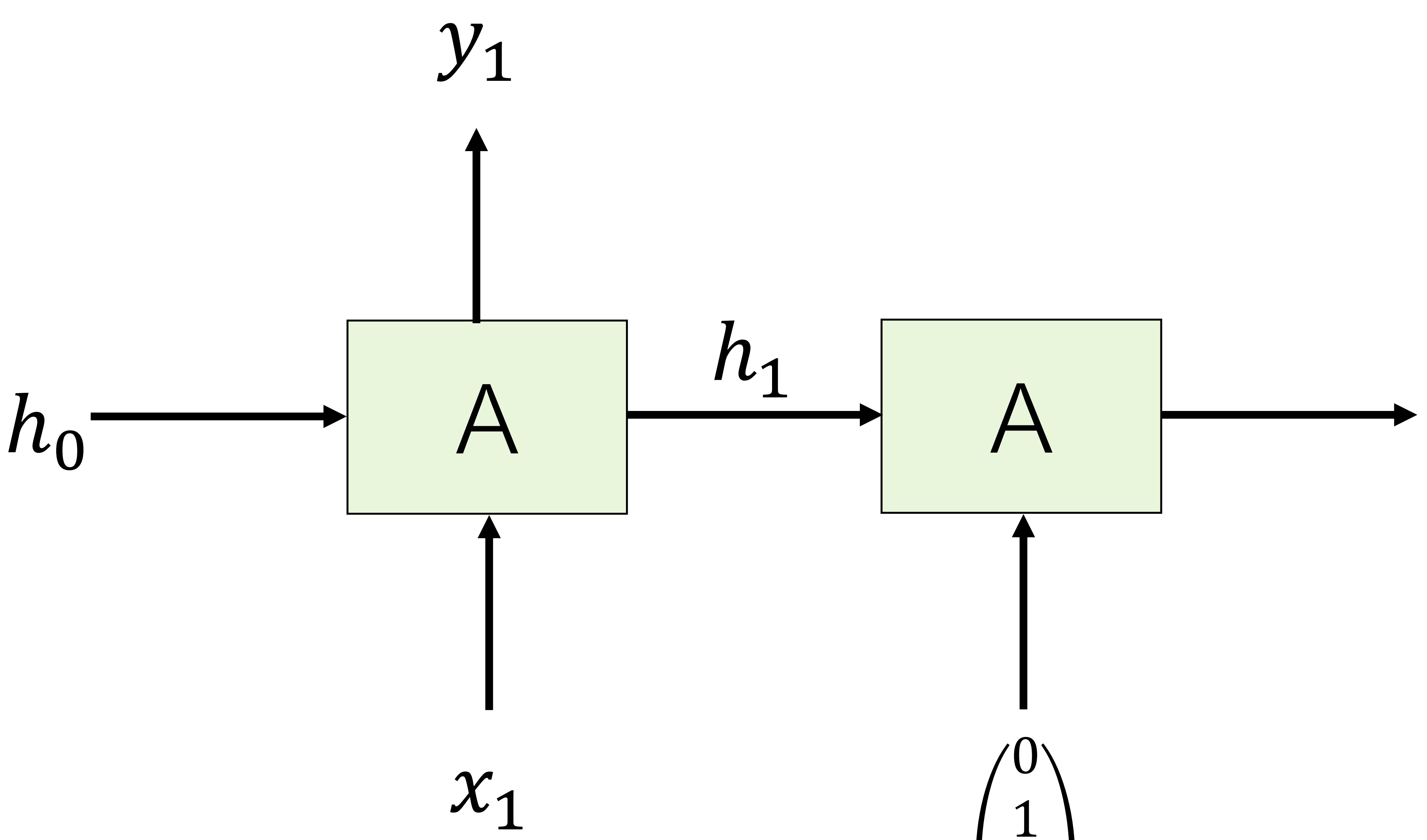
$$y_1 = W_{hy} h_1$$

$$h_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \longrightarrow \boxed{A} \longrightarrow h_1 = A(h_0, x_1) = \tanh(W_{hh} h_0 + W_{xh} x_1)$$

$$x_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

How does training work for the next characters? Back to our previous example:

We concatenate the **same** cell after the output of our previous one:



$$y_2 = W_{hy} h_2$$

$$h_2 = A(h_1, x_2) = \tanh(W_{hh} h_1 + W_{xh} x_2)$$

$$x_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

RNNs

How does training work? The loss function:

$$loss =$$

$$y_1 = W_{hy} h_1 \longrightarrow Cross\text{-}entropy\,(\,Softmax\,(y_1)\,,\;x_2\,)$$

$$+$$

$$Cross\text{-}entropy\,(\,Softmax\,(y_2)\,,\;x_3\,)$$

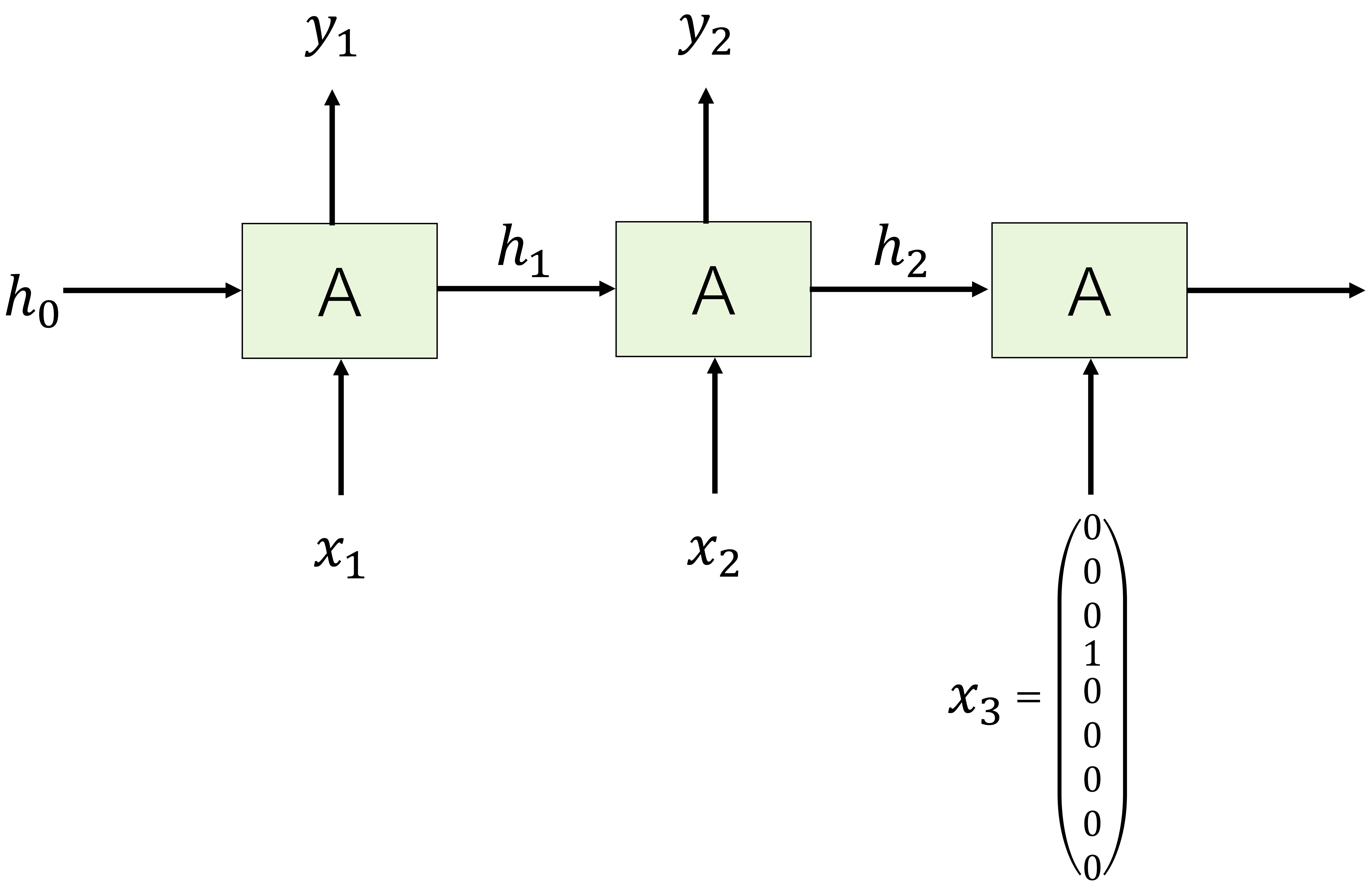
Loss function corresponding to the second predicted letter

remember: hot-encoded

RNNs

How does training work for the next characters? Back to our previous example:

We concatenate the **same** cell after the output of our previous one:



$$y_3 = W_{hy} h_3$$

$$h_3 = A(h_2, x_3) = \tanh(W_{hh} h_2 + W_{xh} x_3)$$

RNNs

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How does training work? The loss function:

$$loss =$$

$$y_1 = W_{hy}h_1 \longrightarrow Cross\text{-}entropy\,(\,Softmax\,(y_1)\,,\;x_2\,)$$

$$+$$

$$Cross\text{-}entropy\,(\,Softmax\,(y_2)\,,\;x_3\,)$$

$$+$$

$$Cross\text{-}entropy\,(\,Softmax\,(y_3)\,,\;x_4\,)$$

$$+\ \ldots$$

We repeat the process for all the characters in our input and add their loss contributions

Loss function corresponding to the third predicted letter

remember: hot-encoded

And then use gradient descent to train our weights in the matrices

Text generation using a trained network

Now we are going to use our trained RNN to generate some text:

output

$$y_1 = W_{hy} h_1$$

$$h_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

A

$$h_1 = A(h_0, x_1) = \tanh(W_{hh} h_0 + W_{xh} x_1)$$

$$x_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Random initialization

Text generation using a trained network

Now we are going to use our trained RNN to generate some text:

$$y_1 = W_{hy} h_1$$

$$Softmax\ (y_1)$$

sample x_2 from $Softmax$

Use this sampled x_2 to generate the next character and so on

Text generation using a trained network

Now we are going to use our trained RNN to generate some text:

$$y_1 = W_{hy} h_1$$

Now we are not training, but we still want to compute the Softmax

$$Softmax\ (y_1)$$

Here we sample values! There is some degree of randomness which generates different text every time

sample x_2 from $Softmax$

Use this sampled x_2 to generate the next character and so on

Recap of training an RNN

1. Take first character x_1, combine with current hidden state h_0 (its dimension is a hyperparameter) to derive h_1, calculate associated output vector y_1 (of size equal to vocabulary), transform y_1 into Softmax vector, calculate cross-entropy with second character x_2 of the sequence.
2. Take the second character as input to repeat the step 1. and calculate loss function again using the value of the third character of the sequence, and so on until we reach the end of the sequence.
3. Add up all the above loss functions.
4. Do backpropagation to calculate gradients according to each trainable parameter.
5. Modify parameters by gradient descent.
6. Move to next sequence of p characters, until end of training set is reached. Size of p is a hyperparameter too!

Recap of generating text with an RNN

Generate a new sequence of n characters:

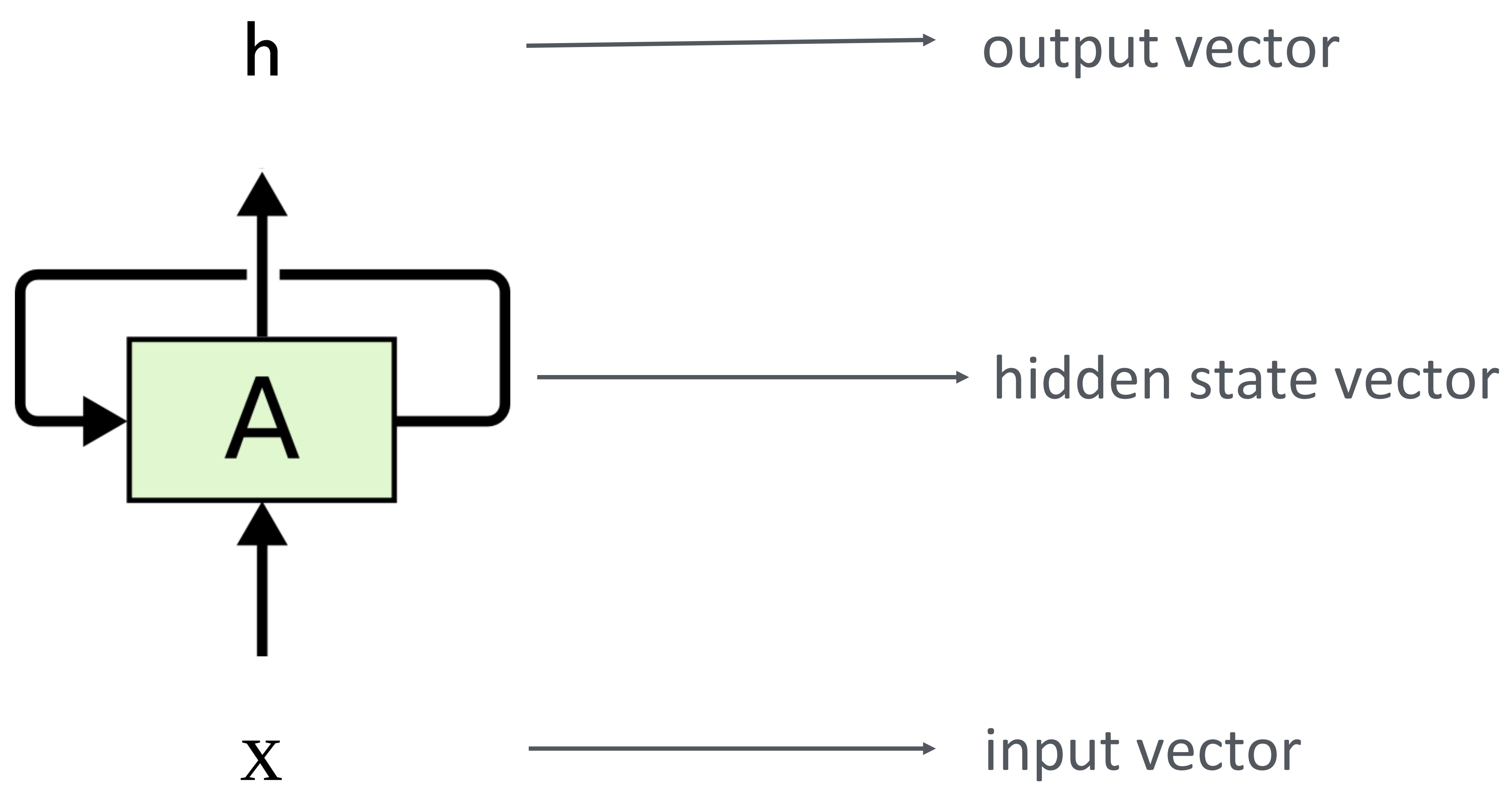
1. Randomly sample first character x_1.
2. Combine with initial (zero) hidden state vector value h_0 to derive new vector h_1.
3. Calculate associated output vector y_1 of size equal to vocabulary size.
4. Transform y_1 into Softmax vector.
5. Sample new character x_2 based on Softmax probability.
6. Continue until the required number of characters have been generated.

