Probability and Statistics course

Introduction

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Week 2

Materials

[Baron, 2014] [Ross, 2012]

Baron, M. (2014).
Probability and statistics for computer scientists.
CRC Press.

Ross, S. (2012).
First Course in Probability.
Pearson.

Conditional probability

Examples

Suppose that we toss 2 dice, and suppose that each of the 36 possible outcomes is equally likely to occur and hence has probability $1/36$. Suppose further that we observe that the first die is a 3. Then, given this information, what is the probability that the sum of the 2 dice equals 8?

Definition (Conditional Probability)

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \quad \text{if } P(B) \neq 0$$

Examples

What is the probability that in two dice rolls, both rolls are 6, given that we know at least one of the rolls is a 6?

Law of Total Probability

Definition (Complete System of Events)

The events $B_1, B_2, \ldots$ form a complete system of events if: 1) $B_i \cap B_j = \emptyset$ for all $i \neq j$; 2) $\bigcup_{i=1}^{\infty} B_i = \Omega$

Theorem (Law of Total Probability)

Let $B_1, B_2, \ldots$ be a complete system of events, and A be any event, with $P(B_j) > 0$ for all j. Then $P(A) = \sum_{j=1}^{\infty} P(A|B_j)P(B_j)$.

Examples

An company believes that people can be divided into 2 classes: those who are accident-prone and those who are not. The statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability 0.4, whereas this probability decreases to 0.2 for a person who is not accident prone. If we assume that 30 percent of the population is accident prone, what is the probability that a new policyholder will have an accident within a year of purchasing a policy?

Bayes Theorem

Theorem (Bayes Theorem)

Let $B_1, \ldots, B_n, \ldots$ be a complete system of events, and A be any event, with $P(B_j) > 0$ for all j. Then

$$P(B_k|A) = \frac{P(A|B_k)P(B_k)}{\sum_{j=1}^{\infty} P(A|B_j)P(B_j)}$$

Examples

In answering a question on a multiplechoice test, a student either knows the answer or guesses. Let p be the probability that the student knows the answer and $1 - p$ be the probability that the student guesses. Assume that a student who guesses at the answer will be correct with probability $1/m$, where m is the number of multiplechoice alternatives. What is the conditional probability that a student knew the answer to a question given that he or she answered it correctly?

Independence

Definition (Independence)

Events A and B are independent if

$$P(A \cap B) = P(A) \cdot P(B)$$

This means that the occurrence of event A does not influence the occurrence of event B, and vice versa.

Examples

A computer program consists of two independent parts. One part has a probability of error of 0.2, and the other has a probability of error of 0.3. If the program indicates an error, what is the probability that both parts are faulty?

Solution

Let A and B be the events that the first and second parts of the program are faulty, respectively. We are given:

$$P(A) = 0.2, \quad P(B) = 0.3$$

Since the parts are independent, the probability that both are faulty is:

$$P(A \cap B) = P(A)P(B) = 0.2 \times 0.3 = 0.06$$

The program indicates an error if at least one part is faulty. This is the union of events A and B:

$$\begin{aligned} P(\text{Error}) &= P(A \cup B) \\ &= P(A) + P(B) - P(A \cap B) \\ &= 0.2 + 0.3 - 0.06 \\ &= 0.44 \end{aligned}$$

We seek the probability that both parts are faulty given that an error is indicated:

$$\begin{aligned} P(\text{Both Faulty} \mid \text{Error}) &= \frac{P(A \cap B)}{P(A \cup B)} \\ &= \frac{0.06}{0.44} \\ &= \frac{3}{22} \approx 0.1364 \end{aligned}$$

The probability that both parts are faulty, given an error occurred, is approximately **13.64%**.

Homework

1. 41 million "Ötöslottó" tickets are filled out independently. What is the probability that there will be at least one five-match?
2. Out of 100 coins, one is fake (it has heads on both sides). We randomly select one coin and flip it 10 times, getting 10 heads. Given this outcome, what is the probability that we were flipping the fake coin?
3. A computer processor is produced in 3 factories. There is a 20% chance it was made in the first, a 30% chance in the second, and a 50% chance in the third. The probabilities of warranty failures in these factories are 10%, 4%, and 1%, respectively. If our computer's processor has broken down, what is the probability that it was made in the first factory?
4. (*Monty Hall problem*) In a television show there are 3 doors, such that there is a present behind one of them, and there is nothing behind the other two. The participant of the game chooses one of the doors. Then, the presenter opens one of the other doors, and shows that there is nothing behind it. Then he asks the participant whether he would like to stay with his original choice, or he would like to chose the other door which has not been opened yet. Is it worth changing?

Homework 2

1. There is a disease which affects 2% of the population. There is a blood test which is positive with probability 95% for people who have the disease, and it is positive with probability 1% for healthy people.
   a What is the probability that Peter's test will be positive? (Peter is a randomly chosen person.)
   b Given that the test of Peter is positive, what is the conditional probability that he has the disease?
   c Now suppose that the test was repeated k times independently for a randomly chosen person, and all results were positive. What is the conditional probability that this person has the disease? Calculate this probability for $k = 2$ and $k = 3$
2. What is the probability that maximum of the numbers that we get is equal to 5 if we throw 2 (generally n) regular dice? (Regular dice: 1, 2, 3, 4, 5, 6 with equal probabilities.)
3. Odesseus arrives at a junction while wandering on the road. One leads to Athens, one to Sparta and one to Mycenae. The athenians tell the truth every third time when asked, the mycanaeans every second time, while the spartans never lie. Odesseus does not know which way leads to which city so he chooses randomly. Arriving in the city he asks a person the question: How much is 2×2. The answer is 4. What is the probability that he is in Athens?

Homework 3

1. A biased coin ($P(H) = 2/3$) is tossed 3 times independently. What is the probability that we get exactly 2 heads, given that at least one head was tossed?
2. Two different suppliers, A and B, provide bolts to a factory. Supplier A provides 60% of the bolts and 5% of them are defective. Supplier B provides 40% of the bolts and 10% are defective. If a bolt is chosen at random and found to be defective, what is the probability it came from Supplier B?
3. In a certain population, 40% of people have brown eyes, 35% have blue eyes, and 25% have green eyes. If two people are selected independently and at random, what is the probability that they have the same eye color?