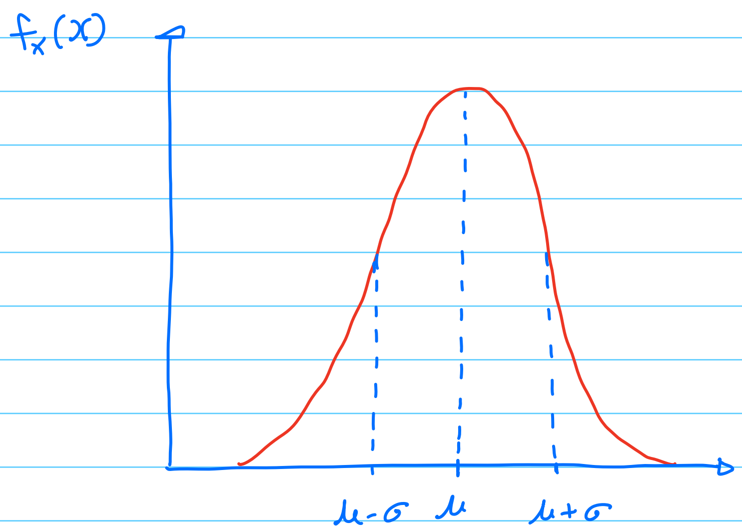
Distribuição Normal

F.D.P: $\dfrac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

com $-\infty < \mu < \infty$ e $\sigma > 0$

Notação: $X \sim N(\mu, \sigma^2)$



$E(X) = \mu$

$Var(X) = \sigma^2$

$m_X(t) = e^{\mu t + \sigma^2 t^2/2}$

Moda: $x = \mu$

Pontos de inflexão: $x = \mu - \sigma$ e $x = \mu + \sigma$

- IMPORTANTE P/ A TEORIA ESTATÍSTICA
- UTILIZADA EM DIVERSAS APLICAÇÕES
- DISTRIBUIÇÃO LIMITE NO TEOREMA CENTRAL DO LIMITE
- SE UMA V.A. $X \sim N(\mu, \sigma^2)$ EM $\mu = 0$ E $\sigma^2 = 1$, ENTÃO ELA É UMA V.A. NORMAL PADRÃO.

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1)$$

DISTR. NORMAL PADRÃO

FDP: $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$

FDA: $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}z^2} \, dz$

IMPORTANTE: $\Phi(z) = 1 - \Phi(-z)$

SE $X \sim N(\mu, \sigma^2)$, ENTÃO:

$$P(a < X < b) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$$

EXEMPLO:

SUPONHA QUE OS DIÂMETROS DE EIXOS FABRICADOS POR UM DETERMINADO PROCESSO PRODUTIVO SEJAM VARIÁVEIS ALEATÓRIAS NORMAIS C/ MÉDIA DE 10 CM E DESVIO PADRÃO DE 0,1 CM. SE P/ UMA DETERMINADA SITUAÇÃO O DIÂMETRO DEVE ESTAR ENTRE 9,9 E 10,2 CM, QUAL A PROPORÇÃO DOS EIXOS FABRICADOS ATENDERÁ AO REQUISITO?

$$X \sim N(\mu = 10,\ \sigma^2 = 0{,}1^2)$$

$$P(9{,}9 < X < 10{,}2) = \Phi\left(\frac{10{,}2 - 10}{0{,}1}\right) - \Phi\left(\frac{9{,}9 - 10}{0{,}1}\right)$$

$$= \Phi(2) - \Phi(-1)$$

$$= 0{,}9772^{*} - 0{,}1587^{*}$$

$$= 0{,}8185$$

* TABELA DA NORMAL PADRÃO $P(Z < z)$

81,85% DOS EIXOS ATENDERÃO AO REQUISITO.

INTEGRANDO "NA MÃO" A FDP DA NORMAL.