RNNs

Let’s look at some code!

RNNs

Synthetic notation for RNNs:



Simple implementation in python:

https://gist.github.com/karpathy/d4dee566867f8291f086

And after a few hours of training, we start to get some decent writing:

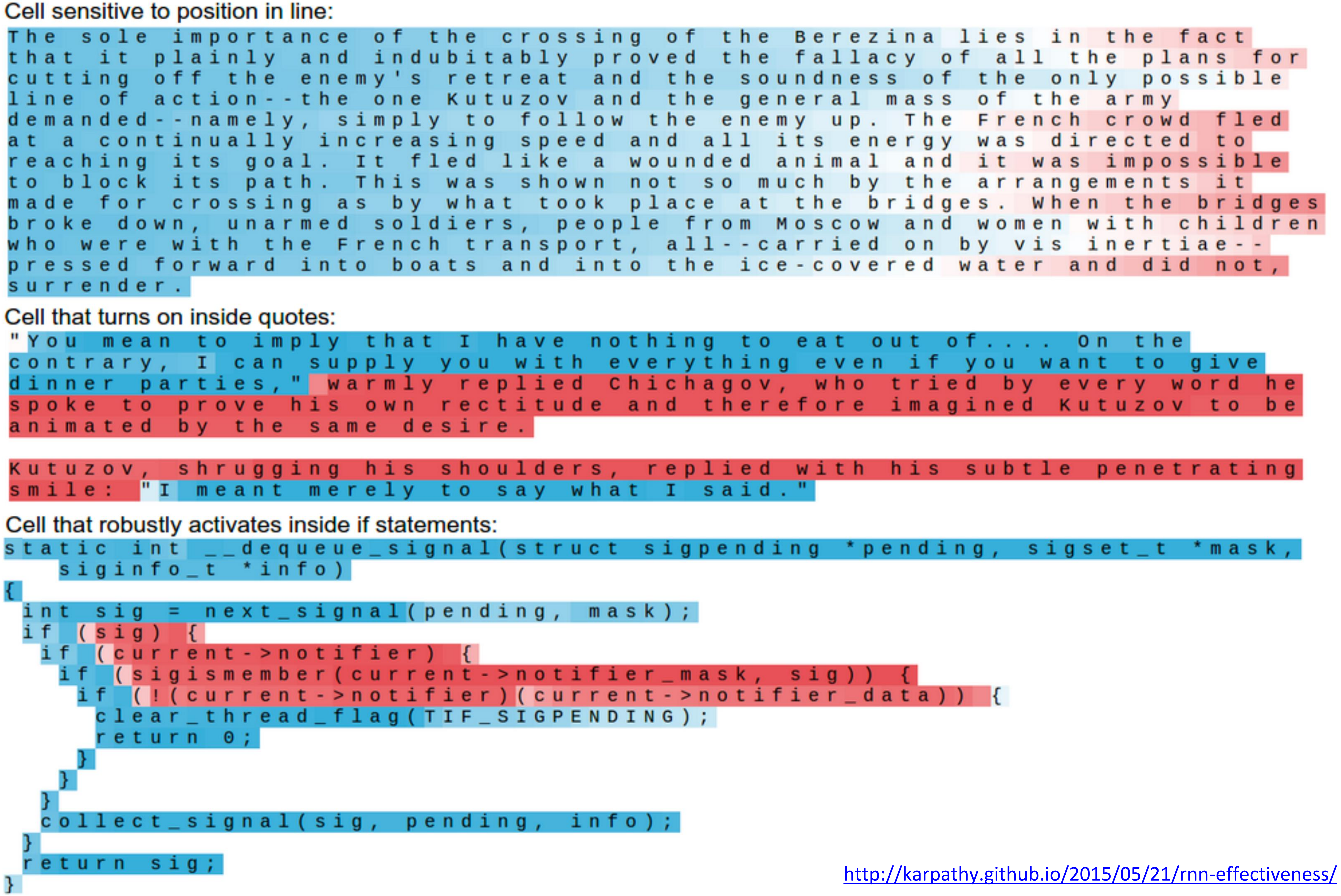
```
VIOLA:
Why, Salisbury must find his flesh and thought
That which I am not aps, not a man and in fire,
To show the reining of the raven and the wars
To grace my hand reproach within, and not a fair are hand,
That Caesar and my goodly father's world;
When I was heaven of presence and our fleets,
We spare with hours, but cut thy council I am great,
Murdered and by thy master's ready there
My power to give thee but so much as hell:
Some service in the noble bondman here,
Would show him to her wine.

KING LEAR:
O, if you were a feeble sight, the courtesy of your law,
Your sight and several breath, will wear the gods
With his heads, and my hands are wonder'd at the deeds,
So drop upon your lordship's head, and your opinion
Shall be against your honour.
```

http://karpathy.github.io/2015/05/21/rnn-effectiveness/

RNNs

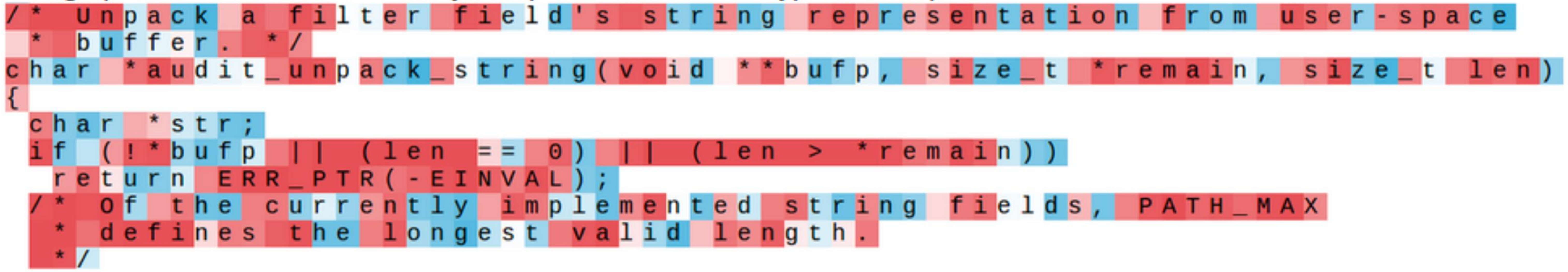
Analysis of the role of h



http://karpathy.github.io/2015/05/21/rnn-effectiveness/

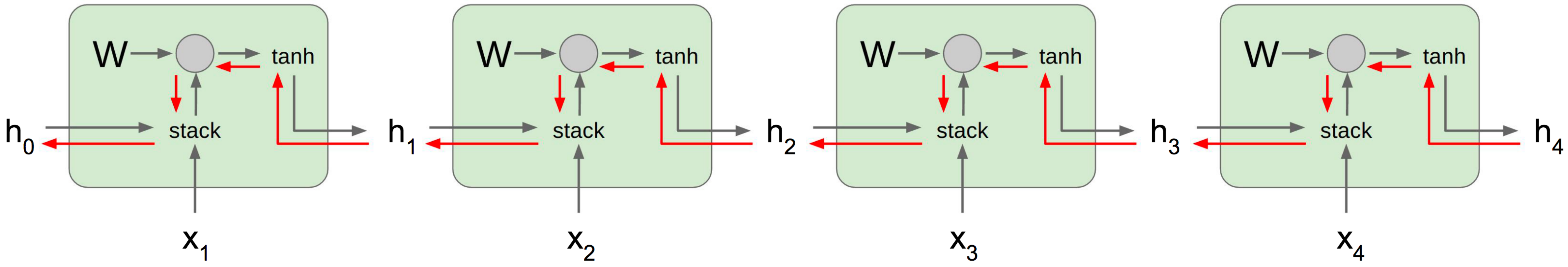
Analysis of the role of h

A large portion of cells are not easily interpretable. Here is a typical example:



In their study, Karpathy et al found that only ~5% of the h positions were interpretable

Backpropagation through RNNs



Computing gradients involves many factors of W and repeated tanh activations and that creates problems:

- Exploding gradients: can be solved by gradient clipping
- Vanishing gradients: requires changes in the RNN architecture

the derivative of tanh is in the range (0, 1) and we apply it multiple times using the chain rule in backprop

RNNs

Issues with RNNs

- Exploding and vanishing gradients
- Lack of long term memory

RNNs

Issues with RNNs

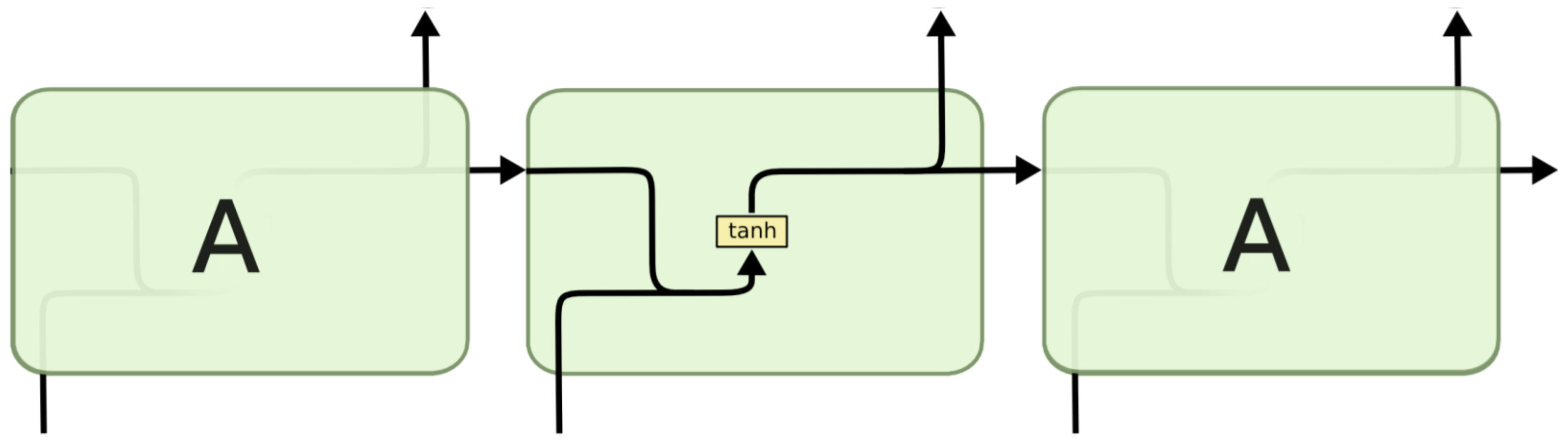
- Exploding and vanishing gradients
- Lack of long term memory

→ LSTMs to the rescue!

