damping 고려 x

$$\rightarrow \begin{cases} (\dot{v})_{x,y} = (u)_{x,y} \qquad (\dot{v})_z = u_z - g \\ (\dot{p})_{xy,z} = (v)_{xyz} \end{cases}$$

달의 중력가속도.

$\Rightarrow$ (i) x, y

$$V_{t+1} = V_t + \Delta t\, u_t$$

$$\begin{aligned} P_{t+1} &= P_t + \Delta t\, V_t \\ &= P_t + \frac{\Delta t}{2}(V_t + V_{t+1}) \\ &= P_t + \frac{\Delta t}{2}(V_t + V_t + \Delta t\, u_t) \\ &= P_t + \Delta t V_t + \frac{\Delta t^2}{2} u_t \end{aligned}$$

(ii) z

$$\begin{aligned} V_{t+1} &= V_t + \Delta t\,(u_t - g) \\ &= V_t + \Delta t\, u_t - g \cdot \Delta t \end{aligned}$$

$$\begin{aligned} P_{t+1} &= P_t + \Delta t\, V_t \\ &= P_t + \frac{\Delta t}{2}(V_t + V_{t+1}) \\ &= P_t + \frac{\Delta t}{2}(2V_t + \Delta t\, u_t - g \cdot \Delta t) \\ &= P_t + V_t \Delta t + \frac{\Delta t^2}{2} u_t - \frac{g}{2}\Delta t^2 \end{aligned}$$

$\therefore$

$$A = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad x_t = \begin{bmatrix} P_x \\ P_y \\ P_z \\ V_x \\ V_y \\ V_z \end{bmatrix}_t \quad B = \begin{bmatrix} \frac{\Delta t^2}{2} & 0 & 0 \\ 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & 0 & \frac{\Delta t^2}{2} \\ \Delta t & 0 & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & \Delta t \end{bmatrix} \quad u = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ -\frac{g}{2}\Delta t^2 \\ 0 \\ 0 \\ -g \cdot \Delta t \end{bmatrix}$$

$\underline{\underline{6 \times 1}}$

$$x_1 = Ax_0 + Bu_0 + b$$

$$x_2 = Ax_1 + Bu_1 + b$$

$$= A(Ax_0 + Bu_0 + b) + Bu_1 + b$$

$$= A^2x_0 + ABu_0 + Bu_1 + Ab + b$$

$$x_3 = A(A^2x_0 + ABu_0 + Bu_1 + Ab + b) + Bu_2 + b$$

$$= A^3x_0 + A^2Bu_0 + ABu_1 + A^2b + Ab + Bu_2 + b$$

$$= A^3x_0 + A^2Bu_0 + ABu_1 + Bu_2 + A^2b + Ab + b$$

$$\vdots$$

3x1 벡터

$$x_n = A^n x_0 + A^{n-1}Bu_0 + \cdots + Bu_{n-1} + A^{n-1}b + \cdots + b$$

$$= \underbrace{\begin{bmatrix} A^{n-1}B & \cdots & B \end{bmatrix}}_{G} \begin{bmatrix} u_0 \\ \vdots \\ u_{n-1} \end{bmatrix} + \overbrace{\underbrace{\begin{bmatrix} A^{n-1} & \cdots & I \end{bmatrix}}_{P} \begin{bmatrix} b \\ \vdots \\ b \end{bmatrix}}^{Q}$$

$$\Downarrow$$

$$\begin{bmatrix} u_{x_0} \\ u_{y_0} \\ u_{z_0} \\ u_{x_1} \\ u_{y_1} \\ \vdots \end{bmatrix}$$

$$Gu = \underline{x_{des} - Q - A^n x_0}$$

$$GU = \underbrace{x_{des} - Q - A^n x_0}_{\to\ 6 \times 1} =: C$$

$6 \times (3 \times n) \times 3n \times 1$

$$\|U\|^2 + \alpha \|GU - C\|^2$$

$$\alpha \left\{ \|GU - C\|^2 + \underbrace{\frac{1}{\alpha}}_{=:\lambda} \|U\|^2 \right\} \Rightarrow \text{minimize} \quad \|GU - C\|^2 + \lambda \|U\|^2$$

$$= \left\| \begin{array}{c} 6 \times 3n \\ \begin{bmatrix} G \\ \sqrt{\lambda} \end{bmatrix} \\ 3n \times 3n \end{array} \begin{array}{c} U \\ 3n \times 1 \end{array} - \begin{bmatrix} C \\ 0 \end{bmatrix} \right\|^2 \quad \left(\because = \left\| \begin{matrix} GU - C \\ \sqrt{\lambda} U \end{matrix} \right\|^2 \right)$$

$$\begin{bmatrix} G \\ \hline \sqrt{\lambda} I \end{bmatrix} \begin{bmatrix} U \end{bmatrix}$$