Schrodinger

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Group Schrodinger

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CHAPTER ONE

SYNOPSIS:

Asteroids entering Earth's atmosphere are subject to extreme drag forces that decelerate, heat and disrupt the space rocks. The fate of an asteroid is a complex function of its initial mass, speed, trajectory angle and internal strength.

Asteroids 10-100 m in diameter can penetrate deep into Earth's atmosphere and disrupt catastrophically, generating an atmospheric disturbance (airburst) that can cause damage on the ground. Such an event occurred over the city of Chelyabinsk in Russia, in 2013, releasing energy equivalent to about 520 kilotons of TNT (1 kt TNT is equivalent to 4.184×10^{12} J), and injuring thousands of people (Popova et al., 2013; Brown et al., 2013). An even larger event occurred over Tunguska, a relatively unpopulated area in Siberia, in 1908.

This simulator predicts the fate of asteroids entering Earth's atmosphere, and provides a hazard mapper for an impact over the UK.

CHAPTER

TWO

PROBLEM DEFINITION

2.1 Equations of motion for a rigid asteroid

The dynamics of an asteroid in Earth's atmosphere prior to break-up is governed by a coupled set of ordinary differential equations:

$$
\begin{aligned}
\frac{dv}{dt} &= \frac{-C_D\rho_a A v^2}{2m} + g\sin\theta \\
\frac{dm}{dt} &= \frac{-C_H\rho_a A v^3}{2Q} \\
\frac{d\theta}{dt} &= \frac{g\cos\theta}{v} - \frac{C_L\rho_a A v}{2m} - \frac{v\cos\theta}{R_P + z} \\
\frac{dz}{dt} &= -v\sin\theta \\
\frac{dx}{dt} &= \frac{v\cos\theta}{1 + z/R_P}
\end{aligned}
$$

In these equations, v, m, and A are the asteroid speed (along trajectory), mass and cross-sectional area, respectively. We will assume an initially **spherical asteroid** to convert from inital radius to mass (and cross-sectional area). θ is the meteoroid trajectory angle to the horizontal (in radians), x is the downrange distance of the meteoroid from its entry position, z is the altitude and t is time; C_D is the drag coefficient, ρ_a is the atmospheric density (a function of altitude), C_H is an ablation efficiency coefficient, Q is the specific heat of ablation; C_L is a lift coefficient; and R_P is the planetary radius. All terms use MKS units.

2.2 Asteroid break-up and deformation

A commonly used criterion for the break-up of an asteroid in the atmosphere is when the ram pressure of the air interacting with the asteroid $\rho_a v^2$ first exceeds the strength of the asteroid Y.

$$\rho_a v^2 = Y$$

Should break-up occur, the asteroid deforms and spreads laterally as it continues its passage through the atmosphere. Several models for the spreading rate have been proposed. In the simplest model, the fragmented asteroid's spreading rate is related to its along trajectory speed (Hills and Goda, 1993):

$$\frac{dr}{dt} = \left[\frac{7}{2}\alpha\frac{\rho_a}{\rho_m}\right]^{1/2} v$$

Where r is the asteroid radius, ρ_m is the asteroid density (assumed constant) and α is a spreading coefficient, often taken to be 0.3. It is conventional to define the cross-sectional area of the expanding cloud of fragments as $A = \pi r^2$ (i.e., assuming a circular cross-section), for use in the above equations. Fragmentation and spreading **ceases** when the ram pressure drops back below the strength of the meteoroid $\rho_a v^2 < Y$.

2.3 Airblast damage

The rapid deposition of energy in the atmosphere is analogous to an explosion and so the environmental consequences of the airburst can be estimated using empirical data from atmospheric explosion experiments (Glasstone and Dolan, 1977).

The main cause of damage close to the impact site is a strong (pressure) blastwave in the air, known as the **airblast**. Empirical data suggest that the pressure in this wave p (in Pa) (above ambient, also known as overpressure), as a function of explosion energy E_k (in kilotons of TNT equivalent), burst altitude z_b (in m) and horizontal range r (in m), is given by:

$$p(r) = 3.14 \times 10^{11} \left(\frac{r^2 + z_b^2}{E_k^{2/3}}\right)^{-1.3} + 1.8 \times 10^7 \left(\frac{r^2 + z_b^2}{E_k^{2/3}}\right)^{-0.565}$$

For airbursts, we will take the total kinetic energy lost by the asteroid at the burst altitude as the burst energy E_k. For cratering events, we will define E_k as the **larger** of the total kinetic energy lost by the asteroid at the burst altitude or the residual kinetic energy of the asteroid when it hits the ground.

The following threshold pressures can then be used to define different degrees of damage.

Damage Level	Description	Pressure (kPa)
1	~10% glass windows shatter	1.0
2	~90% glass windows shatter	3.5
3	Wood frame buildings collapse	27
4	Multistory brick buildings collapse	43

Table 1: Pressure thresholds (in kPa) for airblast damage

2.4 Additional sections

You should expand this documentation to include explanatory text for all components of your tool.

CHAPTER

THREE

FUNCTION API