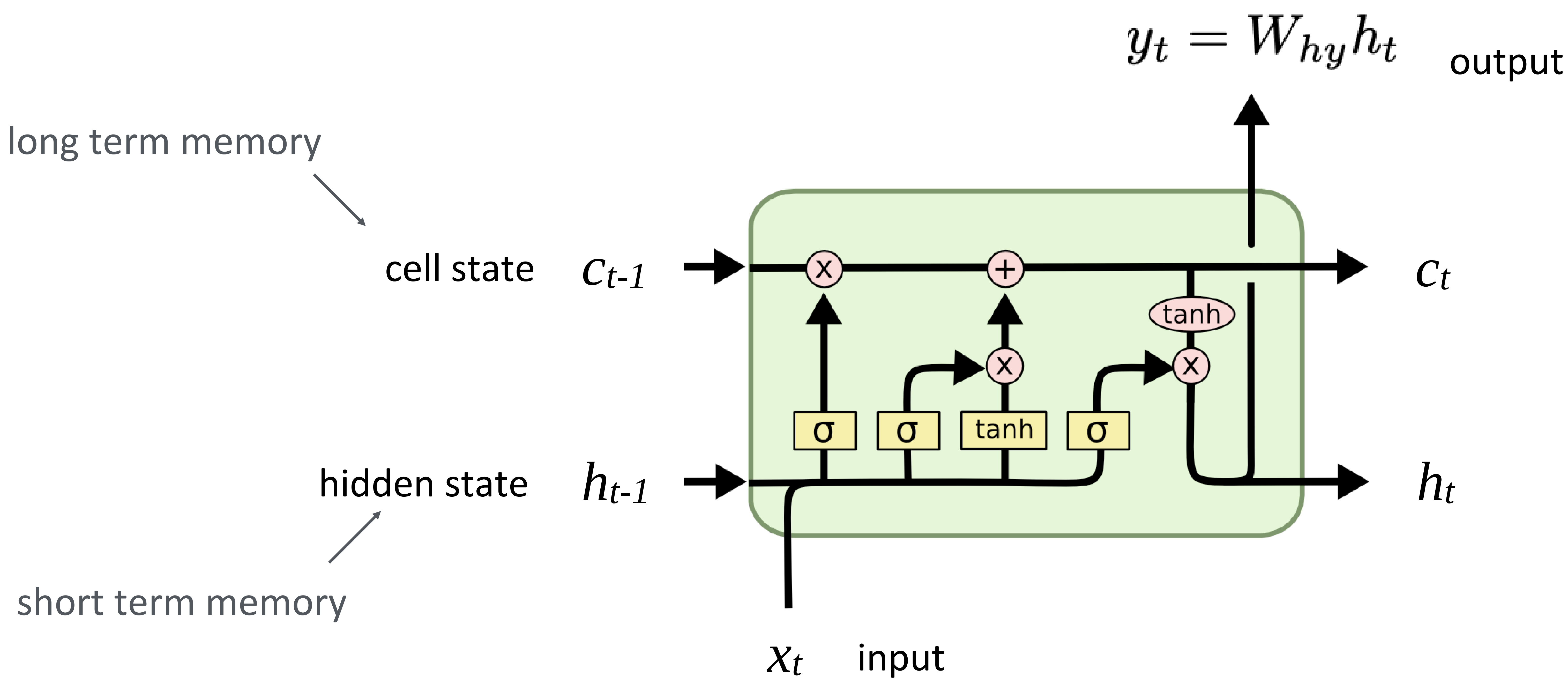
1. Recurrent Neural Networks (RNNs)

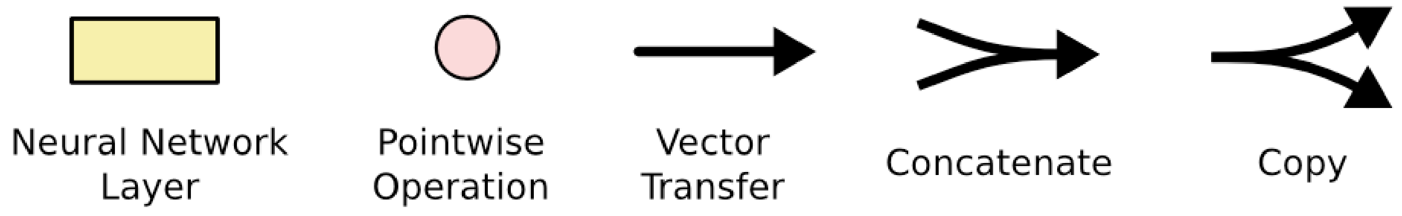
2. **Long Short-Term Memory (LSTM) networks**

Basic structure of an LSTM:



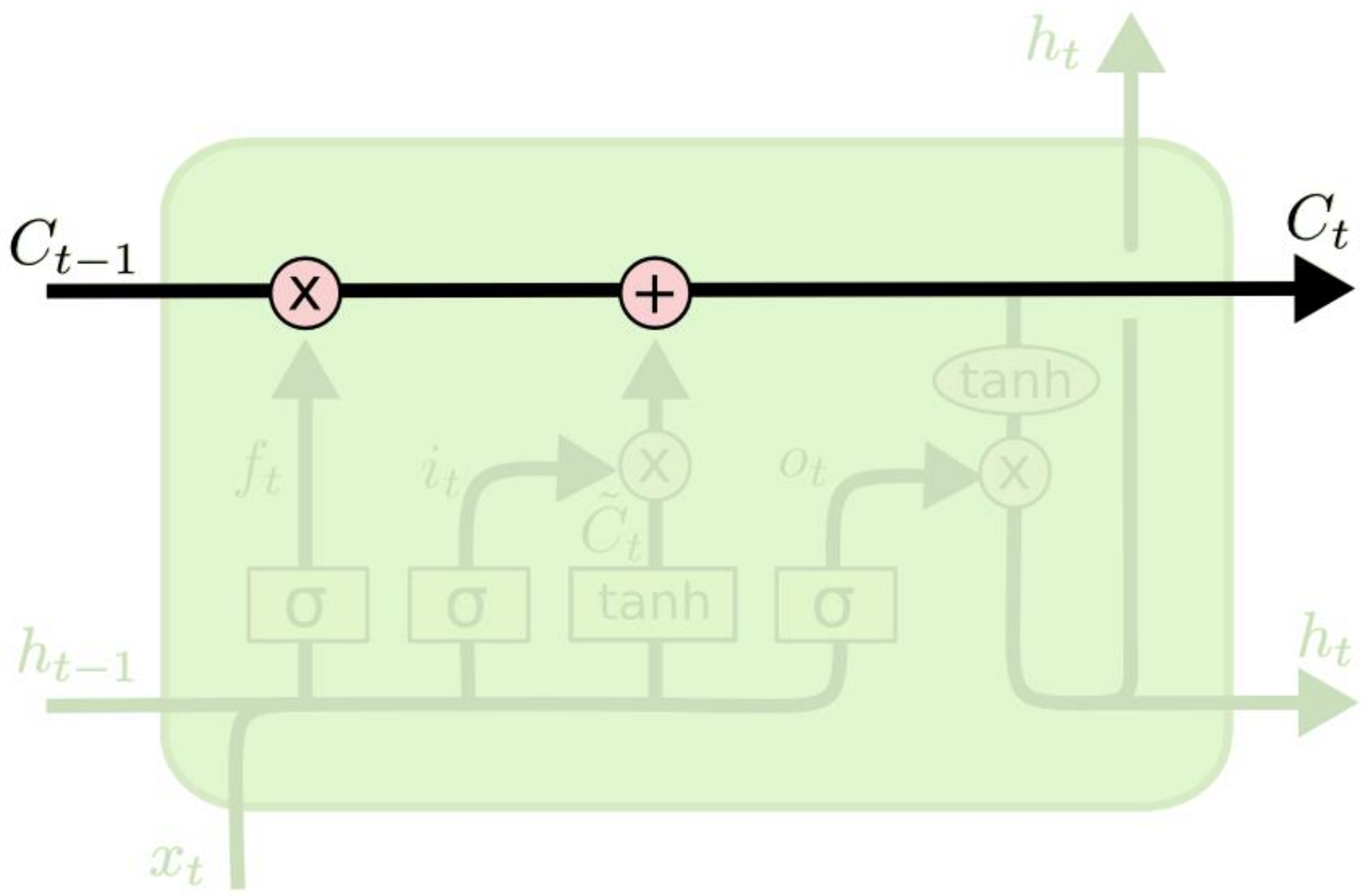
RNN structure

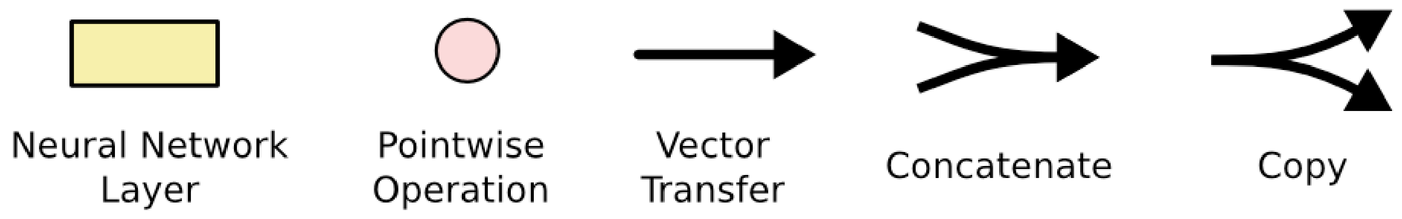




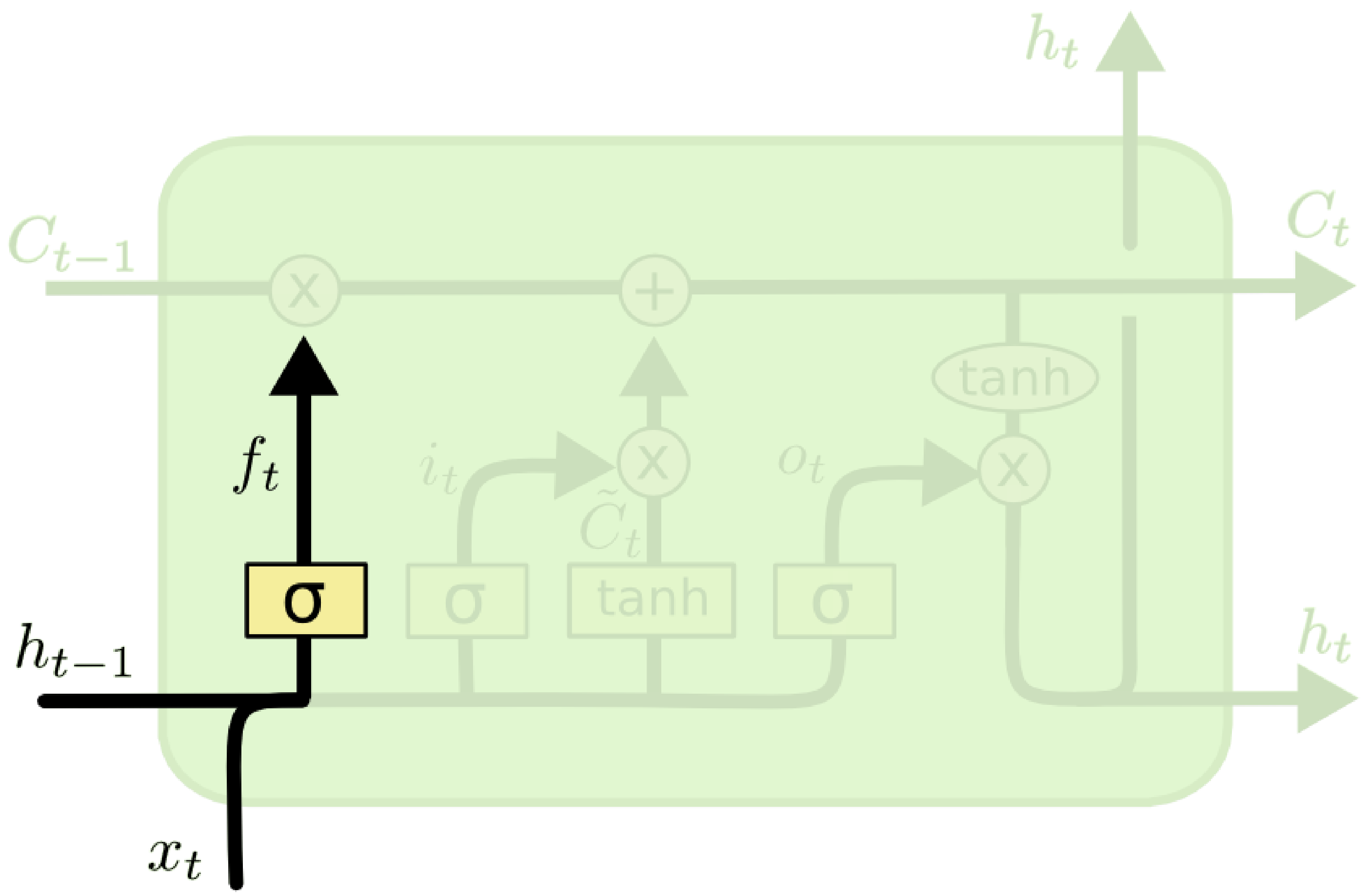
LSTMs

The core idea behind LSTMs: the **cell state** directly connects the whole chain of cells.



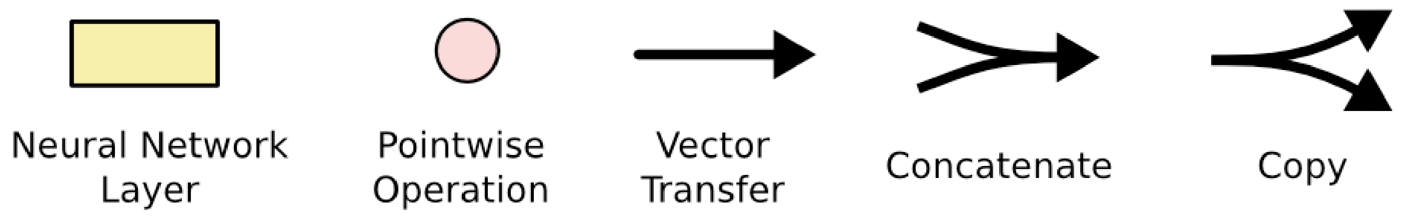


The **forget gate**: is there any information in the cell state that we no longer need?