SE $X \sim N(\mu, \sigma^2)$,

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty \ \text{E} \ \sigma > 0$$

$$① = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$u = \frac{x-\mu}{\sigma}$$

$$u = \frac{1}{\sigma}x - \frac{\mu}{\sigma}$$

$$du = \frac{1}{\sigma}dx$$

$$du = \frac{dx}{\sigma}$$

$$\underline{\underline{dx = \sigma\, du}}$$

$$\Phi = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}u^2} \sigma\, du$$

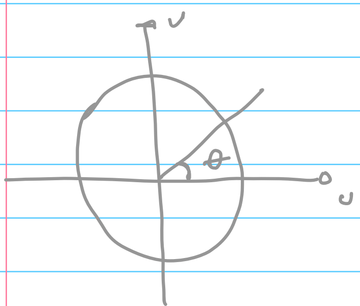
$$\Phi = \int_{-\infty}^{\infty} \frac{\cancel{\sigma}}{\sqrt{2\pi}\,\cancel{\sigma}} e^{-\frac{1}{2}u^2} du$$

$$\Phi = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$$

$$\Phi^2 = \left(\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du\right)\left(\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}v^2} dv\right)$$

$$\Phi^2 = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\infty} e^{-\frac{1}{2}(u^2+v^2)} du\, dv$$

COORDENADAS POLARES



$$u = r.\cos\theta \qquad v = r.\text{sen}\,\theta$$

$$u^2 = r^2.\cos^2\theta \qquad v^2 = r^2.\text{sen}^2\theta$$

$$u^2+v^2 = r^2.\cos^2\theta + r^2.\text{sen}^2\theta$$

$$u^2+v^2 = r^2\underbrace{(\cos^2\theta + \text{sen}^2\theta)}_{}$$

$= 1$ PELO TEOREMA FUNDAMENTAL DA TRIGONOMETRIA

$$\underline{\underline{u^2+v^2 = r^2}}$$

$$\Phi^2 = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\infty} e^{-\frac{1}{2}(r^2\cos^2\theta + r^2\text{sen}^2\theta)} \underbrace{|J|}_{\text{JACOBIANA}}\, dr\, d\theta$$

$$|J| = \begin{vmatrix} \frac{du}{d\theta} & \frac{du}{dr} \\ \frac{dv}{d\theta} & \frac{dv}{dr} \end{vmatrix} = \begin{vmatrix} -r\,sen\,\theta & \cos\theta \\ r\cos\theta & sen\,\theta \end{vmatrix}$$

$$|J| = |(-r\,sen\,\theta).(sen\,\theta) - (r\cos\theta).(\cos\theta)|$$

$$|J| = |-r\,sen^2\theta - r\cos^2\theta|$$

$$|J| = |-r||sen^2\theta + \cos^2\theta| = \underline{\underline{r}}$$

$$Q^2 = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\infty} e^{-\frac{1}{2}[r^2(\cos^2\theta + sen^2\theta)]}\, r\, dr\, d\theta$$

$$Q^2 = \frac{1}{2\pi}\int_0^{2\pi}\left[\int_0^{\infty} e^{-\frac{1}{2}r^2}\, r\, dr\right] d\theta$$

$$Q^2 = \frac{1}{2\pi}\int_0^{2\pi}\left[\int_0^{\infty} e^{-\frac{1}{2}w}\, \frac{dw}{2}\right] d\theta$$

$w = r^2$

$dw = 2r\,dr$

$r\,dr = \frac{dw}{2}$

$$Q^2 = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2}\left[-2e^{-\frac{1}{2}w}\Big|_0^{\infty}\right] d\theta$$

$$Q^2 = \frac{1}{2\pi}\int_0^{2\pi} \frac{-2}{2}\left[e^{-\frac{1}{2}w}\Big|_0^{\infty}\right] d\theta$$

$$\Phi^2 = \frac{1}{2\pi}\int_0^{2\pi} -\left[e^{-\frac{1}{2}\omega}\Big|_0^{\infty}\right] d\theta \qquad -\left[e^{-\frac{1}{2}\infty} - e^{-\frac{1}{2}0}\right]$$

$$\Phi^2 = \frac{1}{2\pi}\int_0^{2\pi} -(-1)\, d\theta \qquad -\left[0 - 1\right] = 1$$

$$\Phi^2 = \frac{1}{2\pi}\int_0^{2\pi} d\theta$$

$$\Phi^2 = \frac{1}{2\pi}\cdot\theta\Big|_0^{2\pi} = \frac{1}{2\pi}\left[2\pi - 0\right] = \frac{2\pi}{2\pi} = 1$$

$$\Phi^2 = 1 \longrightarrow \Phi = \sqrt{1} = 1$$

PORTANTO:

$$\Phi = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = 1$$