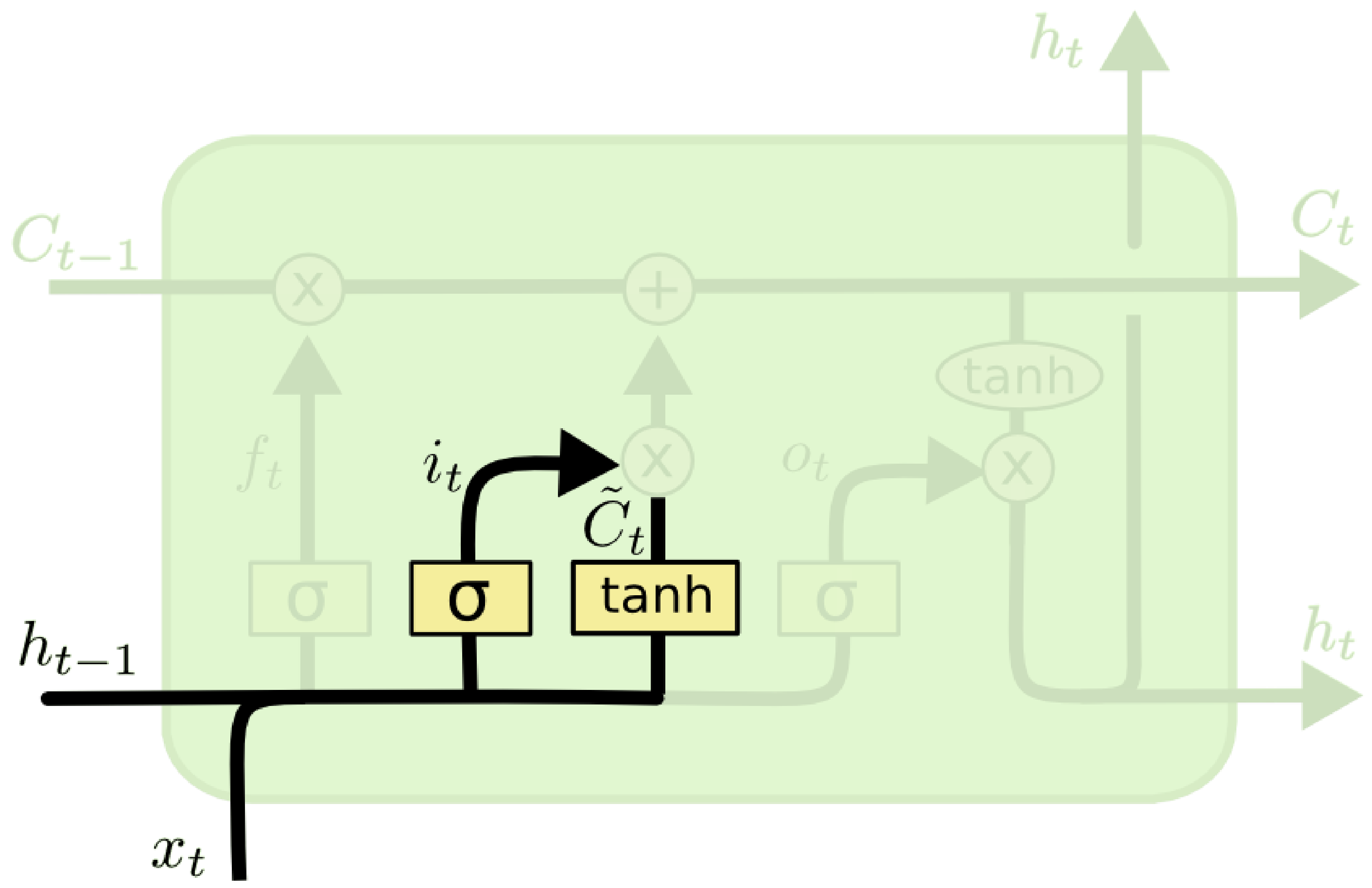
sigmoid normalises between 0 and 1

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$



1EDSML
ASO1E2
CE1MGO
SEMS12
EGOCO2

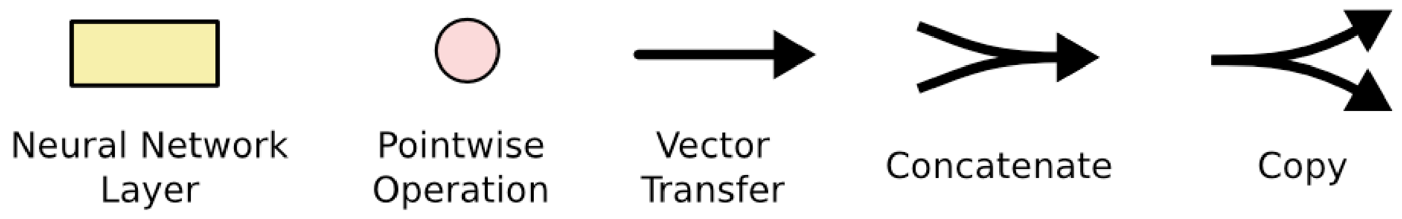
The **input gate** and the **candidate update**: what part of the input should we use to update the cell state?



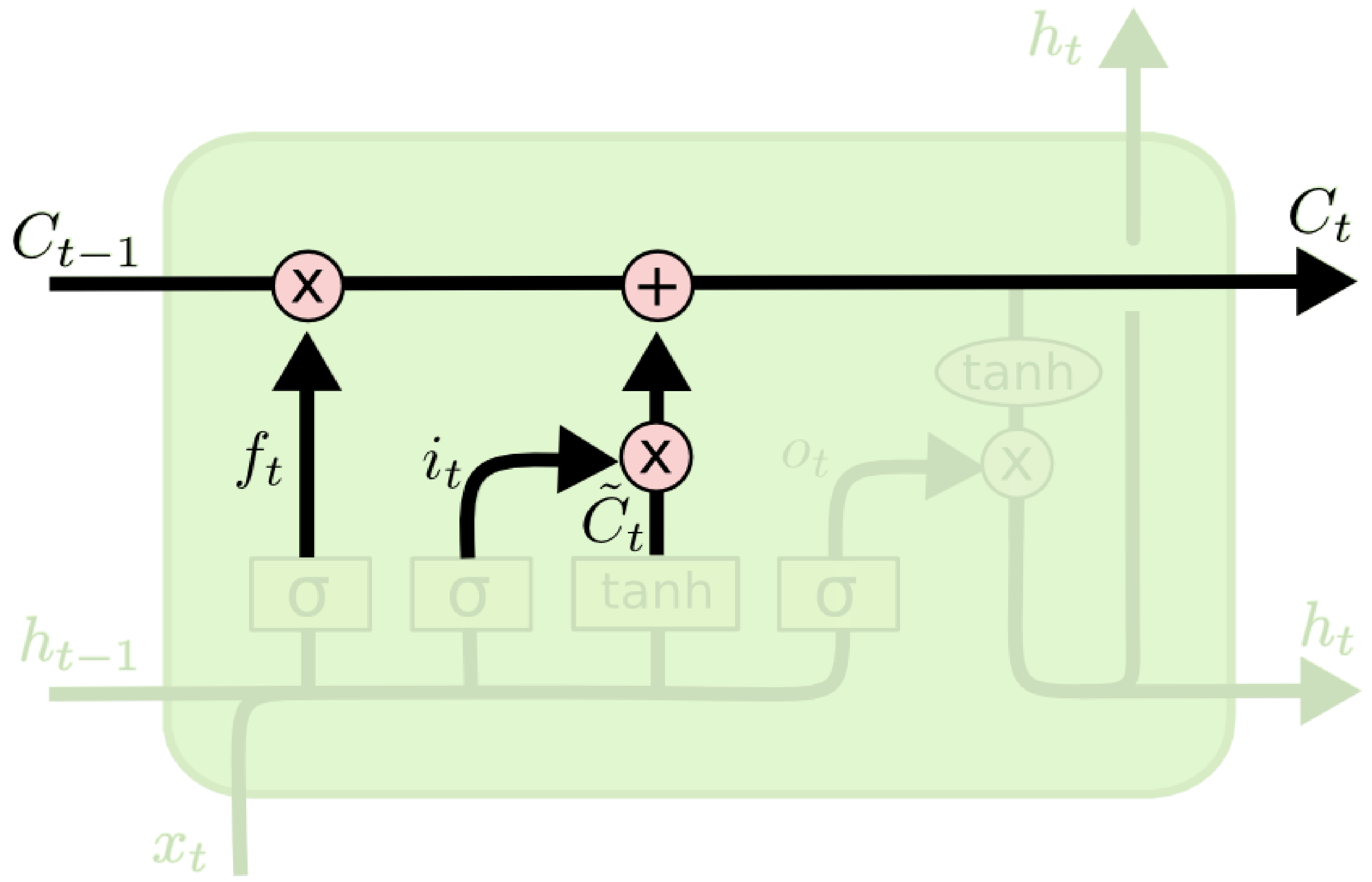
$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_c \cdot [h_{t-1}, x_t] + b_c)$$

tanh normalises between -1 and 1, like cell state



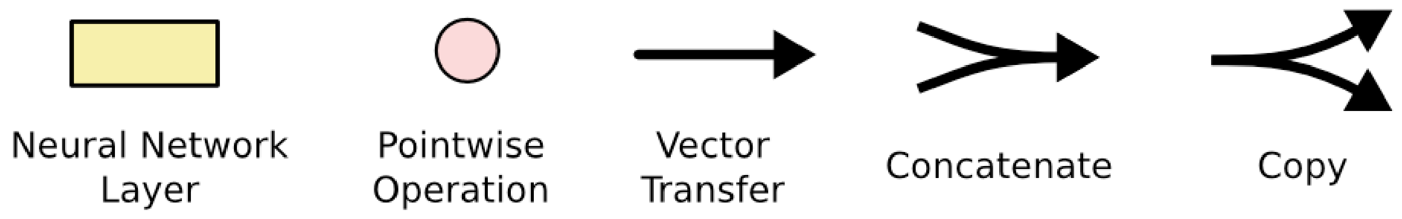
Updating the cell state:



add the gated candidate values

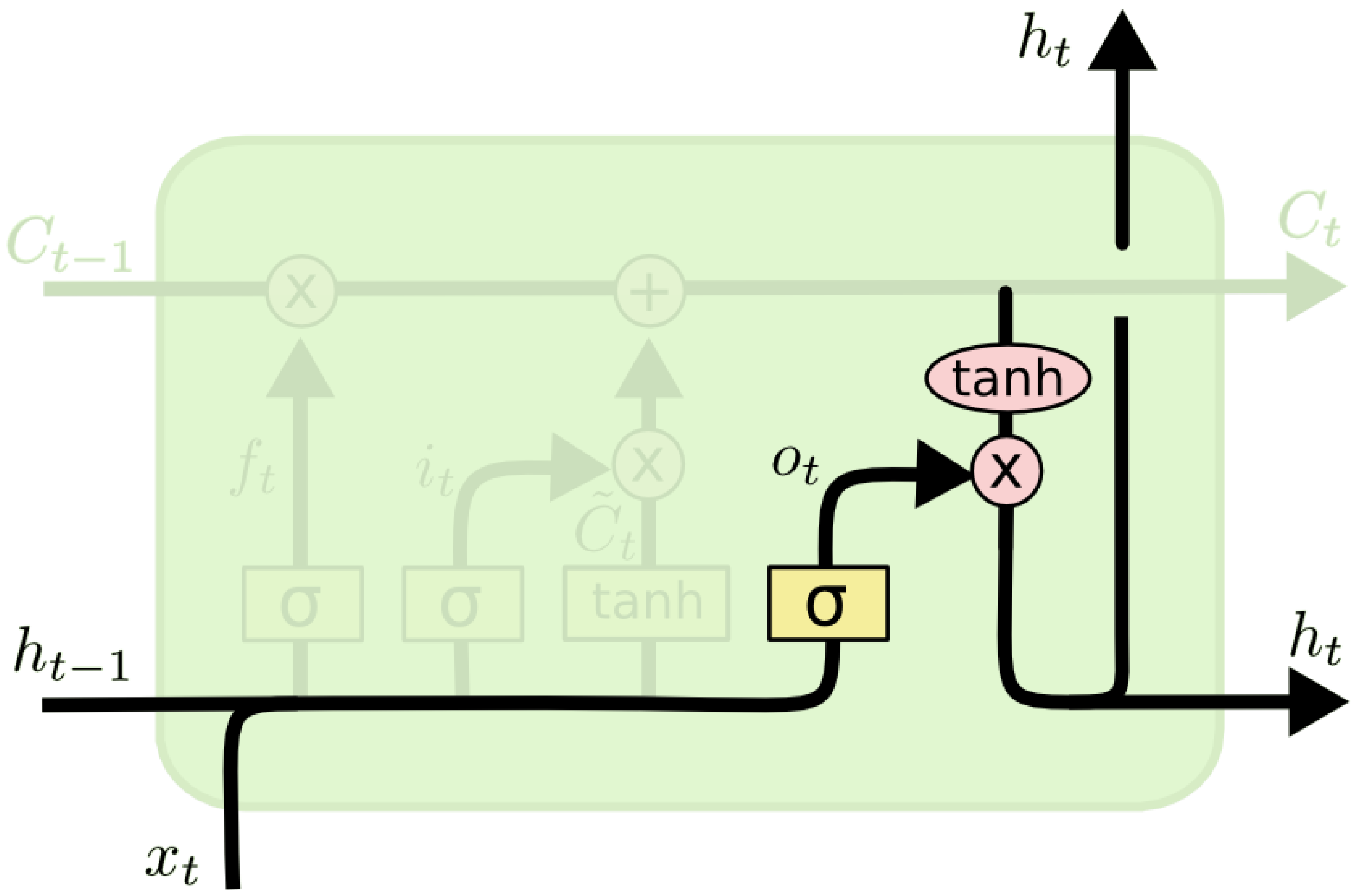
$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$

forget part of the previous cell state



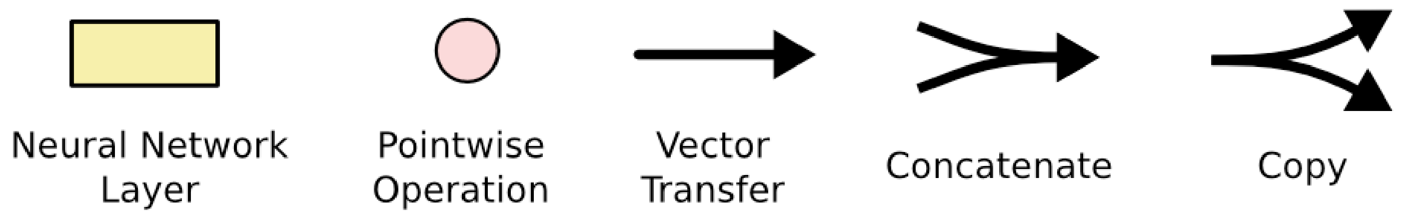
LSTMs

The **output gate**: what part of the cell state should influence the next output?



$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t \odot \tanh(C_t)$$



LSTMs

Basic structure of an LSTM:

vanilla RNN:

$$h_t = tanh\left(W\begin{pmatrix}h_{t-1}\\x_t\end{pmatrix}\right)$$

LSTM:

$$\begin{array}{r}\text{input gate}\\\text{forget gate}\\\text{output gate}\\\text{candidate update}\end{array}\quad\begin{pmatrix}i\\f\\o\\\tilde{c}\end{pmatrix}=\begin{pmatrix}\sigma\\\sigma\\\sigma\\tanh\end{pmatrix}W\left(\begin{pmatrix}h_{t-1}\\x_t\end{pmatrix}\right)$$

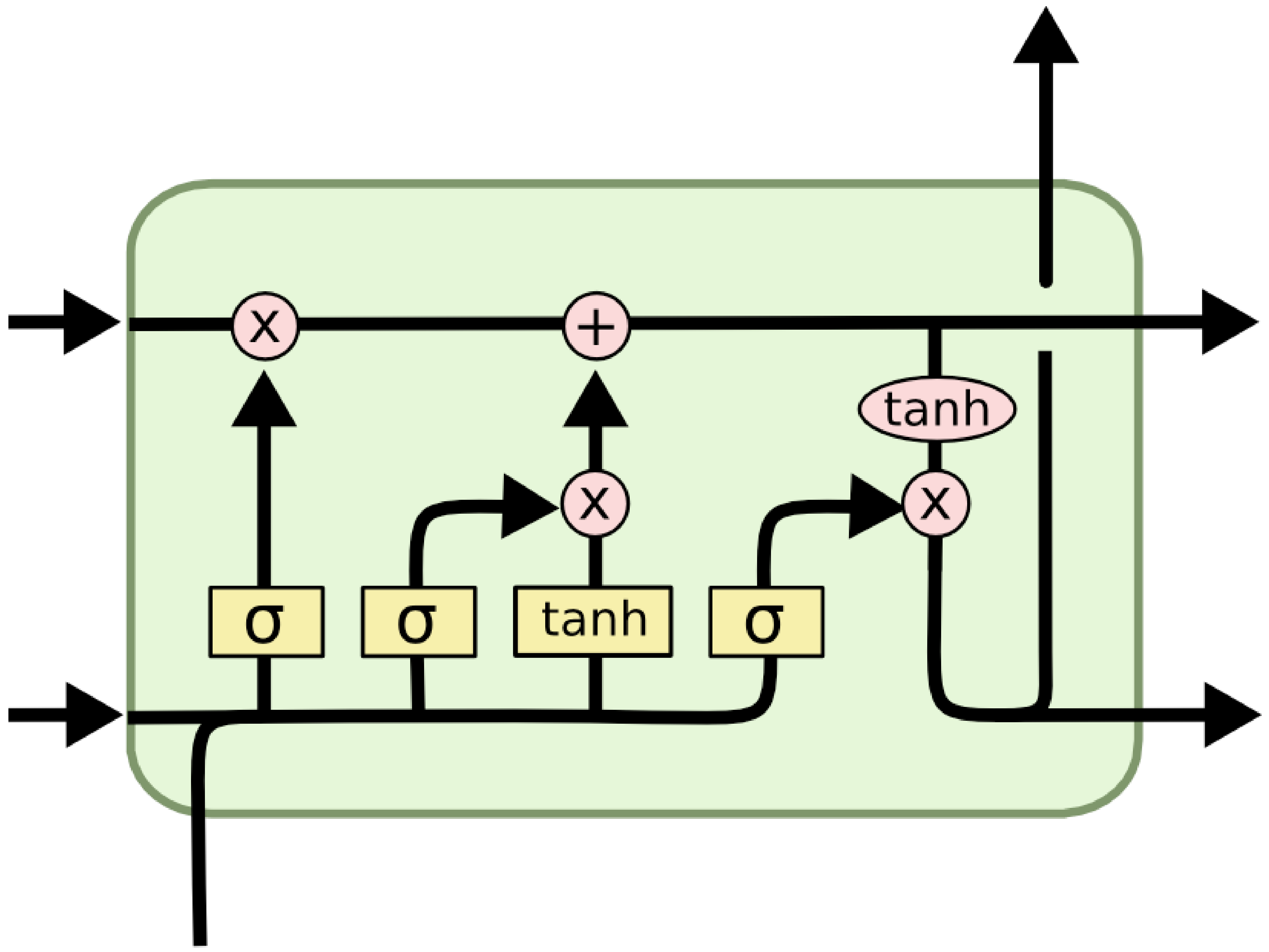
$$c_t = f \odot c_{t-1} + i \odot \tilde{c}$$

$$h_t = o \odot tanh(c_t)$$

Hadamard product (element-wise multiplication)

LSTMs

Basic structure of an LSTM:



$$\begin{pmatrix} i \\ f \\ o \\ \tilde{c} \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ tanh \end{pmatrix} W \left(\begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right)$$

$$c_t = f \odot c_{t-1} + i \odot \tilde{c}$$

$$h_t = o \odot tanh(c_t)$$

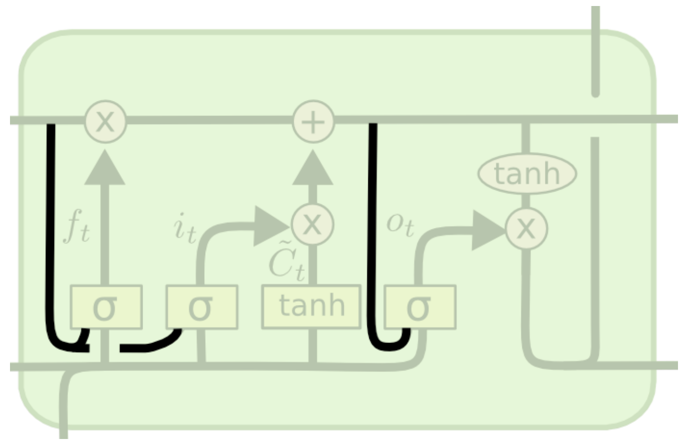
https://colah.github.io/posts/2015-08-Understanding-LSTMs/

LSTMs

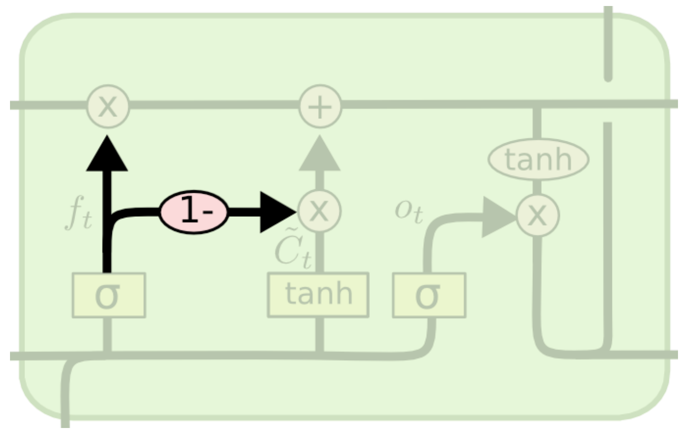
Let’s look at some code!

LSTMs

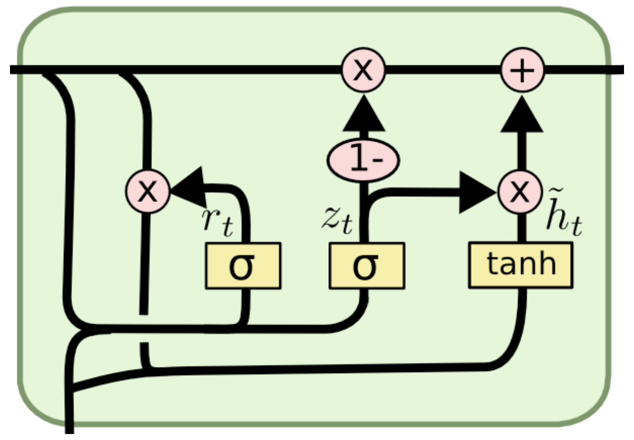
Variations on LSTMs:



peep-hole connections

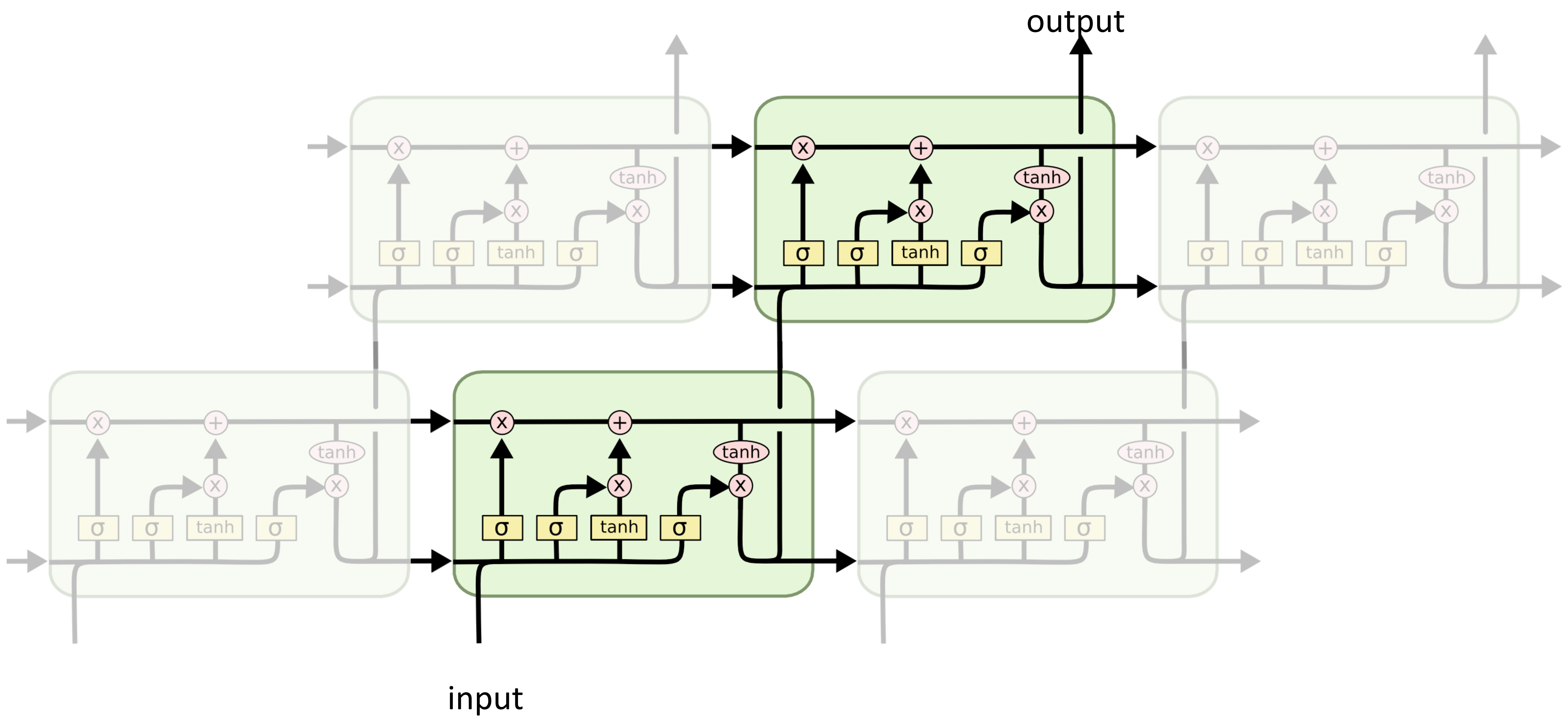


coupled forget and input gates

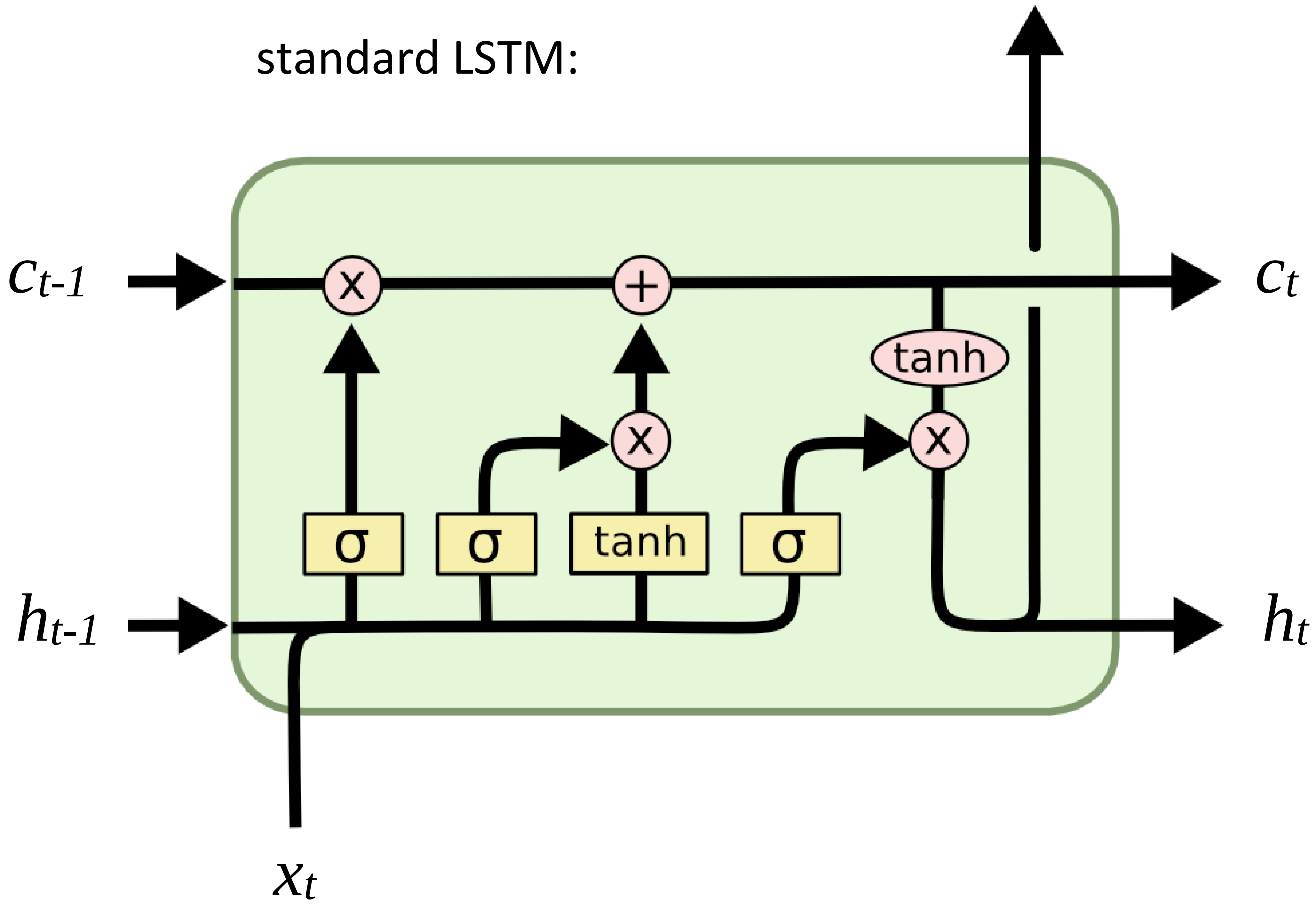


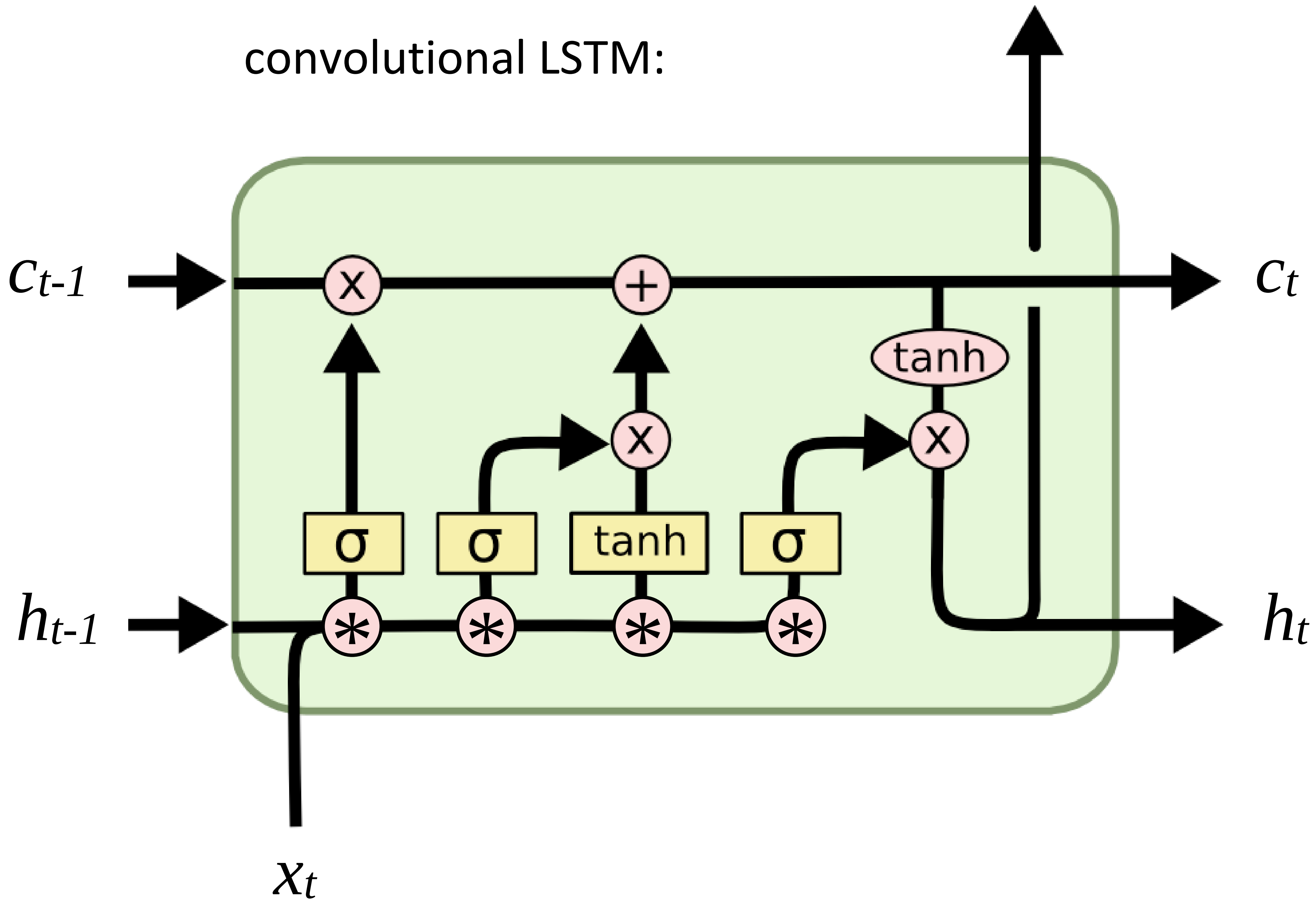
GRU (Gated Recurrent Unit),
combines forget and input gates into
a single update gate

stacked LSTMs



Basic structure of a convolutional LSTM:

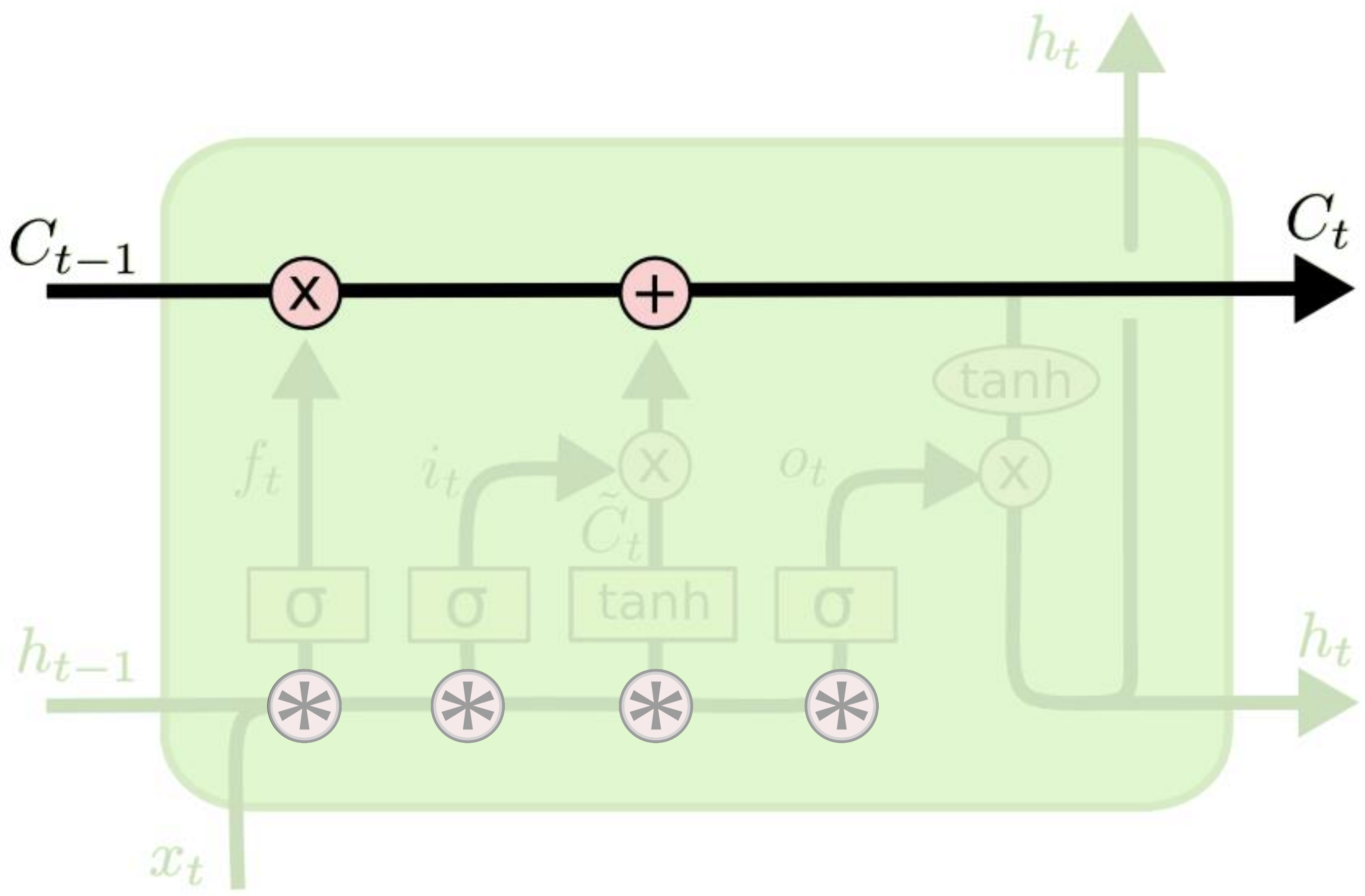




https://papers.nips.cc/paper/2015/hash/07563a3fe3bbe7e3ba84431ad9d055af-Abstract.html

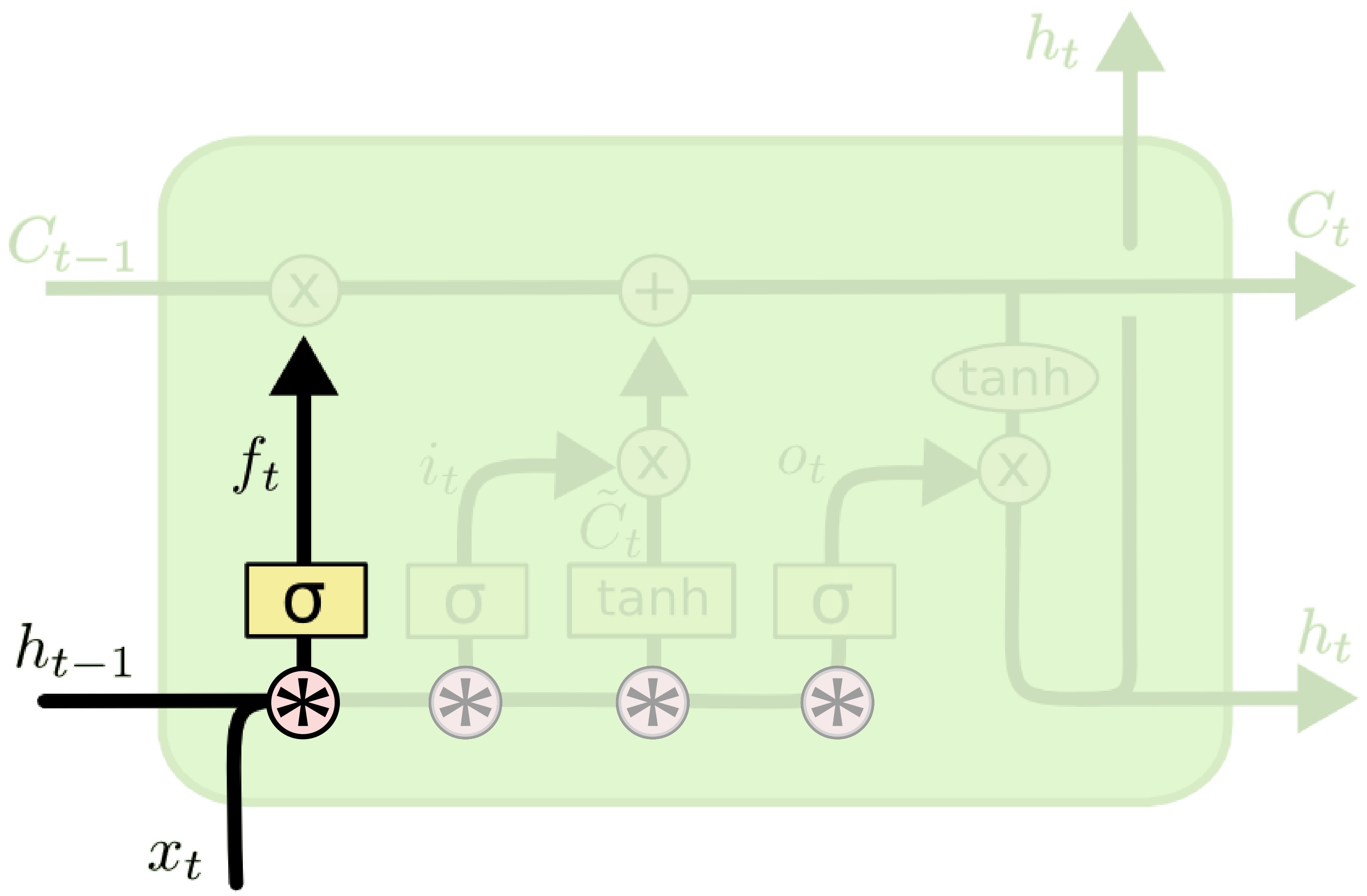
CONVOLUTIONAL LSTMS

The core idea behind convolutional LSTMs is the same:



Convolutional LSTMs

The **forget gate**: is there any information in the cell state that we no longer need?

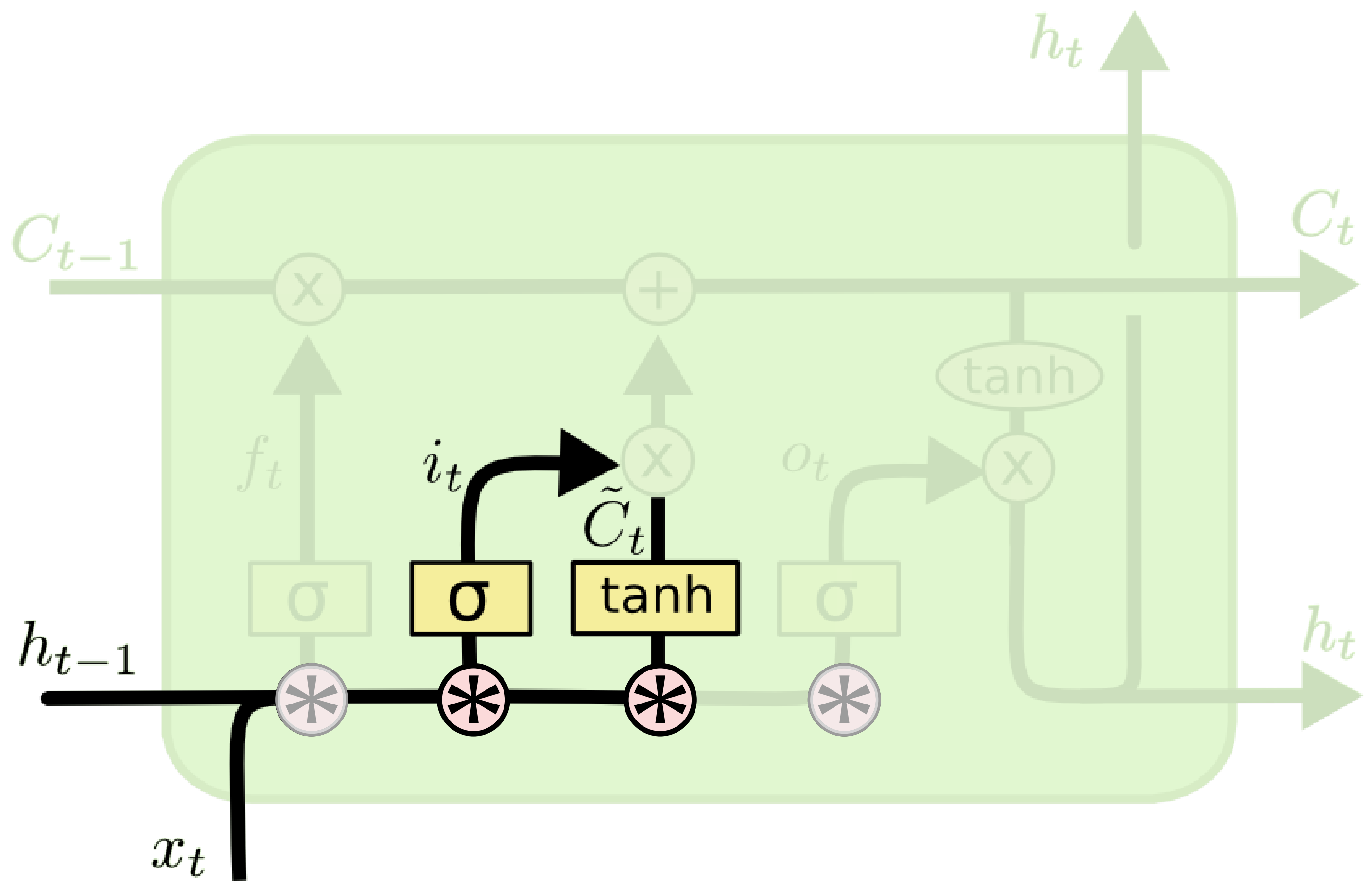


convolution operation

$$f_t = \sigma(W_f * [h_{t-1}, x_t] + b_f)$$

CONVOLUTIONAL LSTMS

The **input gate** and the **candidate update**: what part of the input should we use to update the cell state?



$$i_t = \sigma(W_i * [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_c * [h_{t-1}, x_t] + b_c)$$

1EDSML
ASO1E2
CE1MGO
SEMS12
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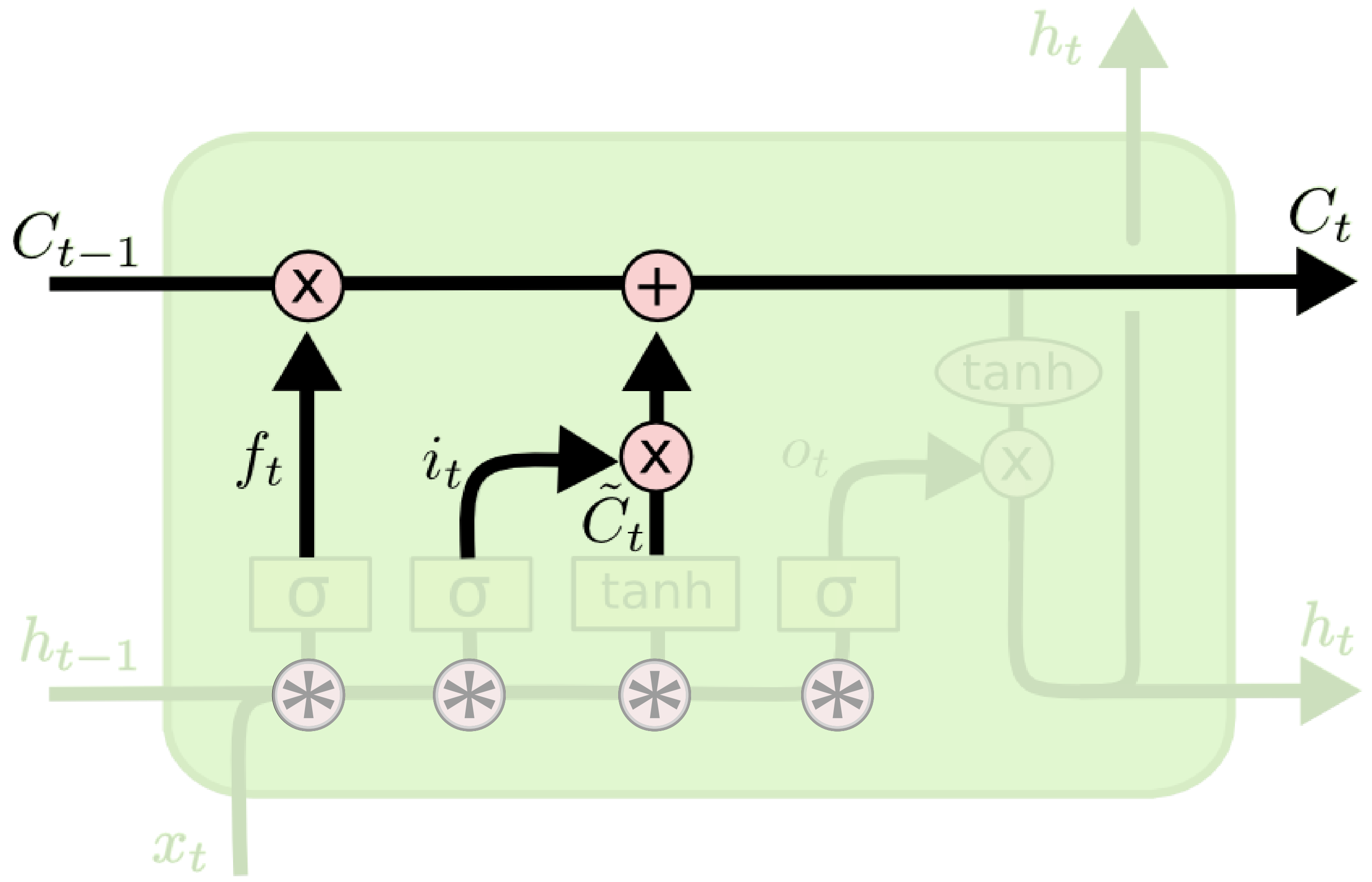
Updating the cell state:

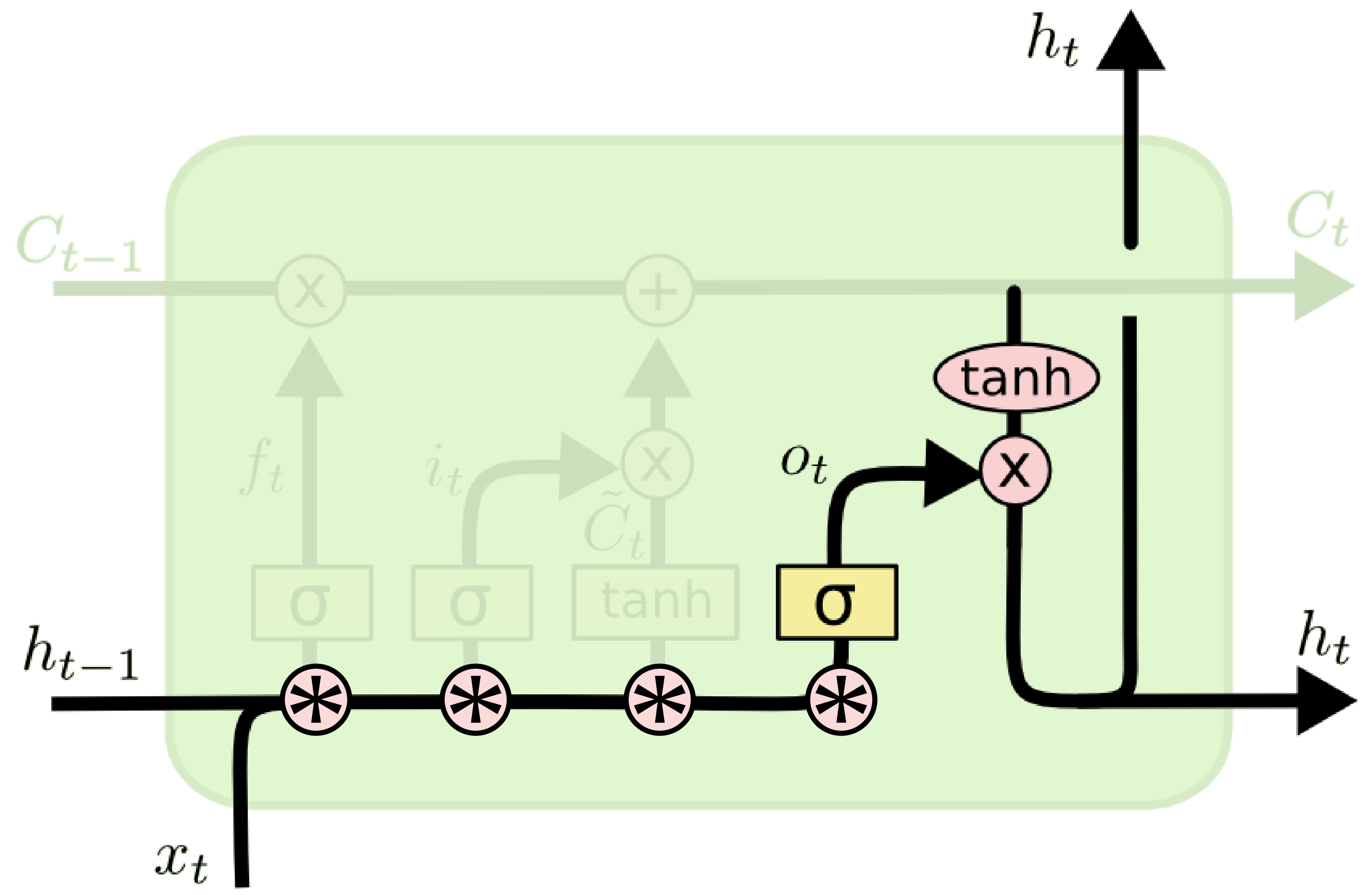


add the gated candidate values

$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$

forget part of the previous cell state

The **output gate**: what part of the cell state should influence the next output?

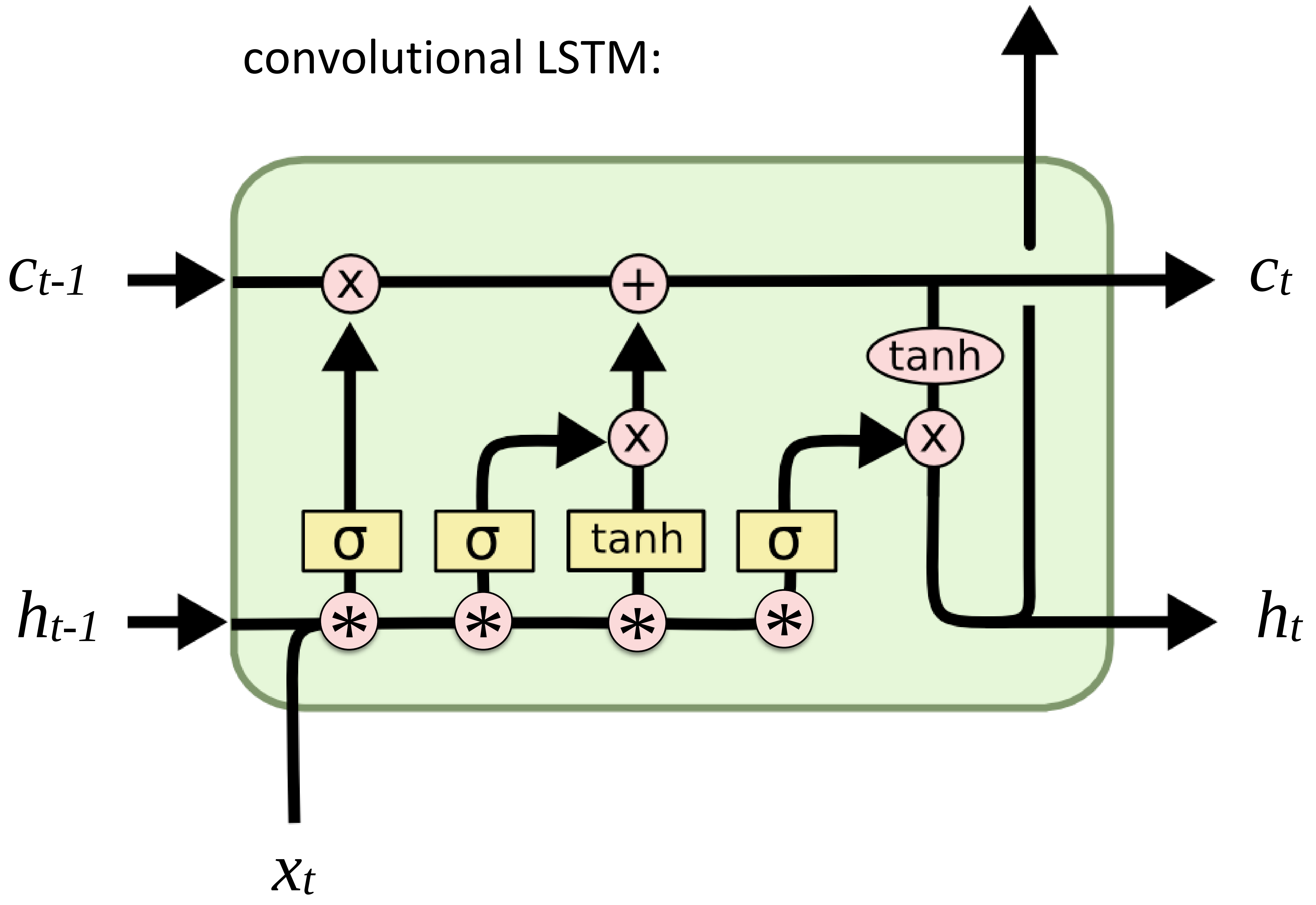


$$o_t = \sigma(W_o * [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t \odot \tanh(C_t)$$

Basic structure of a convolutional LSTM:

$$f_t = \sigma(W_f * [h_{t-1}, x_t] + b_f)$$
$$i_t = \sigma(W_i * [h_{t-1}, x_t] + b_i)$$
$$\tilde{C}_t = \tanh(W_c * [h_{t-1}, x_t] + b_c)$$
$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$
$$o_t = \sigma(W_o * [h_{t-1}, x_t] + b_o)$$
$$h_t = o_t \odot \tanh(C_t)$$



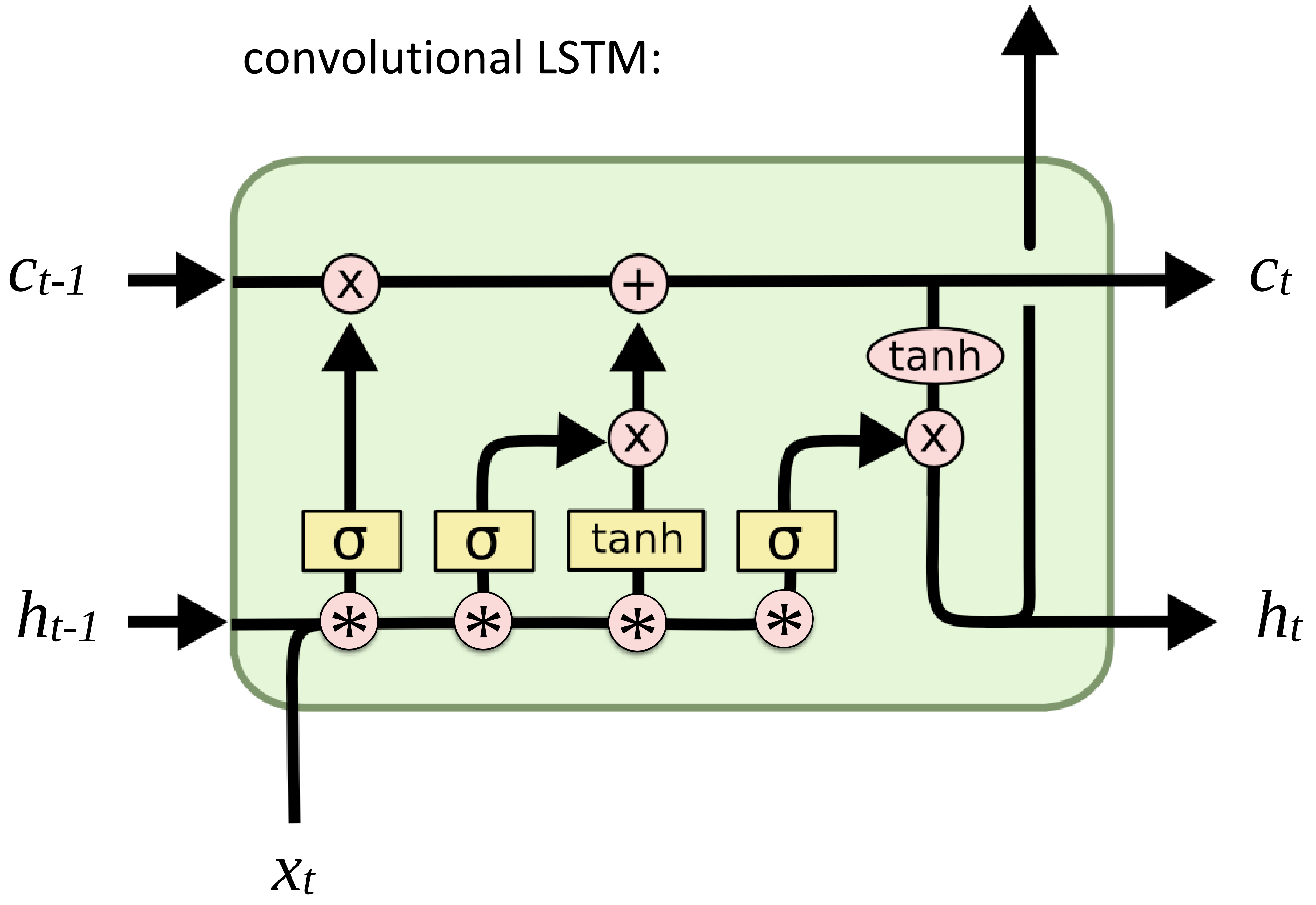
https://papers.nips.cc/paper/2015/hash/07563a3fe3bbe7e3ba84431ad9d055af-Abstract.html

CONVOLUTIONAL LSTMS

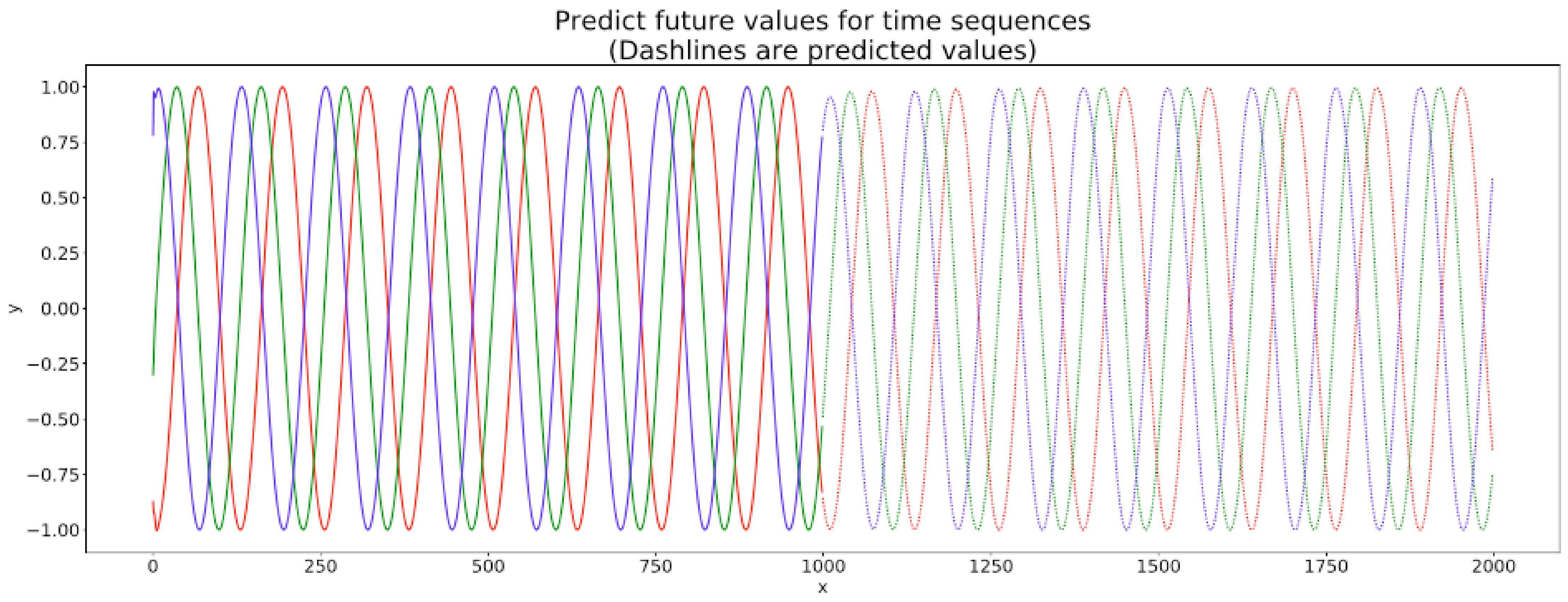
Basic structure of a convolutional LSTM (with peep-hole connections):

$$f_t = \sigma(W_f * [h_{t-1}, x_t] + W_{cf} \odot C_{t-1} + b_f)$$
$$i_t = \sigma(W_i * [h_{t-1}, x_t] + W_{ci} \odot C_{t-1} + b_i)$$
$$\tilde{C}_t = \tanh(W_c * [h_{t-1}, x_t] + b_c)$$
$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$
$$o_t = \sigma(W_o * [h_{t-1}, x_t] + W_{co} \odot C_{t-1} + b_o)$$
$$h_t = o_t \odot \tanh(C_t)$$

convolutional LSTM:



Examples of application:



https://github.com/pytorch/examples/tree/master/time_sequence_prediction

```
VIOLA:
Why, Salisbury must find his flesh and thought
That which I am not aps, not a man and in fire,
To show the reining of the raven and the wars
To grace my hand reproach within, and not a fair are hand,
That Caesar and my goodly father's world;
When I was heaven of presence and our fleets,
We spare with hours, but cut thy council I am great,
Murdered and by thy master's ready there
My power to give thee but so much as hell:
Some service in the noble bondman here,
Would show him to her wine.

KING LEAR:
O, if you were a feeble sight, the courtesy of your law,
Your sight and several breath, will wear the gods
With his heads, and my hands are wonder'd at the deeds,
So drop upon your lordship's head, and your opinion
Shall be against your honour.
```

http://karpathy.github.io/2015/05/21/rnn-effectiveness/

SUMMARY

- Recurrent Neural Networks (RNNs) are the basic approach for sequential data.
- RNNs suffer from exploding and vanishing gradients and only preserve short-term memory.
- Long Short-Term Memory (LSTM) networks address these two issues.